

Evidence for $D^0-\bar{D}^0$ Mixing

Ray F. Cowan
Fermilab Seminar
April 9, 2007



Massachusetts
Institute of
Technology



Topics

Phenomenology of neutral meson mixing and CP violation

$BABAR D^0 \rightarrow K\pi$ mixing analysis

Strategy: Time-dependent study of $D^0 \rightarrow K\pi$ mode

$BABAR$ detector and dataset

Event selection

Signal and background separation

Proper-time fits

Results

Cross-checks and validation studies

Systematics

CP violation

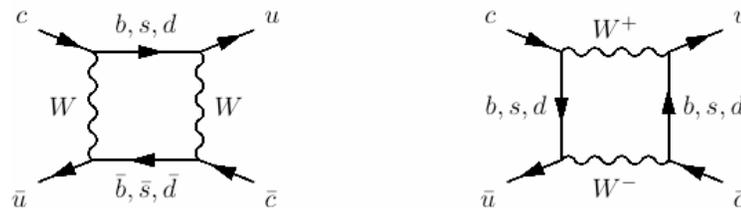
Comparison with recent BELLE results

Summary



Neutral meson mixing

Why search for mixing in the charm sector?



An observation in the charm sector would complete the picture of quark mixing already seen in the K , B , and B_s systems.

It would provide new information on processes involving down-type quarks in the mixing loop diagram.

It would be a significant step along the road to observation of CP violation in the charm sector.

It could be an indication of new physics.



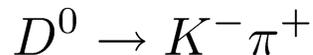
Mixing signatures

Mixing occurs when a meson produced as a D^0 decays as a \bar{D}^0 or vice versa.

This can be studied by tagging the D^0 flavor at production and at decay.

We use the $D^0 \rightarrow K\pi$ decay mode

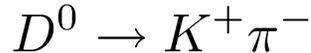
Cabibbo-favored (CF), “right-sign” (RS) decay



Doubly Cabibbo-suppressed (DCS), “wrong-sign” (WS) decay $D^0 \rightarrow \bar{D}^0 \rightarrow K^+ \pi^-$

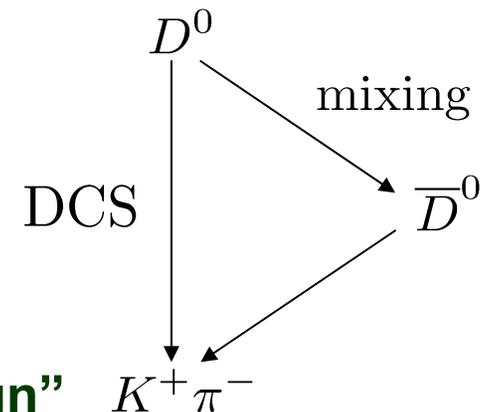
Rate: $\tan^4 \theta_c \approx 0.3\%$

Mixing followed by CF decay (WS)



Rate: 10^{-4} or less

(interference between mixing and DCS can enhance)





Mixing Hamiltonian

Neutral D^0 and \bar{D}^0 mesons are produced as flavor eigenstates of the strong interaction.

Their time development is governed by an effective Hamiltonian

$$i \frac{\partial}{\partial t} \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2} \mathbf{\Gamma} \right) \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix}$$

which has physical eigenstates D_1, D_2 that are linear combinations of the flavor eigenstates

$$\begin{aligned} |D_1\rangle &= p|D^0\rangle + q|\bar{D}^0\rangle \\ |D_2\rangle &= p|D^0\rangle - q|\bar{D}^0\rangle \end{aligned} \quad \text{where} \quad \left(\frac{q}{p} \right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}$$

and $|q|^2 + |p|^2 = 1$.

The states D_1, D_2 possess masses M_1, M_2 and lifetimes Γ_1, Γ_2 .



Masses, lifetimes, and amplitudes

We define mass and lifetime differences and averages of the physical eigenstates D_1, D_2

$$\Delta M = M_1 - M_2 \quad \Delta\Gamma = \Gamma_1 - \Gamma_2 \quad M = \frac{M_1 + M_2}{2} \quad \Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$$

and the parameters $x = \frac{\Delta M}{\Gamma}$ and $y = \frac{\Delta\Gamma}{2\Gamma}$.

We also define the amplitude for decay to CP -conjugate final states $f = K^+\pi^-$, $\bar{f} = K^-\pi^+$ as

$$A_f = \langle f|H|D^0\rangle, \quad \bar{A}_f = \langle f|H|\bar{D}^0\rangle, \quad A_{\bar{f}} = \langle \bar{f}|H|D^0\rangle, \quad \bar{A}_{\bar{f}} = \langle \bar{f}|H|\bar{D}^0\rangle$$

WS

RS

RS

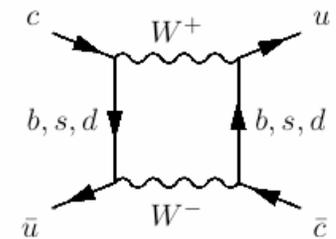
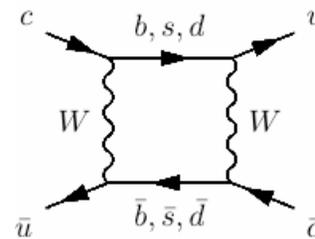
WS



Short- and long-distance effects

Short-distance contributions from mixing box diagrams primarily affect x (expect $O(10^{-5})$ or less)

**b quark is CKM-suppressed
 s and b quarks GIM suppressed**



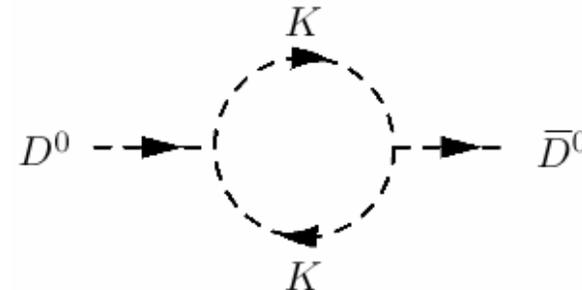
Long-distance contributions primarily affect y (expect $O(10^{-2})$ or less)

Non-perturbative effects

New physics would be indicated if

$x \gg y$

CP violation is observed





Time-dependent decay rate

For $x, y \ll 1$

$$\frac{d\Gamma}{dt} [|D^0(t)\rangle \rightarrow f] \propto e^{-\Gamma t} \left(\underbrace{R_D}_{\text{DCS decay}} + \underbrace{\sqrt{R_D} y' \Gamma t}_{\text{Interference between DCS and mixing}} + \overbrace{\frac{x'^2 + y'^2}{4} (\Gamma t)^2}^{\text{Mixing}} \right)$$

Allows for a strong phase difference $\delta_{K\pi}$ between CF and DCS direct decay

$$x' = x \cos \delta_{K\pi} + y \sin \delta_{K\pi}, \quad y' = -x \sin \delta_{K\pi} + y \cos \delta_{K\pi}$$

This phase may differ between decay modes.



