Introduction to Flavor Physics in and beyond the Standard Model

Lecture 1/4

Enrico Lunghi

Fermilab

References:
The BaBar physics book, http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-r-504-TOC-pdf.pdf
B physics at the Tevatron: Run II and beyond, hep-ph/0201071
The Discovery potential of a Super B Factory, hep-ph/0503261

Outline

- The Standard Model

- The Cabibbo-Kobayashi-Maskawa (CKM) matrix

The Standard Model

The ingredients

- Quantum Field Theory *(special relativity + quantum mechanics)*

- Particle content:
  - down-type quarks (d,s,b)
  - up-type quarks (u,c,t)
  - charged leptons (e,μ,τ)

  } left and right-handed fermions

  - neutrinos (ν_e,ν_μ,ν_τ) → left-handed fermions
  - gauge bosons (γ,$W^\pm$,Z,g) → vectors
  - Higgs boson (h) → scalar

- Local gauge invariance: $SU(3)_c \times SU(2)_L \times U(1)_Y$

  g_a W_i B^0 ($W_{1,2} \to W^\pm$, $[W_3,B^0] \to [\gamma, Z]$)

Phenomenological models contain interactions with vector bosons

These theories make sense only if we impose local gauge invariance

Gauge invariance dictates the form of the interaction

Example of gauge transformations

- The QED lagrangian is:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\gamma^\mu(i\partial_\mu - eQ_\psi A_\mu)\psi - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\ &\equiv \bar{\psi}(i\partial\!\!\!/ - eQ_\psi A\!\!\!/)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

- This lagrangian has a U(1) local gauge symmetry:

$$\psi \longrightarrow e^{i\,e\,Q_\psi\,\omega(x)}\,\psi \qquad A_\mu \longrightarrow A_\mu - \partial_\mu\omega(x)$$

- Let's try to write a similar transformation involving two fermions:

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \longrightarrow U\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = e^{i\,g\,\sigma^a\,\omega_a(x)}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$\sigma^1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\ \sigma^2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\ \sigma^3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$ Pauli matrices

- We need three gauge bosons:

$$\sigma^a W^a_\mu \longrightarrow U(\sigma^a W^a_\mu)U^\dagger + \sigma^a\,\partial_\mu\omega_a(x)$$

Gauge transformations of quarks and leptons

- The electroweak theory is *chiral*: left and right-handed fields transform differently under $SU(2)_L \times U(1)_Y$

- singlets

$$E_R = (e_R, \mu_R, \tau_R)\,, \qquad Y_E = -1$$
$$U_R = (u_R, c_R, t_R)\,, \qquad Y_U = 2/3$$
$$D_R = (d_R, s_R, b_R)\,, \qquad Y_D = -1/3$$

$$\mathcal{L}_R = \bar{E}_R(i\partial\!\!\!/ - g_1 Y_E B\!\!\!/)E_R + \bar{D}_R(iD\!\!\!\!/ - g_1 Y_D B\!\!\!/)D_R + \bar{U}_R(iD\!\!\!\!/ - g_1 Y_U B\!\!\!/)U_R$$

doublets

$$L_L = \left\{ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \right\}, \qquad Y_L = -1/2$$
$$Q_L = \left\{ \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L \right\}, \qquad Y_Q = 1/6$$

$$\mathcal{L}_L = \bar{L}_L(i\partial\!\!\!/ - g_1 Y_L B\!\!\!/ - g_2 W\!\!\!\!/)L_L + \bar{Q}_L(iD\!\!\!\!/ - g_1 Y_Q B\!\!\!/ - g_2 W\!\!\!\!/)Q_L$$

Gauge bosons

- The gauge bosons transforms as the adjoint of the respective group:

$$
\begin{aligned}
U(1)_Y &\rightarrow B^\mu \quad \text{(singlet)} \\
SU(2)_I &\rightarrow W^\mu = \sum_{i=1}^{3} W_i^\mu\, \sigma^i/2 \quad \text{(triplet)} \\
SU(3)_c &\rightarrow g^\mu = \sum_{a=1}^{8} g_a^\mu\, T^a \quad \text{(octet)}
\end{aligned}
$$

