Precision Calculations for the ILC and LHC

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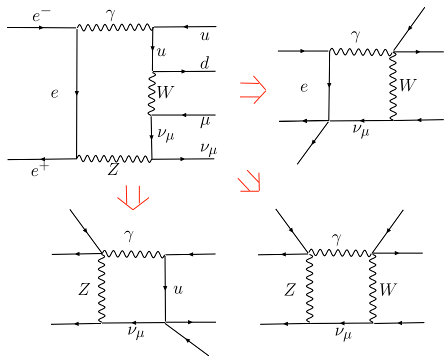

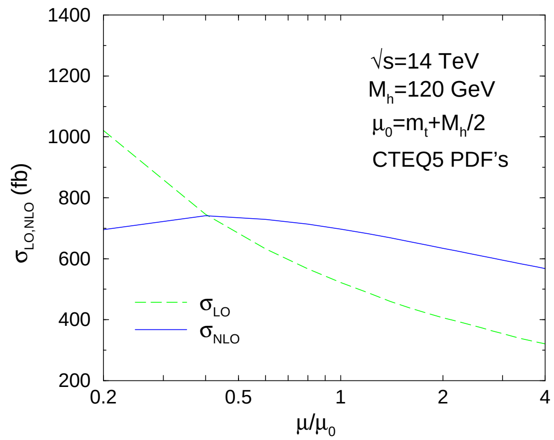

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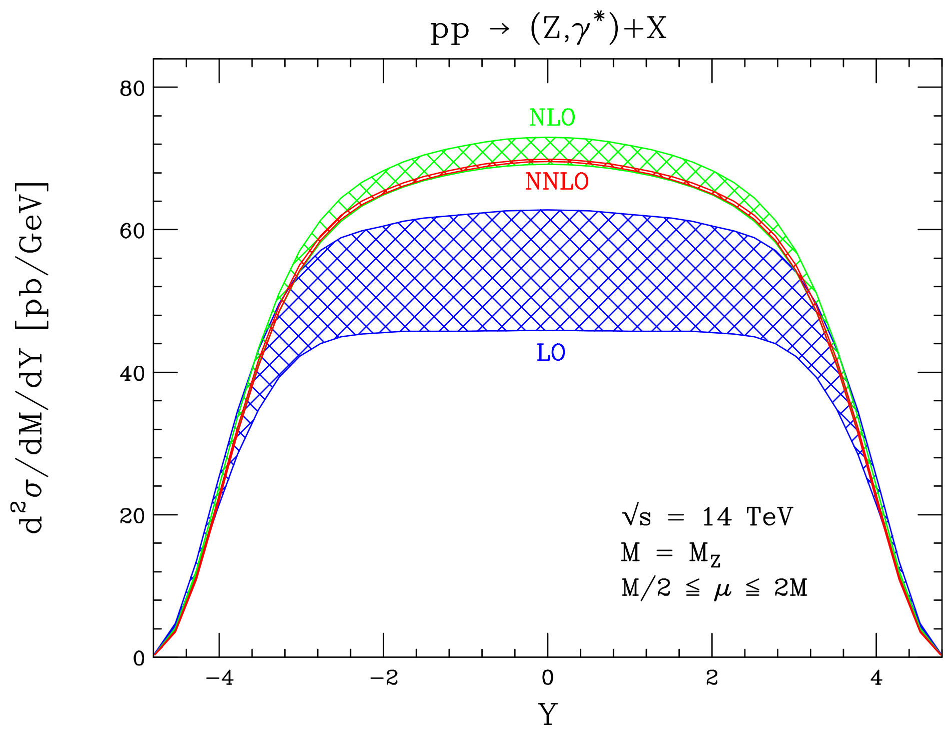
1 – Why do we care?

- the prime goal of the LHC is to shed light on the mechanism of electroweak symmetry breaking and to discover new physics beyond the Standard Model (SM)
- to achieve this, accurate theoretical predictions for SM processes are needed
- many of the interesting final states are complex
  $\rightarrow$ lowest order (LO) predictions strongly depend on the unphysical factorization and renormalization scales which can be traced to the truncation of the perturbation series. $\mathcal{O}(10\% - 100\%)$ uncertainty
- need to calculate higher order QCD and, in some cases, electroweak radiative corrections

- QCD corrections at hadron colliders
  - ☞ instrumental for precise predictions for all processes of interest
  - ☞ taking into account QCD corrections reduces (sometimes dramatically) the renormalization and factorization scale uncertainty
  - ☞ example: Z boson rapidity distribution at LHC (landmark calculation by Anastasiou, Dixon, Melnikov, Petriello)

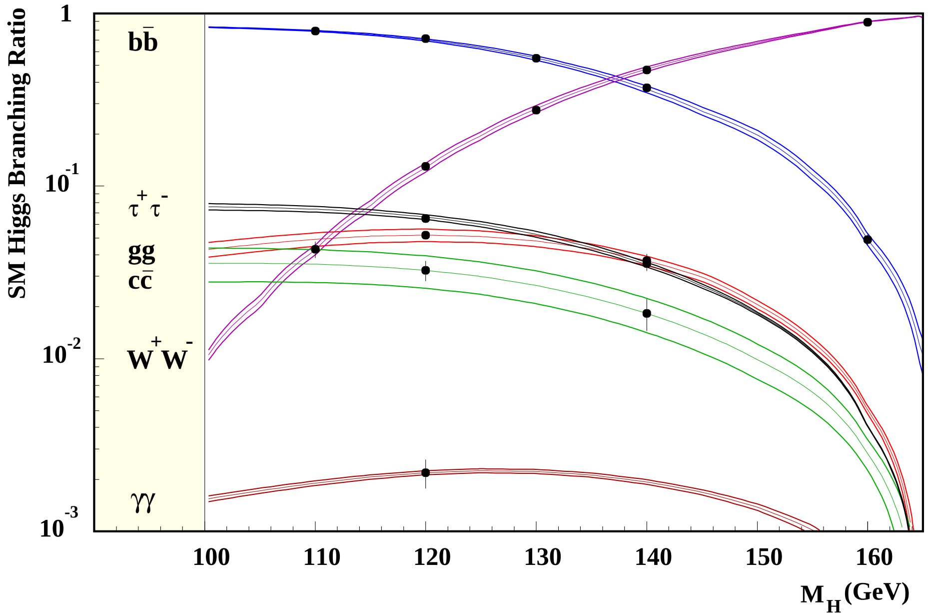


- much has been accomplished in recent years, but much remains to be done
- time ordered LHC shopping list
  - ☞ need for $10-30$ fb^{-1} (2008-2010):
    - → full NLO QCD corrections to $pp \to t\bar{t} \to b\bar{b} + 4f$
    - → NLO QCD corrections to $t\bar{t}j$, $t\bar{t}\gamma$, $W/Z+ \geq 3$ jets production
    - → NNLO QCD corrections to PDF's, 2-jet production
  - ☞ need for 300 fb^{-1} (2012-2013):
    - → NLO QCD corrections to $gg \to HH$, $t\bar{t}W$, $t\bar{t}Z$ production
    - → NLO EW corrections are needed for all hard scattering processes
  - ☞ need for 3000 fb^{-1} (>2015):
    - → NLO QCD corrections to $WWWjj$, $jj\gamma\gamma$, $Q\bar{Q}\gamma j$ production
    - → probably many more processes as time and physics knowledge base evolves

ILC

- new physics at the ILC
  - ☞ through direct searches (eg. supersymmetry)
  - ☞ through the precise measurement of Standard Model parameters
- precise calculations are needed to successfully compare theory and experiment
- in particular electroweak radiative corrections: one-loop, 2-loop, etc.

