

# CP Violation in $B_s$ Mixing

*Brendan Casey  
Brown University  
On behalf of the  
 $D\bar{\Omega}$  collaboration*



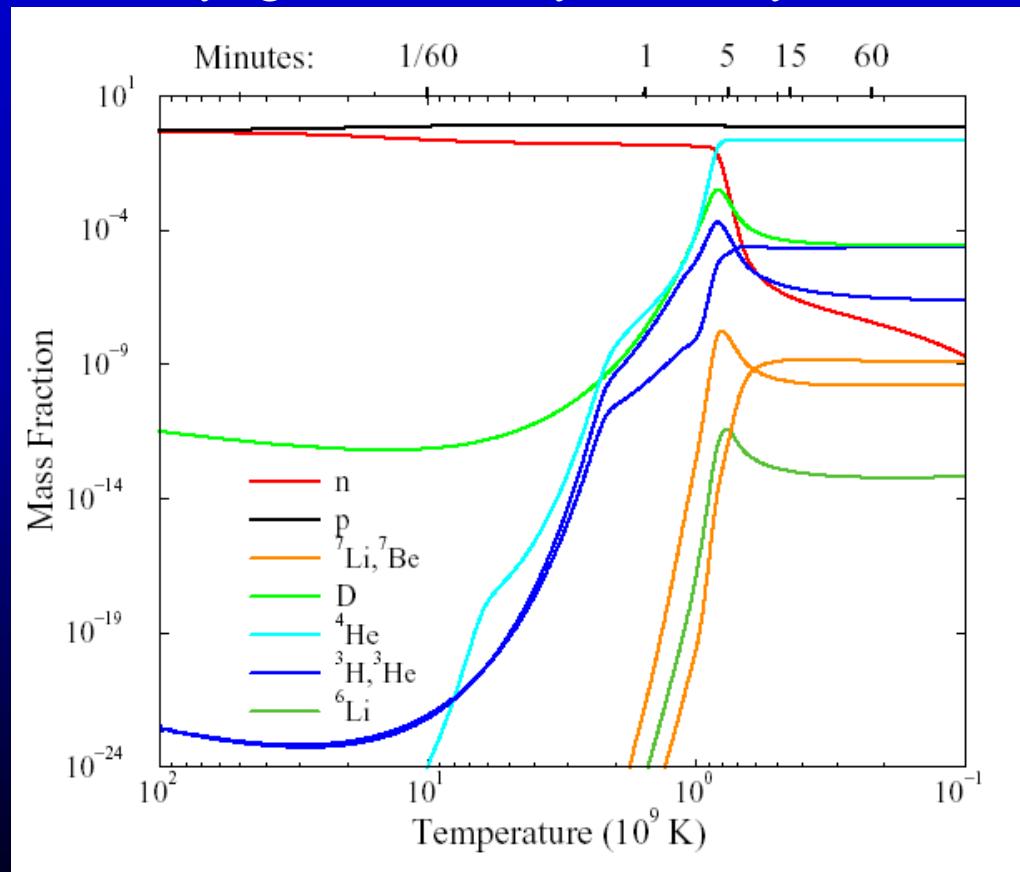
Fermilab W&C Seminar  
February 9, 2007



# Role of CPV in the Universe

GR + Astro + Particle + Nuclear Physics = Big Bang Nucleosynthesis

*Evolution of light elements in first hour of the universe*



Burles, Nollett, Turner astro-ph/99003300

One of the greatest successes of modern phys.

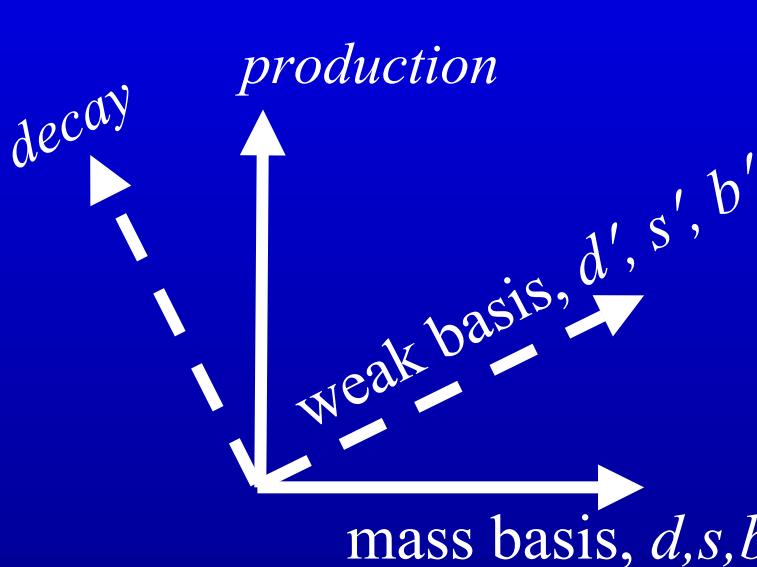
But:  
Small excess of baryons is an initial condition of BBN, not a prediction.

No explanation of the evolution of anti-elements

CP Violation necessary ingredient for our existence

one of the biggest obstacles in explaining the birth of our universe

# CPV: Standard Model to the rescue?



$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

quarks:  $d'_I = V_{ij} d_j$   
antiquarks:  $\bar{d}'_i = V_{ij}^* \bar{d}_j$

$$V_{ub} \neq V_{ub}^*, V_{td} \neq V_{td}^* \Rightarrow \text{CPV}$$

Provides a mechanism to generate a net baryon number through decay of heavy to light particles

# Standard Model CPV

Three properties govern size of CPV in SM:

$$Mag(\text{CPV}) \approx f(m_j^2 - m_i^2) \times f(\theta_{ij}) \times \sin \phi_{CP}$$

- quarks:
  - Large mass splitting  $\uparrow$
  - Small mixing angles  $\downarrow$
  - Large  $\phi_{CP} \uparrow$
- neutrinos:
  - small mass splitting  $\downarrow$
  - large mixing angles  $\uparrow$
  - $\phi_{CP}$  ?

$\sim 15$  orders of magnitude  
too small

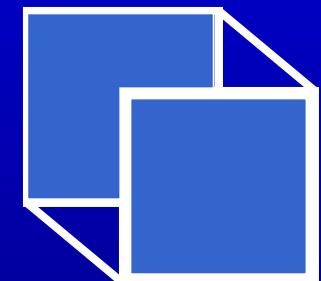
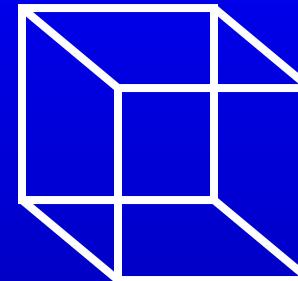
*Huet, Sather PRD51 379 (1995)*

Jury still out

Need new sources of CPV!

# Mesons, Mixing, and CPV

- Ideal laboratory for studying CPV
  - Mesons: matter – antimatter bound state
  - Neutral mesons: continuously transforming between matter and antimatter states



$$i \frac{\partial}{\partial t} |M(t)\rangle = (m - i \frac{\Gamma}{2}) |M(t)\rangle$$

$$i \frac{\partial}{\partial t} \begin{pmatrix} |M(t)\rangle \\ |\overline{M}(t)\rangle \end{pmatrix} = \begin{pmatrix} m_{11} - i \frac{\Gamma_{11}}{2} & m_{12} e^{-i\phi} - i \frac{\Gamma_{12}}{2} \\ m_{21} e^{i\phi} - i \frac{\Gamma_{21}}{2} & m_{11} - i \frac{\Gamma_{11}}{2} \end{pmatrix} \begin{pmatrix} |M(t)\rangle \\ |\overline{M}(t)\rangle \end{pmatrix}$$

# Observables

$$|M(t)\rangle_1 = \left( g^+(t) |M\rangle + g^-(t) e^{-2i\phi} |\bar{M}\rangle \right)$$

$$|M(t)\rangle_2 = \left( g^-(t) e^{2i\phi} |M\rangle + g^+(t) |\bar{M}\rangle \right)$$

$$|g^\pm(t)|^2 = \frac{e^{-\Gamma t}}{2} \left[ \cosh\left(\frac{\Delta\Gamma}{2}t\right) \pm \cos(\Delta m t) \right]$$

$$\Delta m = m_H - m_L = 2|m_{12}|$$

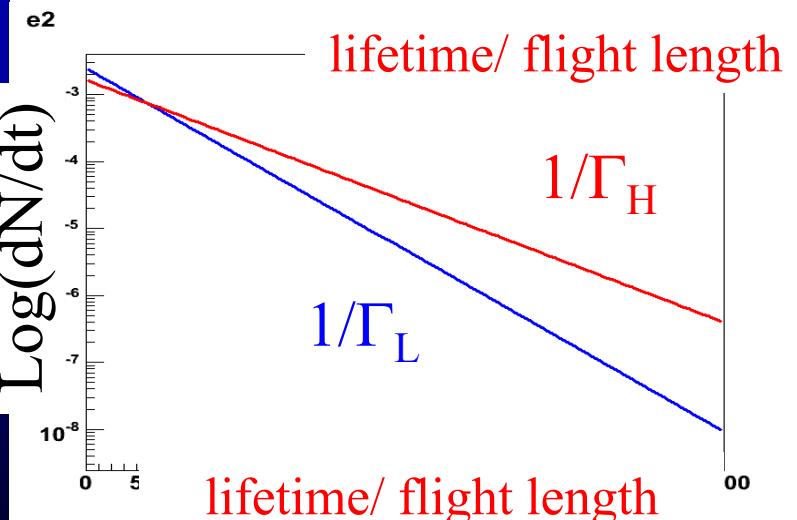
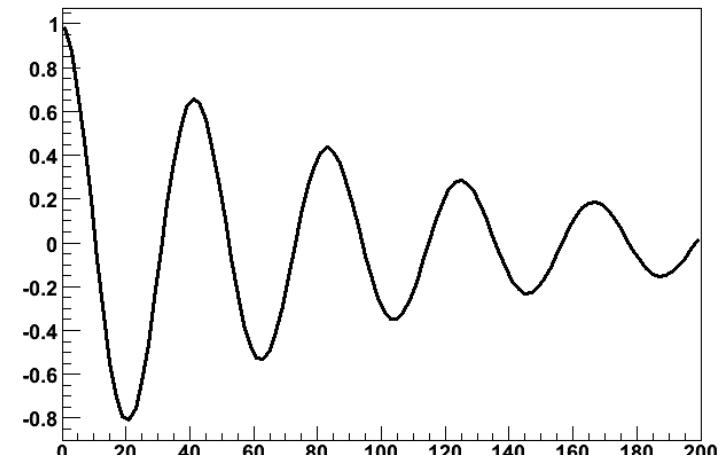
$$\Delta\Gamma = \Gamma_L - \Gamma_H = 2|\Gamma_{12}| \cos\phi$$

$$\phi = \arg\left(-\frac{m_{12}}{\Gamma_{12}}\right)$$

$$\bar{\Gamma} = \frac{1}{\bar{\tau}} = \frac{1}{2}(\Gamma_L + \Gamma_H)$$

$$\Delta\Gamma_{CP} = \Gamma_{even} - \Gamma_{odd} = 2|\Gamma_{12}|$$

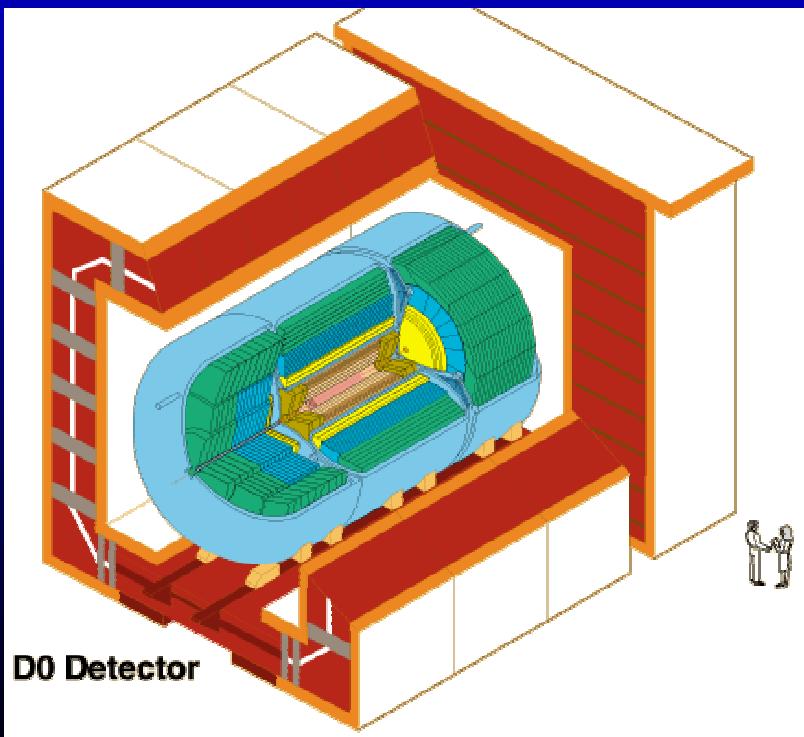
$$\frac{N(unmixed) - N(mixed)}{N(unmixed) + N(mixed)}$$



$$\Gamma(M \Rightarrow \bar{M}) \neq \Gamma(\bar{M} \Rightarrow M)$$



	$ \eta $	$ \theta $
Muon ID	$\sim 2$	$\sim 15^\circ$
tracking	$\sim 3$	$\sim 5^\circ$
EM and hadronic calorimetry	$\sim 4$	$\sim 2^\circ$