- Kinetic and self-interactions of gauge bosons:

$$
\begin{aligned}
\mathcal{L}_{\text{gauge}} &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} F^i_{\mu\nu} F^{i,\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\
F^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_2\, \epsilon^{ijk}\, W^j_\mu\, W^k_\nu \\
G^a_{\mu\nu} &= \partial_\mu g^a_\nu - \partial_\nu g^a_\mu + g_3\, f^{abc}\, g^b_\mu\, g^c_\nu
\end{aligned}
$$

- Structure constants:

$$
\begin{aligned}
SU(2) &: \quad [\sigma^i, \sigma^j] = \epsilon^{ijk}\, \sigma^k \\
SU(3) &: \quad [T^a, T^b] = f^{abc}\, T^c
\end{aligned}
$$

Fermion masses

- Mass terms for fermions (and gauge bosons) break gauge invariance:

$$\mathcal{L}_m = -m\bar{\psi}\psi = -m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$$

SU(2) singlet SU(2) doublet

- To construct a gauge invariant interaction between ψ_L and ψ_R, we have to introduce a new field (Higgs):
  Lorentz invariance requires it to be a scalar
  SU(2) invariance requires it to be a doublet
  U(1) invariance fixes its hypercharge

- For the electron we can write the following Yukawa interaction:

$$\mathcal{L}_Y = -y_e \left(\bar{\ell}_L \, \phi \, e_R + \bar{e}_R \, \phi^\dagger \, \ell_L \right) \, ,$$

  where Y_ϕ = $Y_L - Y_E$ = ½ and, using $Q=Y+I_3$, we can write $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

- The scalar field ϕ^0 has no color and no electric charge and can acquire a non-zero vacuum expectation value (vev) without breaking $SU(3)_c$, $U(1)_{em}$ and Lorentz invariance

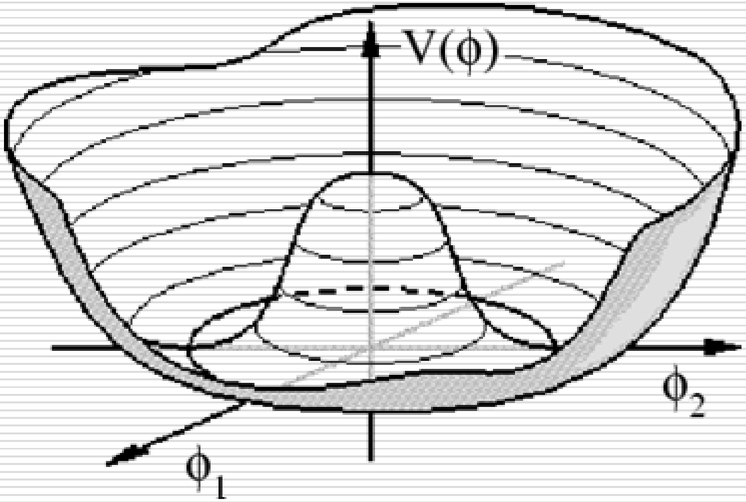
Spontaneous Symmetry Breaking (SSB)

- *Reminder: the vacuum of the theory is the configuration that minimizes the energy*

- We cannot force a field to acquire a non-zero vev, it has to come from the minimization of the potential:

$$\mathcal{L}_{Higgs} = |D_\mu \phi|^2 - V(\phi) = |D_\mu \phi|^2 - \left(\lambda v^2\, \phi^\dagger \phi + \lambda\, (\phi^\dagger \phi)^2\right)$$
$$D_\mu \phi = \left(\partial_\mu - g_1 Y_\phi B_\mu - g_2 W_\mu\right) \phi$$

- λ has to be positive in order to bound from below the potential energy. If $v^2 < 0$ we have one miminum at $|\langle\phi\rangle| = 0$
If $v^2 > 0$ we have a family of minima at $|\langle\phi\rangle| = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$



Generation of mass

- The Higgs doublet can be written as

$$\phi(x) = e^{i\sum_{a=1}^{3}\sigma_a \xi^a(x)/2v}\begin{pmatrix} 0 \\ [v+h(x)]/\sqrt{2} \end{pmatrix}$$

