Introduction to Flavor Physics in and beyond the Standard Model

Lecture 3/4

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References:

The BaBar physics book, http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-r-504-TOC-pdf.pdf

B physics at the Tevatron: Run II and beyond, hep-ph/0201071

The Discovery potential of a Super B Factory, hep-ph/0503261

Outline

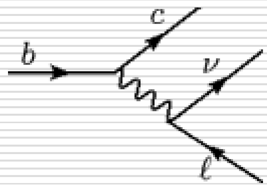
- Summary of the previous lecture
- CP violation: meson mixing, classification of CPV effects
- Examples: $\sin(2\beta)$, $\sin(2\alpha)$, γ

Main points from lecture 2

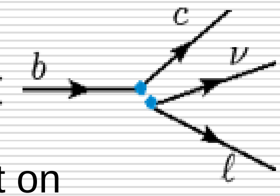
Effective Hamiltonian approach

m_W

perturbative → integrate out:



$\longrightarrow \underbrace{C(\mu)} \times$



not dependent on
the external states

m_b

dependence on external states (possibly non-perturbative)

$$A(B \to X) \;=\; \sum C_i(\mu)\,\langle X\,|O_i(\mu)|\,B\rangle + O\left(\frac{m_b^2}{m_W^2}\right)$$

Advantage 1: improved perturbation theory

- A generic amplitude at the 1-loop level looks like:

$$A(i \to f) \sim 1 + \alpha_s(1+L) + \alpha_s^2(1+L+L^2) + O(\alpha_s^3)$$

where $L = \log m_W^2 / p_{\rm ext}^2$

- We can write $\alpha_s \log \frac{m_W^2}{p_{\rm ext}^2} = \underbrace{\alpha_s \log \frac{m_W^2}{\mu^2}}_{C(\mu)} + \underbrace{\alpha_s \log \frac{\mu^2}{p_{\rm ext}^2}}_{\langle f|O(\mu)|i\rangle}$

- Using the fact that A(i→f) is μ independent, we can write a RGE for C(μ), whose solution resums all these logarithms:

$$\frac{\mathrm{d}C(\mu)}{\mathrm{d}\log\mu} = \gamma\, C(\mu) \longrightarrow C(\mu) = C(\mu_0)\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{\frac{\gamma_0}{2\beta_0}}$$

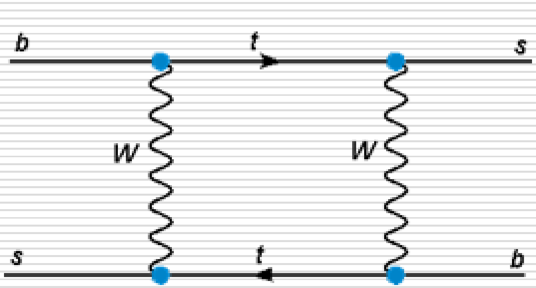
- Now we can choose $\mu_0 \sim O(m_W)$ in order to minimize the logs in $C(\mu_0)$ and $\mu_b \sim O(m_b)$ to minimize the logs in $\langle f|O(\mu_b)|i\rangle$:

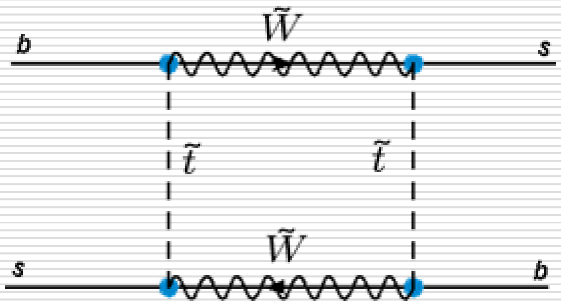
$$A(i \to f) = C(\mu_0)\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu_b)}\right)^{\frac{\gamma_0}{2\beta_0}} \langle f|O(\mu_b)|i\rangle$$

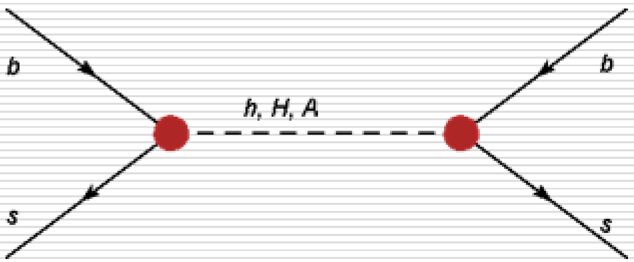
Advantage 2: separation of short and long distance

- The Wilson coefficients do not depend on the external states

- Since the matrix elements of the various operators are dominated by large distance physics, new physics can enter only by:
  1. modifying the Wilson coefficients
  2. inducing new operators







SM | Supersymmetry | SUSY at large tan(β)

$$O^{VLL} = (\bar{b}_L \gamma_\mu q_L)(\bar{b}_L \gamma^\mu q_L)$$

$$O_2^{LR} = (\bar{b}_R q_L)(\bar{b}_L q_R)$$
$$O_1^{SLL} = (\bar{b}_R q_L)(\bar{b}_R q_L)$$

CP Violation

Introduction

- Associated to complex parameters:

$$\underbrace{\begin{aligned} \mathcal{L} &= -\frac{g_2}{\sqrt{2}}\left[V_{ub}\bar{u}_L\gamma^\mu b_L W^+_\mu + V^*_{ub}\bar{b}_L\gamma^\mu u_L W^-_\mu\right] \\ CP\,\mathcal{L}\,(CP)^{-1} &= -\frac{g_2}{\sqrt{2}}\left[V_{ub}\bar{b}_L\gamma^\mu u_L W^-_\mu + V^*_{ub}\bar{u}_L\gamma^\mu b_L W^+_\mu\right] \end{aligned}}_{\mathcal{L} = CP\,\mathcal{L}\,(CP)^{-1} \;\Leftrightarrow\; V_{ub} = V^*_{ub}}$$

- CPV appears only in weak interactions and together with flavor changing interactions

- The CPV phase in the Standard Model is large

CP violation in meson decays

1. CPV in mixing

$$|B^0\rangle \quad \ldots\ldots t \ldots\ldots \quad \beta\,|\bar{B}^0\rangle$$
$$|\bar{B}^0\rangle \quad \ldots\ldots t \ldots\ldots \quad \bar{\beta}\,|B^0\rangle$$

if $|\beta| \neq |\bar{\beta}|$, it is possible to measure CPV in decays to final states accessible only to B^0or $\bar{B}^0$

2. CPV in decay

$$\Gamma(B \to f) \neq \Gamma(\bar{B} \to \bar{f})$$

$$A(B \to f) = \sum A_k e^{i(\delta_k + \phi_k)} \longrightarrow A(\bar{B} \to \bar{f}) = \sum A_k e^{i(\delta_k - \phi_k)}$$

from QCD from CKM

3. CPV in the interference of decays with and without mixing

$$\begin{tikzcd}
B^0 \arrow[rr, bend left] \arrow[dr] & & f_{CP} \\
& \bar{B}^0 \arrow[ur] &
\end{tikzcd}$$

Meson mixing

- Let's consider two states that at t = 0 are pure B^0 and $\bar{B}^0$:

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}|B^0(t)\rangle\\|\bar{B}^0(t)\rangle\end{pmatrix}=\left(\underbrace{M}_{\text{mass matrix}}-i\underbrace{\frac{\Gamma}{2}}_{\text{decay matrix}}\right)\begin{pmatrix}|B^0(t)\rangle\\|\bar{B}^0(t)\rangle\end{pmatrix}$$

- CPT: $M_{11}=M_{22}$ and $\Gamma_{11}=\Gamma_{22}$
  Hermiticity: $M_{12}=M_{21}^*$ and $\Gamma_{12}=\Gamma_{21}^*$