- examples for precision measurements at the ILC
  - ☞ can measure $Hf\bar{f}$ and HVV ($V = \gamma$, Z, W) couplings to a few percent



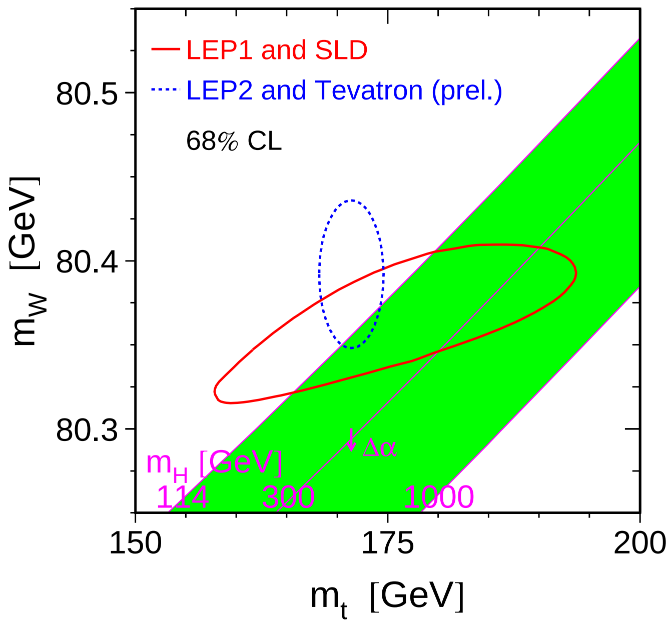
and the HHH self-coupling to $\sim 20 - 30\%$

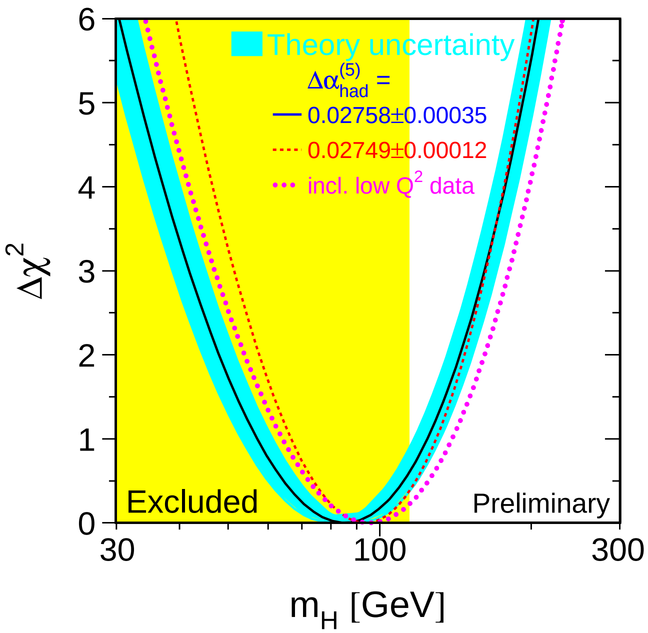
➞ need 1-loop electroweak corrections to $e^+e^- \to ZH$, $e^+e^- \to \nu\bar{\nu}H$, $e^+e^- \to e^+e^-H$, $e^+e^- \to ZHH$, $e^+e^- \to \nu\bar{\nu}HH$, $e^+e^- \to e^+e^-HH$

☞ M_W, $\sin^2\theta_{eff}$ and M_H:

1-loop corrections to M_W and $\sin^2\theta_{eff}$ depend quadratically on the top quark mass, m_t, and logarithmically on M_H

→ measuring M_W ($\sin^2\theta_{eff}$) and m_t one can extract information on M_H





→ $M_H < 199$ GeV @ 95% CL

- fit results depend on
  - experimental uncertainties
  - and theoretical uncertainties
    - primordial theoretical uncertainties:

      associated with the extraction of (pseudo)observable from measured quantities

      example: M_W from transverse mass distribution
    - intrinsic theoretical uncertainties:

      from unknown higher order corrections

      example: “blueband” (dominated by uncertainty of $\sin^2\theta_{eff}$)

- ☞ at GigaZ one expects to determine the effective weak mixing angle with a precision of

$$\delta \sin^2 \theta_{eff} = 1.3 \times 10^{-5}$$

  - ➞ current theoretical prediction includes 1-loop and 2-loop corrections
  - ➞ full 2-loop corrections have recently been completed (Awramik, Czakon and Freitas)
  - ➞ estimated present theoretical uncertainty from unknown higher order corrections (mostly $\mathcal{O}(\alpha_s \alpha^2)$)

$$\delta \sin^2 \theta_{eff}^{theor} \approx (4-5) \times 10^{-5}$$

  - ➞ need 3-loop $\mathcal{O}(\alpha_s \alpha^2)$ corrections…

☞ The W mass can be measured with a precision of $\delta M_W \approx 7$ MeV in a threshold scan ($\sqrt{s} \approx 161$ GeV) at a LC (Wilson, Sitges)

- ➞ $e^+e^- \to 4$ fermion cross section is sensitive to M_W in threshold region
- ➞ theoretical uncertainties: if one wishes to achieve $\delta M_W \approx 7$ MeV, one needs $\delta M_W^{theor} \sim 1$ MeV
- ➞ need to know cross section in threshold region with

$$\frac{\Delta\sigma}{\sigma} \approx 0.05\%$$

- uncertainty of Born cross section in threshold region (CERN LEP2 Yellow Report):

$$\frac{\Delta\sigma}{\sigma} \approx 1.4\%$$

- need full $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to 4f$ in threshold region
- without knowing these corrections:

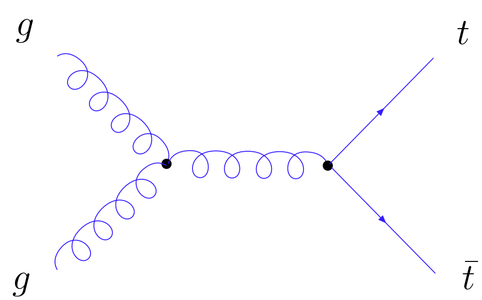
$$\delta M_W \approx \delta M_W^{theor} \approx 24 \text{ MeV}$$

sic transit gloria mundi

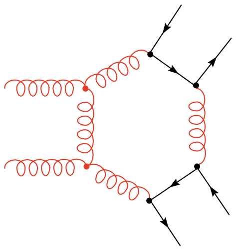
- a calculation of the full $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to 4f$ has been completed recently (more later)