- Inserting this expression into the Yukawa interaction we obtain a mass term:

$$y_e\left(\bar{\ell}_L\,\phi\,e_R + \bar{e}_R\,\phi^\dagger\,\ell_L\right) \longrightarrow y_e\frac{v}{\sqrt{2}}(\bar{e}_L e_R + \bar{e}_R e_L) = m_e\,\bar{e}e$$

- The interactions of ϕ with the SM gauge bosons are completely determined by its SU(3)xSU(2)xU(1) quantum numbers:

$$\phi = (\ \mathbf{1}_{SU(3)}\ ,\ \mathbf{2}_{SU(2)}\ ,\ Y_\phi = 1/2\ )$$

$$D_\mu\phi \ = \ \left(\partial_\mu - g_2 W_\mu - g_1 Y_\phi B_\mu\right)\phi$$

- After exanding around the vev, the term $|D_\mu\phi|^2$ induces masses for the W and Z bosons and the term $-\lambda v^2\phi^\dagger\phi$ induces a mass for h:

$$\begin{aligned} m_W &= g_2\, v/2 \\ m_Z &= \sqrt{g_1^2 + g_2^2}\, v/2 \\ m_h &= \sqrt{2\lambda}\, v \end{aligned}$$

$$\boxed{\begin{aligned} v &= 246\ \text{GeV} \\ \lambda &= ? \end{aligned}}$$

The Yukawa matrices

- Yukawa interactions for leptons:

$$\mathcal{L}_{Y_e} = -\sum_{i,j=1}^{3} \widehat{y}^{e}_{ij}\, \bar{L}^{i}_{L}\, \phi\, E^{j}_{R} + \text{h.c.} = -\bar{L}_L\, \phi\, \widehat{y}^{e}\, E_R + \text{h.c.}$$

$Y_L = -\frac{1}{2}$ $\quad Y_\phi = \frac{1}{2}$ $\quad Y_E = -1$

- Yukawa interactions for right-handed down quarks:

$$\mathcal{L}_{Y_d} = -\bar{Q}_L\, \phi\, \widehat{y}^{d}\, D_R + \text{ h.c.}$$

$Y_Q = \frac{1}{6}$ $\quad Y_\phi = \frac{1}{2}$ $\quad Y_D = -\frac{1}{3}$

- Yukawa interactions for right-handed up quarks:

$$\mathcal{L}_{Y_u} = -\bar{Q}_L\, \tilde{\phi}\, \widehat{y}^{u}\, U_R + \text{ h.c.}$$

$Y_Q = \frac{1}{6}$ $\quad Y_{\tilde{\phi}} = -\frac{1}{2}$ $\quad Y_D = \frac{2}{3}$

in the SM we can use $\tilde{\phi} \equiv i\sigma_2 \phi^* = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix}$

The complete Lagrangian

$$
\begin{aligned}
\mathcal{L}_{SM} \;=\; & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}F^{i}_{\mu\nu}F^{i,\mu\nu} - \frac{1}{4}G^{a}_{\mu\nu}G^{a,\mu\nu} + \theta_{\mathrm{QCD}}\frac{g_s^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^{a}_{\mu\nu}G^{a}_{\rho\sigma} \\
& + \bar{L}_L\, i\not{D}\, L_L + \bar{Q}_L\, i\not{D}\, Q_L + \bar{E}_R\, i\not{D}\, E_R + \bar{D}_R\, i\not{D}\, D_R + \bar{U}_R\, i\not{D}\, U_R \\
& + |D_\mu\phi|^2 - V(\phi) \\
& - \left[\bar{L}_L\, \phi\, \widehat{y}^{e}\, E_R + \bar{Q}_L\, \phi\, \widehat{y}^{d}\, D_R + \bar{Q}_L\, \tilde{\phi}\, \widehat{y}^{u}\, U_R + \text{h.c.}\right]
\end{aligned}
$$