- The mass eigenstates are ($|p|^2+|q|^2=1$):

$$\begin{aligned}M_L,\Gamma_L &: |B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle\\ M_H,\Gamma_H &: |B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle\end{aligned}$$

$$\begin{aligned}m &= (M_H+M_L)/2 = M_{11} & \Delta m &= M_H - M_L\\ \Gamma &= (\Gamma_L+\Gamma_H)/2=\Gamma_{11} & \Delta\Gamma &= \Gamma_H-\Gamma_L\end{aligned}$$

Time evolution of weak eigenstates

- The time evolution of these states is:

$$|B_{H,L}(t)\rangle \;=\; e^{-(iM_{H,L}+\Gamma_{H,L}/2)t}|B_{H,L}\rangle$$

- Let us consider a state that is a pure B^0 at t = 0:

$$\begin{aligned} |B^0\rangle &= (|B_L\rangle + |B_H\rangle)/2p \\ |\bar{B}^0\rangle &= (|B_L\rangle - |B_H\rangle)/2q \end{aligned}$$

- At some time t it will be:

$$|B^0(t)\rangle \;=\; \frac{1}{2p}\left[e^{-iM_Lt-\Gamma_Lt/2}|B_L\rangle + e^{-iM_Ht-\Gamma_Ht/2}|B_H\rangle\right]$$

- Expressing everything in terms of the weak eigenstates:

$$|B^0(t)\rangle \;=\; g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\bar{B}^0\rangle$$

Time evolution of weak eigenstates *cont'd*

- Similarly:

$$
\begin{aligned}
|B^0(t)\rangle &= g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\bar{B}^0\rangle \\
|\bar{B}^0(t)\rangle &= \frac{p}{q}g_-(t)|B^0\rangle + g_+(t)|\bar{B}^0\rangle
\end{aligned}
$$

where

$$
\begin{aligned}
g_+(t) &= e^{-imt}\,e^{-\Gamma t/2}\left[\cosh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta m t}{2} - i\sinh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta m t}{2}\right] \\
g_-(t) &= e^{-imt}\,e^{-\Gamma t/2}\left[-\sinh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta m t}{2} + i\cosh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta m t}{2}\right]
\end{aligned}
$$

- CPV in mixing:

$$
\begin{aligned}
&|B^0\rangle \quad \ldots\ldots t \ldots\ldots \quad \beta\,|\bar{B}^0\rangle \quad \Rightarrow \beta = (q/p)\,g_-(t) \\
&|\bar{B}^0\rangle \quad \ldots\ldots t \ldots\ldots \quad \bar{\beta}\,|B^0\rangle \quad \Rightarrow \bar{\beta} = (p/q)\,g_-(t)
\end{aligned}
$$

$$
|\beta| \neq |\bar{\beta}| \Leftrightarrow \left|\frac{q}{p}\right|^2 \neq 1
$$

Meson mixing parameters

- Dispersive and absorbitive parts of ΔB=2 diagrams yield M_{12} and Γ_{12}

- Explicit solution of the eigenvalue equation for M-i Γ/2 are:

$$\begin{aligned}
(\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2 &= 4|M_{12}|^2 - |\Gamma_{12}|^2 \\
\Delta m\, \Delta\Gamma &= -4\mathrm{Re}(M_{12}\Gamma_{12}^*) \\
\frac{q}{p} &= -\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m + i\Delta\Gamma/2}
\end{aligned}$$

- These expressions are valid for any kind of meson mixing: K, D, B_d and B_s

The B mixing case

- In the B mixing system, there is empirical evidence for:

$$|\Gamma_{12}| \ll |M_{12}| \quad \text{and} \quad \Delta\Gamma \ll \Delta m$$

- In general $|\Gamma_{12}| < \Gamma$, because the former stems from final states that are common to B^0 and $\bar{B}^0$

- B$_s$ case.
  Experimental evidence shows that $\Gamma_{B_s} \ll \Delta m_{B_s} \Rightarrow |\Gamma_{12}^s| \ll \Delta m_{B_s}$

- B$_d$ case.
  Decays to final states common to B^0 and $\bar{B}^0$ have BR in the 10^{-3} range. We can assume that their combined effect will be in the 10^{-2} range:

$$|\Delta\Gamma_{B_d}|/\Gamma_{B_d} = O(10^{-2})$$

Using $\Delta m_{B_d} \sim 0.75\Gamma_{B_d}$ we obtain:

$$|\Gamma_{12}^d| \ll \Delta m_{B_d}$$

The B mixing case *cont'd*

$$
\begin{aligned}
(\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2 &= 4|M_{12}|^2 - |\Gamma_{12}|^2 \\
\Delta m\, \Delta\Gamma &= -4\mathrm{Re}(M_{12}\Gamma_{12}^*) \\
\frac{q}{p} &= -\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m + i\Delta\Gamma/2}
\end{aligned}
$$

$+$

$$
\begin{aligned}
|\Gamma_{12}| &\ll |M_{12}| \\
\Delta\Gamma &\ll \Delta m
\end{aligned}
$$

$$
\Delta m = 2|M_{12}|
$$

$$
\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|}\left[1 - \frac{1}{2}\mathrm{Im}\frac{\Gamma_{12}}{M_{12}}\right]
$$

$$
\Delta\Gamma = 2\,\mathrm{Re}\left(\frac{M_{12}}{|M_{12}|}\Gamma_{12}^*\right)
$$

Time-dependent decay rates

- Given a final state f, let us introduce the two amplitudes $A_f = \langle f|\mathcal{H}_d|B^0\rangle$ and $\bar{A}_f = \langle f|\mathcal{H}_d|\bar{B}^0\rangle$, and the quantity:

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f} = -\frac{M_{12}^*}{|M_{12}|}\frac{\bar{A}_f}{A_f}\left[1 - \frac{1}{2}\underbrace{\mathrm{Im}\frac{\Gamma_{12}}{M_{12}}}_{a}\right]$$

- Master equations:

$$\begin{aligned}\Gamma(B^0(t)\to f) &= \mathcal{N}_f|A_f|^2 e^{-\Gamma t}\Big\{\frac{1+|\lambda_f|^2}{2}\cosh\frac{\Delta\Gamma t}{2} + \frac{1-|\lambda_f|^2}{2}\cos(\Delta m t)\\ &\quad -\mathrm{Re}\lambda_f \sinh\frac{\Delta\Gamma t}{2} - \mathrm{Im}\lambda_f \sin(\Delta m t)\Big\}\end{aligned}$$

$$\begin{aligned}\Gamma(\bar{B}^0(t)\to f) &= \mathcal{N}_f|A_f|^2(1+a)e^{-\Gamma t}\Big\{\frac{1+|\lambda_f|^2}{2}\cosh\frac{\Delta\Gamma t}{2} - \frac{1-|\lambda_f|^2}{2}\cos(\Delta m t)\\ &\quad -\mathrm{Re}\lambda_f \sinh\frac{\Delta\Gamma t}{2} + \mathrm{Im}\lambda_f \sin(\Delta m t)\Big\}\end{aligned}$$

The three types of CP violation

- There are three quantities that drive CPV effects:

$$\left|\frac{q}{p}\right|, \quad \left|\frac{\bar{A}_{\bar{f}}}{A_f}\right|, \quad \lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

- CPV in mixing: $|q/p| \neq 1$
- CPV in decay: $|\bar{A}_{\bar{f}}/A_f| \neq 1$
- CPV in the interference between decay and mixing: $\lambda_f \neq \pm 1$

CPV in mixing

$$\left|\frac{q}{p}\right|^2 = \left|\frac{2M_{12}^* - i\Gamma_{12}^*}{2M_{12} - i\Gamma_{12}}\right| \neq 1$$