2 – Tools for Loopies

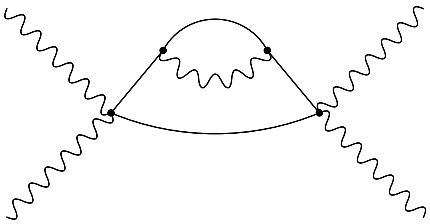
- theoretical framework for calculations: perturbation theory
  - ☞ expand cross section (which links experiment and quantum field theory) in powers of a (small) coupling constant
  - ☞ lowest order (LO): tree level



  - ☞ next-to-leading order (NLO): 1-loop



☞ NNLO: 2-loop



☞ and so on ...

- How to do a loop calculation?

Example: 1-loop QCD for $e^+e^- \to \bar{q}q$ (2 jets)

☞ need $e^+e^- \to \bar{q}q$ at 1-loop, and

☞ $e^+e^- \to \bar{q}q + gluon$ at tree level

☞ the cross section for each process diverges, however, their sum is finite

- $e^+e^- \to \bar{q}q + gluon$: the easy part
  - ☞ tree level calculations are technically straightforward
  - ☞ tools for automated tree level calculations exist
  - ☞ most general and flexible (e^+e^-, pp, $p\bar{p}$): `MadEvent` (Maltoni, Stelzer) [NEW: `MadEvent` for MSSM at `http://madgraph.roma2.infn.it`], `GRACE` (Bélanger et al.) and `ComHep` (Boos et al.)
  - ☞ also popular in LC studies: `Whizard` (Kilian)
  - ☞ capabilities of these programs are limited only by computing power available
  - ☞ for multi-particle final states thousands of diagrams can contribute Example: $e^+e^- \to W^+W^-\bar{b}bjj$: 4896 diagrams
  - ☞ general rule: number of diagrams grows factorially with number of final state particles

- the hard part: $e^+e^- \to \bar{q}q$ at 1-loop
  - ☞ recipe for loop calculations:

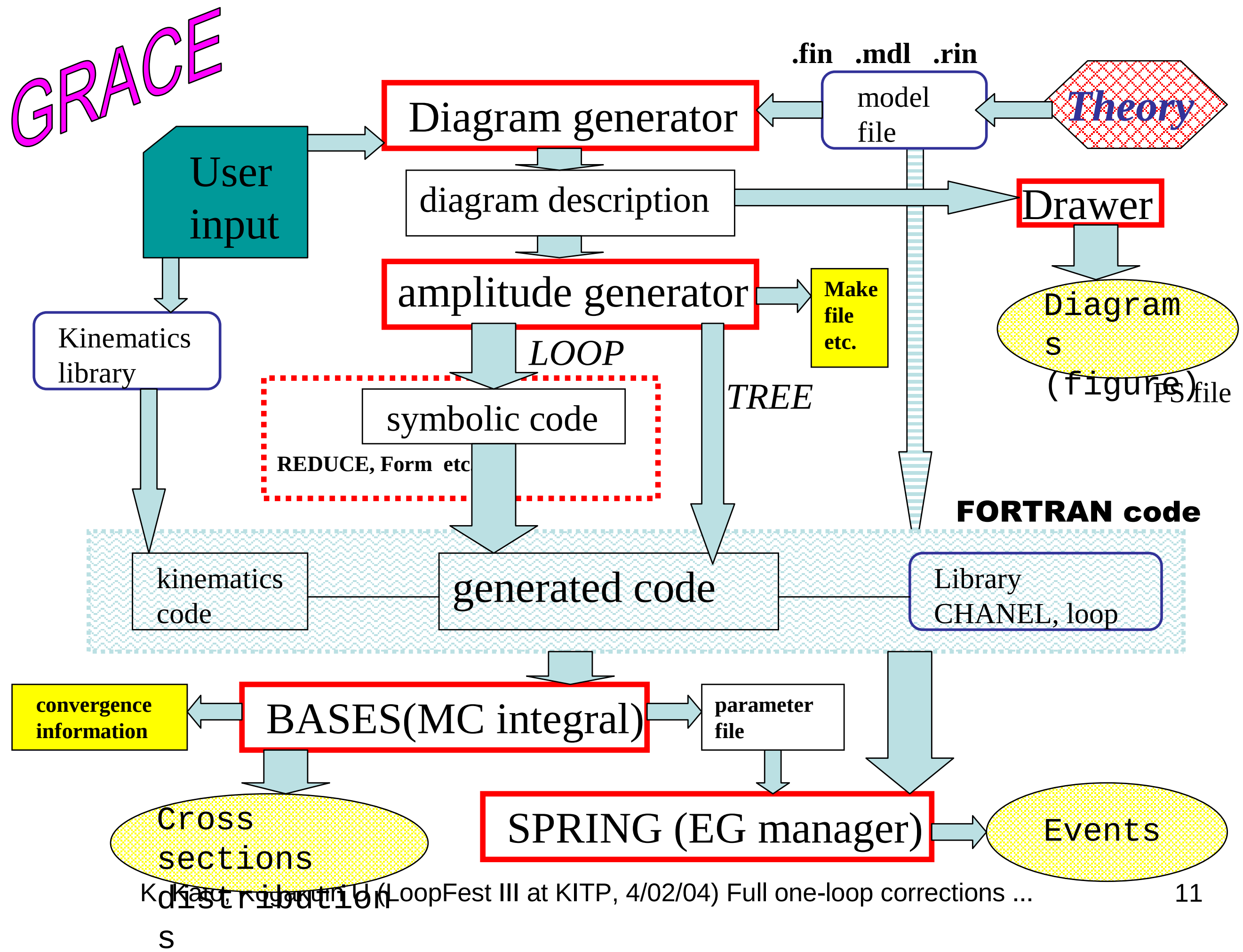
draw all possible diagrams	topological task
which particles run in given diagram	combinatorial task
translate diagrams into formulas via Feynman rules	database look-up
contract Lorentz indices; take traces	algebraic manipulation
reduce to known/master integrals	algebraic manipulation
cancel IR and/or UV singularities	algebraic manipulation
translate output into computer program	programming
run program	wait, drink coffee

  - ☞ large cancellations may occur between certain Feynman diagrams → need to be careful with numerical implementation to achieve reliable results
  - ☞ even for moderately complicated processes ($e^+e^- \to 4$ fermions) the number of Feynman diagrams can be extremely large ($\mathcal{O}(10^4)$)