- Parameters:

$$
\left.
\begin{array}{llr}
g_1,\ g_2,\ g_3,\ \theta_{\mathrm{QCD}} & & 4 \\
\lambda,\ v & & 2 \\
\widehat{y}^{e},\ \widehat{y}^{d},\ \widehat{y}^{u} \;\longrightarrow & m_e,\ m_\mu,\ m_\tau, & \\
 & m_d,\ m_s,\ m_b, & 13 \\
 & m_u,\ m_c,\ m_t, & \\
 & V_{\mathrm{CKM}}\,(\lambda, A, \rho, \eta) &
\end{array}
\right\}\ \text{19 parameters}
$$

The CKM matrix

Physical parameters in the Yukawas

A single electron example:

$$\mathcal{L} \;=\; \underbrace{\bar{e}_R\, i\not{D}\, e_R + \bar{\ell}_L\, i\not{D}\, \ell_L}_{\text{invariant under } e_R \to e^{i\phi_R} e_R \text{ and } \ell_L \to e^{i\phi_L} \ell_L} - \left(y_e e^{i\delta}\, \bar{\ell}_L\, \phi\, e_R + y_e e^{-i\delta}\, \bar{e}_R\, \phi^\dagger\, \ell_L \right) . \qquad (1)$$

Let's define $e_R = e^{-i\delta} e'_R$.

The Lagrangian becomes:

$$\mathcal{L} \;=\; \bar{e}'_R\, i\not{D}\, e'_R + \bar{\ell}_L\, i\not{D}\, \ell_L - y_e \left(\bar{\ell}_L\, \phi\, e'_R + \bar{e}'_R\, \phi^\dagger\, \ell_L \right) . \qquad (2)$$

This means that physical observables calculated using the lagrangian in (1) will not depend on δ.

Physical parameters in the Yukawas *(cont'd)*

The three leptons case:

$$\mathcal{L} = \underbrace{\bar{E}_R\, i\not{D}\, E_R + \bar{L}_L\, i\not{D}\, L_L}_{} - \left(\bar{L}_L\, \phi\, \widehat{y}^e E_R + \text{h.c.}\right). \tag{1}$$

invariant under $E_R \to R_e\, E_R$ and $L_L \to S_e\, L_L$
where $S_e\, ,\, R_e \in U(3)$

Let's define $E_R = R_e E'_R$ and $L_L = S_e L'_L$

The Lagrangian becomes:

$$\mathcal{L} = \bar{E}'_R\, i\not{D}\, E'_R + \bar{L}'_L\, i\not{D}\, L'_L - \left(\bar{L}'_L\, \phi\, \underbrace{S_e^\dagger \widehat{y}^e R_e}_{\equiv\, y^e}\, E'_R + \text{h.c.}\right). \tag{2}$$

With proper choice of S_e and R_e , y^e can be made diagonal, real and non-negative.

This means that physical observables calculated using the lagrangian in (1) will depend only on three real parameters (no CP violation).

Physical parameters in the Yukawas *(cont'd)*

The six quarks case:

$$\mathcal{L} = \underbrace{\bar{U}_R\, i\not{D}\, U_R + \bar{D}_R\, i\not{D}\, D_R + \bar{Q}_L\, i\not{D}\, Q_L}_{} - \left(\bar{Q}_L\, \tilde{\phi}\, \hat{y}^u U_R + \bar{Q}_L\, \phi\, \hat{y}^d D_R + \text{h.c.}\right) .$$

invariant under $U_R \to R_u\, U_R$, $D_R \to R_d\, D_R$ and $Q_L \to S_u\, Q_L$

where R_u , R_d , $S_u \in U(3)$

Let's define $U_R = R_u U'_R$, $D_R = R_d D'_R$ and $Q_L = S_u Q'_L$

The Lagrangian becomes:

$$\begin{aligned}\mathcal{L} \;=\; & \bar{U}'_R\, i\not{D}\, U'_R + \bar{D}'_R\, i\not{D}\, D'_R + \bar{Q}'_L\, i\not{D}\, Q'_L \\ & -\Big(\bar{Q}'_L\, \tilde{\phi}\, \underbrace{S_u^\dagger \hat{y}^u R_u}_{\equiv\, y^u}\, U'_R + \bar{Q}'_L\, \phi\, \underbrace{S_u^\dagger \hat{y}^d R_d}_{\equiv\, S_u^\dagger S_d\, y^d \,\equiv\, V\, y^d}\, D'_R + \text{h.c.}\Big) .\end{aligned}$$