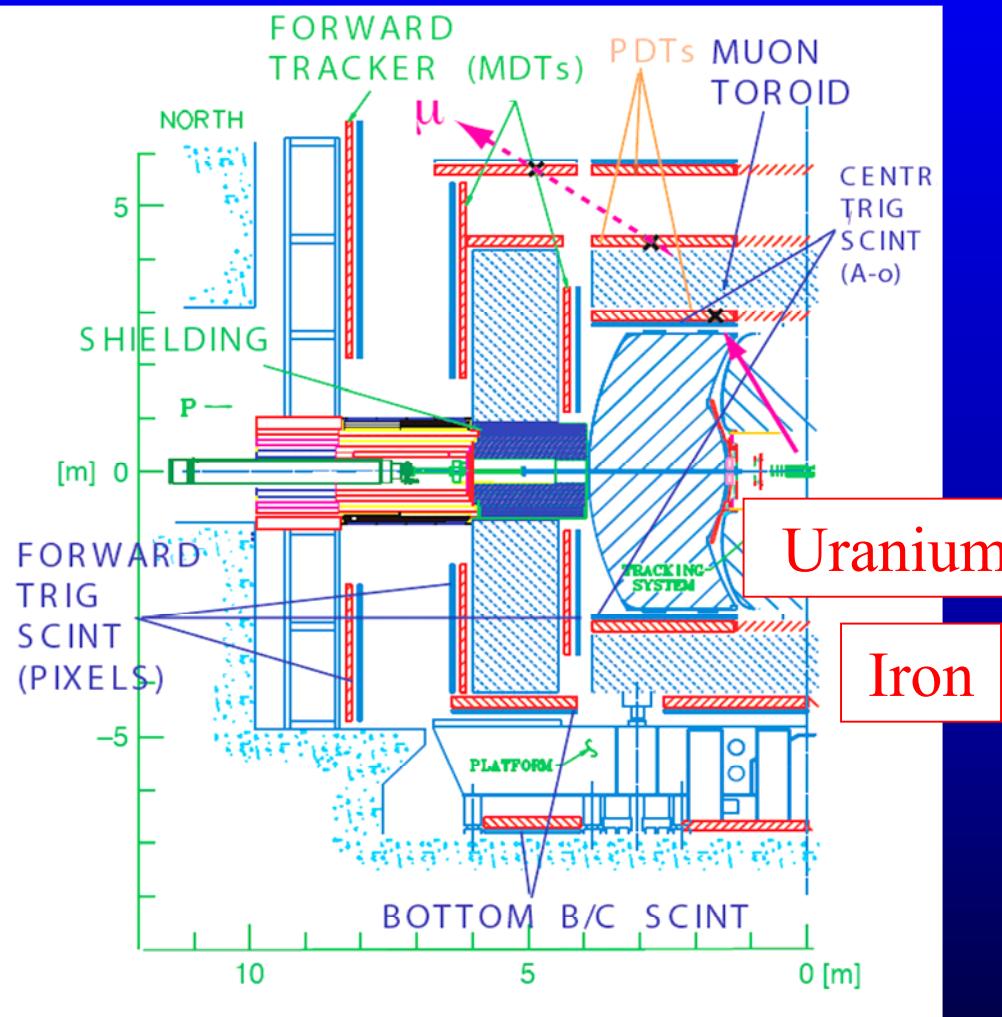


D0 Detector



629 scientists,  
81 institutions,  
20 countries

# Heart of the DØ Program



Toroid & solenoid polarity  
flipped regularly

~equal exposure to 4  
possible field configurations

Excellent, large angle, muon spectrometer and trigger

Large  $B \rightarrow \mu$  semileptonic and  $B \rightarrow J/\psi$  samples

# Tevatron Data Set

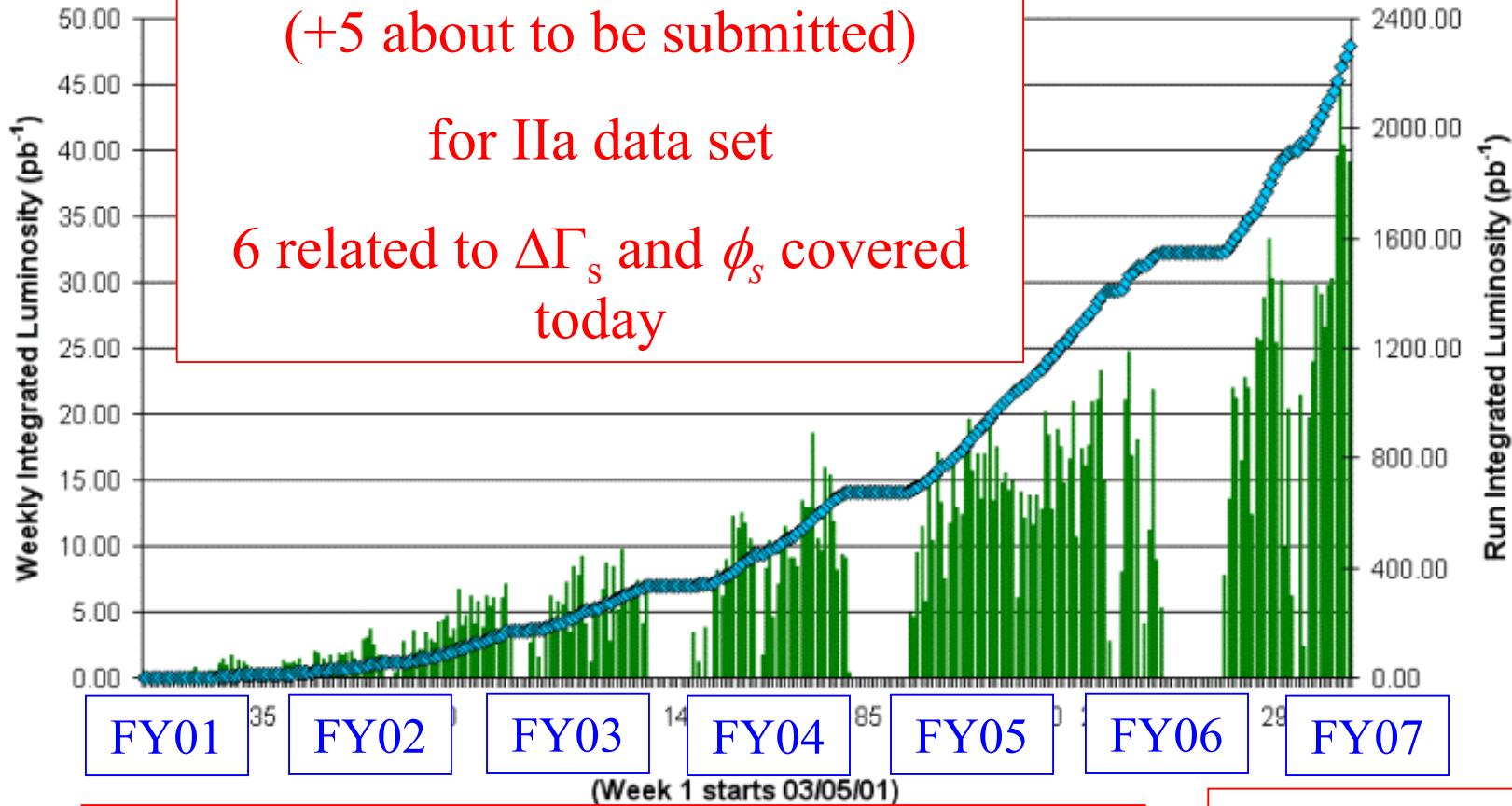
>2.3  $\text{fb}^{-1}$   
delivered

15 Heavy Flavor Publications

(+5 about to be submitted)

for IIa data set

6 related to  $\Delta\Gamma_s$  and  $\phi_s$  covered  
today

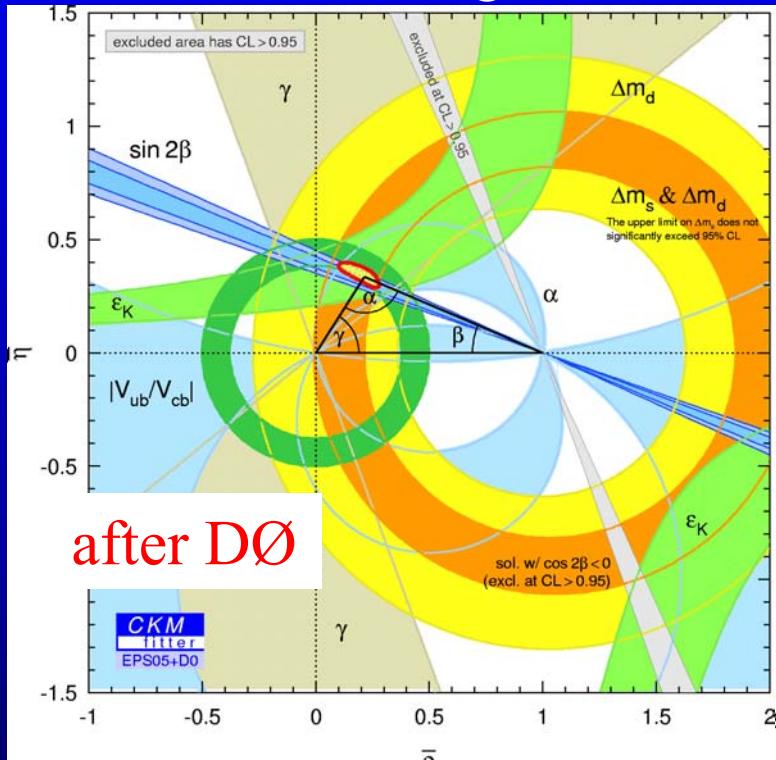


Run IIa: 1.3  $\text{fb}^{-1}$  on tape

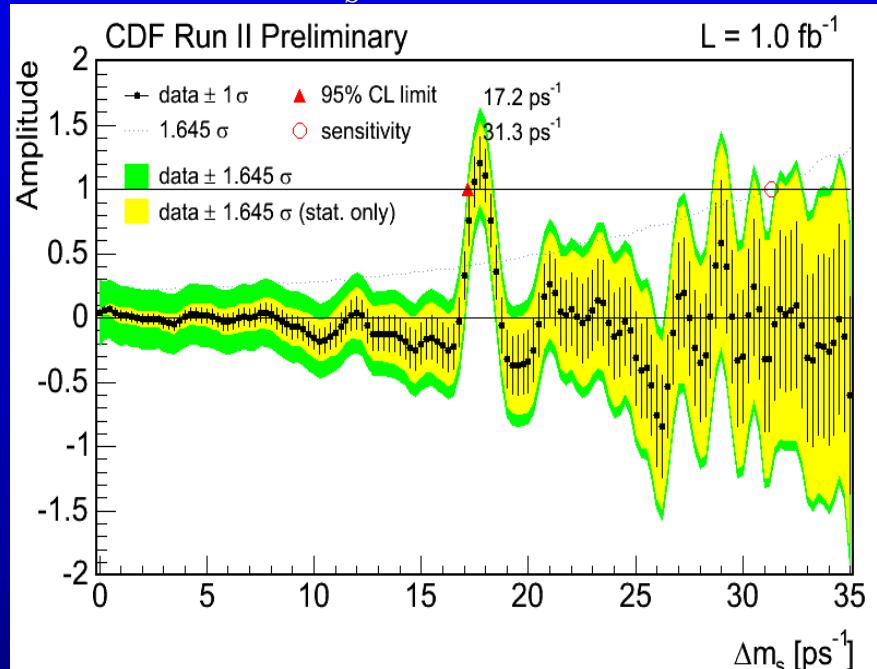
Run IIb:  
>700  $\text{pb}^{-1}$   
on tape

# First Parameter: $\Delta m_s$

CKM triangle



CDF  $\Delta m_s$  Amplitude Scan



$$\left| \frac{V_{td}}{V_{ts}} \right| = 0.2060 \pm 0.0007 (\text{ex.})^{+0.0081}_{-0.0060} (\text{theo.})$$

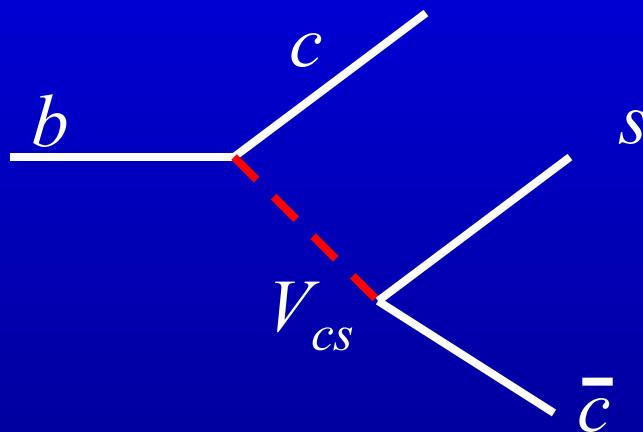
Rule out large new physics: DØ Collab. PRL 97 021802 (2006)

Precision SM measurement: CDF Collab. PRL 97 062003 (2006)

Two top cited experimental HEP papers of 2006

# Second Parameter: $\Delta\Gamma$

*Governed by decay  
rates to CP eigenstates*



Tree process,  
don't expect new physics

Importance: defines the phenomenology

$$\Gamma_L(t) + \Gamma_H(t) \approx [e^{-\Gamma_L t} - e^{-\Gamma_H t}] \sin \phi$$

Large  $\Delta\Gamma$ : CPV observables do not cancel out, info in all events

Small  $\Delta\Gamma$ : CP studies require initial state tagging  $\varepsilon_{eff} \approx 2.5\%$

# Second Parameter: $\Delta\Gamma$

- First step:
  - Determine  $\Delta\Gamma$  assuming SM level CPV in  $B_s$  mixing:  $\cos\phi_s \equiv 1$
  - $\Gamma_L - \Gamma_H = \Gamma_{CP+} - \Gamma_{CP-}$
- Three main channels:
  - $B_s \rightarrow D_s^{(*)} D_s^{(*)}$
  - Flavor specific  $B_s$  lifetime
  - $B_s \rightarrow J/\psi \phi$