- The mass eigenstates are not CP eigenstates
- An example is the neutral semileptonic decay asymmetry to wrong sign leptons:

$$\begin{aligned} a_{sl}(t) &= \frac{\Gamma(\bar{B}^0(t) \to \ell^+\nu X) - \Gamma(B^0(t) \to \ell^-\bar{\nu} X)}{\Gamma(\bar{B}^0(t) \to \ell^+\nu X) + \Gamma(B^0(t) \to \ell^-\bar{\nu} X)} \\ &= \frac{1 - |q/p|^4}{1 + |q/p|^4} \simeq \mathrm{Im}\frac{\Gamma_{12}}{M_{12}} \end{aligned}$$

- Large hadronic uncertainties in the calculation of Γ_{12}

CPV in decay

- In charged B decays this is the only source of CP asymmery

- We need two amplitudes with different weak and strong phases:

$$
\begin{aligned}
A_f &= \sum A_k e^{i(\delta_k+\phi_k)} \\
\bar{A}_{\bar{f}} &= \sum A_k e^{i(\delta_k-\phi_k)}
\end{aligned}
$$

$$
\begin{aligned}
A &= \frac{\Gamma(B^- \to f) - \Gamma(B^+ \to \bar{f})}{\Gamma(B^- \to f) + \Gamma(B^+ \to \bar{f})} \\
&= -\frac{1-|\bar{A}_{\bar{f}}/A_f|^2}{1+|\bar{A}_{\bar{f}}/A_f|^2} \\
&= -\frac{2A_1A_2\sin(\delta_2-\delta_1)\sin(\phi_2-\phi_1)}{A_1^2+A_2^2+2A_1A_2\cos(\delta_2-\delta_1)\cos(\phi_2-\phi_1)}
\end{aligned}
$$

- We need the amplitude ratio and the difference of strong phases: both are difficult to calculate

CPV in the interference between decay and mixing

- Similar to CPV in decay: the two amplitudes are $B^0 \to f_{CP}$ and $B^0 \to \bar{B}^0 \to f_{CP}$

$$
\begin{aligned}
A_{f_{CP}} &= \frac{\Gamma(\bar{B}^0(t) \to f) - \Gamma(B^0(t) \to f)}{\Gamma(\bar{B}^0(t) \to f) + \Gamma(B^0(t) \to f)} \\
&= \underbrace{\frac{2\mathrm{Im}\lambda_f}{1+|\lambda_f|^2}}_{S_f} \sin(\Delta m\, t) - \underbrace{\frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}}_{C_f} \cos(\Delta m\, t)
\end{aligned}
$$

- In the limit $|\lambda_f| = 1$ we have:

$$
A_{f_{CP}} = \mathrm{Im}\lambda_f \sin(\Delta m\, t)
$$

Flavor specific decays and Δm

- Let us consider a final state accessible to B^0 but not to $\bar{B}^0$
- Examples are $B_s^0 \to D_s^- \pi^+$ and $B^0 \to X\ell^+\nu$
- The time dependent rates are:

$$\Gamma(B^0(t) \to f) = \mathcal{N}_f |A_f|^2 e^{-\Gamma t} \frac{1}{2}\left[\cosh\frac{\Delta\Gamma t}{2} + \cos(\Delta m t)\right]$$
$$\Gamma(B^0(t) \to \bar{f}) = \mathcal{N}_f |A_f|^2 e^{-\Gamma t} \frac{1}{2}\left[\cosh\frac{\Delta\Gamma t}{2} - \cos(\Delta m t)\right]$$

- The asymmetry is:

$$\mathcal{A}_0 = \frac{\cos(\Delta m\, t)}{\cosh(\Delta\Gamma\, t/2)} + \frac{1}{2}\mathrm{Im}\frac{\Gamma_{12}}{M_{12}}\left[1 - \frac{\cos^2(\Delta m\, t)}{\cosh^2(\Delta\Gamma\, t/2)}\right]$$

- Used to measure Δm

Untagged B mesons and $\Delta\Gamma$

- Let us consider the time evolution of an equal admixture of B^0 and $\bar{B}^0$

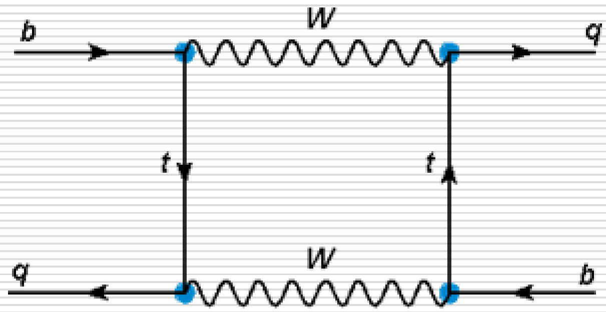
$$\begin{aligned}\Gamma[f,t] &= \Gamma(B^0(t)\to f)+\Gamma(\bar{B}^0(t)\to f)\\ &= \underbrace{\mathcal{N}_f|A_f|^2(1+|\lambda_f|^2)e^{-\Gamma t}}_{\text{can be eliminated if the total branching ratio is known}}\left[\cosh\frac{\Delta\Gamma\, t}{2}-\frac{2\mathrm{Re}\lambda_f}{1+|\lambda_f|^2}\sinh\frac{\Delta\Gamma\, t}{2}\right]\end{aligned}$$

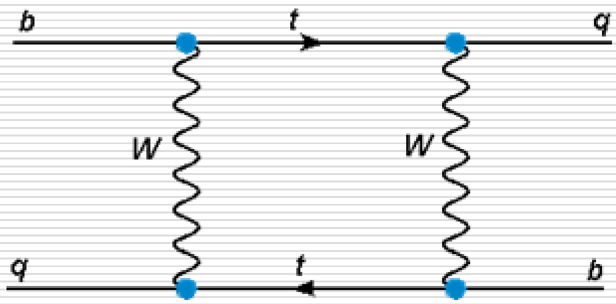
can be eliminated if the total
branching ratio is known

- Useful to extract information on $\Delta\Gamma$ and λ_f at hadronic machines

Calculation of M_{12}

- $\Delta B=2$ transitions need to go through a W loop → short distance physics is involved





- As usual we write an effective Hamiltonian that captures the SD physics:

$$H_{\text{eff}} = \underbrace{\frac{G_F^2 m_W^2}{4\pi^2}(V_{tb}^* V_{tq})^2 S_0\left(\frac{m_t^2}{m_W^2}\right)\eta_B b_B(\mu)}_{\text{NLO Wilson coefficient: } C(\mu)} \underbrace{(\bar{q}_L \gamma_\mu b_L)(\bar{q}_L \gamma^\mu b_L)}_{\text{Local operator: } Q(\mu)} + \text{h.c.}$$

Calculation of M_{12} *cont'd*

- Now we must specify the external states:

$$M_{12} = C(\mu)\,\langle B^0|Q(\mu)|\bar{B}^0\rangle$$

- From lattice-QCD we obtain:

$$\langle B^0|Q(\mu)|\bar{B}^0\rangle = \frac{2}{3} f_B^2 \frac{\hat{B}_B}{b_B(\mu)}$$

- Putting everything together:

$$\begin{aligned} M_{12} &= \frac{\langle B^0|\mathcal{H}_{eff}|\bar{B}^0\rangle}{2m_B} \\ &= \frac{G_F^2 m_W^2}{12\pi^2}\eta_B m_B\,(f_B^2 \underbrace{\hat{B}_B)}_{\text{theory uncertainty}}\,S_0\left(\frac{m_t^2}{m_W^2}\right)(V_{tb}V_{tq}^*)^2 \end{aligned}$$