CKM matrix

- The quarks mass matrices are:

$$m_u = \frac{v}{\sqrt{2}} y_u, \quad m_d = \frac{v}{\sqrt{2}} y_d$$

Mass eigenstate basis

- In phenomenological applications is more convenient to work in a basis in which the mass terms are diagonal:

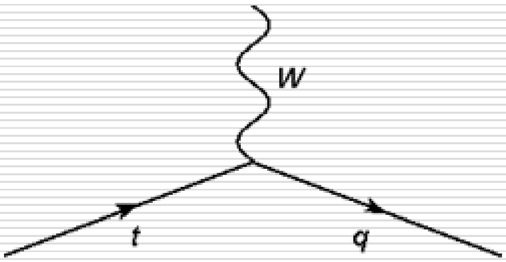
$$\begin{pmatrix} U_L \\ D_L \end{pmatrix} \to \begin{pmatrix} U_L \\ V D_L \end{pmatrix}$$

$$-\bar{U}_L m^u U_R - \bar{D}_L V m^d D_R + \text{h.c.} \to -\bar{U}_L m^u U_R - \bar{D}_L m^d D_R + \text{h.c.}$$

- Flavor changing charged currents:

$$-\frac{g_2}{\sqrt{2}}\left[\bar{U}_L \not{W}^+ D_L + \bar{D}_L \not{W}^- U_L\right] \to -\frac{g_2}{\sqrt{2}}\left[\bar{U}_L \not{W}^+ V D_L + \bar{D}_L V^\dagger \not{W}^- U_L\right]$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Neutral currents

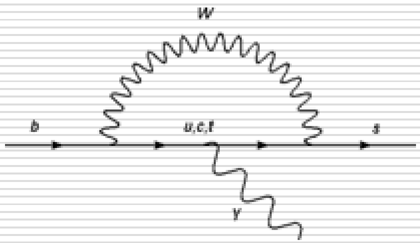
- Tree-level neutral currents remain flavor diagonal:

$$\begin{pmatrix} U_L \\ D_L \end{pmatrix} \to \begin{pmatrix} U_L \\ V D_L \end{pmatrix}$$

$$a_R\, \bar{D}_R \not{Z} D_R + a_L\, \bar{D}_L \not{Z} D_L \to a_R\, \bar{D}_R \not{Z} D_R + a_L\, \bar{D}_L \underbrace{V^\dagger \not{Z} V}_{\not{Z}} D_L$$

- What happens at the 1-loop level?

$\sim 10^{-4}$ ~ 4



$$\begin{aligned} &= V_{ub}^* V_{us} f(\frac{m_u^2}{m_W^2}) + V_{cb}^* V_{cs} f(\frac{m_c^2}{m_W^2}) + V_{tb}^* V_{ts} f(\frac{m_t^2}{m_W^2}) \\ &= V_{tb}^* V_{ts} \left(f(\frac{m_t^2}{m_W^2}) - f(\frac{m_u^2}{m_W^2}) \right) + V_{cb}^* V_{cs} \left(f(\frac{m_c^2}{m_W^2}) - f(\frac{m_u^2}{m_W^2}) \right) \\ &\simeq V_{tb}^* V_{ts} \left(f(\frac{m_t^2}{m_W^2}) - f(0) \right) \propto V_{tb}^* V_{ts} \frac{m_t^2}{m_W^2} \end{aligned}$$

Unitarity: $V_{tb}^* V_{ts} + V_{cb}^* V_{cs} + V_{ub}^* V_{us} = (V^\dagger V)_{32} = 0$