CP violation

CP violation can be classified as occurring

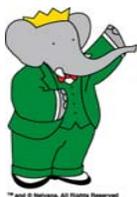
- ◆ In direct decay $|\bar{A}_{\bar{f}}/A_f| \neq 1$
- ◆ In mixing $|q/p| \neq 1$
- ◆ In the interference between them $\text{Im} \left(\frac{q}{p} \frac{\bar{A}_f}{A_f} \right) \neq 0$

This introduces an asymmetry

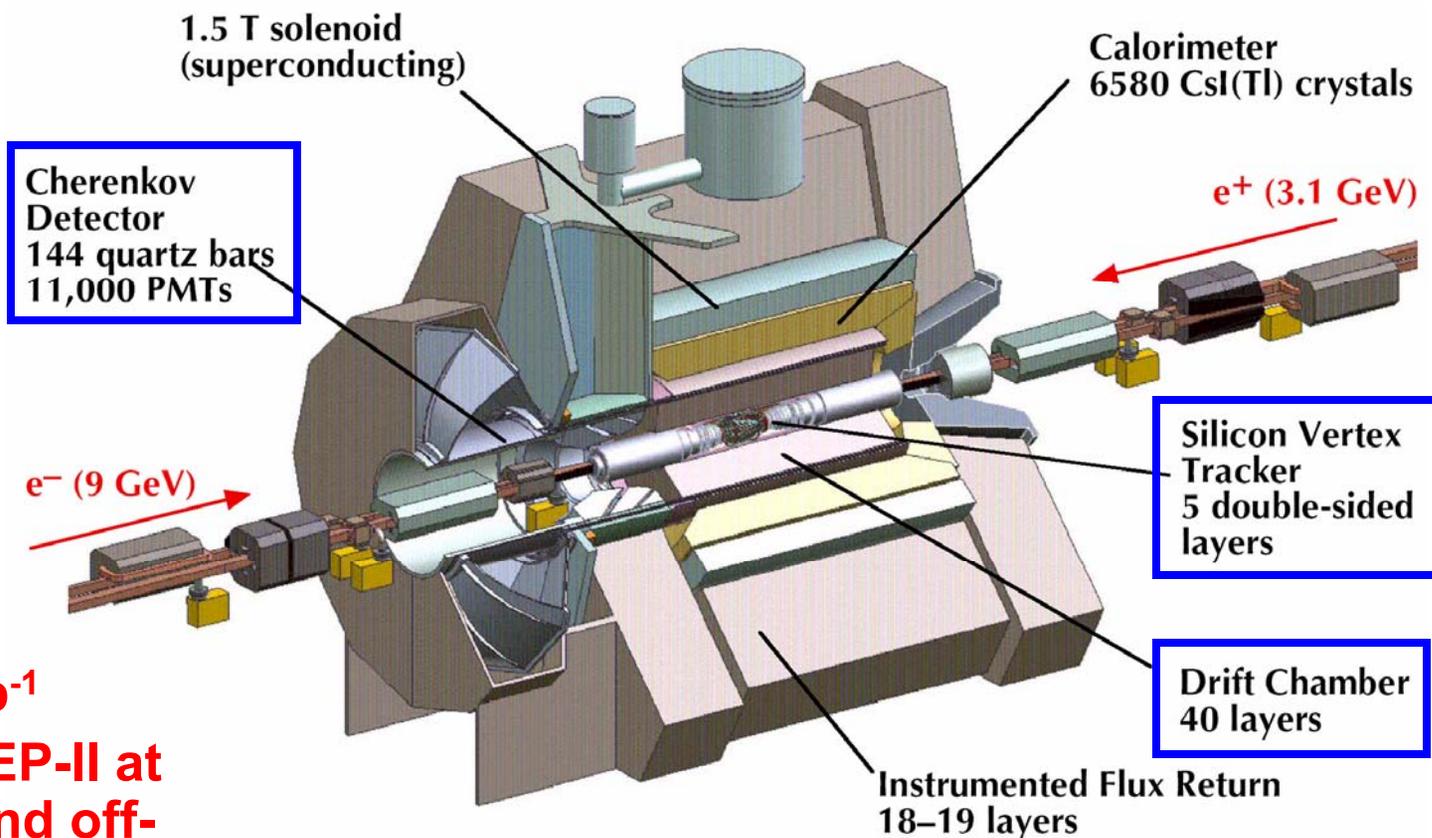
in the time-dependence between D^0 and \bar{D}^0 decays

$$\frac{d\Gamma}{dt} [|D^0(t)\rangle \rightarrow f] \propto e^{-\Gamma t} \times R_D + \sqrt{R_D} \left| \frac{q}{p} \right| (y' \cos \varphi - x' \sin \varphi) \Gamma t + \left| \frac{q}{p} \right|^2 \frac{x'^2 + y'^2}{4} (\Gamma t)^2$$
$$\frac{d\Gamma}{dt} [|\bar{D}^0(t)\rangle \rightarrow \bar{f}] \propto e^{-\Gamma t} \times R_D + \sqrt{R_D} \left| \frac{p}{q} \right| (y' \cos \varphi + x' \sin \varphi) \Gamma t + \left| \frac{p}{q} \right|^2 \frac{x'^2 + y'^2}{4} (\Gamma t)^2$$

where ϕ is the phase angle of $\lambda_f = \left(\frac{q}{p} \frac{\bar{A}_f}{A_f} \right)$



BABAR detector and dataset



Dataset: 384 fb^{-1}
Collected at PEP-II at SLAC on- and off- the $\Upsilon(4S)$ resonance

NIM A479, 1 (2002)



Analysis Method

Identify the D^0 charge conjugation state at prod. & decay using

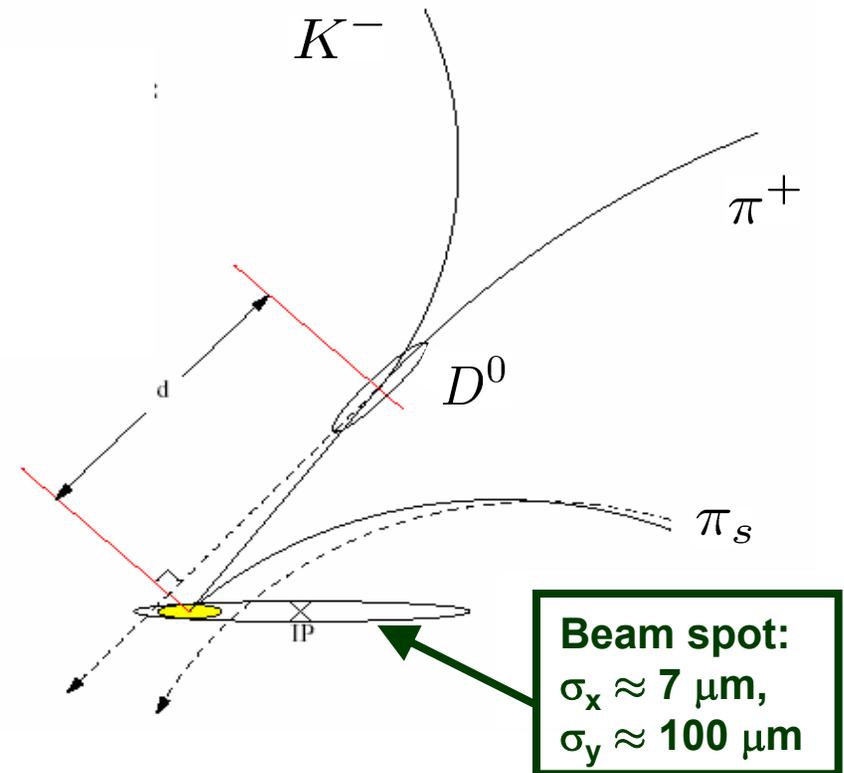
$$D^{*\pm} \rightarrow \pi_s^\pm D^0, \quad D^0 \rightarrow K^\mp \pi^\pm$$