Calculation of M_{12} *cont'd*

- Mass difference:

$$\begin{aligned} \Delta m_B &= 2|M_{12}| \\ &= \frac{G_F^2 m_W^2}{6\pi^2} \eta_B m_B \left(f_B^2 \hat{B}_B\right) S_0\left(\frac{m_t^2}{m_W^2}\right) \left|V_{tb}^* V_{tq}\right|^2 \end{aligned}$$

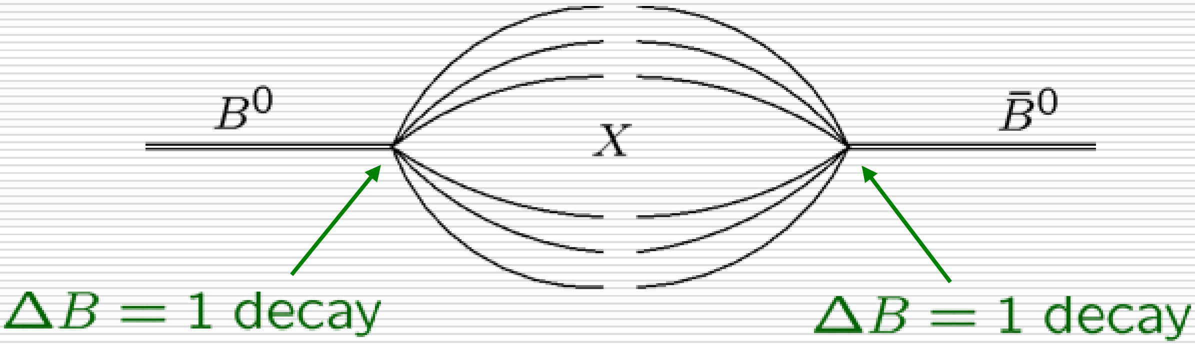
- Mixing parameter:

$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|} = -\frac{(V_{tb}^* V_{tq})^2}{|V_{tb}^* V_{tq}|^2} = \begin{cases} -e^{-2i\beta} & \text{q=d} \\ -e^{-2i\beta_s} & \text{q=s} \end{cases}$$

up to corrections proportional to Γ_{12}/M_{12}

Calculation of Γ_{12}

- Γ_{12} is the absorbitive part of the $B^0 \to \bar{B}^0$ amplitude: it originates from decays that are common to B^0 and $\bar{B}^0$



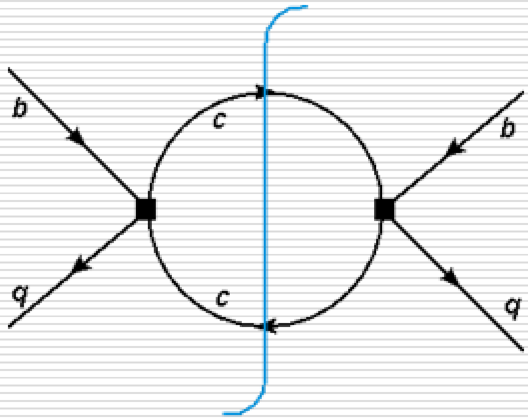
- In formulae:

$$\begin{aligned}\Gamma_{12} &= \frac{1}{2m_B}\delta^4(p_B - p_X)\langle B|\mathcal{H}^{\Delta B=1}|X\rangle\langle X|\mathcal{H}^{\Delta B=1}|\bar{B}\rangle \\ &= \frac{1}{2m_B}\mathrm{Im}\langle B|i\int \mathrm{d}^4x T\left\{\mathcal{H}^{\Delta B=1}(x)\,\mathcal{H}^{\Delta B=1}(0)\right\}|\bar{B}\rangle\end{aligned}$$

Calculation of Γ_{12} *cont'd*

- At leading order we have:

$\Gamma_{12} =$



$$= -\frac{G_F^2}{24\pi} m_B m_b^2 f_B^2 B_B \left(V_{tb} V_{tq}^*\right)^2 \; [\ldots]$$

- Note that: $\dfrac{\Gamma_{12}}{M_{12}} \sim O\left(\dfrac{m_b^2}{m_W^2}\right) \ll 1$

Calculation of the amplitudes (A_f)

- We are plagued by non-perturbative QCD effects
- In general we must take matrix elements of a decay effective Hamiltonian:

$$
\begin{aligned}
A_f = A(B \to f) &= \langle f|\mathcal{H}^d|B\rangle = \sum C_i \, \langle f|O_i|B\rangle \\
\bar{A}_{\bar{f}} = A(\bar{B} \to \bar{f}) &= \langle \bar{f}|\mathcal{H}^d|\bar{B}\rangle = \sum C_i^* \, \underbrace{\langle \bar{f}|O_i^\dagger|\bar{B}\rangle}
\end{aligned}
$$

Form factors
QCD factorization
Lattice QCD

- If only one operator contributes ($C = |C|e^{i\varphi}$):

$$\frac{\bar{A}_{\bar{f}}}{A_f} = \frac{C^*}{C} = e^{-2i\varphi}$$

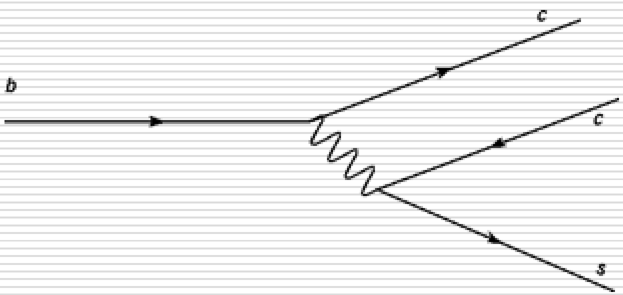
- If all the Wilson coefficients come with the same phase (φ):

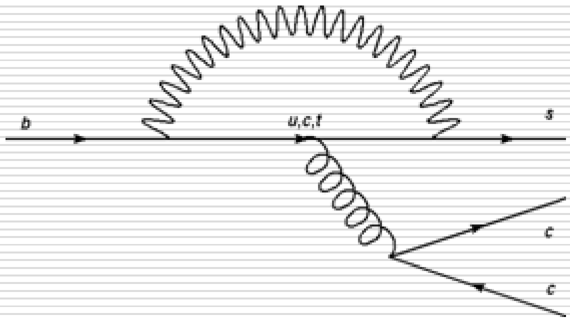
$$\frac{\bar{A}_{\bar{f}}}{A_f} = e^{-2i\varphi} \frac{\sum |C_i| \, \langle \bar{f}|O_i^\dagger|\bar{B}\rangle}{\sum |C_i| \, \langle f|O_i|B\rangle} = e^{-2i\varphi}$$

Examples

$\sin(2\beta)$ from $B \to J/\Psi K_s$

- The final state is a CP eigenstate (with eigenvalue -1)
- This process is mediated by both tree and penguin operators:





$$\begin{aligned} A &= T_{c\bar{c}s}\, V_{cb}V_{cs}^{*} + P_s^u\, V_{ub}V_{us}^{*} + P_s^c\, V_{cb}V_{cs}^{*} + P_s^t\, V_{tb}V_{ts}^{*} \\ &= (T_{c\bar{c}s} + P_s^c - P_s^t)\, V_{cb}V_{cs}^{*} + \underbrace{(P_s^u - P_s^t)\, V_{ub}V_{us}^{*}}_{\text{penguin pollution}} \end{aligned}$$

- The last term is doubly suppressed:

$$\frac{V_{ub}V_{us}^{*}}{V_{cb}V_{cs}^{*}} \simeq 10^{-2}$$

$$P/T \sim O(0.1)$$

$$\frac{A(B^0 \to J/\psi K^0)}{A(\bar{B}^0 \to J/\psi \bar{K}^0)} \simeq \frac{V_{cb}V_{cs}^{*}}{V_{cb}^{*}V_{cs}}$$