- ☞ tools which automate loop calculations are essential to accomplish goals

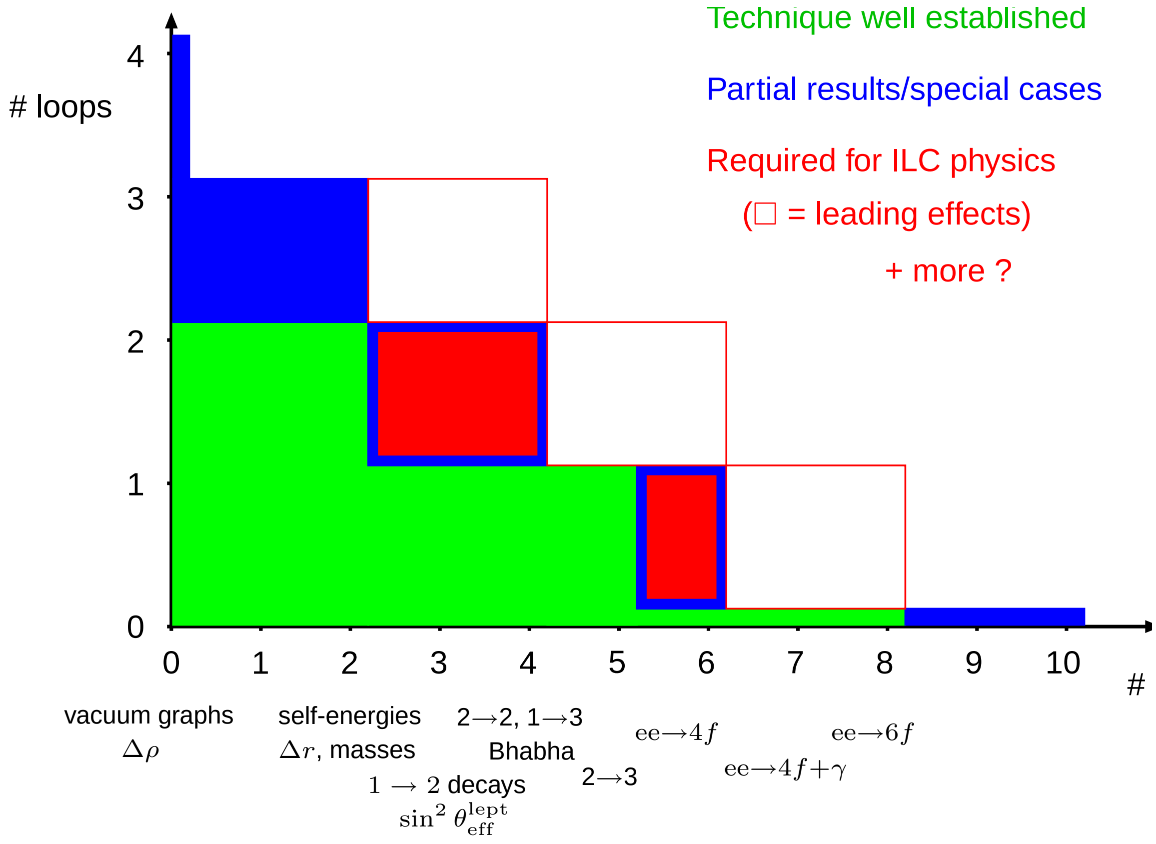
- program packages for automated calculations of loop processes
  - ☞ so far, such packages exist only for electroweak 1-loop corrections. Most widely used:
    - → `GRACE-loop` (Bélanger et al.)
    - → `FeynArts/FeynCalc/FormCalc/LoopTools` (Hahn, Böhm, Denner, Eck, Küblbeck)
    - → `DIANA` (Fleischer, Tentyukov, Tarasov)
  - ☞ packages for 1-loop QCD corrections are under development
    - → `Samper` (Ellis, Giele et al.)
    - → `Grace-QCD` (Bélanger et al.)
  - ☞ not surprisingly, the structure of these packages is fairly complex

    Example: `GRACE-loop`


GRACE
.fin .mdl .rin
Diagram generator
model file
Theory
User input
diagram description
Drawer
amplitude generator
Make file etc.
Diagrams (figure)
PS file
Kinematics library
LOOP
TREE
symbolic code
REDUCE, Form etc
FORTRAN code
kinematics code
generated code
Library CHANEL, loop
convergence information
BASES(MC integral)
parameter file
Cross sections distributions
SPRING (EG manager)
Events
K. Kato, Kogakuin U (LoopFest III at KITP, 4/02/04) Full one-loop corrections ...
11

- what we know and what we need to know for the ILC (and LHC)

  cross-sections (event rates) fall quickly with increasing number of particles in the final state
  → less precise calculations are needed for $2 \to n$ $(n > 2)$ reactions than for $2 \to 2$ processes

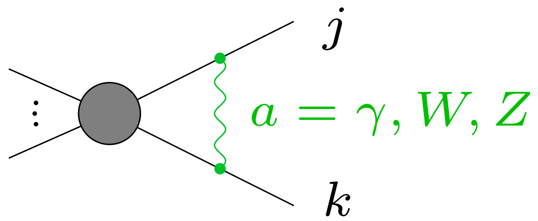


3 – Recent Results

- LHC: the 6-gluon amplitude at one loop (Ellis, Giele, Zanderighi)
  - ☞ needed for $pp \to 4$ jets at NLO QCD
  - ☞ 6-gluon amplitude is the most complicated ingredient
  - ☞ about 10^4 Feynman diagrams
  - ☞ semi-numerical evaluation of loop integrals (Ellis, Giele, Glover, Zanderighi)

    about 9 sec CPU time per phase space point
  - ☞ numerical comparison with existing analytical results

- Electroweak radiative corrections at high energies at the LHC
  - ☞ Sudakov logarithms induced by soft W, Z exchange