# $B_s \rightarrow D_s^{(*)} D_s^{(*)}$ and $\Delta\Gamma_{CP}$

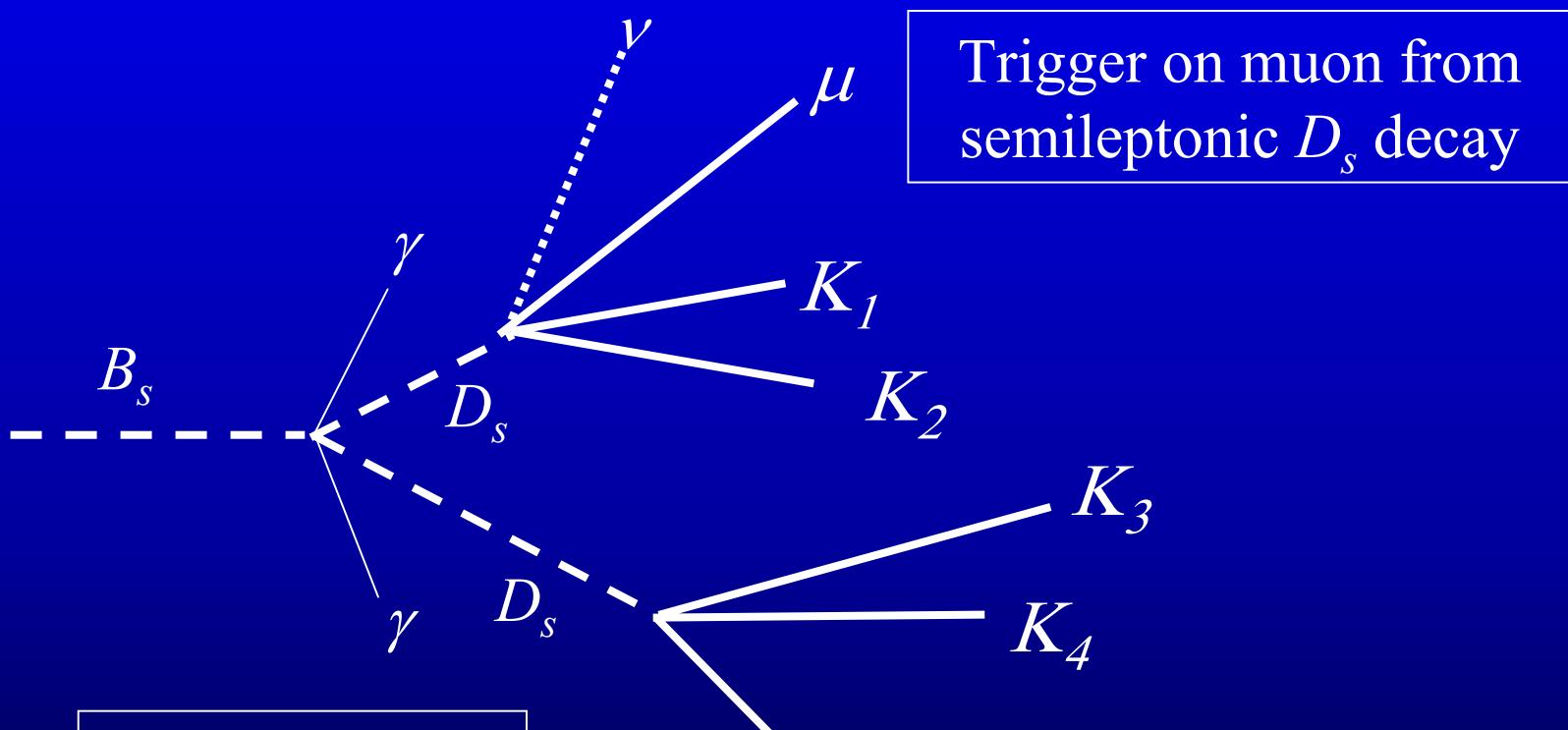
- Three channels
  - $D_s D_s$  (PP),  $D_s^* D_s$  (VP),  $D_s^* D_s^*$  (VV)
- Heavy quark limit + factorization
  - $B_s^{\text{odd}} \rightarrow D_s^* D_s$  forbidden
- $D_s^* D_s^*$  in S-wave

*R. Alexan et al., PLB 316, 567 (1993) ~5% odd  
Others: maybe as much as 30% odd*

$\Rightarrow Ds^{(*)} Ds^{(*)}$  pure CP even

$$BF(B_s \rightarrow D_s^{(*)} D_s^{(*)}) = \left( \frac{\Delta\Gamma_{CP}}{2\Gamma} \right) \left( 1 + O\left(\frac{\Delta\Gamma}{\Gamma}\right) \right)$$

$$B_s \rightarrow D_s^* D_s^*$$

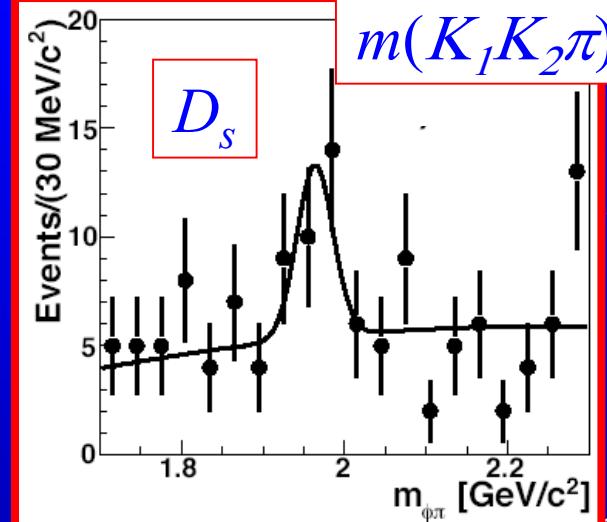


Ignore any photons

Trigger on muon from semileptonic  $D_s$  decay

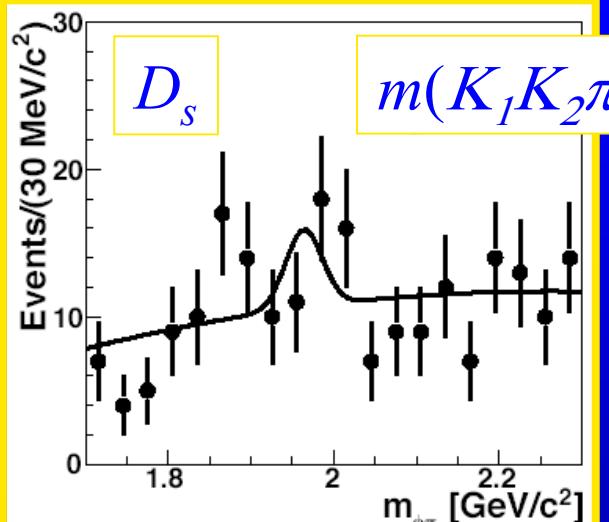
Look for correlated production of  $D_s \rightarrow \phi \pi$  and  $D_s \rightarrow \phi \mu$

$m(K_1 K_2 \pi)$



$D_s \mu \phi$   
candidates

$m(K_1 K_2 \pi)$

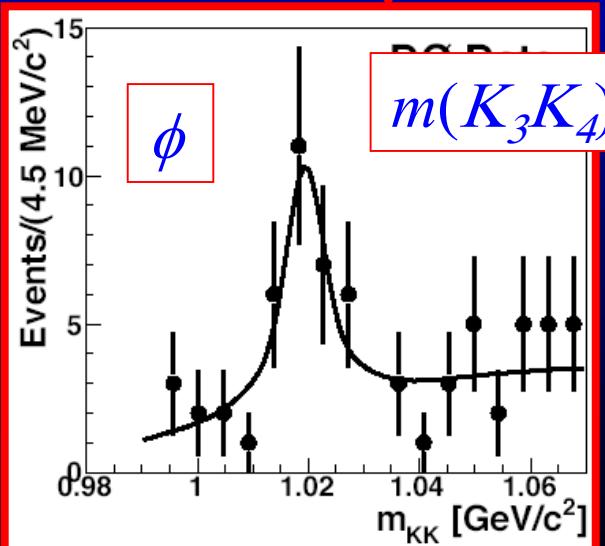


signal

$m(K_1 K_2 \pi)$

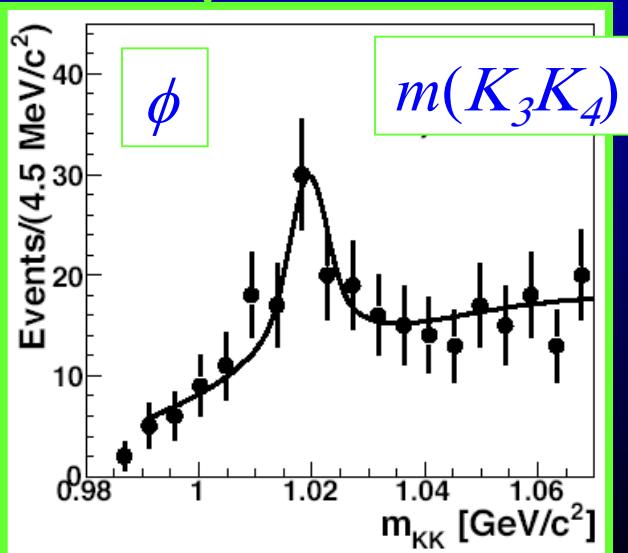
sidebands

$m(K_3 K_4)$



$$N(D_s^{(*)} D_s^{(*)}) = \\ 13.4^{+6.6}_{-6.0}$$

$m(K_3 K_4)$



$B_s \rightarrow D_s^{(*)} D_s^{(*)}$

$$BF(B_s \rightarrow D_s^{(*)} D_s^{(*)}) = \left( \frac{\Delta\Gamma_{CP}}{2\Gamma} \right) \left( 1 + O\left(\frac{\Delta\Gamma}{\Gamma}\right) \right)$$

$$BF(B_s \rightarrow D_s^{(*)} D_s^{(*)}) = 0.039^{+0.019}_{-0.017} \quad {}^{+0.016}_{-0.015}$$

$$\frac{\Delta\Gamma_{CP}}{\Gamma} = 0.079^{+0.038}_{-0.035} \quad {}^{+0.031}_{-0.030}$$

# Flavor Specific $B_s$ Lifetime

*Flavor specific decays carry equal amounts of  $B_H$  and  $B_L$*

$$|B_s \rightarrow D_s \mu \nu\rangle = \frac{1}{\sqrt{2}}(|B_H\rangle + |B_L\rangle)$$

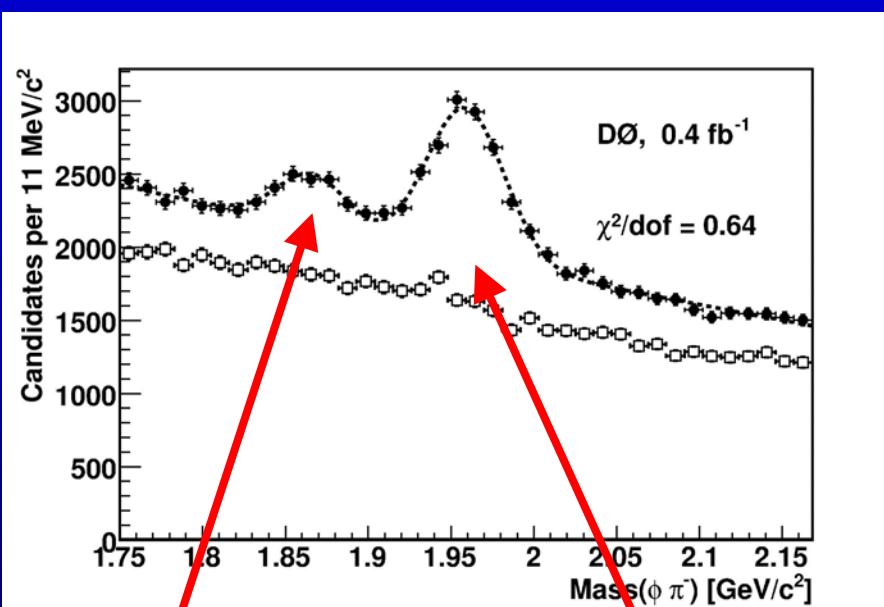
$$e^{-t/\tau_{FS}} \equiv \frac{1}{2} \left( e^{-t/\tau_H} + e^{-t/\tau_L} \right)$$

*Get the flavor specific lifetime when you fit FS data with single exponential*

*Maps out a 2-D constraint on the average width and the width difference*

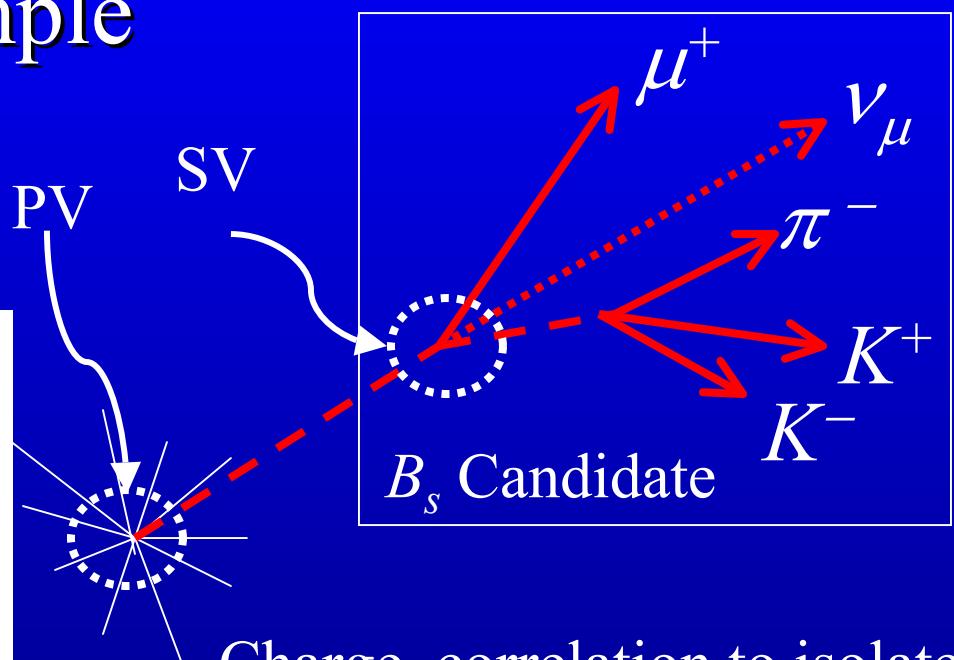
$$\begin{aligned}\tau_{FS} &= \frac{1}{\Gamma_s} \left( \frac{1+y^2}{1-y^2} \right) \\ y &= \frac{\Delta\Gamma}{2\Gamma}\end{aligned}$$