$\sin(2\beta)$ from $B \to J/\Psi K_s$

- There is a problem: $B^0 \to K^0$ and $\bar{B}^0 \to \bar{K}^0$
- K_s is the lighter mass eigenstate: $|K_s\rangle = p_K|K^0\rangle + q_K|\bar{K}^0\rangle$
- Interference between $B^0 \to J/\psi K_s$ and $\bar{B}^0 \to J/\psi K_s$ is only possible through K mixing:

$$\frac{A(B^0 \to J/\psi K_s)}{A(\bar{B}^0 \to J/\psi K_s)} = \frac{A(B^0 \to J/\psi K^0)}{A(\bar{B}^0 \to J/\psi \bar{K}^0)}\frac{p_K}{q_K} = -\eta_{\psi K_s}\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}$$

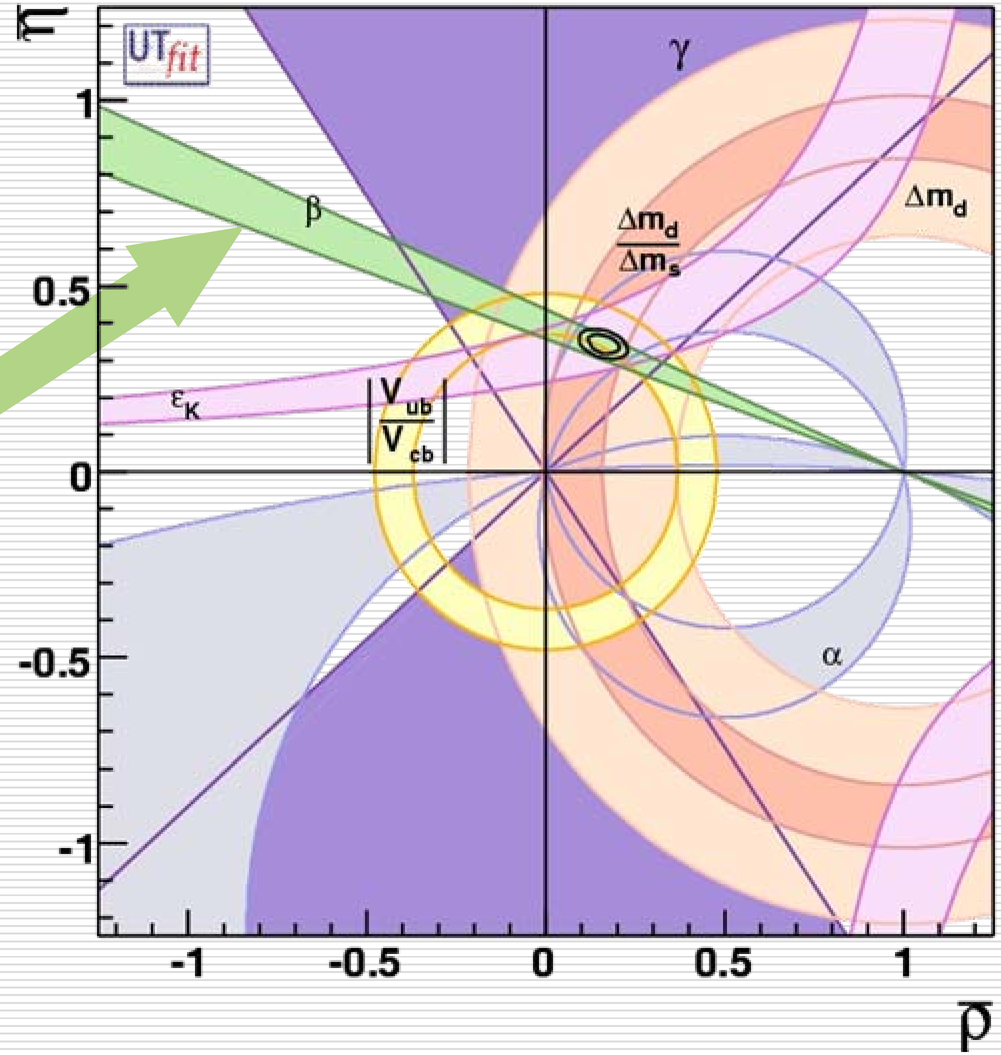
- Putting everything together:

$$\lambda_{\psi K_s} = \frac{q}{p}\frac{A_{\psi K^0}}{\bar{A}_{\psi \bar{K}^0}}\frac{p_K}{q_K} = \eta_{\psi K_s}\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}} = \eta_{\psi K_s}e^{-2i\beta}$$

- and the time dependent CP asymmetry is

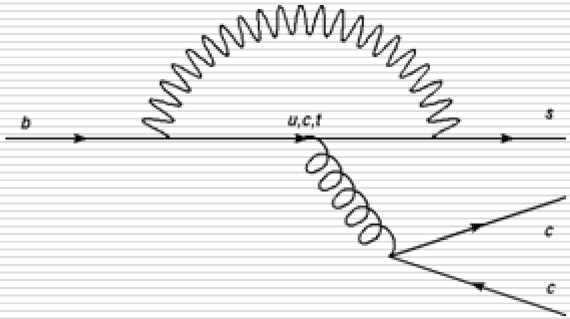
$$\begin{aligned}\mathcal{A}_{\psi K_s} &= \frac{\Gamma(\bar{B}^0(t) \to J/\psi K_s) - \Gamma(B^0(t) \to J/\psi K_s)}{\Gamma(\bar{B}^0(t) \to J/\psi K_s) + \Gamma(B^0(t) \to J/\psi K_s)} \\ &= -\eta_{\psi K_s}\sin(2\beta)\ \sin(\Delta m_{B_d}\, t)\end{aligned}$$

$$a_{\psi K_s}^{\text{exp}} = 0.759 \pm 0.037$$



$\sin(2\beta)$ from $B \to \phi K_s$?

- This process is mediated by penguin operators only:



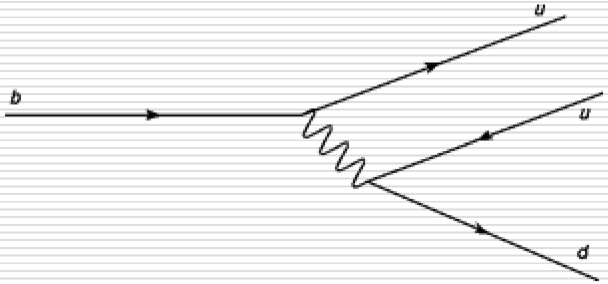
$$\begin{aligned} A &= +P_s^u\, V_{ub}V_{us}^* + P_s^c\, V_{cb}V_{cs}^* + P_s^t\, V_{tb}V_{ts}^* \\ &= \underbrace{(P_s^c - P_s^t)\, V_{cb}V_{cs}^*}_{\text{penguin pollution}} + \underbrace{(P_s^u - P_s^t)\, V_{ub}V_{us}^*}_{\text{penguin pollution}} \end{aligned}$$

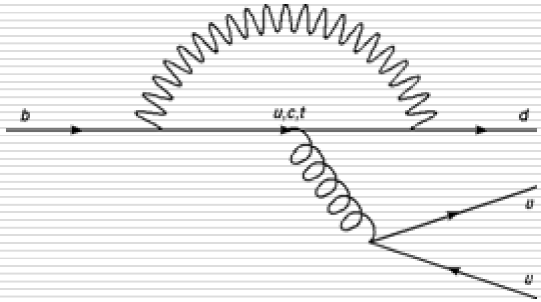
- The last term is only CKM suppressed:

$$\frac{V_{ub}V_{us}^*}{V_{cb}V_{cs}^*} \simeq 10^{-2} \quad \Rightarrow \quad \frac{A(B^0 \to \phi K^0)}{A(\bar{B}^0 \to \phi \bar{K}^0)} \simeq \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}} + O(??)$$