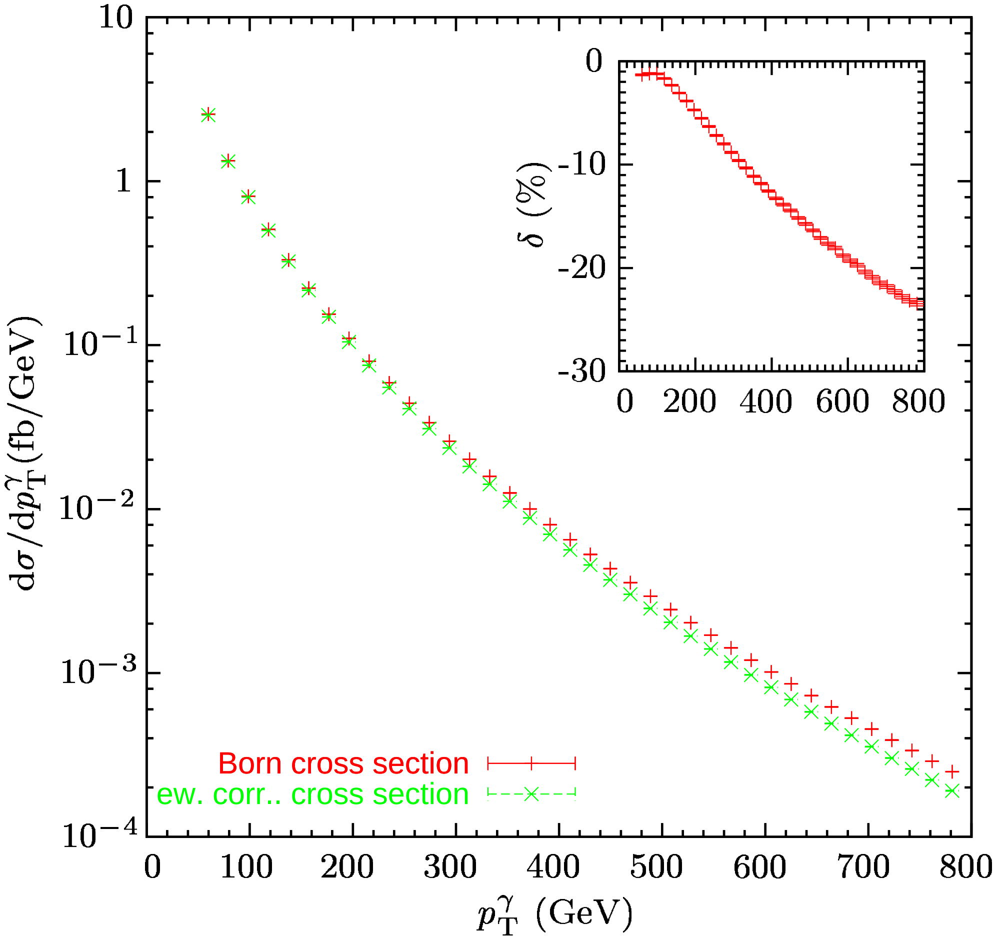
  - ☞ relative correction to cross section for $2 \to 2$ reactions at $\sqrt{\hat{s}} = 1$ TeV:

$$\delta_{1-loop} \sim -\frac{\alpha}{\pi \sin^2\theta_W} \log^2\left(\frac{\hat{s}}{M_W^2}\right) \approx -26\%$$

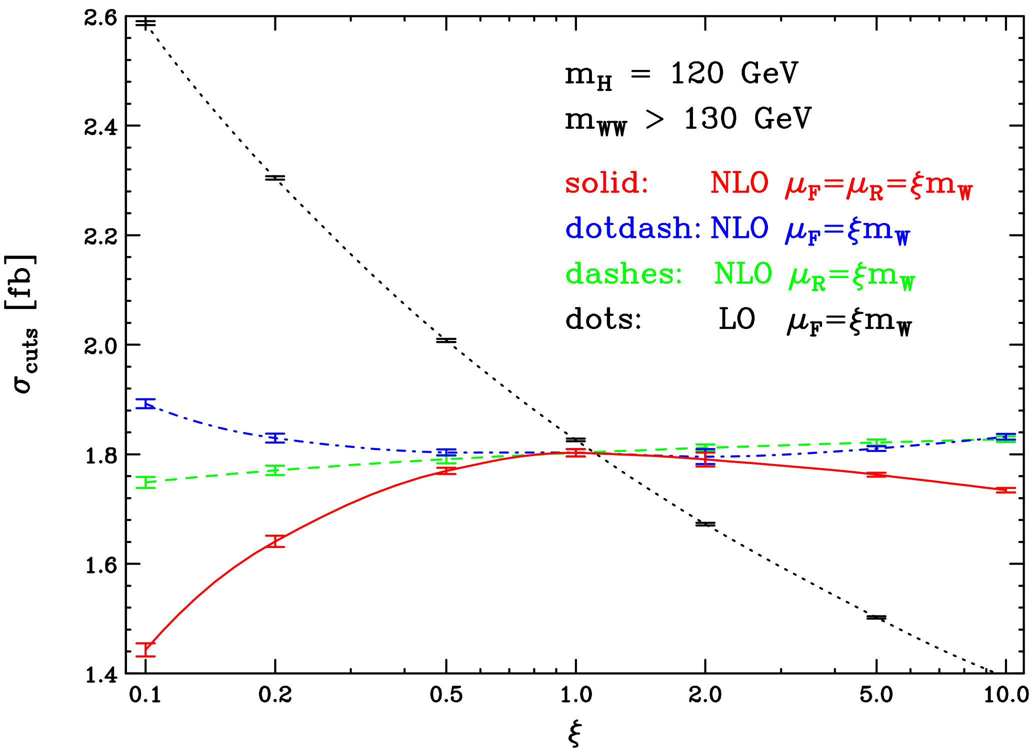
  - ☞ NLO EW corrections become as large as NLO QCD corrections in the TeV region

- ☞ origin of Sudakov logs:
  - → corresponding logs in QED/QCD cancel between virtual and real corrections
  - → massive W and Z bosons are infrared regulators for loops
  - → no technical reason to include real W and Z radiation
- ☞ The NLO EW corrections have been calculated to a number of processes recently, *eg*
  - → Drell-Yan production (UB, Wackeroth, Hollik, Brein, Schappacher, Krämer, Dittmaier)
  - → inclusive jet, isolated photon production (Nolten, Moretti, Ross, Kühn, Pozzorini, Kulesza, Schulze)
  - → di-boson production (Accomando, Denner, Pozzorini, Meier, Kaiser)