# $B_s$ Semileptonic Sample



$$D^+ \rightarrow \phi \pi$$

$$D_s^- \rightarrow \phi \pi$$



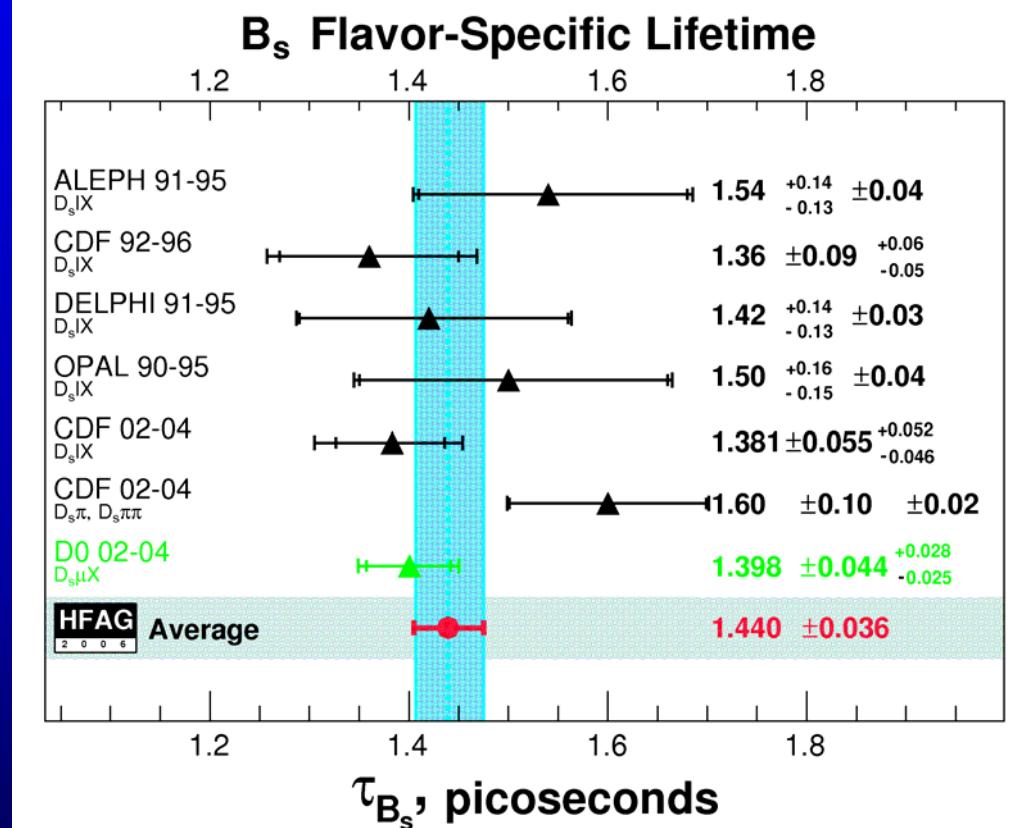
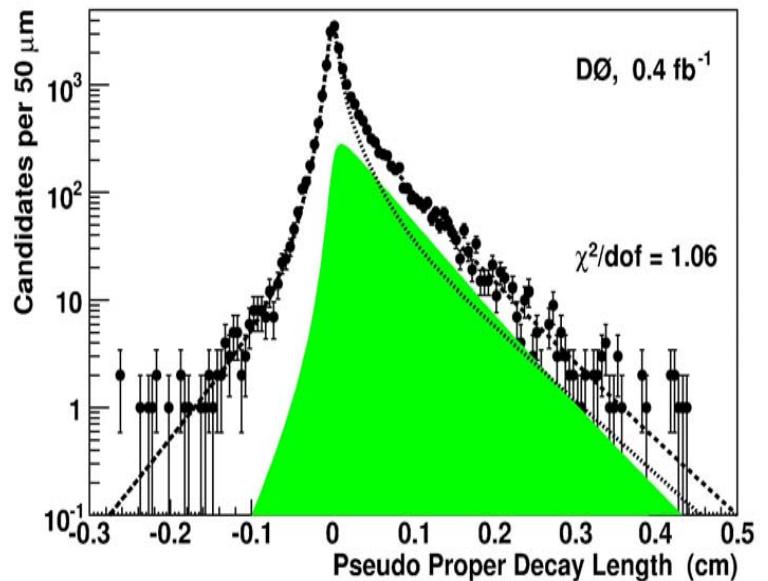
Charge correlation to isolate  
 $B_s$  sample

Transverse decay length  
determined in lab frame

Boost back using MC to  
estimate neutrino momentum

# $B_s$ Flavor Specific Lifetime

PRL 97, 241801 (2006)



$$\tau_{FS}(B_s) = 1.398 \pm 0.044^{+0.028}_{-0.025} \text{ ps}$$

$$\tau_{FS}(B_s, WA) = 1.440 \pm 0.036 \text{ ps}$$

$$B_s \rightarrow J/\psi \phi: P \rightarrow VV$$

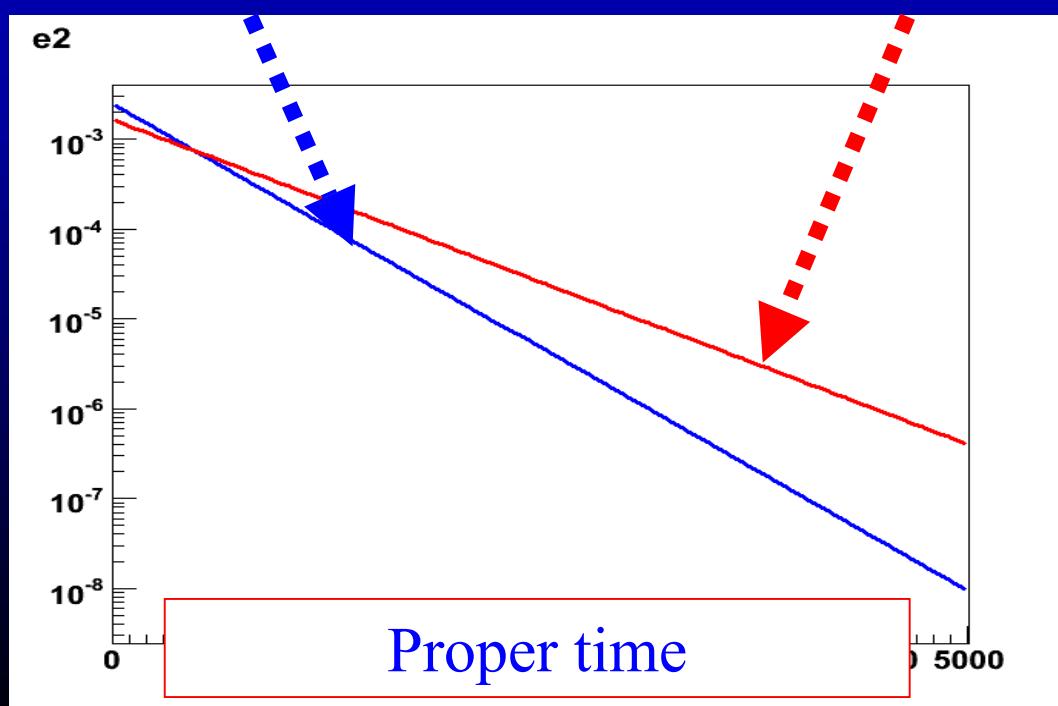
Even and odd paths distinguishable with angular analysis of final state particles

$$A_0 \ A_{||}$$

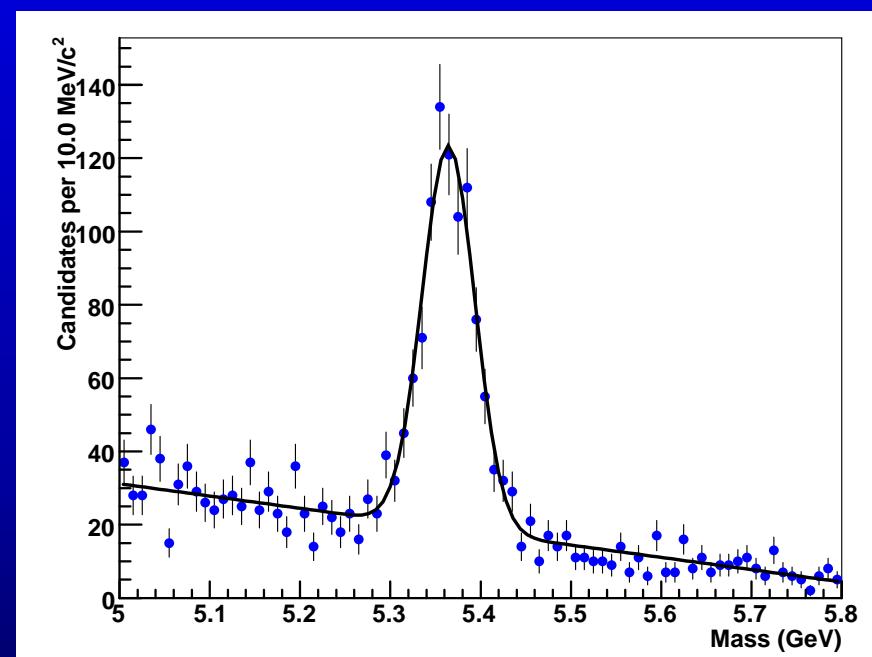
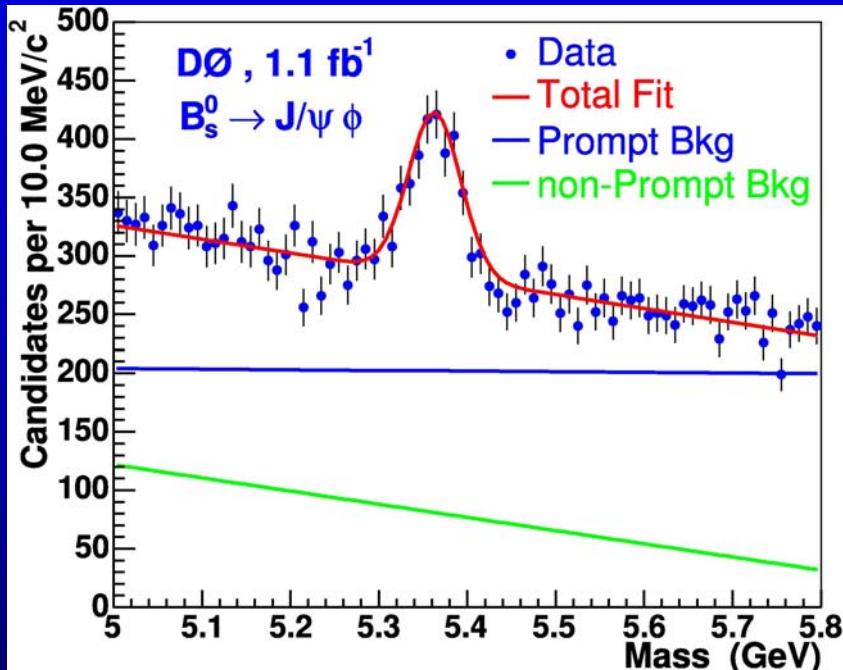
Even waves