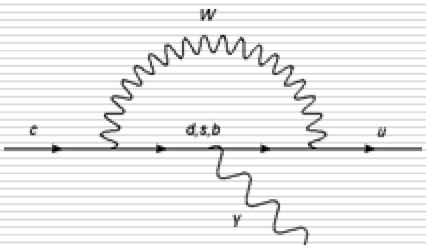
Neutral currents *(cont'd)*

- FCNC in the down-quark sector:

$$\sim \text{(combination of CKM entries)}\ \frac{m_t^2}{m_W^2}$$

- FCNC in the up-quark sector:

$$\sim \times 10^{-6} \qquad \sim 4\times 10^{-3}$$



$$\begin{aligned}
&= V_{ud}^{*}V_{cd}f(\frac{m_d^2}{m_W^2}) + V_{us}^{*}V_{cs}f(\frac{m_s^2}{m_W^2}) + V_{ub}^{*}V_{cb}f(\frac{m_b^2}{m_W^2}) \\
&= V_{ub}^{*}V_{cb}\left(f(\frac{m_b^2}{m_W^2}) - f(\frac{m_d^2}{m_W^2})\right) + V_{us}^{*}V_{cs}\left(f(\frac{m_s^2}{m_W^2}) - f(\frac{m_d^2}{m_W^2})\right) \\
&\simeq V_{ub}^{*}V_{cb}\left(f(\frac{m_b^2}{m_W^2}) - f(0)\right) \propto V_{ub}^{*}V_{cb}\frac{m_b^2}{m_W^2} \\
&\sim \text{(combination of CKM entries)}\ \frac{m_b^2}{m_W^2}
\end{aligned}$$

Physical parameters in the CKM matrix

- We started with two arbitrary complex matrices ($\hat{y}^u$, $\hat{y}^d$): $4n_f^2$

- We used three unitary matrices: $3n_f^2$

  $S_u^\dagger \hat{y}^u R_u = y^u \quad , \quad S_u^\dagger \hat{y}^d R_d = V y^d$

- The matrices $e^{i\varphi} S_u$, $e^{i\varphi} R_u$, $e^{i\varphi} R_d$ achive the same structure: $3n_f^2 - 1$

- Total number of parameters: $4n_f^2 - (3n_f^2 - 1) = n_f + n_f + (n_f - 1)^2$

  $y^u \quad y^d \quad V$

- $(n_f - 1)^2 = \underbrace{n_f(n_f - 1)/2}_{\text{real}} + \underbrace{(n_f - 1)(n_f - 2)/2}_{\text{imaginary}}$

A global look at the SM

- The gauge invariant part of the Lagrangian depends on 4 parameters

- The Yukawa sector depends on 54 parameters ($\widehat{y}^e,\ \widehat{y}^d,\ \widehat{y}^u$) out of which only 13 are observables.

$$U(3)_{L_L}\otimes U(3)_{E_R}\otimes U(3)_{Q_L}\otimes U(3)_{U_R}\otimes U(3)_{D_R}$$

$$\Big\downarrow\ \widehat{y}^e,\ \widehat{y}^d,\ \widehat{y}^u$$

$$\underbrace{U(1)_e\otimes U(1)_\mu\otimes U(1)_\tau}_{\text{Lepton numbers}}\otimes \underset{\substack{\downarrow\\ \text{baryon number}}}{U(1)_B}$$

- Homework. How many physical parameters are there if we introduce a second Higgs doublet? How many phases? Where do they show up?

$$\sum_{i=1}^{2}\left(\bar{L}_L\,\phi_i\,\widehat{y}_i^e E_R+\bar{Q}_L\,\tilde{\phi}_i\,\widehat{y}_i^u U_R+\bar{Q}_L\,\phi_i\,\widehat{y}_i^d D_R\right)$$

Parametrizations of the CKM matrix

- Standard parametrization:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$

1) $s_{12} \simeq |V_{us}|\ ,\ s_{23} \simeq |V_{cb}|\ ,\ s_{13} \simeq |V_{ub}|$
2) The CP phase is always multiplied by the very small s_{13}

- Wolfenstein parametrization:

$$\lambda \equiv s_{12}\ ,\ A \equiv s_{23}/\lambda^2\ ,\ \rho + i\eta \equiv s_{13}e^{i\delta}/A\lambda^3$$

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

The Jarlskog invariant

- It can be shown that, in the SM, CPV effects are present if and only if

$$F_u \, F_d \, J \neq 0$$

where:

$$\begin{aligned} F_u &= (m_u^2 - m_c^2)(m_c^2 - m_t^2)(m_t^2 - m_u^2) \\ F_d &= (m_d^2 - m_s^2)(m_s^2 - m_b^2)(m_b^2 - m_d^2) \\ J &= \mathrm{Im}[V_{us} V_{cd} V_{cs}^* V_{ub}^*] \\ &= c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} \sin\delta \\ &= A^2 \lambda^6 \eta \end{aligned}$$