Vertex fits with beamspot constraint is important

Reduces the size of the decay-time error resolution

Improves Δm resolution

Right-sign (RS) decay



$$\Delta m = m(D_{\text{rec.}}^{*+}) - m(D_{\text{rec.}}^0)$$



Event selection details

Perform a beam-constrained fit to the full decay chain

$$D^{*\pm} \rightarrow \pi_s^\pm D^0, D^0 \rightarrow K^\mp \pi^\pm$$

Require fit probability > 0.001

$$\delta t < 0.5 \text{ ps}$$

$$-2 < t < 4 \text{ ps}$$

Select the D^0

$$\text{CM } p_D > 2.5 \text{ GeV}/c$$

K, π particle identification

$$1.81 < m_{K\pi} < 1.92 \text{ GeV}/c^2$$

Select the D^{*+}

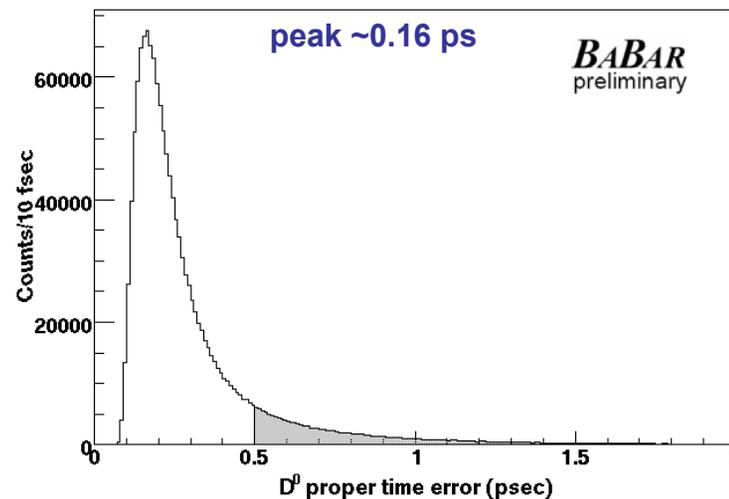
$$\text{CM } p_\pi < 0.45 \text{ GeV}/c$$

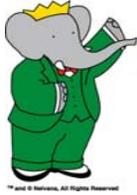
$$p_\pi > 0.1 \text{ GeV}/c \text{ in lab frame}$$

$$0.14 < \Delta m < 0.16 \text{ GeV}/c^2$$

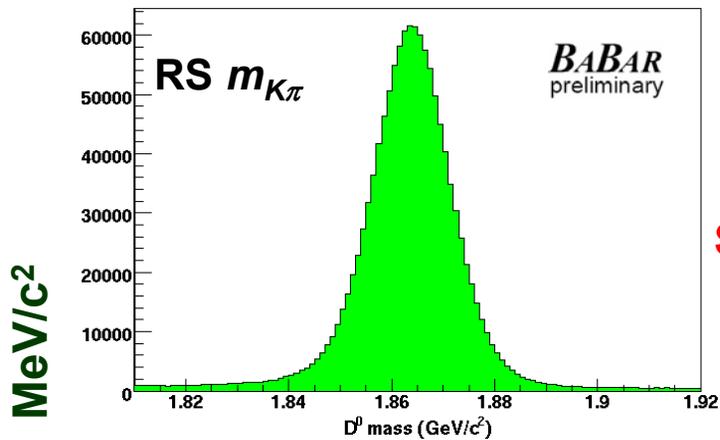
Select candidate with greatest fit probability for multiple D^{*+} candidates sharing tracks

Event selection, fitting procedures finalized before examining the mixing results

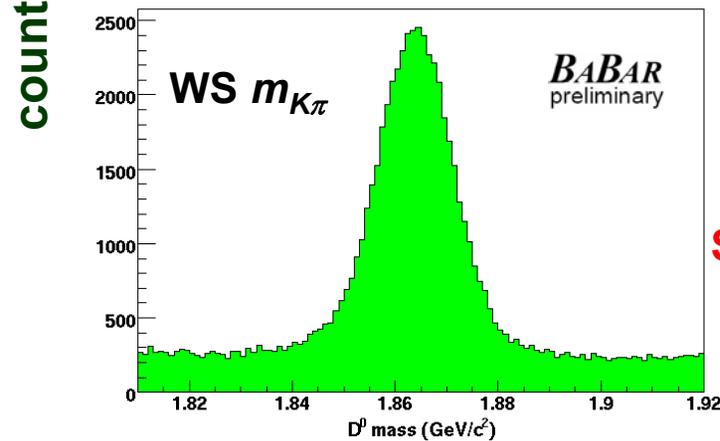
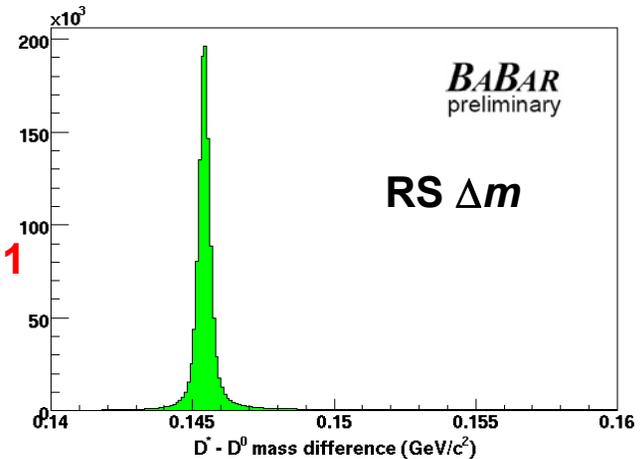




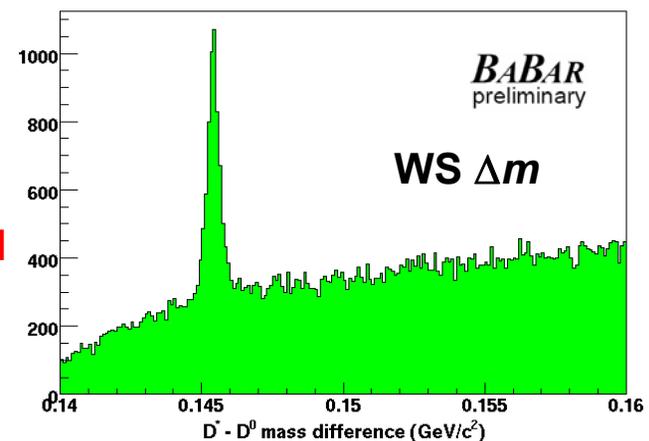
RS & WS $m_{K\pi}$, Δm projections



1,229,000
RS candidates
Signal:bkgd \approx 99:1



64,000
WS candidates
Signal:bkgd \approx 1:1





Separating signal and backgrounds

Signal and backgrounds have differing behavior in $m_{K\pi}$ and Δm .

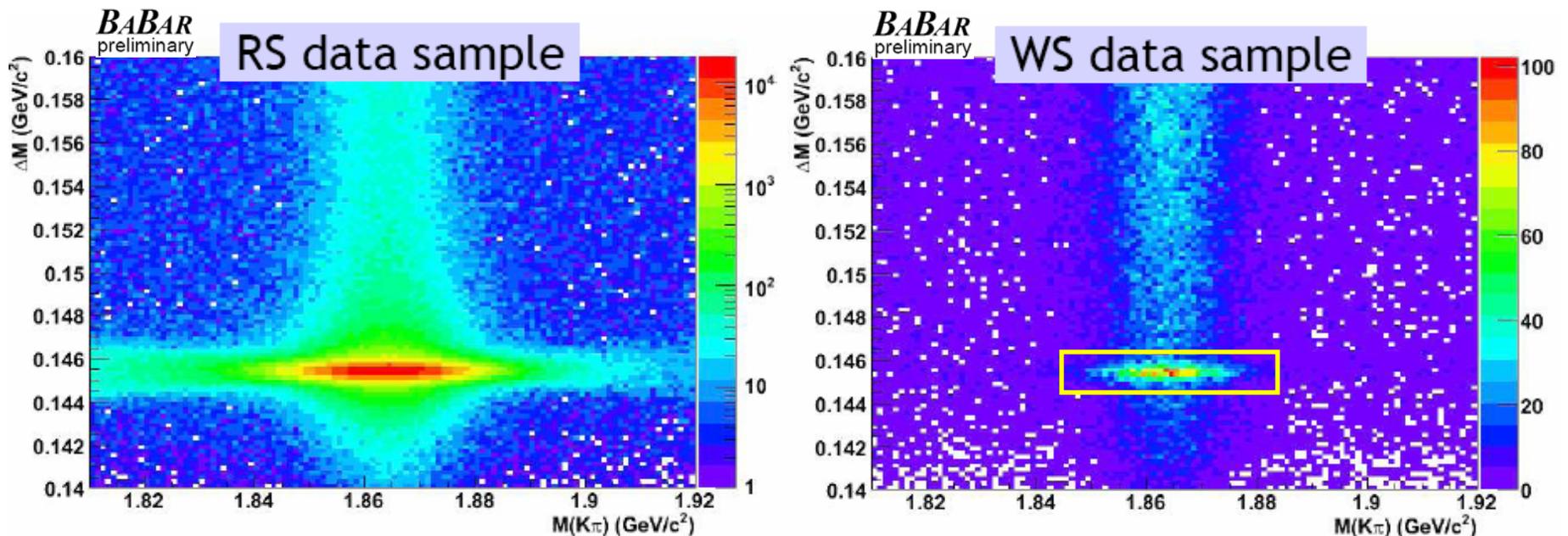
We define four categories:

Signal, random π_s , mis-reconstructed D^0 , and combinatoric.