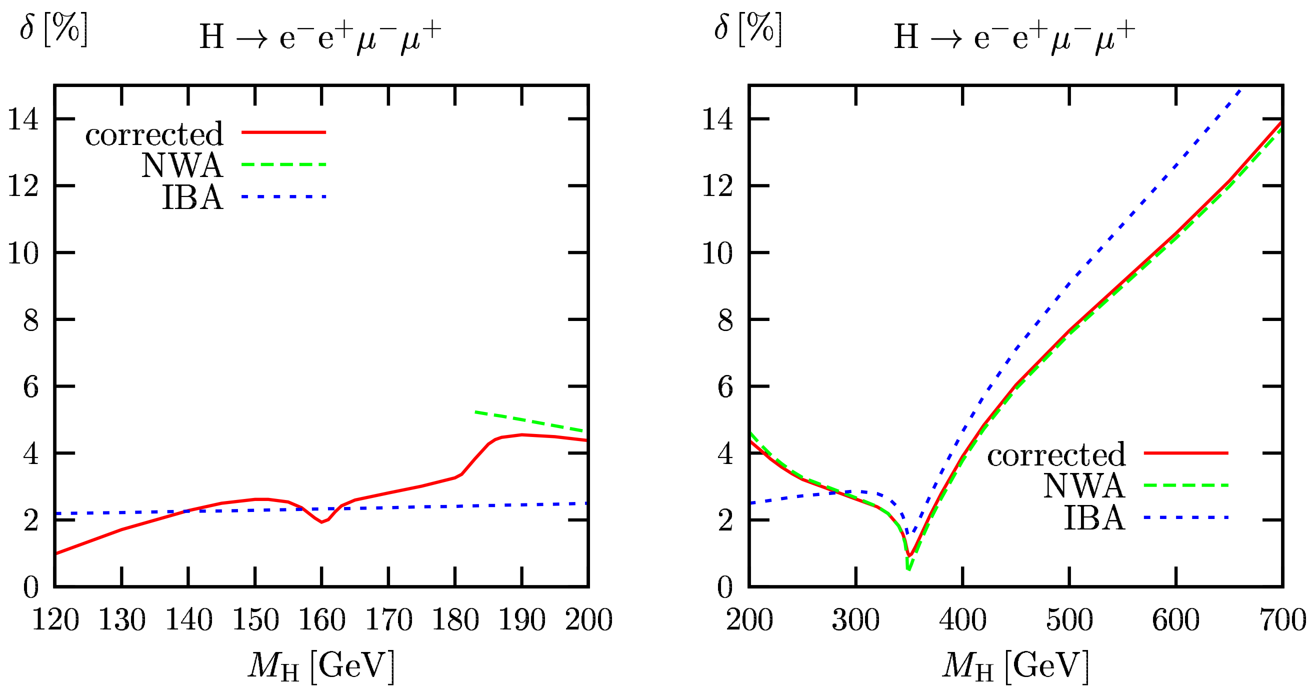
Example: $pp \to W\gamma \to e\nu\gamma$ at the LHC



- NLO QCD corrections to weak boson fusion at the LHC ($qq' \to W^+W^-qq'$) (Jäger, Oleari, Zeppenfeld)
  - ☞ important background to Higgs production via vector boson fusion ($qq' \to Hqq'$)
  - ☞ greatly reduced dependence on factorization (μ_F) and renormalization scale (μ_R)



- EW corrections to $H \to WW, ZZ \to 4f$ (Bredenstein, Denner, Dittmaier, Weber)
- important for a precise measurement of the Higgs boson mass at the LHC
- corrections of up to 14% to partial decay widths

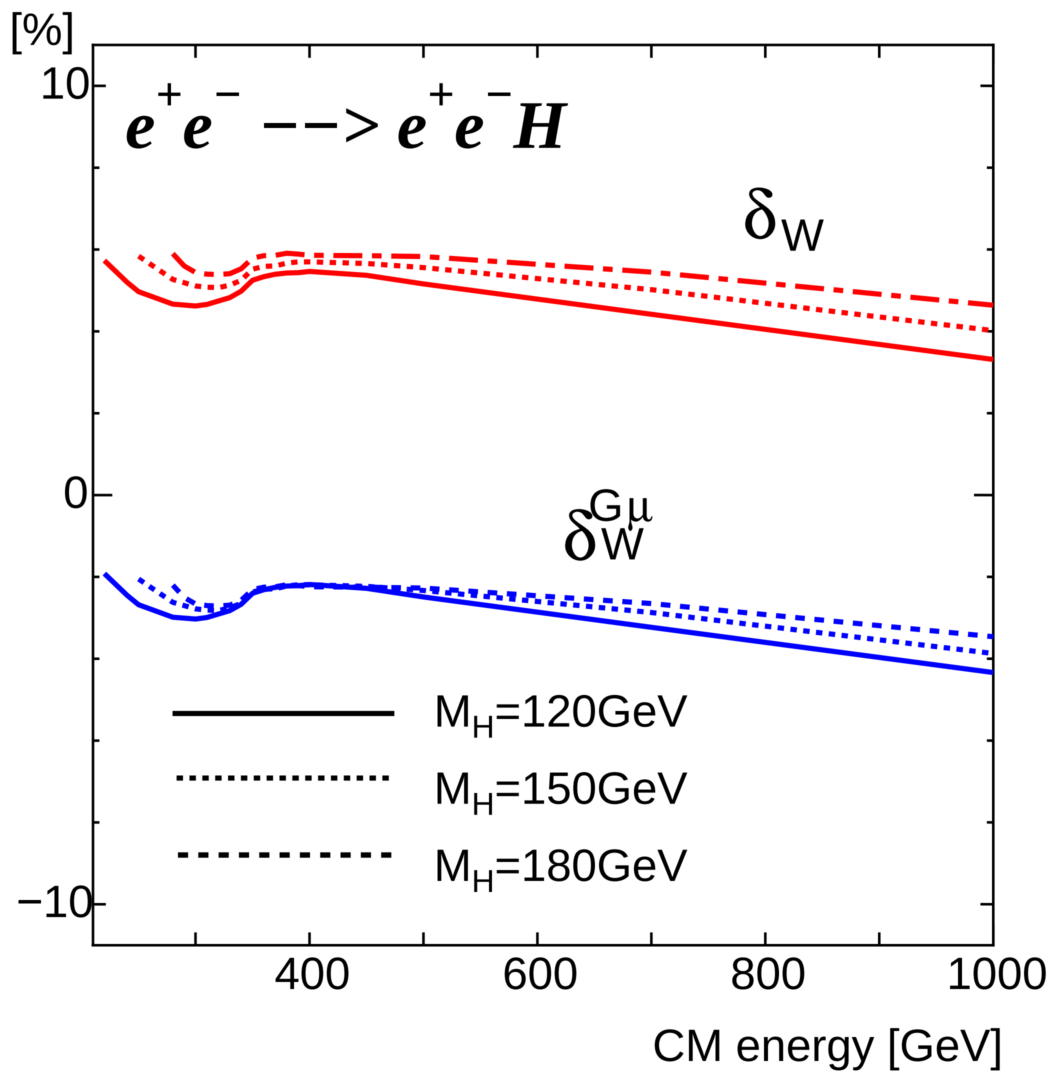


Results relevant for the ILC

- 1-loop level:
  - ☞ The 1-loop corrections to $2 \to 2$ processes, such as $e^+e^- \to ZH$ (Denner et al., Kniehl), have all been calculated more than a decade ago
  - ☞ The frontier now are $2 \to 3$ processes, and we are beginning to tackle $2 \to 4$ processes