- If there is a degeneracy either in the up or down quark mass matrices we have more freedom to choose the unitary transformations S_u, R_d and R_u and we can eliminate the CPV phase δ

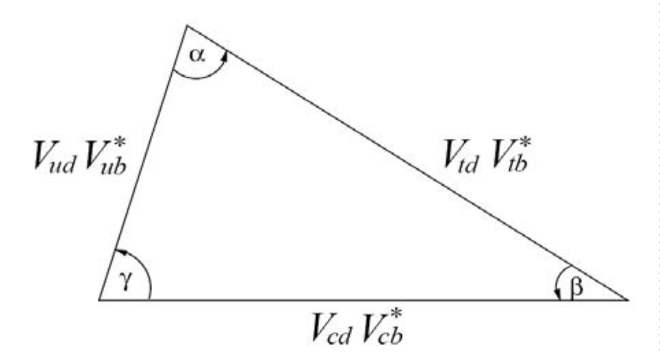
- All CPV effects are proportional to J, hence they are small even if the phase δ is large

Direct measurement of CKM entries

- $|V_{ud}|=0.97377\pm0.00027$: nuclear β decay
- $|V_{us}|=0.2257\pm0.0021$: $K^+\to\pi^0 l\nu$, $k^0\to\pi^- l\nu$
- $|V_{cd}|=0.230\pm0.011$: $\nu_\mu N\to\mu^- X$ vs $\nu_\mu N\to\mu^-\mu^+\nu_\mu X$, $D\to Kl\nu$, $D\to\pi l\nu$
- $|V_{cs}|=0.957\pm0.017\pm0.093$: $D\to Kl\nu$, $D\to\pi l\nu$, $W\to cs$
- $|V_{cb}|=(41.6\pm0.6)\ 10^{-3}$: $B\to X_c l\nu$, $B\to D^{(*)}l\nu$
- $|V_{ub}|=(4.31\pm0.30)\ 10^{-3}$: $B\to X_u l\nu$, $B\to\pi l\nu$
- $|V_{tb}|=1.3\pm0.2$: single top production (!)
- V_{ts} and V_{td}: no direct measurements

Unitarity triangle

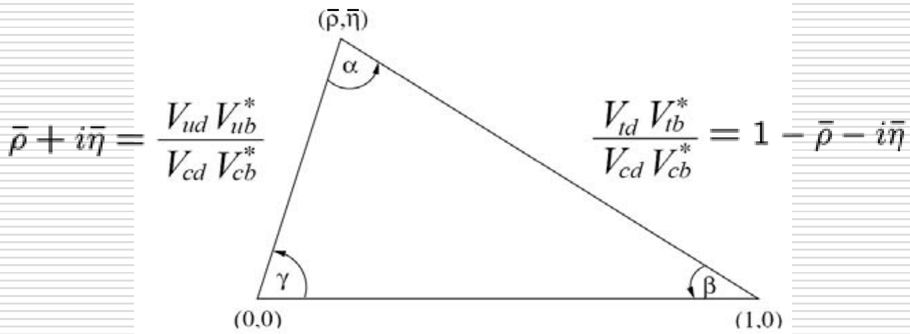
- B_d triangle: $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$



$$\alpha = \phi_2 \quad = \quad \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right]$$

$$\beta = \phi_1 \quad = \quad \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

$$\gamma = \phi_3 \quad = \quad \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$



$$\left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right| = \sqrt{\bar\rho^2 + \bar\eta^2} + O(\lambda^6)$$

$$\left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right| = \sqrt{(1-\bar\rho)^2 + \bar\eta^2} + O(\lambda^6)$$