$\sin(2\alpha)$ from $B \to \pi\pi$

- This process is mediated by both tree and penguin operators:





$$
\begin{aligned}
A &= T_{u\bar{u}d}\, V_{ub}V_{ud}^{*} + P_d^u\, V_{ub}V_{ud}^{*} + P_d^c\, V_{cb}V_{cd}^{*} + P_d^t\, V_{tb}V_{td}^{*} \\
&= (T_{u\bar{u}d} + P_d^u - P_d^c)\, V_{ub}V_{ud}^{*} \;+\; (P_d^t - P_d^c)\, V_{tb}V_{td}^{*} \\
&= T_{\pi\pi}\, V_{ub}V_{ud}^{*} + P_{\pi\pi}\, V_{tb}V_{td}^{*} \\
&= T_{\pi\pi}\, V_{ub}V_{ud}^{*}\left(1 + r_{\pi\pi}\frac{V_{tb}V_{td}^{*}}{V_{ub}V_{ud}^{*}}\right) = T_{\pi\pi}\, V_{ub}V_{ud}^{*}\,\underbrace{(1 + r_{\pi\pi}\,\kappa)}_{\text{penguin pollution}}
\end{aligned}
$$

- Is the last term suppressed?

$$\left|\frac{V_{tb}V_{td}^{*}}{V_{ub}V_{ud}^{*}}\right| \sim 2$$

$$r_{\pi\pi} \sim O(0.1)$$

$$\Rightarrow \quad \frac{A(B^0 \to \pi\pi)}{A(\bar{B}^0 \to \pi\pi)} \simeq \frac{V_{ub}V_{ud}^{*}}{V_{ub}^{*}V_{ud}}\,\frac{1 + r_{\pi\pi}\,\kappa}{1 + r_{\pi\pi}\,\kappa^{*}}$$

$\sin(2\alpha)$ from $B \to \pi\pi$

- $$\lambda_{\pi\pi} = \frac{q}{p}\frac{A_{\pi\pi}}{\bar{A}_{\pi\pi}} = \underbrace{\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}}_{e^{-2i\beta}} \underbrace{\frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}}}_{e^{-2i\gamma}} \frac{1 + r_{\pi\pi}\,\kappa}{1 + r_{\pi\pi}\,\kappa^*} = e^{2i\alpha}\frac{1 + r_{\pi\pi}\,\kappa}{1 + r_{\pi\pi}\,\kappa^*}$$

- The time-dependent CP asymmetry is:

$$\begin{aligned} \mathcal{A}_{\pi\pi} &= -C_{\pi\pi}\cos(\Delta m_{B_d} t) + S_{\pi\pi}\sin(\Delta m_{B_d} t) \\ S_{\pi\pi} &= \sin(2\alpha) + O(r_{\pi\pi}) \\ C_{\pi\pi} &= O(r_{\pi\pi}) \end{aligned}$$

- What do we get from experiments (Belle & BaBar)?

$$\begin{aligned} S_{\pi\pi}^{\text{exp}} &= -0.59 \pm 0.09 \\ C_{\pi\pi}^{\text{exp}} &= -0.39 \pm 0.07 \longleftarrow \text{!!!!} \end{aligned}$$

$\sin(2\alpha)$ from $B \to \pi\pi$

- Solutions:
  - use effective theories to calculate $r_{\pi\pi}$
  - use SU(3) flavor symmetry to relate $B \to K\pi$ and $B \to \pi\pi$
  - isospin analysis

- Up to isospin breaking corrections we can describe

$$A(B^+ \to \pi^+\pi^0),\ A(B^0 \to \pi^+\pi^-),\ A(B^0 \to \pi^0\pi^0)$$

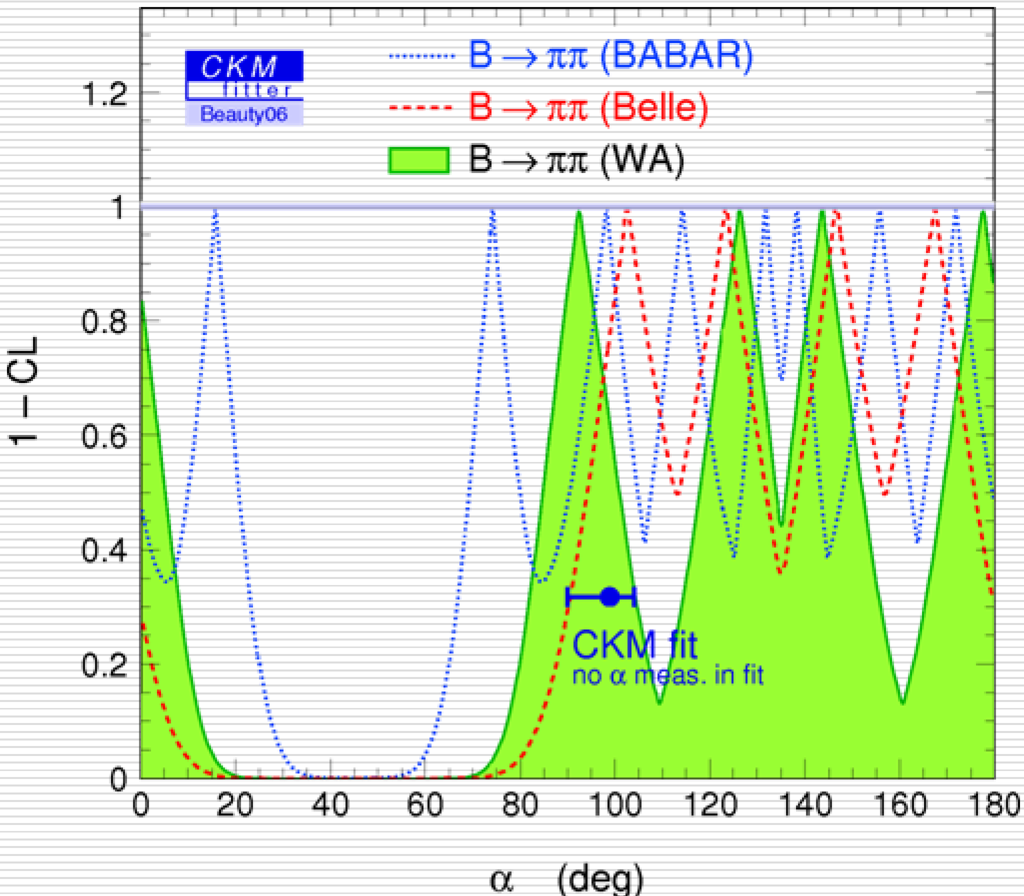
in terms of two isospin amplitudes

$$A(B \to [\pi\pi]_0),\ A(B \to [\pi\pi]_2)$$

- At the end of the day we are able to eliminate $r_{\pi\pi}$ and extract $\sin(2\alpha)$