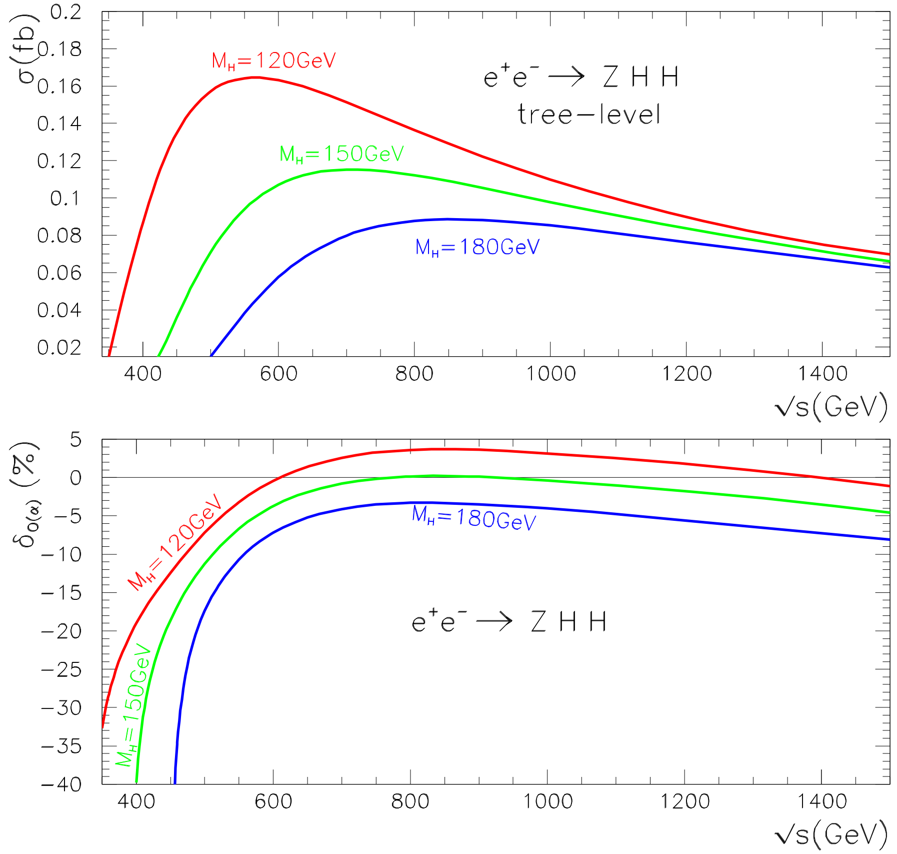
    main complication: pentagon and hexagon diagrams
  - ☞ $e^+e^- \to e^+e^- H$ (Boudjema et al.)
    - ➞ 4470 1-loop diagrams
    - ➞ $e^+e^- \to e^+e^- H$ important for $\sqrt{s} > 800$ GeV

→ EWK radiative corrections are typically of $\mathcal{O}(\text{few}\%)$



☞ $e^+e^- \to ZHH$

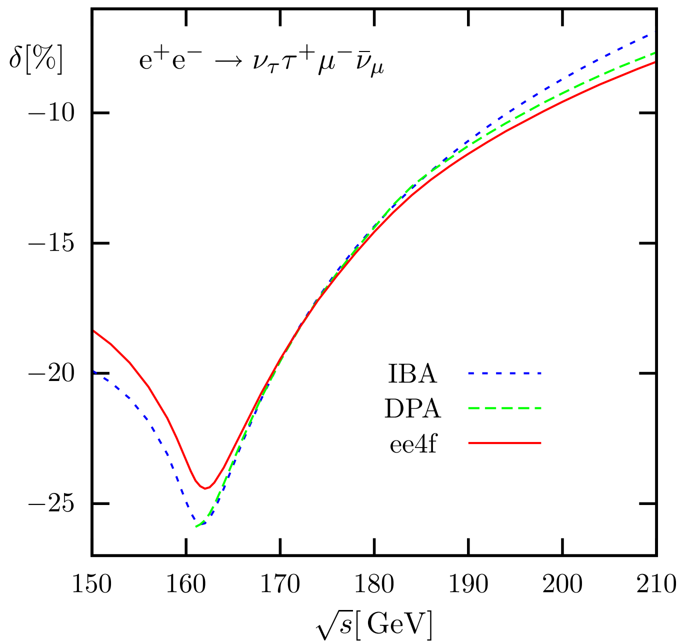
- ➞ about 1600 1-loop diagrams
- ➞ two independent calculations have been performed by Bélanger et al. and You et al. both calculations agree within 1%
- ➞ corrections large at threshold

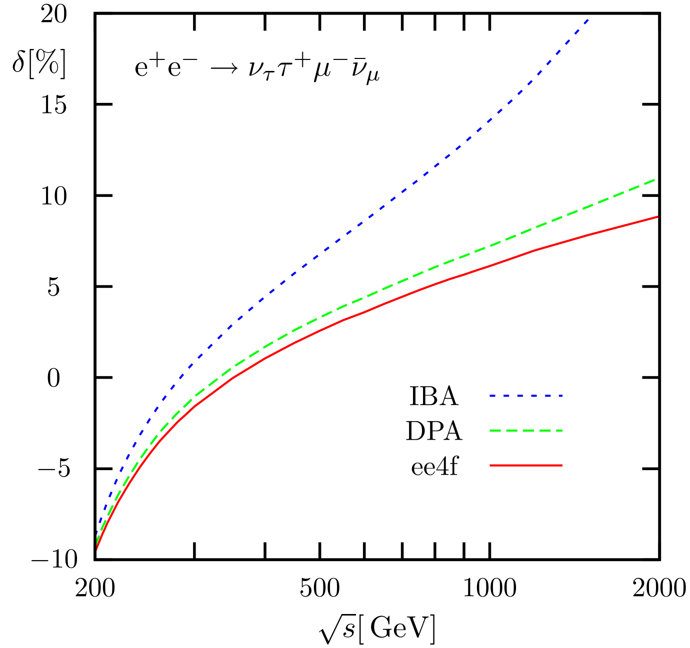


☞ $e^+e^- \to 4$ fermions (Denner, Dittmaier, Roth, Wieders)

- ➞ landmark calculation which introduces novel methods, opening the door to many $2 \to 4$ calculations
- ➞ major complication: include finite W width effects while maintaining gauge invariance
- ➞ solved by using complex mass for W bosons and complex renormalization
- ➞ use `FeynArts/FeynCalc/FormCalc` for generating Feynman diagrams and algebraic manipulations
- ➞ main technical challenge: reduction of hexagon diagrams to box diagrams may lead to numerical instabilities
  - ➞ overcome by using so-called Cayley determinants

→ results: difference between full and approximate $\mathcal{O}(\alpha)$ corrections is 2% in W threshold region