Category	Description	Peaking Behavior
RS signal	$D^0 \rightarrow K^- \pi^+$ signal	$m_{K\pi}$ and Δm
RS random π_s	Correctly-reconstructed D^0 combined with an incorrect slow pion	$m_{K\pi}$
RS mis-recon. D^0	Mis-reconstructed D^0 from $D^0 \rightarrow Kl^+\nu$, $D^0 \rightarrow \pi l^+\nu$, $D^0 \rightarrow \pi^+\pi^-$, $D^0 \rightarrow K^+K^-$	Δm
RS combinatoric	Combinatoric background	non-peaking
WS signal	$D^0 \rightarrow K^+\pi^-$ signal	$m_{K\pi}$ and Δm
WS random π_s	Correctly-reconstructed D^0 combined with an incorrect slow pion	$m_{K\pi}$
WS mis-recon. D^0	Doubly mis-identified $D^0 \rightarrow K^- \pi^+$ decays and $D^0 \rightarrow \pi^+\pi^-$, $D^0 \rightarrow K^+K^-$ reflections	Δm
WS combinatoric	Combinatoric background	non-peaking



RS & WS $m_{K\pi}$, Δm distributions



All fits are over the full range

$1.81 \text{ GeV}/c^2 < m_{K\pi} < 1.92 \text{ GeV}/c^2$ and $0.1445 \text{ GeV}/c^2 < \Delta m < 0.1465 \text{ GeV}/c^2$

A small correlation can be seen between $m_{K\pi}$ and Δm

Signal region:

$1.843 \text{ GeV}/c^2 < m_{K\pi} < 1.883 \text{ GeV}/c^2$ and $0.1445 \text{ GeV}/c^2 < \Delta m < 0.1465 \text{ GeV}/c^2$



Fitting strategy

Fitting is performed in stages to reduce demand on computing resources
All stages are unbinned, extended maximum-likelihood fits.

1. **RS & WS $m_{K\pi}$ Δm fit.**
Yields PDF shape parameters $m_{K\pi}$, Δm categories.
2. **RS lifetime fit.**
 $m_{K\pi}$, Δm category shape parameters held constant.
Yields D^0 lifetime τ_D and proper-time resolution parameters.
Constrained by the large statistics of the RS sample.
3. **WS lifetime fit.**
Yields parameters describing the WS time dependence.

Small correlations in parameters in the different stages justifies the staged approach.

The WS fit is performed under three different assumptions.

Mixing and CP violation (CPV); mixing but no CPV; and no mixing or CPV.

Monte Carlo (MC) simulations are not used directly in the data fits.

MC simulations used only to motivate the fit PDFs

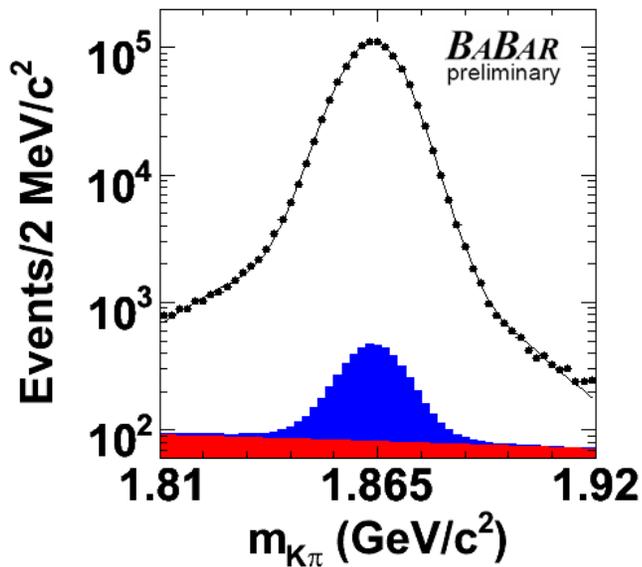
WS mis-reconstructed D^0 category studied in swapped $K \leftrightarrow \pi$ data.



Right-sign $m_{K\pi}$, Δm fit

Shown are the fits to right-sign data for $m_{K\pi}$ (left) and Δm (right).

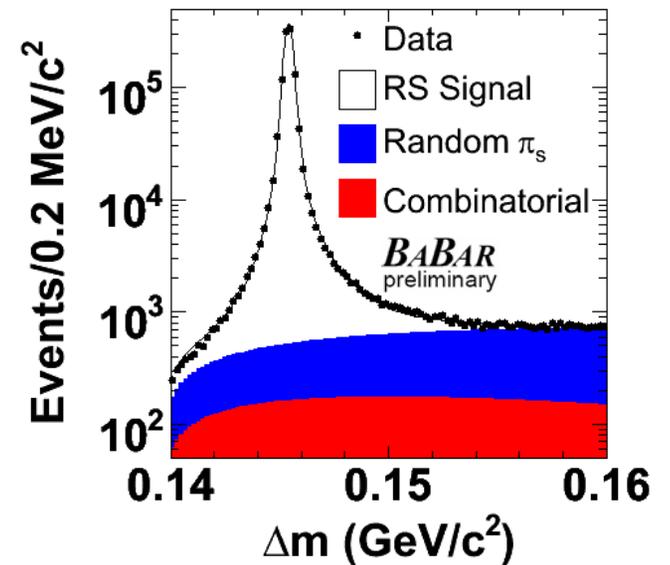
1,141,500 \pm 1,200
RS signal events

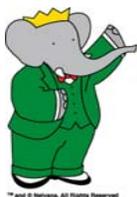


The mis-reconstructed D^0 category is not included in the RS fit.

This background is too small to be reliably determined.