$$A_{\perp}$$

Odd waves



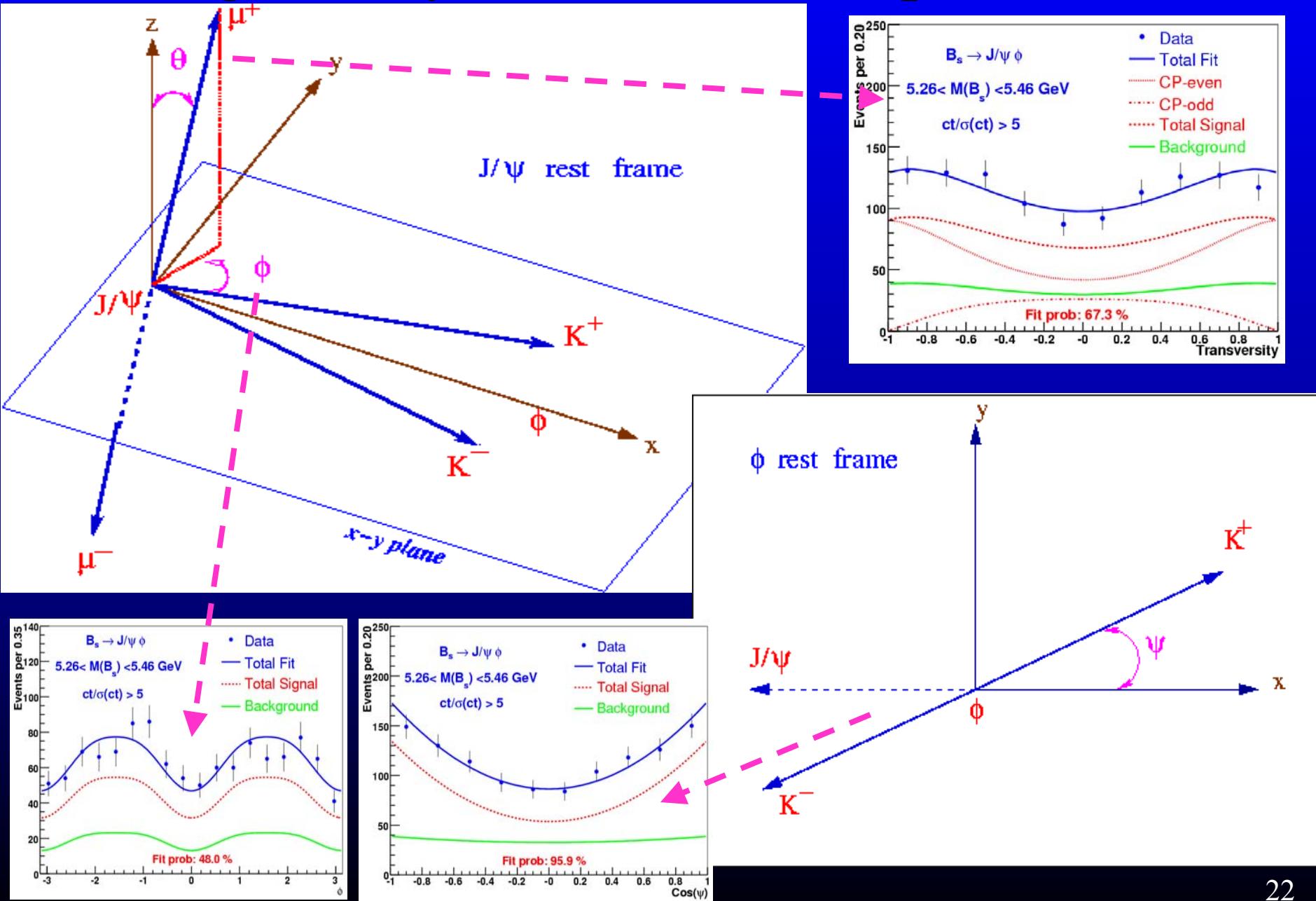
# $B_s \rightarrow J/\psi \phi$



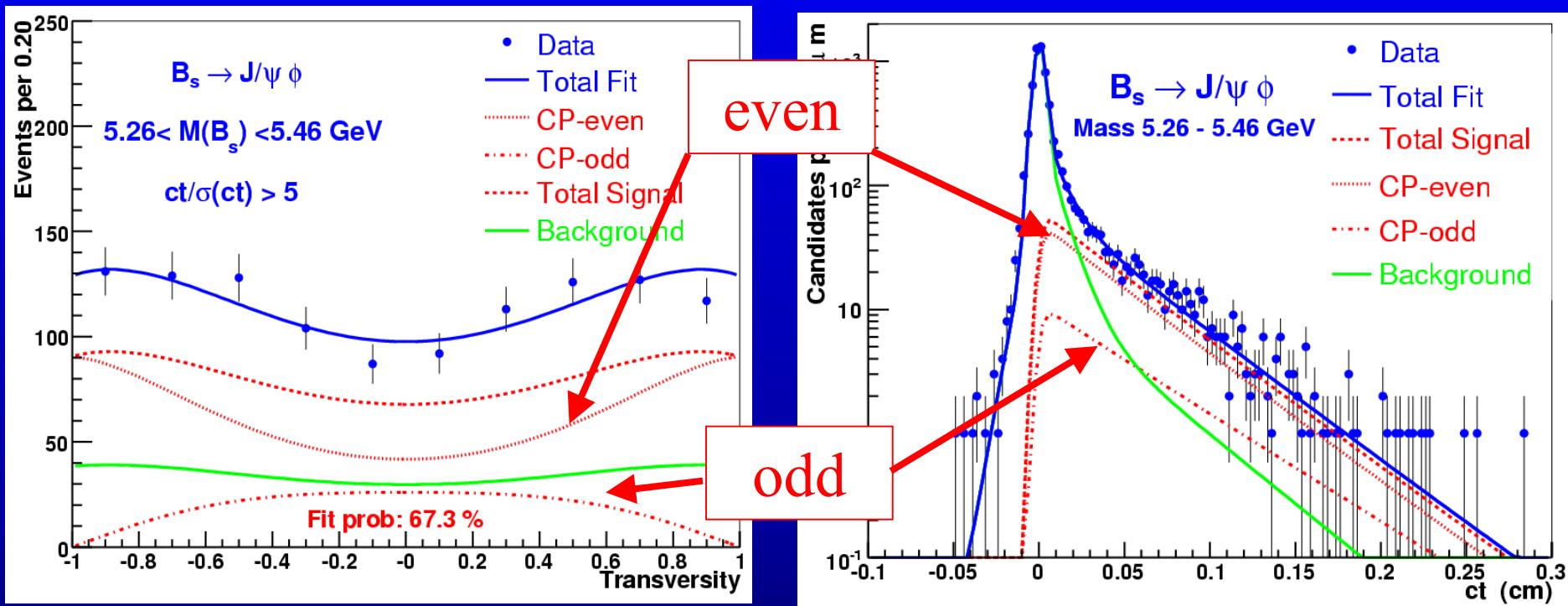
Flight length  
significance  $> 5$

$1039 \pm 45$   $B_s$   
Candidates

# 3 angle analysis to determine polarization



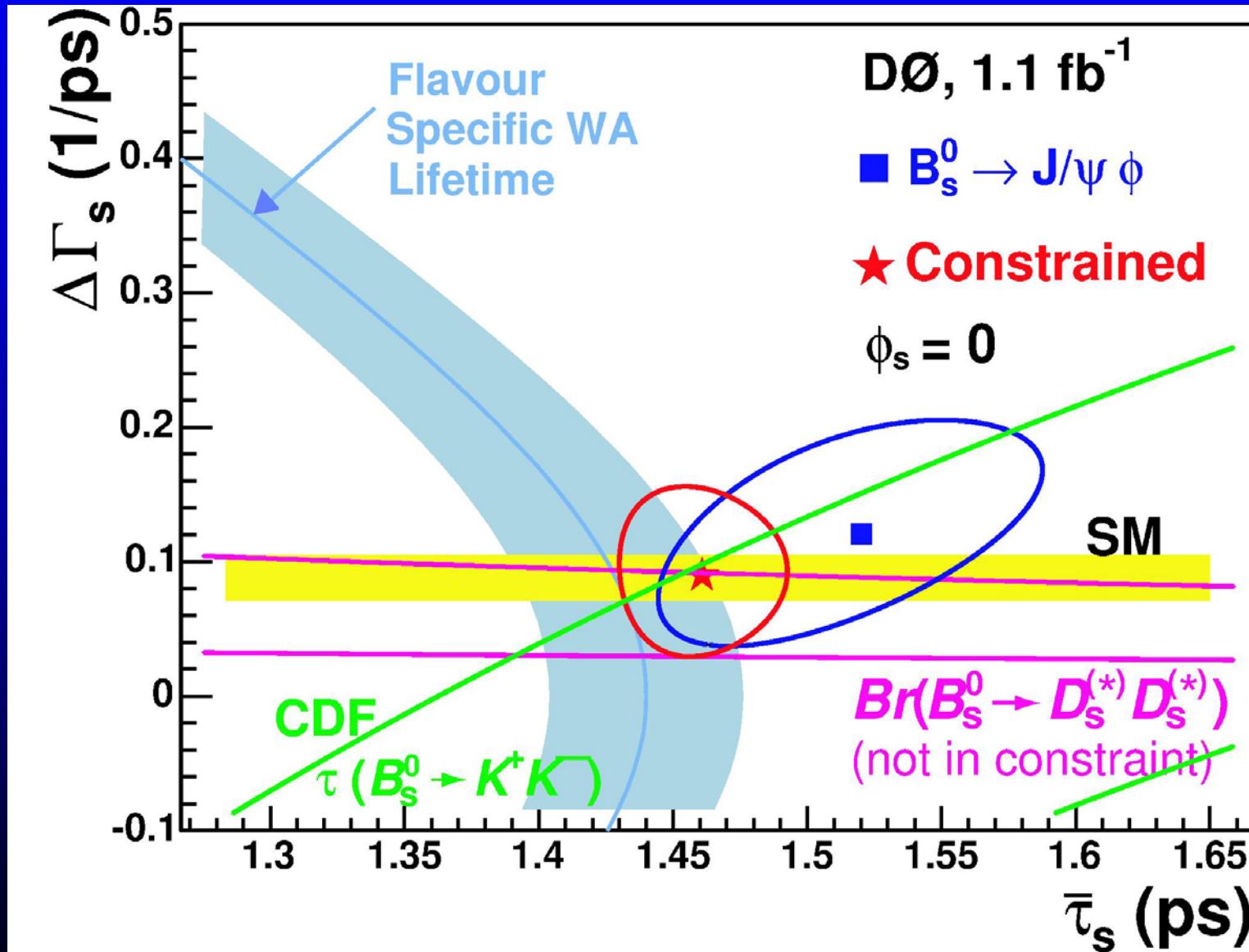
# $B_s \rightarrow J/\psi \phi$



$$\Delta\Gamma_s = 0.12^{+0.08}_{-0.10} \pm 0.02 \text{ ps}^{-1}$$

$$\bar{\tau}_s = \frac{1}{\Gamma_s} = 1.52 \pm 0.08^{+0.01}_{-0.03} \text{ ps}$$

# Combined $\Delta\Gamma$ ( $\cos\phi_s \equiv 1$ )

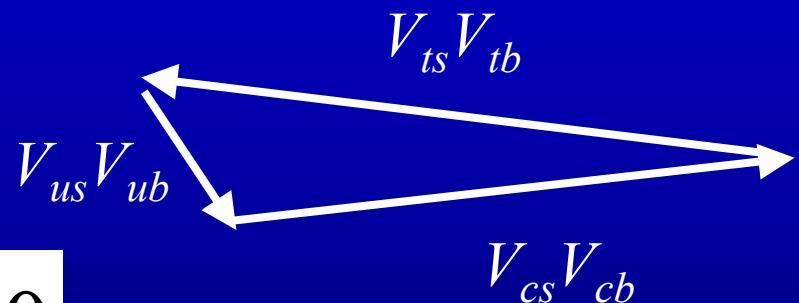


# Third Parameter: $\phi_s$

In Standard Model:  $\phi_s \approx \arg(-V_{ts})$

Size limited by unitarity of CKM matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$

Standard Model  $\phi_s \approx 0.004$  rad.

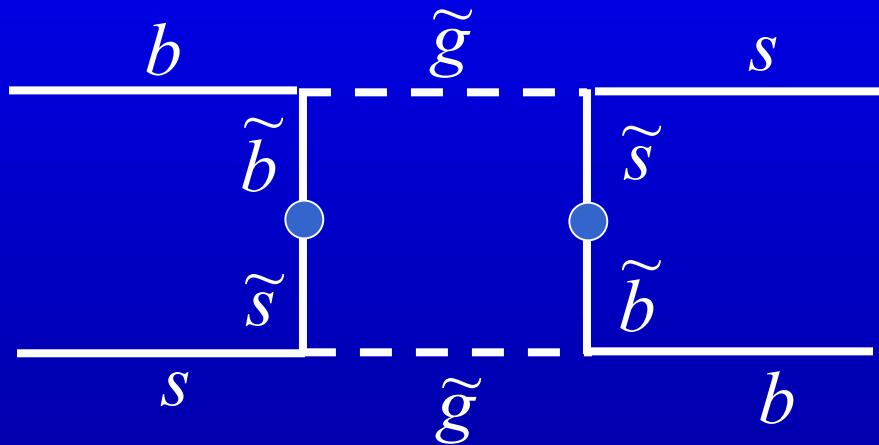
*A. Lenz, U. Nierste hep-ph/0612167*

Observables:

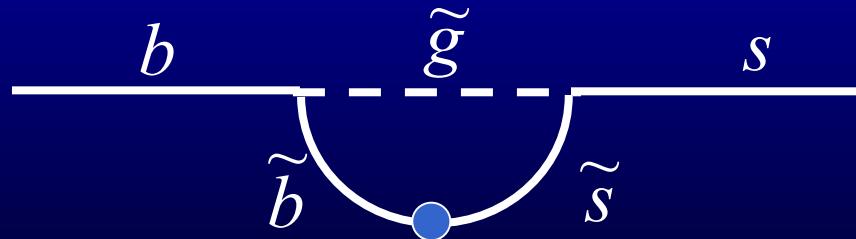
Semileptonic asymmetries, interference in decays to CP eigenstates

# $\Delta B = 2$ versus $\Delta B = 1$

*We are looking for something like this*



*B factories are looking for something like this*



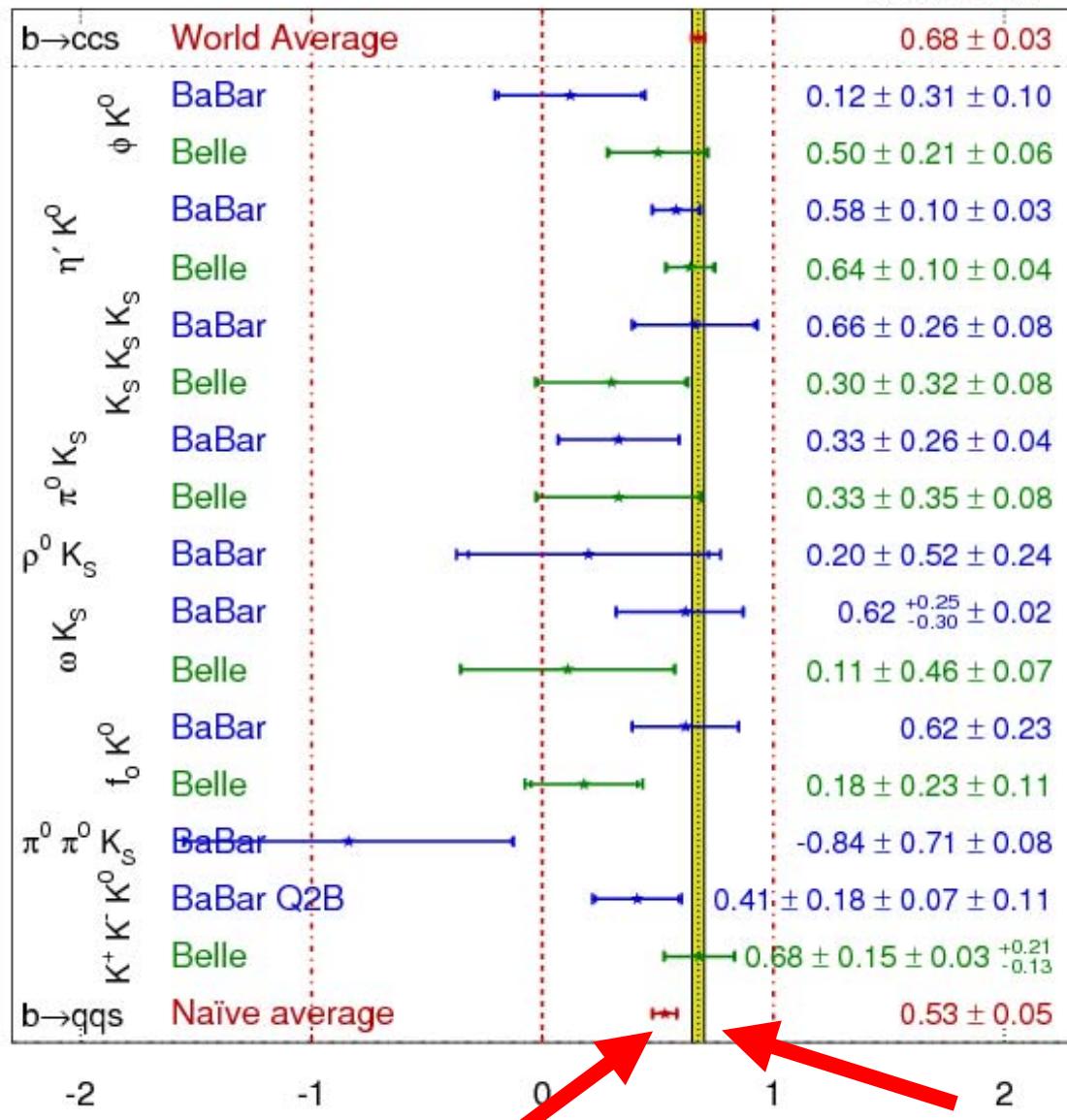
In absence of super-weak models, we are looking at the same thing