$\sin(2\alpha)$ from $B \to \pi\pi$

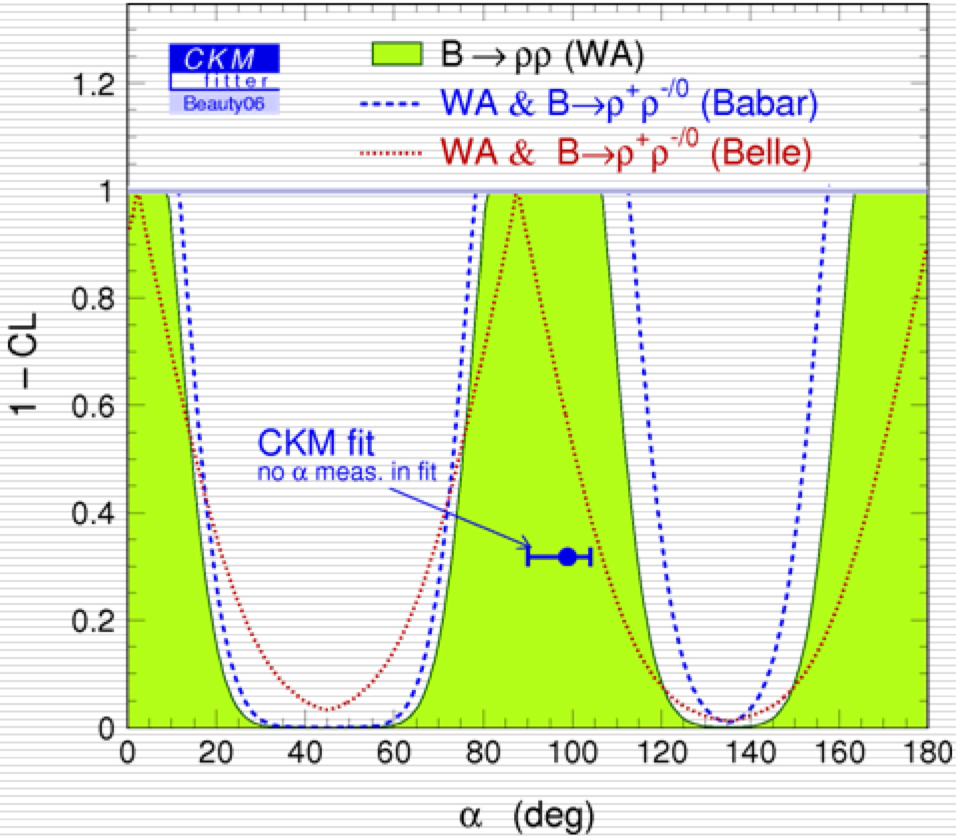
$$S^{exp}_{\pi^+\pi^-} = -0.59 \pm 0.09 \qquad BR^{exp}_{\pi^+\pi^-} = (5.2 \pm 0.2) \times 10^{-6}$$
$$C^{exp}_{\pi^+\pi^-} = -0.39 \pm 0.07 \qquad BR^{exp}_{\pi^+\pi^0} = (5.7 \pm 0.4) \times 10^{-6}$$
$$C^{exp}_{\pi^0\pi^0} = -0.36 \pm 0.33 \qquad BR^{exp}_{\pi^0\pi^0} = (1.3 \pm 0.2) \times 10^{-6}$$

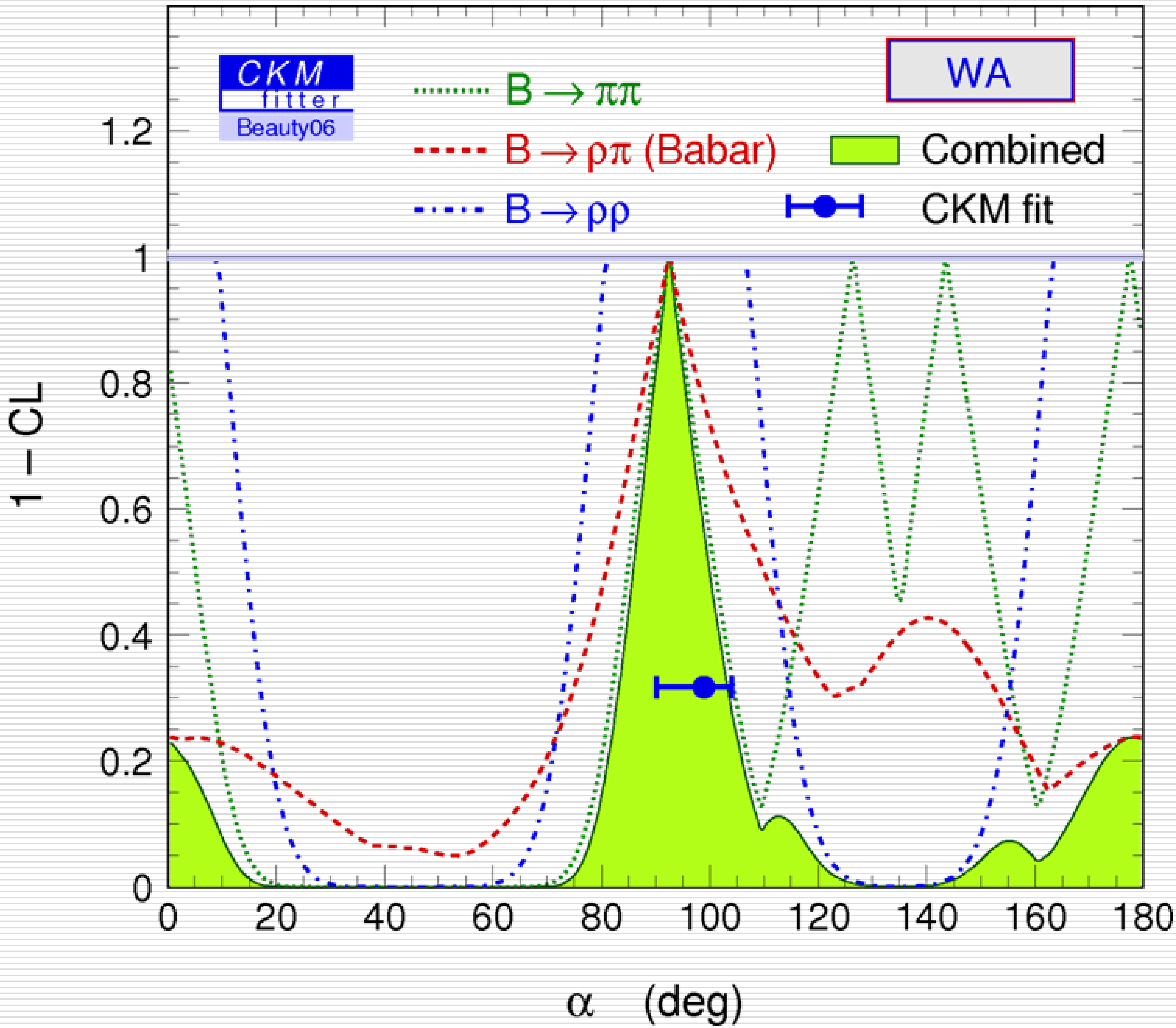


$\sin(2\alpha)$ from $B \to \rho\rho$

- Analysis is identical to the $\pi\pi$ case
- Experimental measurements have larger errors
- The constraint on α is stonger!

$$S^{\exp}_{\rho^+\rho^-} = -0.13 \pm 0.19$$
$$C^{\exp}_{\rho^+\rho^-} = -0.06 \pm 0.14$$
$$BR^{\exp}_{\rho^+\rho^-} = (23.1 \pm 3.3) \times 10^{-6}$$
$$BR^{\exp}_{\rho^+\rho^0} = (18.2 \pm 3.0) \times 10^{-6}$$
$$BR^{\exp}_{\rho^0\rho^0} = (1.2 \pm 0.5) \times 10^{-6}$$