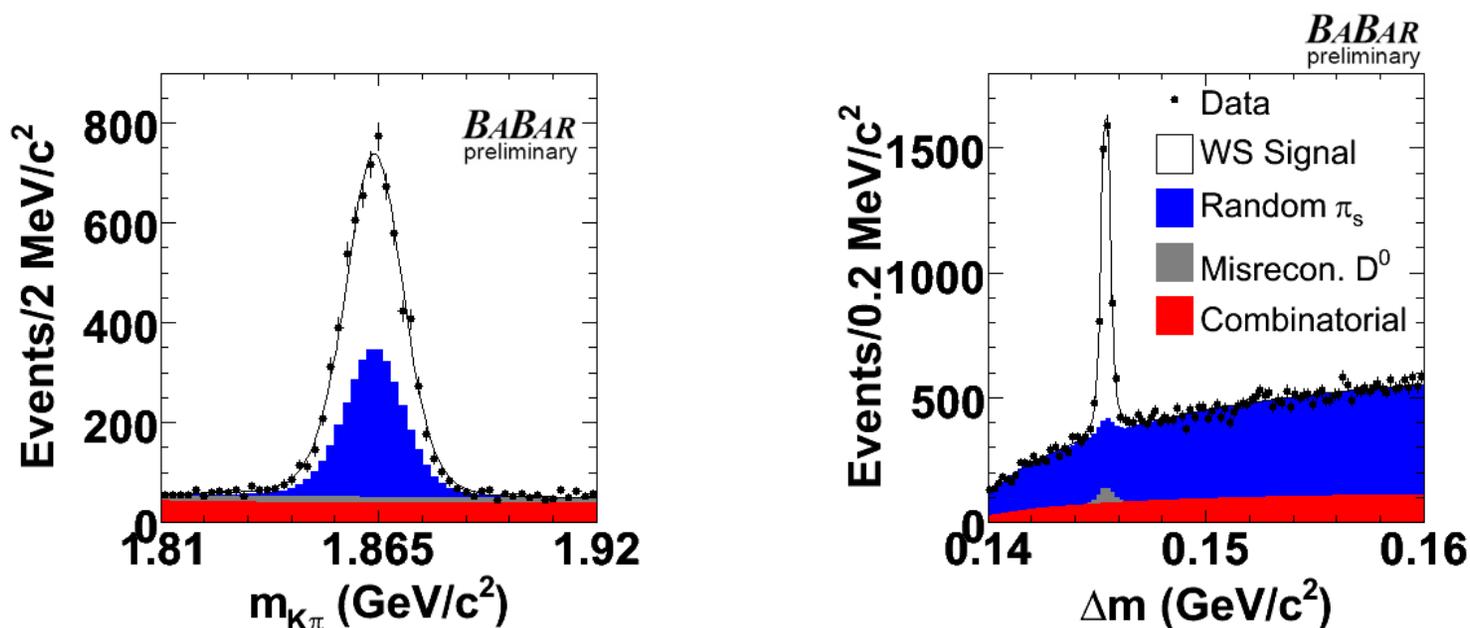
4,030 \pm 90
WS signal events





Wrong-sign $m_{K\pi}$, Δm fit

The $m_{K\pi}$, Δm fit determines the WS b.r. $R_{WS} = N_{WS}/N_{RS}$



BABAR (384 fb⁻¹): $R_{WS} = (0.353 \pm 0.008 \pm 0.004)\%$ (hep-ex/0703020)

BELLE (400 fb⁻¹): $R_{WS} = (0.377 \pm 0.008 \pm 0.005)\%$ (PRL 96, 151801 (2006))



RS proper decay-time fit

The parameters varied are

D^0 lifetime τ_D

Resolution parameters

Including a 3.6 fsec offset

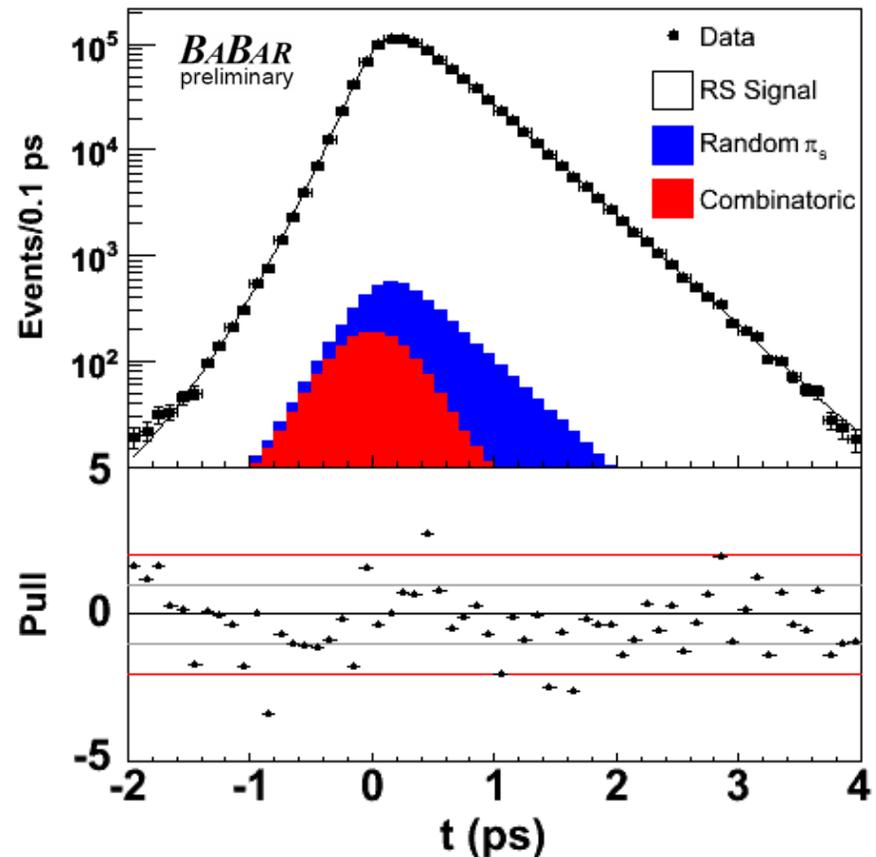
Signal, background category yields

Consistency check

Fitted $\tau_D = (410.3 \pm 0.6)$ fsec

(statistical error only)

(PDG 2006: 410.1 ± 1.5 fsec)



RS fit projection in the signal region
 $1.843 \text{ GeV}/c^2 < m < 1.883 \text{ GeV}/c^2$
 $0.1445 \text{ GeV}/c^2 < \Delta m < 0.1465 \text{ GeV}/c^2$



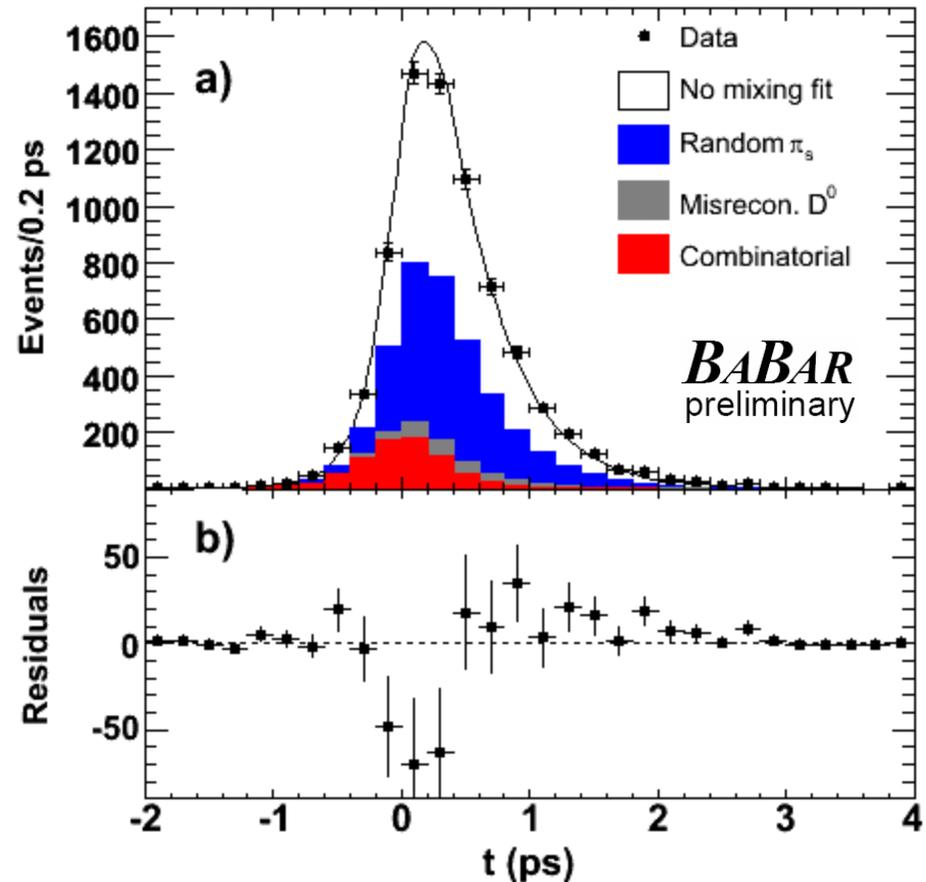
No-mixing WS decay time fit

The parameters varied are

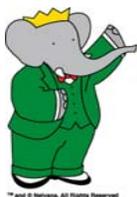
WS category yields
WS combinatoric
shape parameter

As can be seen in the residual plot, there are large residuals.