# Hints from $\Delta B=1$

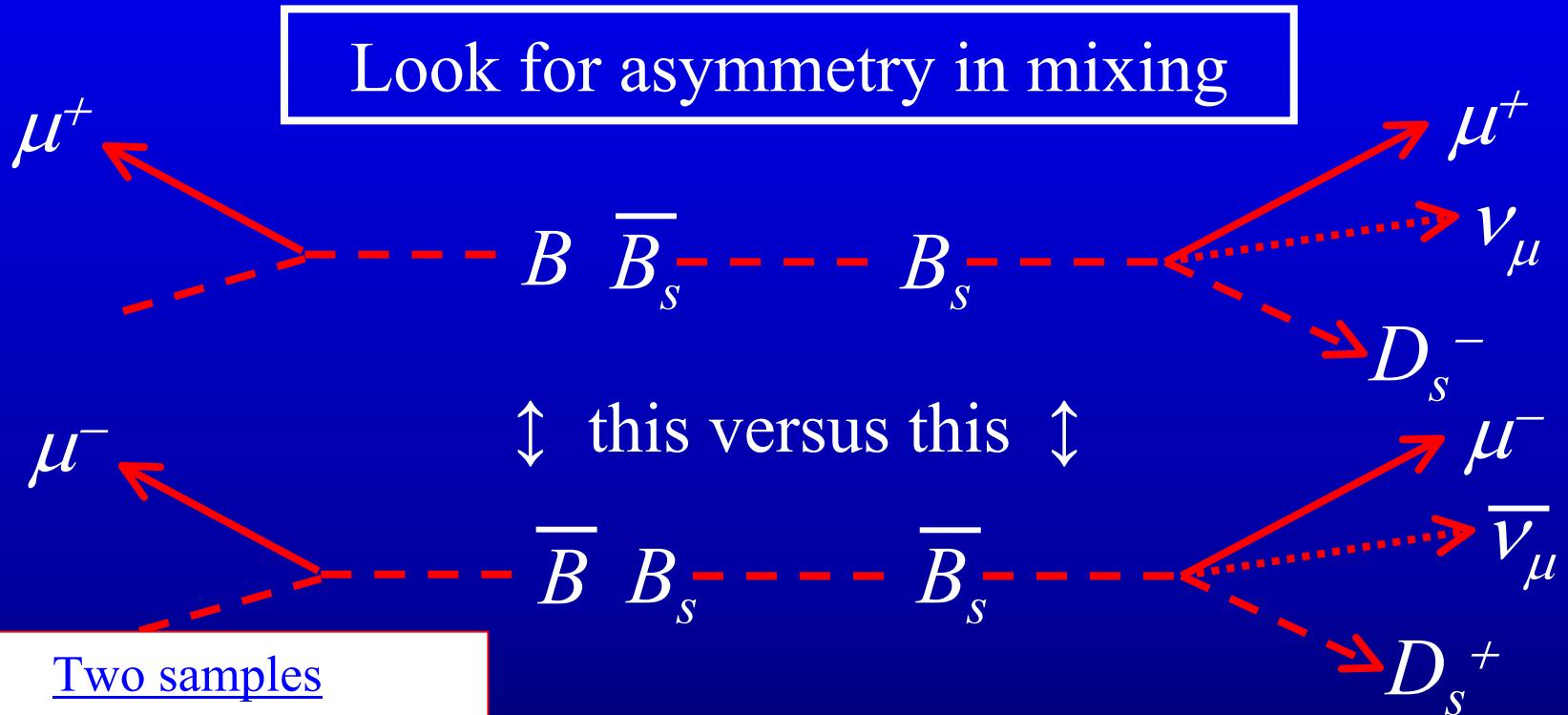
*Comparison of CPV  
in  $b \rightarrow c$  and  $b \rightarrow s,d$*

$$\sin(2\beta^{\text{eff}}) \equiv \sin(2\phi_1^{\text{eff}})$$

**HFAG**  
DPF/JPS 2006  
PRELIMINARY



# Semileptonic Asymmetry Measurements



Exclusive untagged  $D_s\mu$

Inclusive like-sign  $\mu\mu$

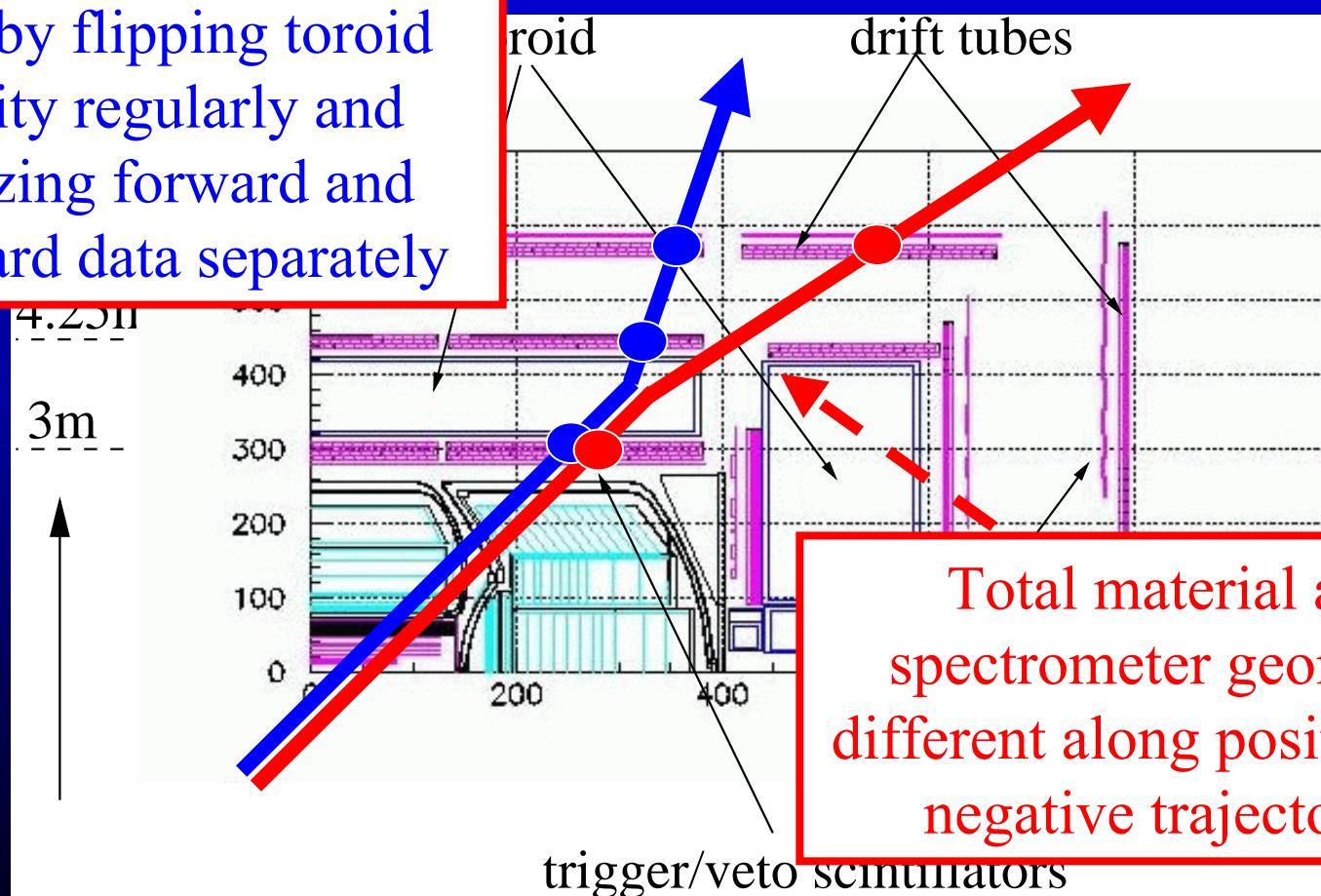
$$\frac{N(D_s\mu^+) - N(D_s\mu^-)}{N(D_s\mu^+) + N(D_s\mu^-)} = A_{SL}(\text{untagged}) \approx \frac{\Delta\Gamma}{\Delta m} \tan\phi$$

$$\frac{N(\mu^+\mu^+) - N(\mu^-\mu^-)}{N(\mu^+\mu^+) + N(\mu^-\mu^-)} = A_{SL}(\text>tagged) = 2A_{SL}(\text{untagged})$$

# Detector Systematics

Main detector related systematic is due to geometry in muon spectrometer

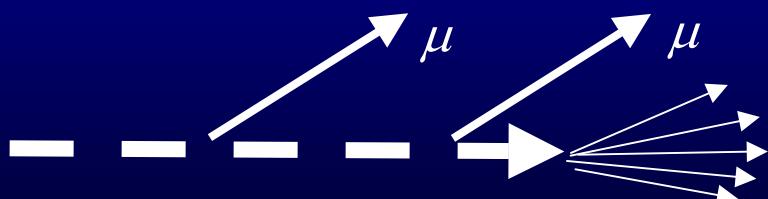
Solve by flipping toroid polarity regularly and analyzing forward and backward data separately



# Physics Systematics



$K^+$  produce  $\mu^+$  from weak decay  $N(\mu^+) \sim \exp(-x/c \tau(K))$



Measured directly in data using kaons tagged from  $D^*$  decay

$K^-$  stop more frequently in calorimeter from  $K^- + N \rightarrow Y + \pi$  nuclear interactions

# Like-sign Dimuons

N(same sign)  $\approx 310K$

$$A_{SL} = -0.0092 \pm 0.0044 \pm 0.0032$$

PRD 74, 092001 (2006)

$\sim$ 60/40 mix of  $B_d$  and  $B_s$

$$A_{SL} = A_{SL}(B_d) + \frac{f_s Z_s}{f_d Z_d} A_{SL}(B_s) \quad Z \sim 2\chi$$

$A_{SL}(B_d)$  from B-factories:

$A_{SL}(B_d) = -0.0047 \pm 0.0046$  (HFAG)

$$A_{SL}(B_s, \mu\mu) = -0.0064 \pm 0.0101$$

# 20 years of like-sign dileptons

- 1987: excess of like-sign dileptons first seen at a hadron collider (UA1)
  - Input from  $\Upsilon(4S)$  allows extraction of  $B_s$  mixing parameters

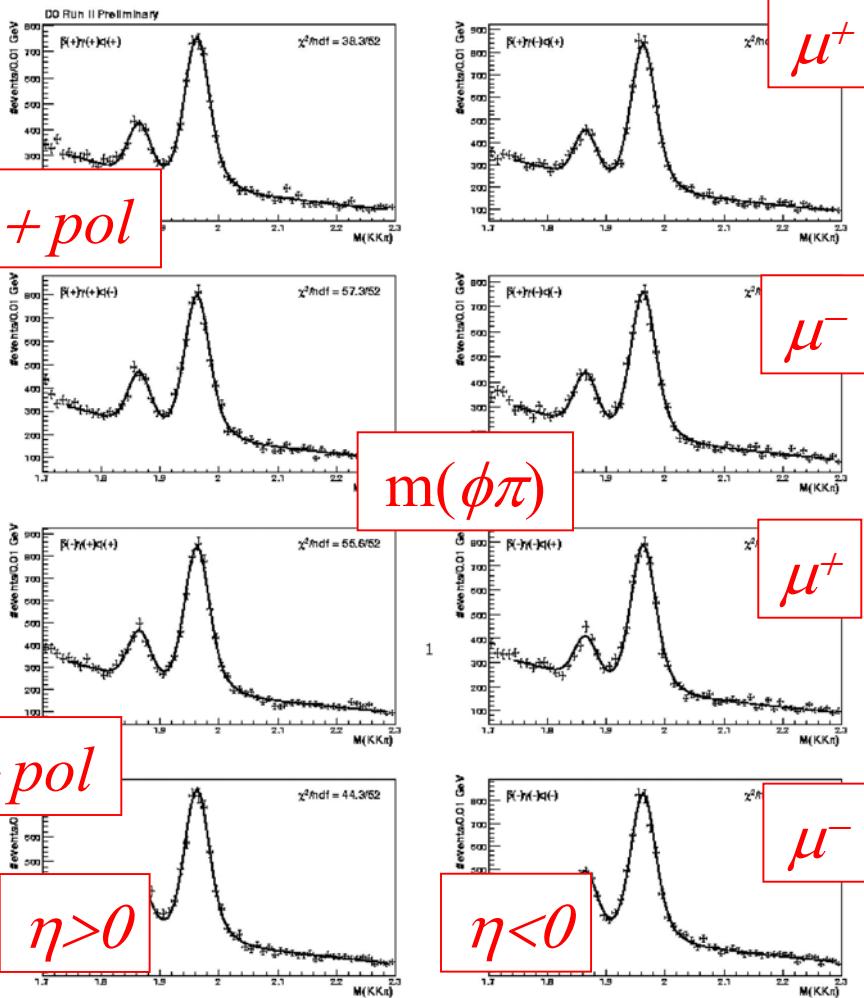
$$\chi = f_d \chi_d + f_s \chi_s$$

- 2006: best limits on CPV in like-sign dileptons determined at the next generation hadron collider
  - Input from next generation  $\Upsilon(4S)$  experiments allows extraction of CPV parameters in  $B_s$  mixing

$$A_{SL} = A_{SL}(B_d) + \frac{f_s Z_s}{f_d Z_d} A_{SL}(B_s)$$

# Exclusive $B_s \rightarrow D_s^\pm \mu\nu$ Results

## Exclusive $D_s \mu$



$$A_{SL}(B_s, D_s \mu) =$$

$$0.0245 \pm 0.0193 \pm 0.0035$$

hep-ex/0701007 submitted to PRL

Combined:

$$A_{SL}(B_s, \mu\mu + D_s \mu) =$$

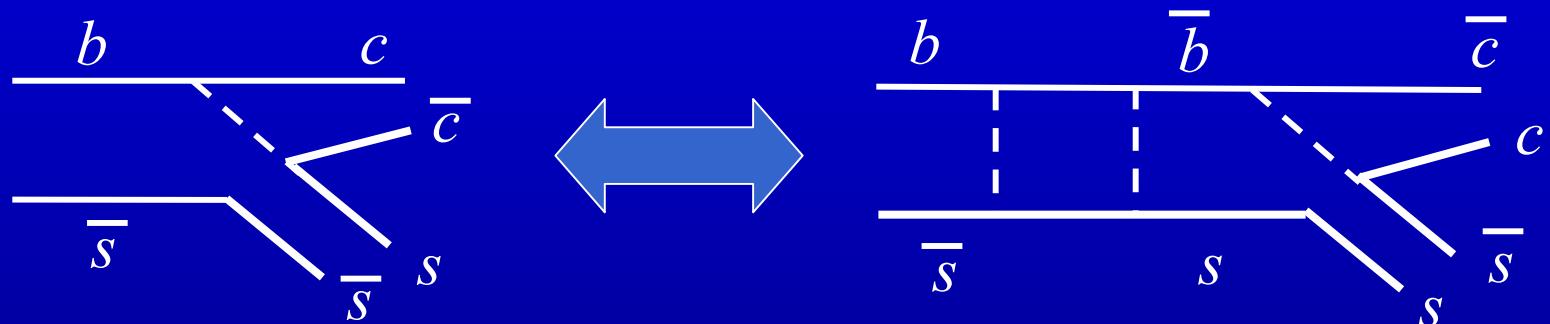
$$0.0001 \pm 0.0090$$

Using  $\Delta m_s$  from CDF:

$$\Delta\Gamma_s \cdot \tan\phi_s = 0.02 \pm 0.16 \text{ ps}^{-1}$$

$$B_s \rightarrow J/\psi \phi$$

Analog to  $B^0 \rightarrow J/\psi K_S$  but  $VV$  instead of  $VP$



$$J/\psi + \phi$$

Even waves

$$J/\psi + \phi$$

Odd waves

# $B_s \rightarrow J/\psi \phi$

Time dependent angular analysis of untagged sample

Time evolution: pure even case

$$\Gamma(t) \approx |A_{even}(\theta, \psi, \varphi, t)|^2$$

$$f(t, even) \approx e^{-\Gamma_L t}$$

# $B_s \rightarrow J/\psi \phi$

Time dependent angular analysis of untagged sample

Time evolution: even plus odd components

$$\Gamma(t) \approx |A_{even}(\theta, \psi, \varphi, t)|^2 + |A_{odd}(\theta, \psi, \varphi, t)|^2$$

$$+ A^* A(CPC)$$

CP conserving interference

$$f(t, even) \approx e^{-\Gamma_L t}$$

$$f(t, odd) \approx e^{-\Gamma_H t}$$

CP states = heavy, light states

# $B_s \rightarrow J/\psi \phi$

Time dependent angular analysis of untagged sample

Time evolution: even plus odd plus CPV

$$\Gamma(t) \approx |A_{even}(\theta, \psi, \varphi, t)|^2 + |A_{odd}(\theta, \psi, \varphi, t)|^2$$

$$+ A^* A(CPC)$$

CP conserving interference

$$+ A^* A(CPV)(e^{-\Gamma_L t} - e^{-\Gamma_H t}) \sin \phi_s$$

CP violating interference  
between two paths

$$f(t, even) \approx (1 + \cos \phi_s) e^{-\Gamma_L t} + (1 - \cos \phi_s) e^{-\Gamma_H t}$$

$$f(t, odd) \approx (1 + \cos \phi_s) e^{-\Gamma_H t} + (1 - \cos \phi_s) e^{-\Gamma_L t}$$

Heavy and light states  
are mixed CP

# $B_s \rightarrow J/\psi \phi$

Time dependent angular analysis of untagged sample

Time evolution: even plus odd plus CPV

$$\Gamma(t) \approx |A_{even}(\theta, \psi, \varphi, t)|^2 + |A_{odd}(\theta, \psi, \varphi, t)|^2$$

+  $A^* A(CPC)$  CP conserving interference

+  $A^* A(CPV)(e^{-\Gamma_L t} - e^{-\Gamma_H t}) \sin \phi_s$  CP violating interference  
between two paths

$$f(t, even) \approx (1 + \cos \phi_s) e^{-\Gamma_L t} \quad \boxed{+ (1 - \cos \phi_s) e^{-\Gamma_H t}}$$

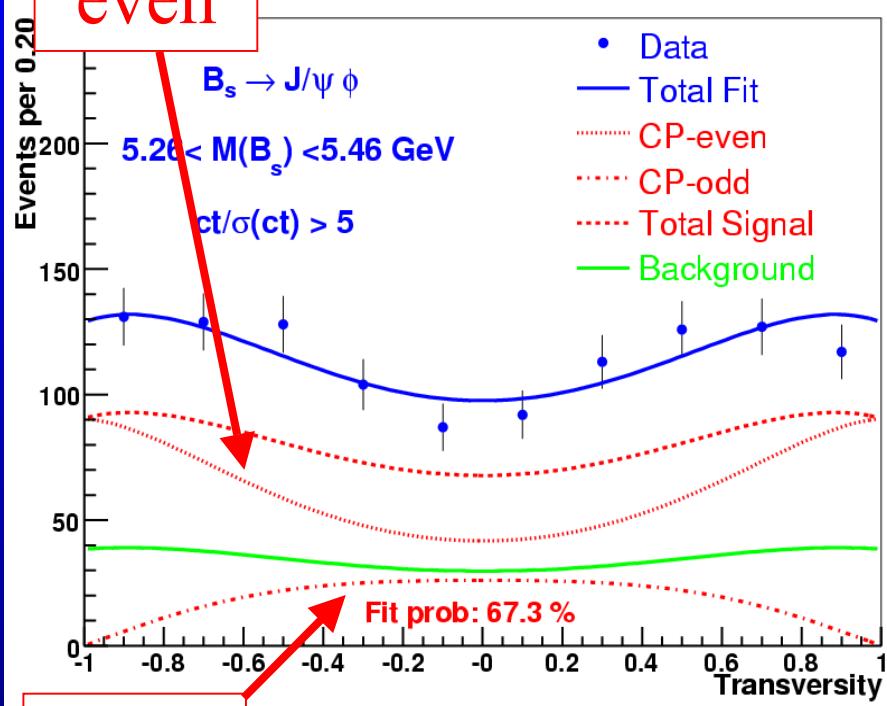
$$f(t, odd) \approx (1 + \cos \phi_s) e^{-\Gamma_H t} + (1 - \cos \phi_s) e^{-\Gamma_L t}$$

Heavy and light states  
are mixed CP

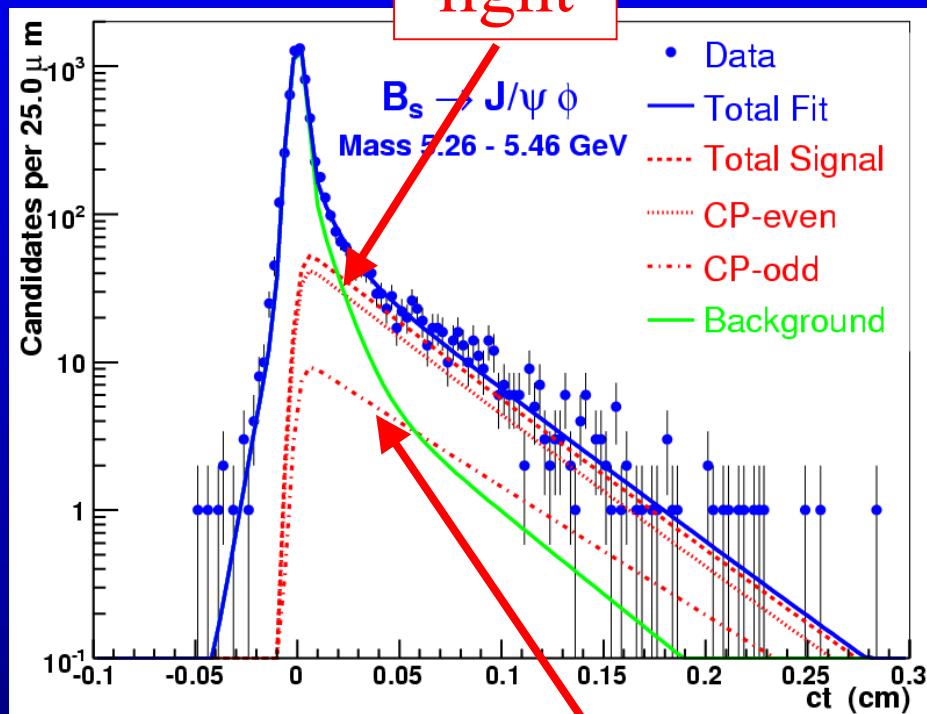
Same as seeing  
 $K_L \rightarrow \pi^+ \pi^-$