$$(\bar\rho, \bar\eta) \equiv (\rho, \eta)(1 - \lambda^2/2)$$

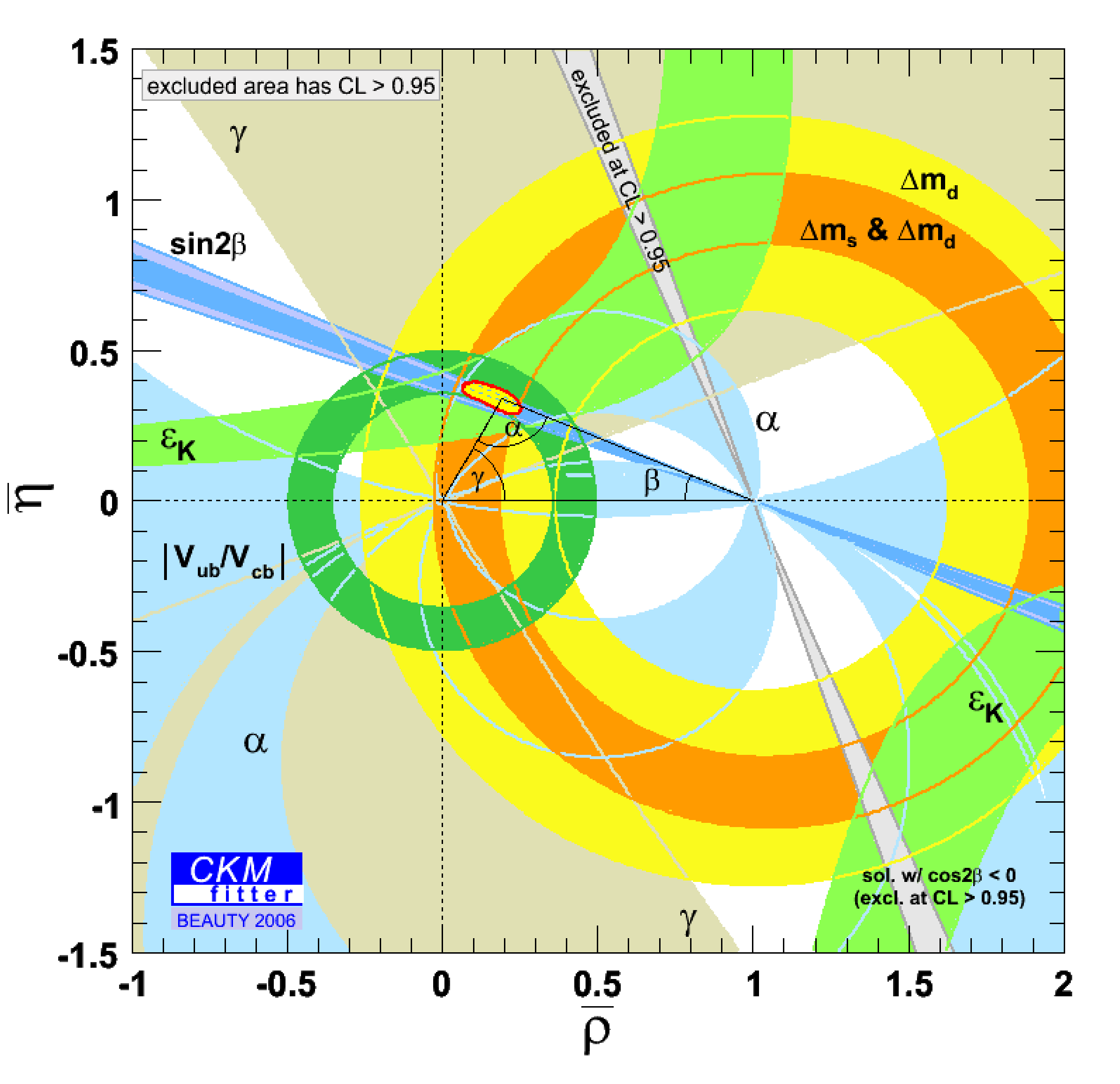
Unitarity triangle *(cont'd)*

- B_s triangle: $V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$

$A\lambda^4\sqrt{\rho^2+\eta^2}$ $\qquad$ $A\lambda^2$

$$\beta_s \;=\; \arg\left[-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right] = \lambda^2\eta + O(\lambda^4)$$

Unitarity triangle fit (CKMfitter)



Unitarity triangle fit (UTfit)