Residuals = data - fit



WS no-mixing fit projection in signal region
 $1.843 \text{ GeV}/c^2 < m < 1.883 \text{ GeV}/c^2$
 $0.1445 \text{ GeV}/c^2 < \Delta m < 0.1465 \text{ GeV}/c^2$



Mixing WS decay time fit

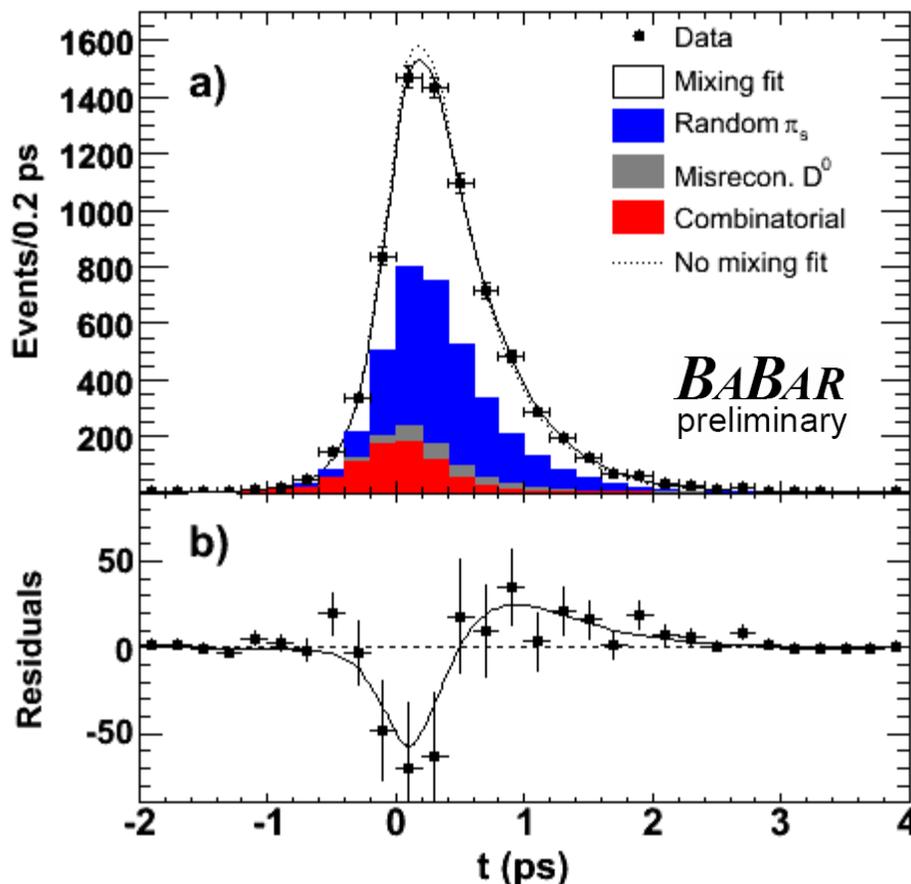
The difference between the no-mixing fit and the fit with mixing is shown in the residuals plot.

The dotted line is the no-mixing fit.

The solid line is the mixing fit.

The fit is significantly improved by allowing for mixing.

Note that the apparent oscillation is not due solely to the mixing component.



WS mixing fit projection in signal region
 $1.843 \text{ GeV}/c^2 < m < 1.883 \text{ GeV}/c^2$
 $0.1445 \text{ GeV}/c^2 < \Delta m < 0.1465 \text{ GeV}/c^2$



Mixing fit likelihood contours

Contours in y' , x'^2
computed from
 $-2\Delta \ln L$

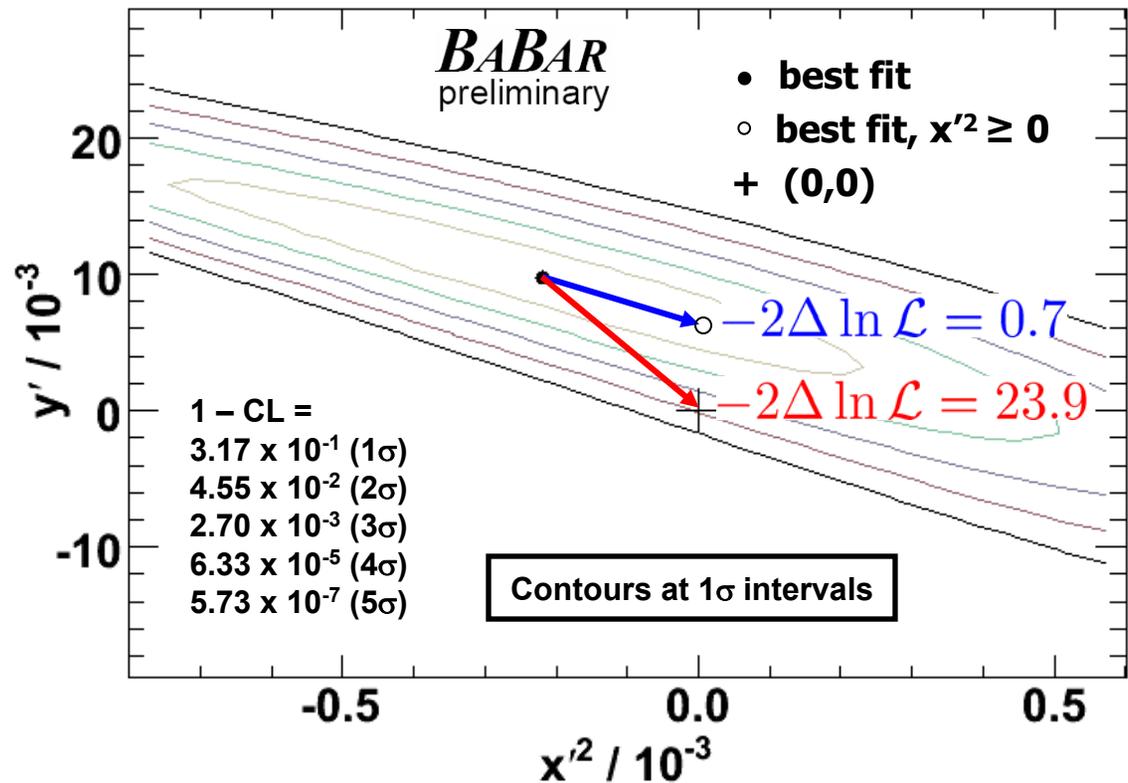
Best-fit point is in the
non-physical region
 $x'^2 < 0$