# $B_s \rightarrow J/\psi \phi$

even



odd

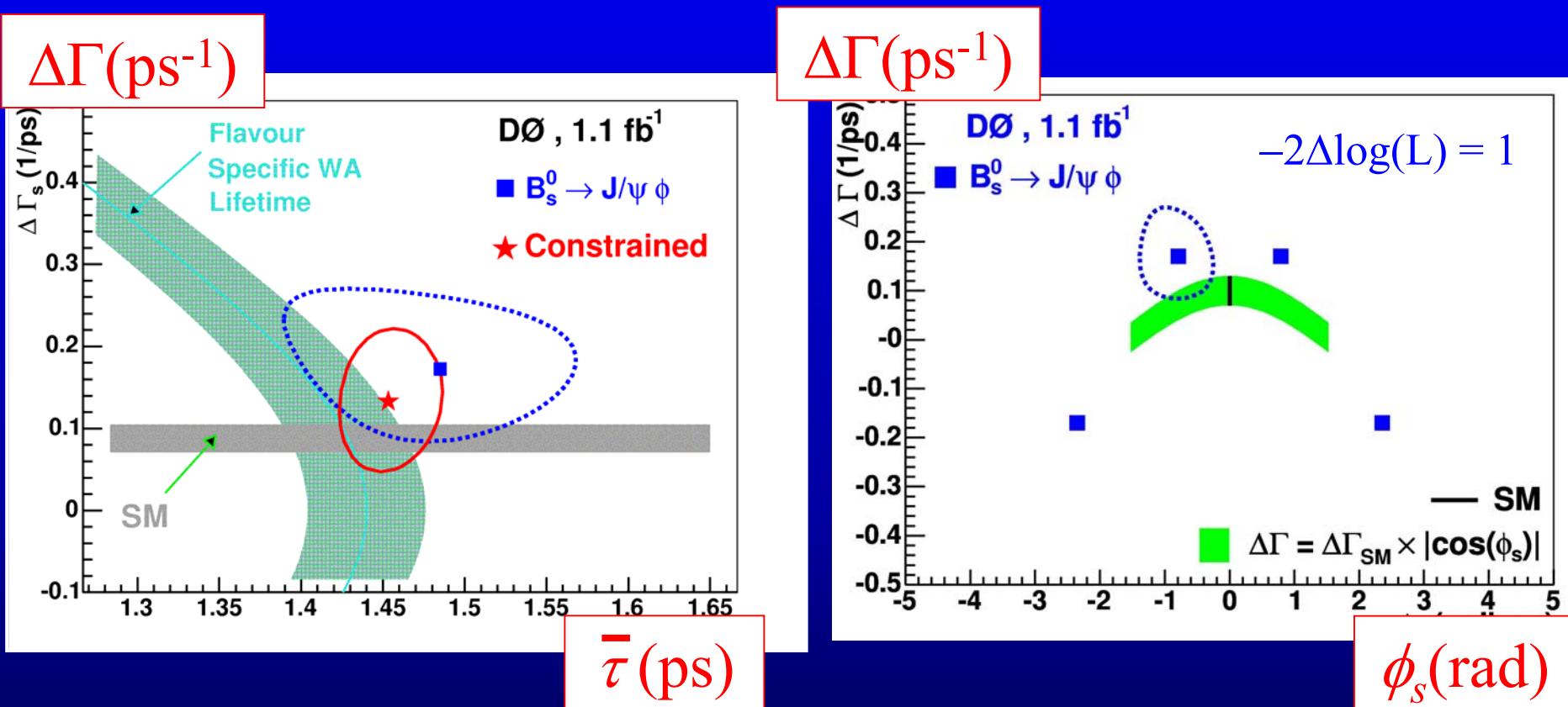


heavy

$$\Delta\Gamma_s = 0.17 \pm 0.09 \pm 0.02\text{ ps}^{-1}$$

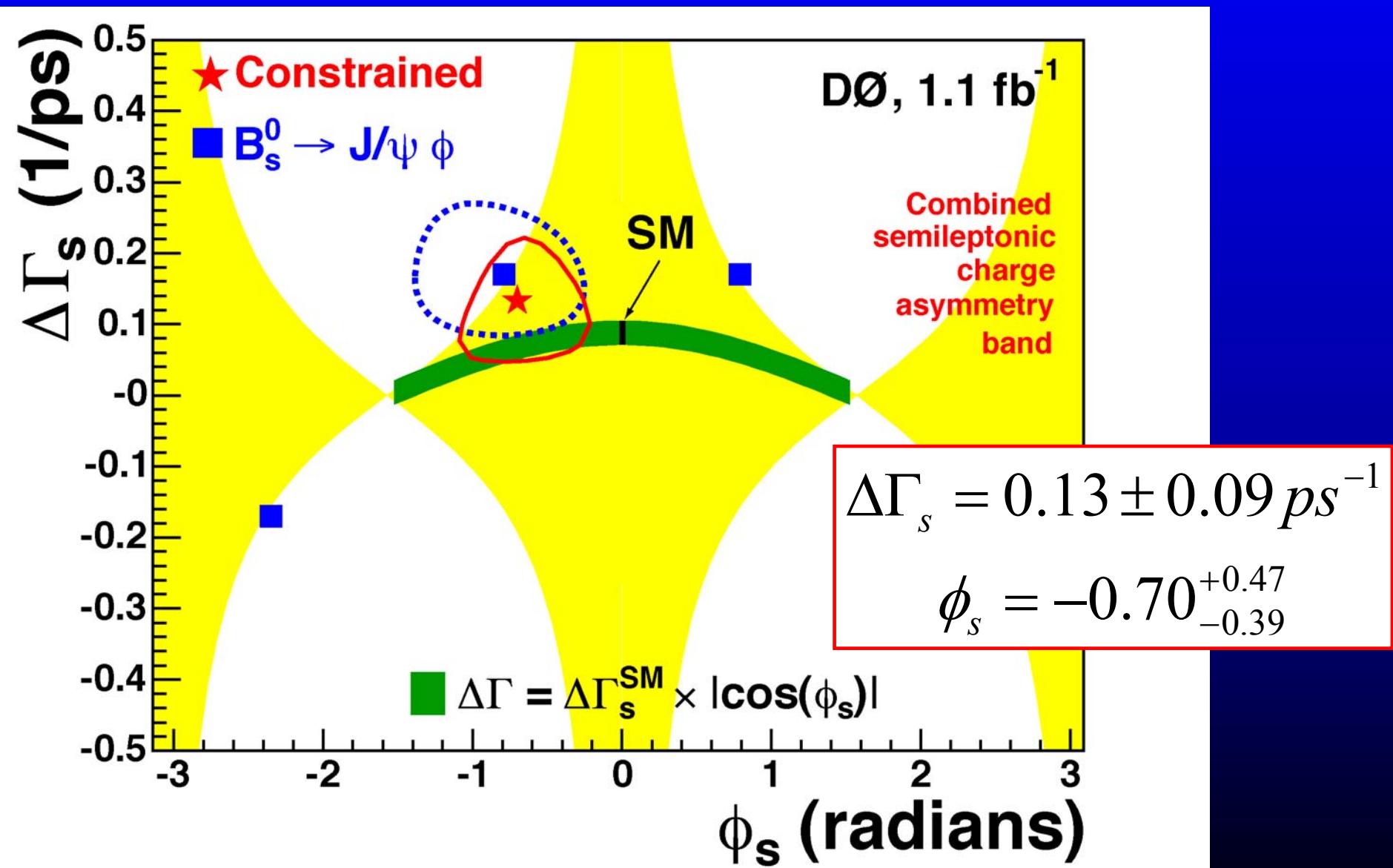
$$\phi_s = -0.79 \pm 0.56^{+0.14}_{-0.01}$$

# $B_s \rightarrow J/\psi \phi$ Results



Likelihood invariant to simultaneous flip of sign of  $\Delta\Gamma$  and even-odd strong phase difference  $\Rightarrow$  4-fold ambiguity

# $\phi_s$ Results



# Outlook

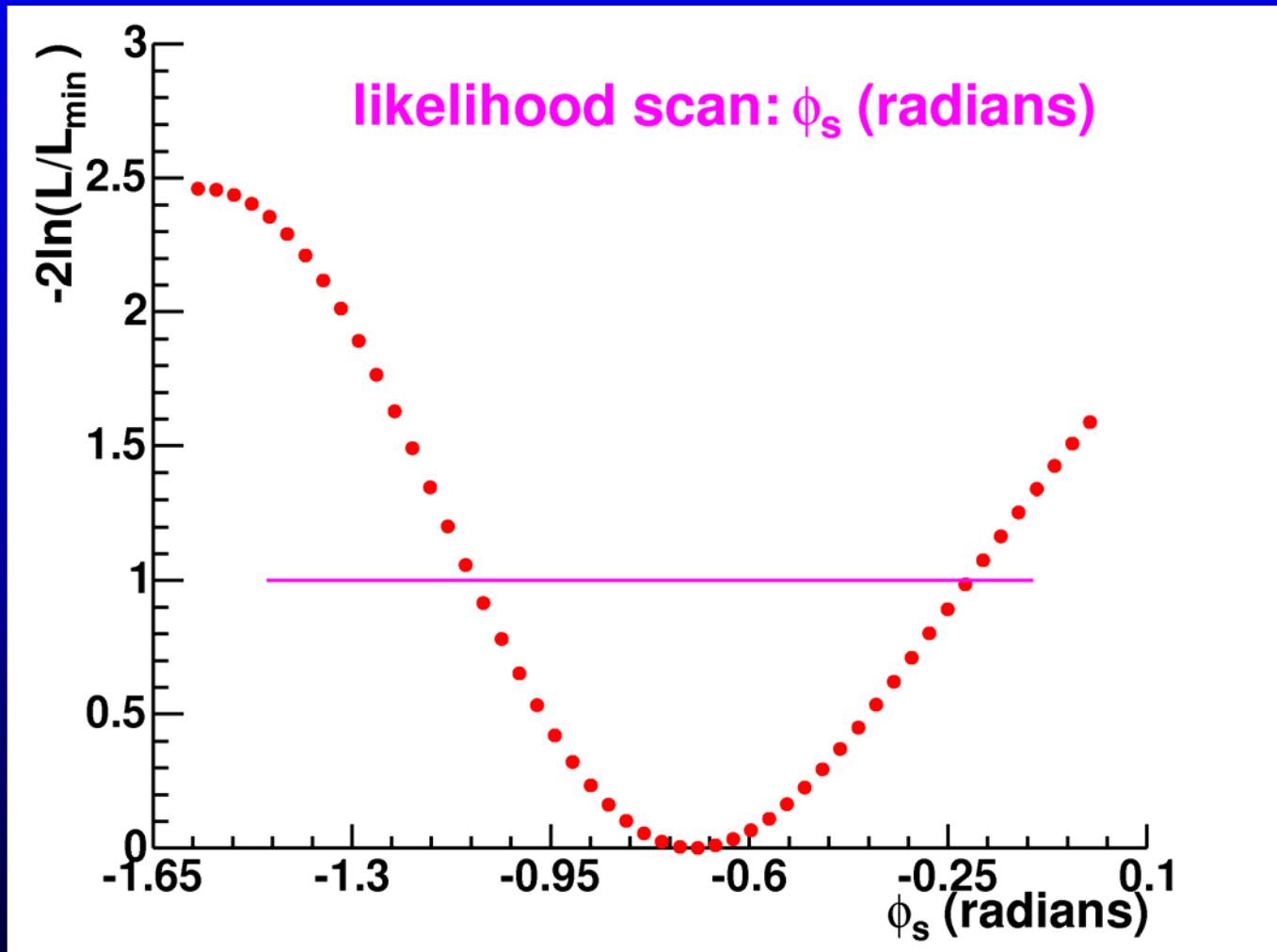
- Right now: Pioneering measurements of CPV parameters in  $B_s$  mixing
  - Agreement with SM expectations at 1.2 sigma
- With more luminosity, we will clearly be able to probe for large new physics effects  $\Rightarrow$  very interesting
- Goal: beat  $\sqrt{\text{Lum}}$ 
  - $J/\psi \phi$ : still use full sample but include initial flavor information in likelihood for tagged events
  - Like-sign  $\mu\mu \Rightarrow$  like-sign  $\mu\mu\phi$ : enhance  $B_s$  component, reduce systematics
- If something is there, we want to find it before the LHC
- But need LHCb to reach SM level

# Conclusions

- Now have information on all  $B_s$  mixing parameters from the DØ.
  - Very diverse program giving consistent results:
    - $\Delta m_s$  consistent with SM/CKM expectations
    - $\Delta\Gamma$  consistent with SM calculations
    - Interesting  $\phi_s$  parameter space accessible at the Tevatron
- Expect rapid improvements in precision on  $\phi_s$ 
  - Doubling data set
  - Results from both experiments based on full data set
  - New modes and new techniques

# Backup Slides

# Likelihood Scan



# Angular Distribution

*Decay amplitude*

$$\frac{d^3\Gamma(t)}{dt d\cos\theta d\varphi d\cos\psi} \propto$$

$$2|A_0(0)|^2 \mathcal{T}_+ \cos^2\psi (1 - \sin^2\theta \cos^2\varphi)$$

$$+ |A_{||}(0)|^2 \mathcal{T}_+ \sin^2\psi (1 - \sin^2\theta \sin^2\varphi)$$

$$+ |A_{\perp}(0)|^2 \mathcal{T}_- \sin^2\psi \sin^2\theta$$

$$+ \frac{1}{\sqrt{2}} |A_0(0)||A_{||}(0)| \cos(\delta_2 - \delta_1) \mathcal{T}_+ \sin 2\psi \sin^2\theta \sin 2\varphi$$

$$+ \left\{ \frac{1}{\sqrt{2}} |A_0(0)||A_{\perp}(0)| \cos\delta_2 \sin 2\psi \sin 2\theta \cos\varphi \right.$$

$$\left. - |A_{||}(0)||A_{\perp}(0)| \cos\delta_1 \sin^2\psi \sin 2\theta \sin\varphi \right\} \frac{1}{2} (e^{-\Gamma_H t} - e^{-\Gamma_L t}) \sin(\delta\phi)$$

$$\mathcal{T}_+ = \frac{1}{2} \left\{ (1 + \cos\delta\phi) e^{-\Gamma_L t} + (1 - \cos\delta\phi) e^{-\Gamma_H t} \right\}$$

$$\mathcal{T}_- = \frac{1}{2} \left\{ (1 - \cos\delta\phi) e^{-\Gamma_L t} + (1 + \cos\delta\phi) e^{-\Gamma_H t} \right\}$$

✓  $\delta\phi \rightarrow$  CP-violating weak phase ; in SM  $\sim 0.004$

✓  $\delta_1, \delta_2 \rightarrow$  CP-conserving strong phase ;  $\sim |\pi|$  and 0

✓  $A_0(0), A_{||}(0) \rightarrow$  CP-even linear polarization amplitude at  $t=0$

✓  $A_{\perp}(0) \rightarrow$  CP-odd linear polarization amplitude at  $t=0$

# Correlations

- Statistical: < 1%
- Systematic:
  - Usually 1 dominant systematic
  - Different for all analysis
    - Kaon decay in flight for  $\mu\mu$
    - $m(\phi\pi)$  fitting for  $D_s\mu$
- All combinations done assuming no correlations
  - No change when correlations included

# Ambiguities

$$\begin{aligned}
& \frac{d^3\Gamma(t)}{d\cos\theta \, d\varphi \, d\cos\psi} \propto 2|A_0(0)|^2 \, T_+ \, \cos^2\psi (1 - \sin^2\theta \cos^2\varphi) + \sin^2\psi \{ |A_{\parallel}(0)|^2 \, T_+ \, (1 - \sin^2\theta \sin^2\varphi) + |A_{\perp}(0)|^2 \, T_- \, \sin^2\theta \} \\
& + \frac{1}{\sqrt{2}} \sin 2\psi |A_0(0)| |A_{\parallel}(0)| \cos(\delta_2 - \delta_1) \, T_+ \, \sin^2\theta \sin 2\varphi \\
& + \left\{ \frac{1}{\sqrt{2}} |A_0(0)| |A_{\perp}(0)| \cos\delta_2 \sin 2\psi \sin 2\theta \cos\varphi \right. \\
& \left. - |A_{\parallel}(0)| |A_{\perp}(0)| \cos\delta_1 \sin^2\psi \sin 2\theta \sin\varphi \right\} \frac{1}{2} (e^{-\Gamma_H t} - e^{-\Gamma_L t}) \sin\phi_s . \quad (2)
\end{aligned}$$

$T_{+-} = \frac{1}{2} ((1 \pm \cos\phi_s)e^{-\Gamma_L t} + (1 \mp \cos\phi_s)e^{-\Gamma_H t})$

$\Delta\Gamma$	$\cos\delta_1$	$\cos\delta_2$	$\phi_s$
$> 0$	$< 0$	$> 0$	-0.79
$< 0$	$< 0$	$> 0$	2.35
$> 0$	$> 0$	$< 0$	0.79
$< 0$	$> 0$	$< 0$	-2.35

# Systematic Errors: dimuons

TABLE VI: Systematic uncertainties of the dimuon charge asymmetry  $A$  for standard cuts.

Source of error	$\Delta A$
detector	0.00015
$e = \epsilon^+/\epsilon^-$	0.00018
prompt $\mu + K^\pm$ decay	0.00083
dimuon cosmic rays	0.00010
prompt $\mu +$ cosmic $\mu$	0.00001
wrong charge sign	0.00018
punch-through	0.00001
Total	0.00089

TABLE VII: Systematic uncertainties of  $f$ .

Source of error	$\Delta f$
$P_2$	0.084
$P_T$	0.002
$P'_8$	0.056
$\chi_{d0}$	0.012
$f_d$	0.014
$\chi_{s0}$	0.0002
$f_s$	0.019
$\rho'$	0.008
$P_{13}$	0.0007
Total	0.105

# Systematic Errors: $J/\psi \phi$

Observable	CP conserved	free $\phi_s$
$\Delta\Gamma$ (ps $^{-1}$ )	$0.12^{+0.08}_{-0.10}$	$0.17^{+0.09}_{-0.09}$
$\frac{1}{\Gamma} = \bar{\tau}$ (ps)	$1.52^{+0.08}_{-0.08}$	$1.49 \pm 0.08$
$\phi_s$	$\equiv 0$	$-0.79 \pm 0.56$
$ A_0(0) ^2 -  A_{\parallel}(0) ^2$	$0.38 \pm 0.05$	$0.37 \pm 0.06$
$A_{\perp}(0)$	$0.45 \pm 0.05$	$0.46 \pm 0.06$
$\delta_1 - \delta_2$	$2.6 \pm 0.4$	$2.6 \pm 0.4$
$\delta_1$	—	$3.3 \pm 1.0$
$\delta_2$	—	$0.7 \pm 1.1$

TABLE II: Sources of systematic uncertainty in the results of the analysis of the decay  $B_s^0 \rightarrow J/\psi \phi$ .

Source	$c\tau(B_s^0)$ μm	$\Delta\Gamma$ ps $^{-1}$	$R_{\perp}$	$\phi_s$
Procedure test	$\pm 2.0$	$\pm 0.02$	$\pm 0.01$	—
Acceptance	$\pm 0.5$	$\pm 0.001$	$\pm 0.003$	$\pm 0.01$
Reco. algorithm	$-8.0, +1.3$	$+0.001$	$\pm 0.01$	$-0.01$
Background model	$+1.0$	$+0.01$	$-0.01$	$+0.14$
Alignment	$\pm 2.0$	—	—	—
Total	$-8.8, +3.3$	$\pm 0.02$	$\pm 0.02$	$-0.01, +0.14$