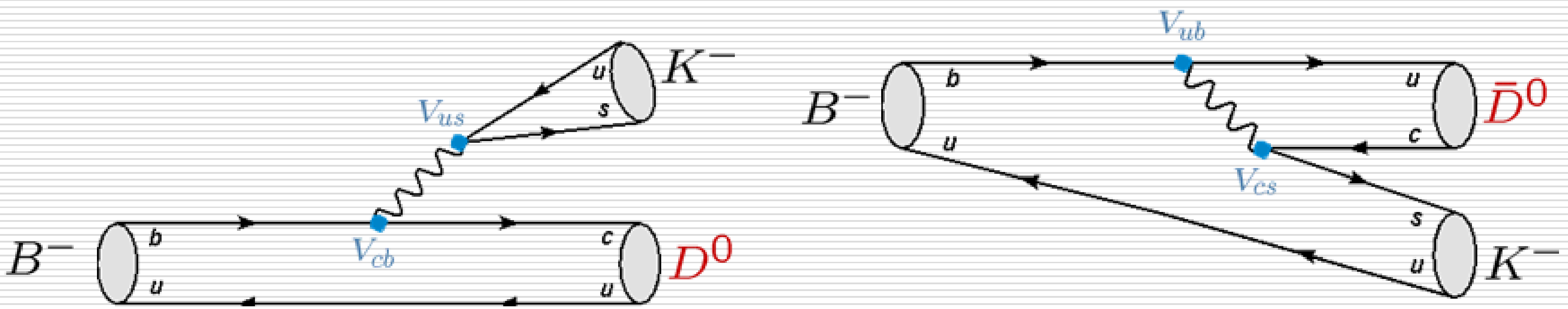




CKM
fitter
Beauty06
$B \to \pi\pi$
$B \to \rho\pi$ (Babar)
$B \to \rho\rho$
WA
Combined
CKM fit
1 – CL
1.2
1
0.8
0.6
0.4
0.2
0
20
40
60
80
100
120
140
160
180
α (deg)

γ from $B^- \to D^{(*)}K^-$

- The decays $B^- \to K^- D^0$ and $B^- \to K^- \bar{D}^0$ are both allowed and have similar CKM suppression ($V_{ub}V_{cs}^*/V_{cb}V_{us}^* = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$):



$$A_-(B^- \to f) = \overbrace{V_{cb}V_{us}^*}^{\kappa_1} A_1 e^{i\alpha_1} + \overbrace{V_{ub}V_{cs}^*}^{\kappa_2} A_2 e^{i\alpha_2}$$

$$A_+(B^+ \to f) = V_{cb}^*V_{us} A_1 e^{i\alpha_1} + V_{ub}^*V_{cs} A_2 e^{i\alpha_2}$$

- The CP eigenstates of the neutral D mesons are:

$$|D^0_{CP\pm}\rangle = \frac{|D^0\rangle \pm |\bar{D}^0\rangle}{\sqrt{2}}$$

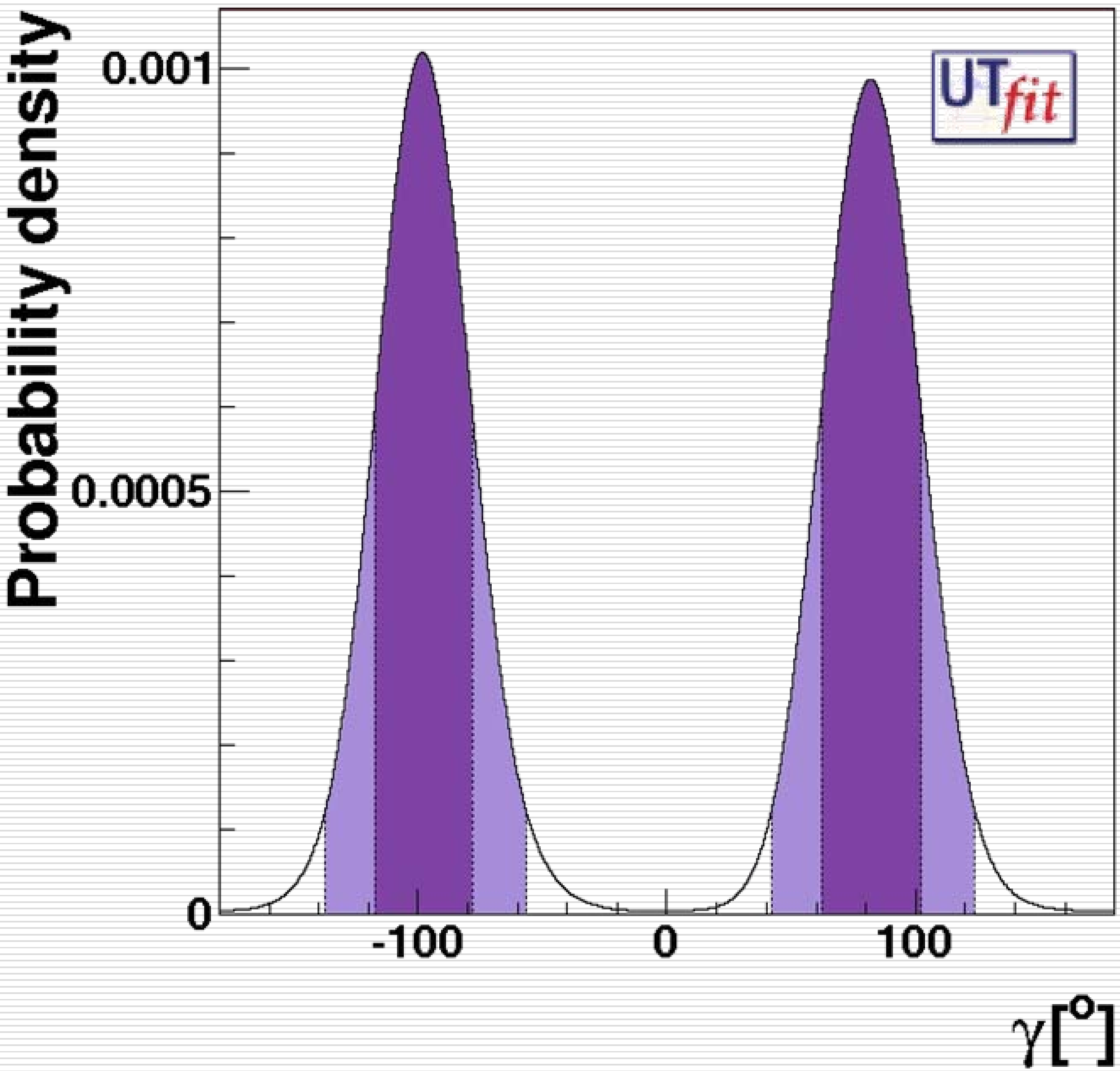
γ from $B^- \to D^{(*)}K^-$

- It is possible to identify the neutral D in the final state:
  - D^0 or $\bar{D}^0$ are identified by semileptonic decays containing a μ^+ or a μ^-, respectively
  - $D_{CP\pm}$ are identified by final states that are CP eigenstates

- Using the following four observables, we can extract γ:

$$
\begin{aligned}
A_{CP\pm} &= \frac{\Gamma(B^+ \to D_{CP\pm}K^+) - \Gamma(B^- \to D_{CP\pm}K^-)}{\Gamma(B^+ \to D_{CP\pm}K^+) + \Gamma(B^- \to D_{CP\pm}K^-)} \\
&= \frac{\pm 2r \sin\gamma \ \sin\delta}{1 + r^2 \pm 2r\cos\gamma \ \cos\delta} \\
R_{CP\pm} &= \frac{\Gamma(B^+ \to D_{CP\pm}K^+) - \Gamma(B^- \to D_{CP\pm}K^-)}{\Gamma(B^+ \to \bar{D}^0K^+) + \Gamma(B^- \to D^0K^-)} \\
&= 1 + r^2 \pm 2r\cos\gamma \ \cos\delta
\end{aligned}
$$

where $\delta = \alpha_1 - \alpha_2$ and $r = A_1\kappa_1/A_2\kappa_2$


Probability density
0.001
0.0005
0
-100
0
100
γ[°]
UTfit