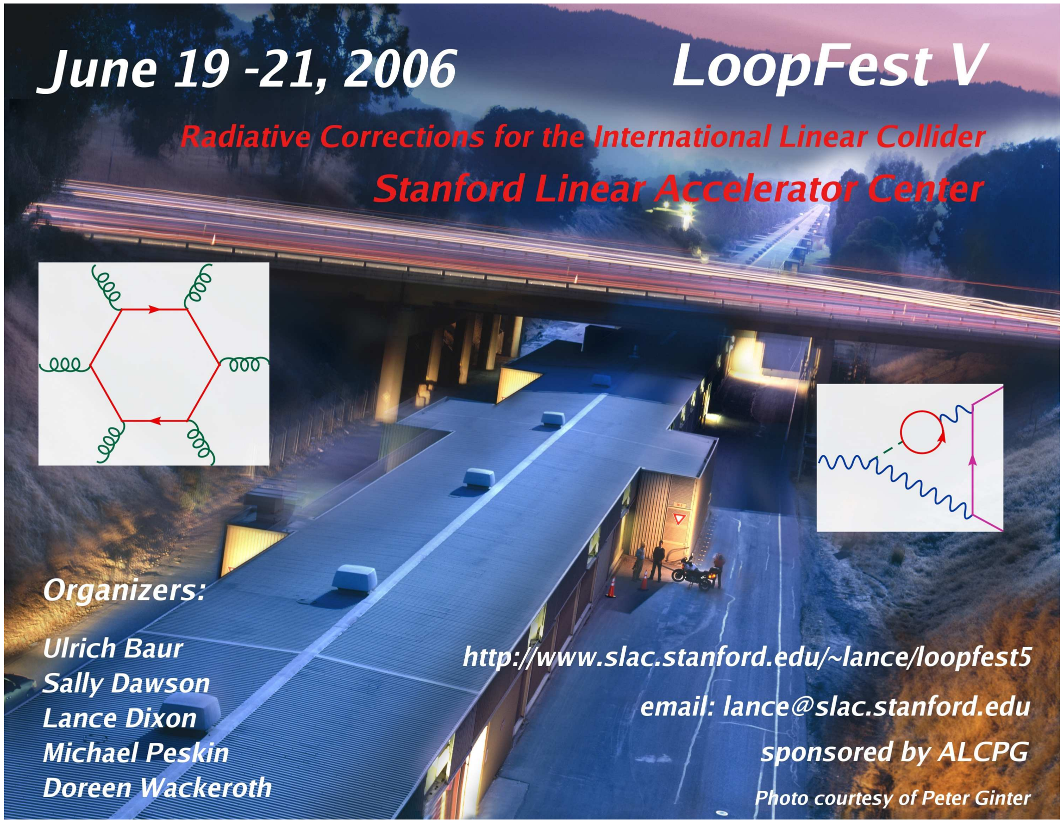
- remaining theoretical uncertainties for W mass measurement at ILC (GigaZ):
  - NLL corrections ($(\alpha/\pi)^2 \log(m_e^2/s)$): $\mathcal{O}(0.1\%)$
  - higher order effects of coulomb singularity: $\sim 0.2\%$ (Fadin et al., Bardin et al.)
- recall goal: 0.05%

- recent results from the 2-loop frontier:
  - ☞ fully differential W production at NNLO (Melnikov, Petriello) for Tevatron and LHC ($\sigma(W)$ can be used as luminosity monitor)
  - ☞ fully differential $pp \to H \to \gamma\gamma$ at NNLO (Anastasiou, Melnikov, Petriello)
  - ☞ enormous progress in calculating 2-loop corrections in the last few years
    - → algebraic reduction to master integrals by integration by parts, Lorentz invariance identities (Anastasiou, Gehrmann, Glover, Laporta, Lazopoulos, Oleari, Remiddi, Smirnov, Veretin,...)
    - → calculation of master integrals using Mellin-Barnes technique, differential equations, and numerical techniques (Anastasiou, Smirnov, Tausk, Tejeda-Yeomans, Binoth, Heinrich)

- leading to explicit results for:
  - 2-loop amplitudes for massless $2 \to 2$ processes (such as Bhabha scattering, $e^+e^- \to e^+e^-$) (Anastasiou, Bern, Dixon, Wong, ...)
  - 2-loop QCD amplitudes for $e^+e^- \to 3$ jets (Garland, Gehrmann, Glover, Moch, Uwer, Weinzierl, ...)
- still to be done: (example: $2 \to 2$ reaction)
  - combine with 1-loop $2 \to 3$ amplitudes and
  - tree-level $2 \to 4$ amplitudes using
  - suitable subtraction method (not trivial!) and
  - put all together in working MC program
- long term goal: fully automate 2-loop calculations

- ☞ twistor inspired techniques:
  bootstrapping 1-loop QCD amplitudes and recursion relations (Berger, Bern, Dixon, Forde, Kosower, Dunbar,…)
  goes beyond calculating Feynman diagrams
- ☞ what about supersymmetry?
  - ➞ precision observables in the MSSM (M_W, $\sin^2\theta_{eff}$) are known to 1-loop for 10 years
  - ➞ the Yukawa corrections to the ρ-parameter are known (Heinemeyer, Weiglein)
  - ➞ precision calculations of MSSM Higgs masses and widths have recently been completed `[FeynHiggs]` (Frank, Hahn, Heinemeyer, Hollik)
  - ➞ 1-loop corrections to chargino and neutralino pair production are known (Blank, Hollik, Öller, Eberl, Majerotto, Diaz, Ross)
- ☞ apologies to all those whose work I forgot to mention

- the ALCPG working group on precision calculations (aka LoopVerein) participates in the global effort to provide calculations for future collider (LHC, LC) experiments
  ☞ annual meetings (LoopFest) are being held, the most recent (2006) at SLAC



- LoopFest VI will be held at Fermilab in Spring 2007
- precision calculations are also one of the main goals of the

  LHC Theory Initiative ;

  details at

  http://www.pas.rochester.edu/∼orr/LHC-TI.html

4 – Quiz

- What fraction of the people doing precision calculations which were cited in this talk are affiliated with a North American institution?

  A 5%?

  B 15%?

  C 50%?

  D 90%?

- The correct answer is B (15%): most work on precision calculations is done outside of the US and Canada
- increased support for theorists providing the tools to successfully compare theory and data (aka "loopies") must be an integral part of the North American High Energy Physics program if we aim for more balance between the regions in supporting precision calculations (and we should aim for that!)
  - ☞ this includes support at the student and postdoc level from the funding agencies and
  - ☞ support from the Physics Departments around the country for faculty lines in this field
  - ☞ one should hope that, with the LHC approaching and the plans for the ILC continuing to develop, this will happen

5 – Conclusions

- Precise calculations, which include higher order corrections, are needed in order to interpret data from LHC and ILC
- in the last few years enormous progress has been made in developing new techniques for loop calculations
- loop calculations are complex, long-term projects which require young, talented and enthusiastic researchers
- automating loop calculations is necessary but very difficult (in particular beyond the 1-loop level)
- much has been accomplished (1-loop corrections for $2 \to 2$, $2 \to 3$ and some $2 \to 4$ processes)
- but much remains to be done (1-loop corrections for $2 \to 4$ processes; 2-loop corrections for $2 \to 2$ and $2 \to 3$ reactions ...)
- support from the funding agencies and the community is essential

- this presentation was prepared in a 100% Microsoft free environment