1σ contour extends
into physical region

Correlation: -0.94

Contours include
systematic errors

The no-mixing point
is at the 3.9σ
contour



$$R_D: (3.03 \pm 0.16 \pm 0.10) \times 10^{-3}$$

$$x'^2: (-0.22 \pm 0.30 \pm 0.21) \times 10^{-3}$$

$$y': (9.7 \pm 4.4 \pm 3.1) \times 10^{-3}$$

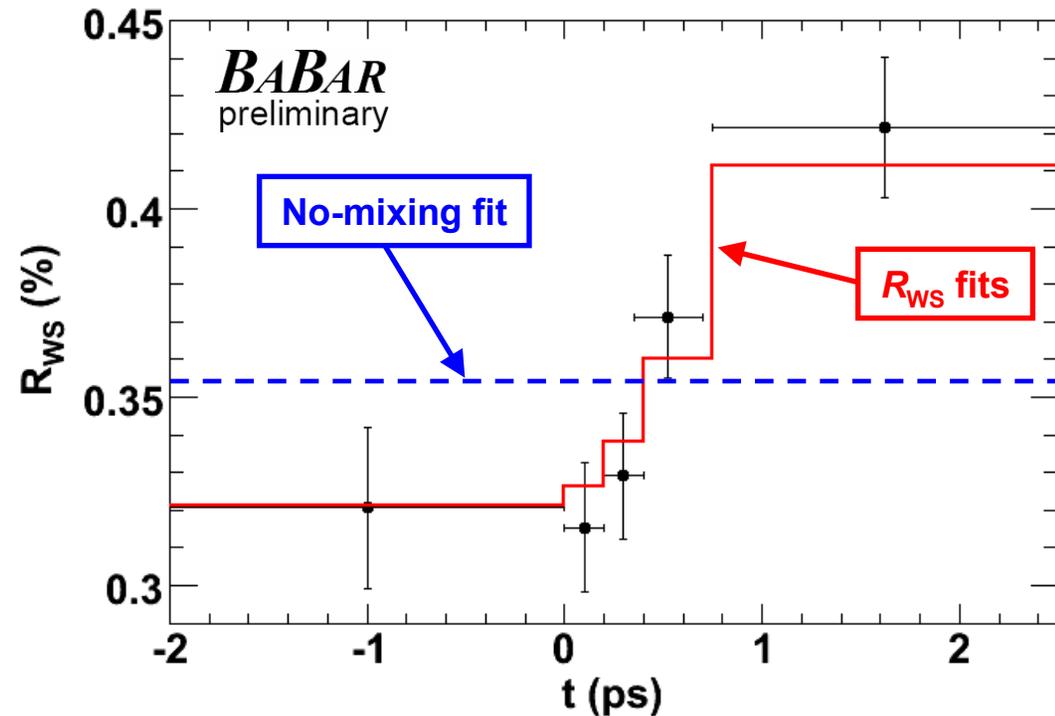


R_{WS} vs. decay-time slices

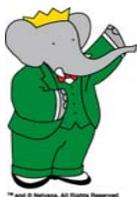
If mixing is present, it should be evident in an R_{WS} rate that increases with decay-time.

Perform the R_{WS} fit in five time bins with similar RS statistics.

Cross-over occurs at $t \approx 0.5$ psec as in residuals plot.



Dashed line: standard R_{WS} fit ($\chi^2=24$).
Solid, red line: independent R_{WS} fits to each time bin ($\chi^2 = 1.5$).



Validation: fit for mixing in RS sample

Fit the RS data using the WS mixing PDF

$$x'^2 = (-0.01 \pm 0.01) \times 10^{-3}$$

$$y' = (0.26 \pm 0.24) \times 10^{-3}$$

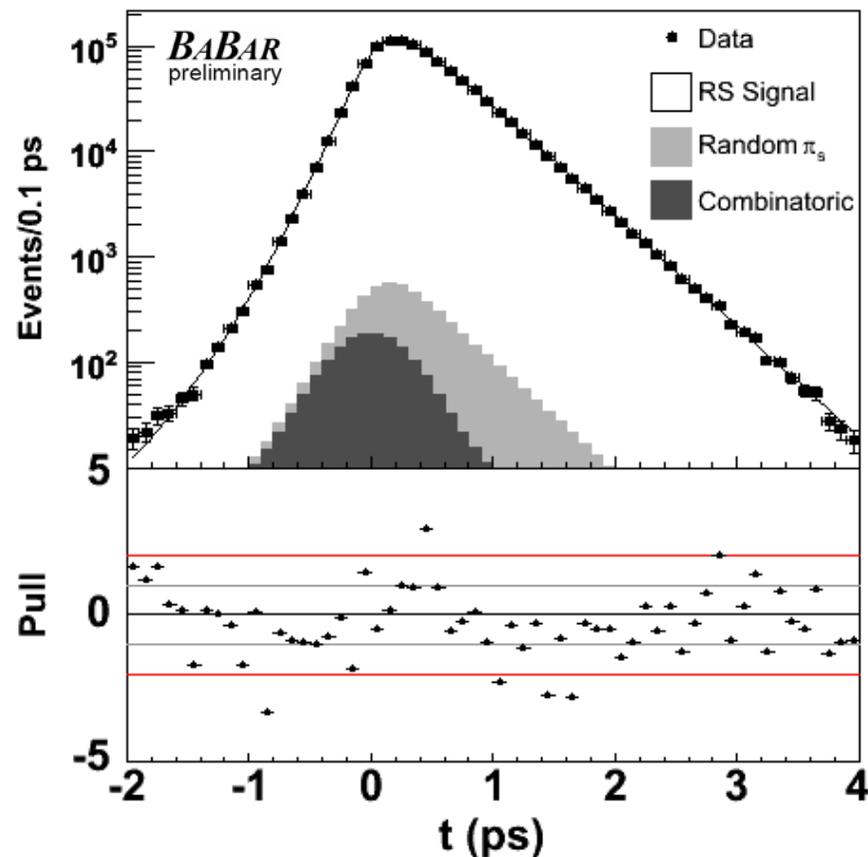
The change in $-2\Delta\ln L$ is 1.4

A very stringent test

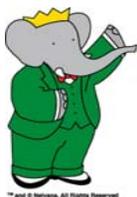
RS sample $270\times$ larger than WS sample

Conclusion:

D^0 decay-time distribution is properly described.



RS mixing fit projection in signal region
 $1.843 \text{ GeV}/c^2 < m < 1.883 \text{ GeV}/c^2$
 $0.1445 \text{ GeV}/c^2 < \Delta m < 0.1465 \text{ GeV}/c^2$



Validation: fit for mixing in MC

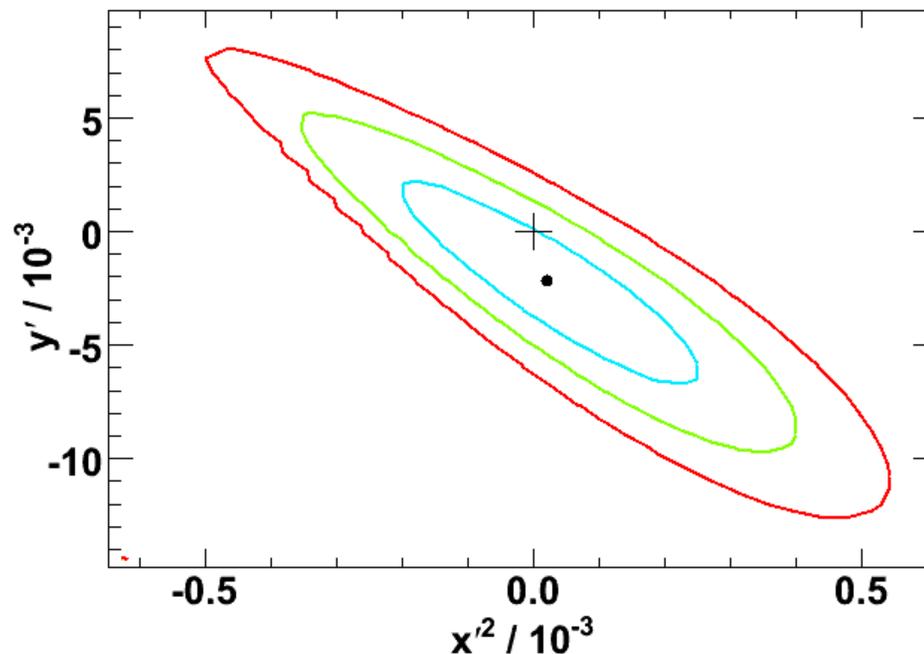
Fit MC for mixing

MC generated with no mixing

Fit finds no mixing signal:

$$x'^2 = (-0.02 \pm 0.18) \times 10^{-3}$$

$$y' = (2.2 \pm 3.0) \times 10^{-3}$$

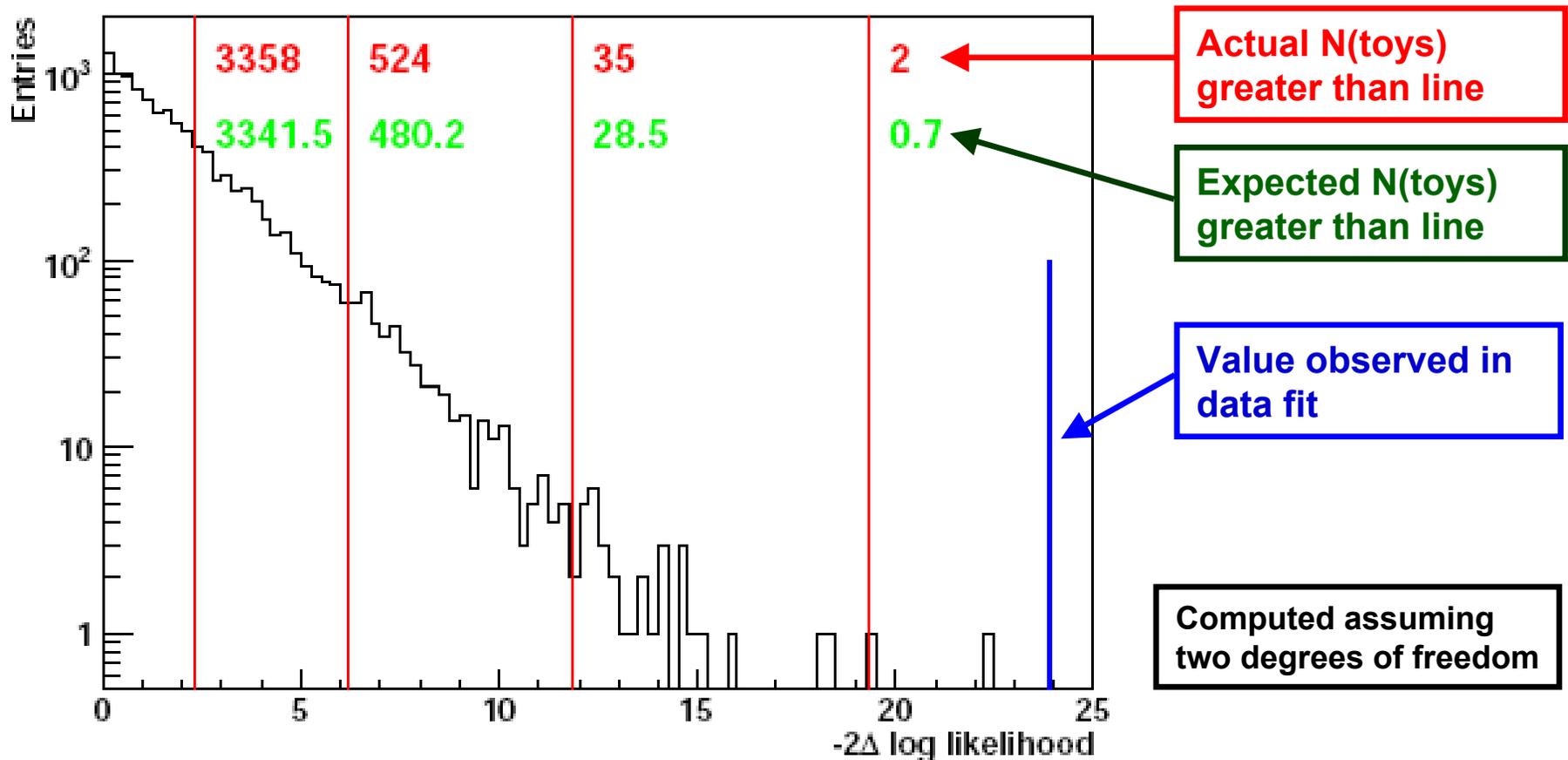


Result of mixing fit to MC
(which has no mixing).
Contours are at 1σ , 2σ , and 3σ



Validation: $-2\Delta\ln L$ frequentist coverage

Generated >10000 toys without mixing to test frequentist coverage





Systematics

Sources:

Variations in functional forms of PDFs

Variations in the fit parameters

Variations in the event selection

Computed using *full* difference with original value

Results are expressed in units of the statistical error

Systematic source	R_D	y'	x'^2
PDF:	0.59σ	0.45σ	0.40σ
Selection criteria:	0.24σ	0.55σ	0.57σ
Quadrature total:	0.63σ	0.71σ	0.70σ

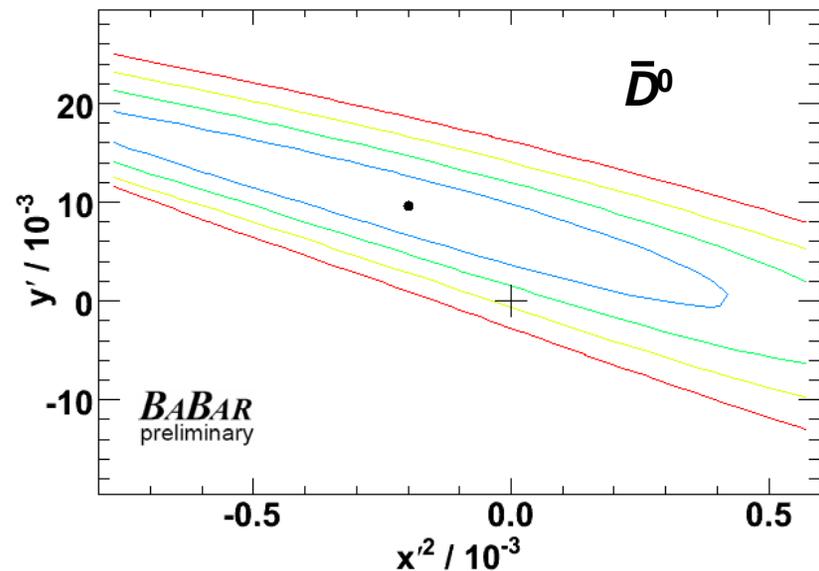
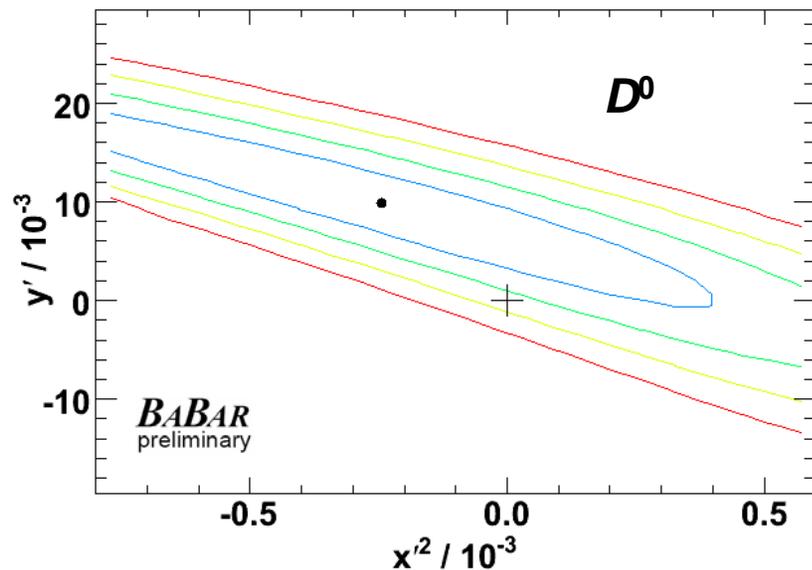


Fits allowing for CP violation

Fit D^0 and \bar{D}^0 decay-time dependence separately.

$$x'^{2+} = (-0.24 \pm 0.43 \pm 0.30) \times 10^{-3}$$
$$y'^+ = (9.8 \pm 6.4 \pm 4.5) \times 10^{-3}$$

$$x'^{2-} = (-0.20 \pm 0.41 \pm 0.29) \times 10^{-3}$$
$$y'^- = (9.6 \pm 6.1 \pm 4.3) \times 10^{-3}$$





Mixing and CPV fit results

Fit results for all three cases:

No mixing or CPV; mixing but no CPV; and CPV and mixing.

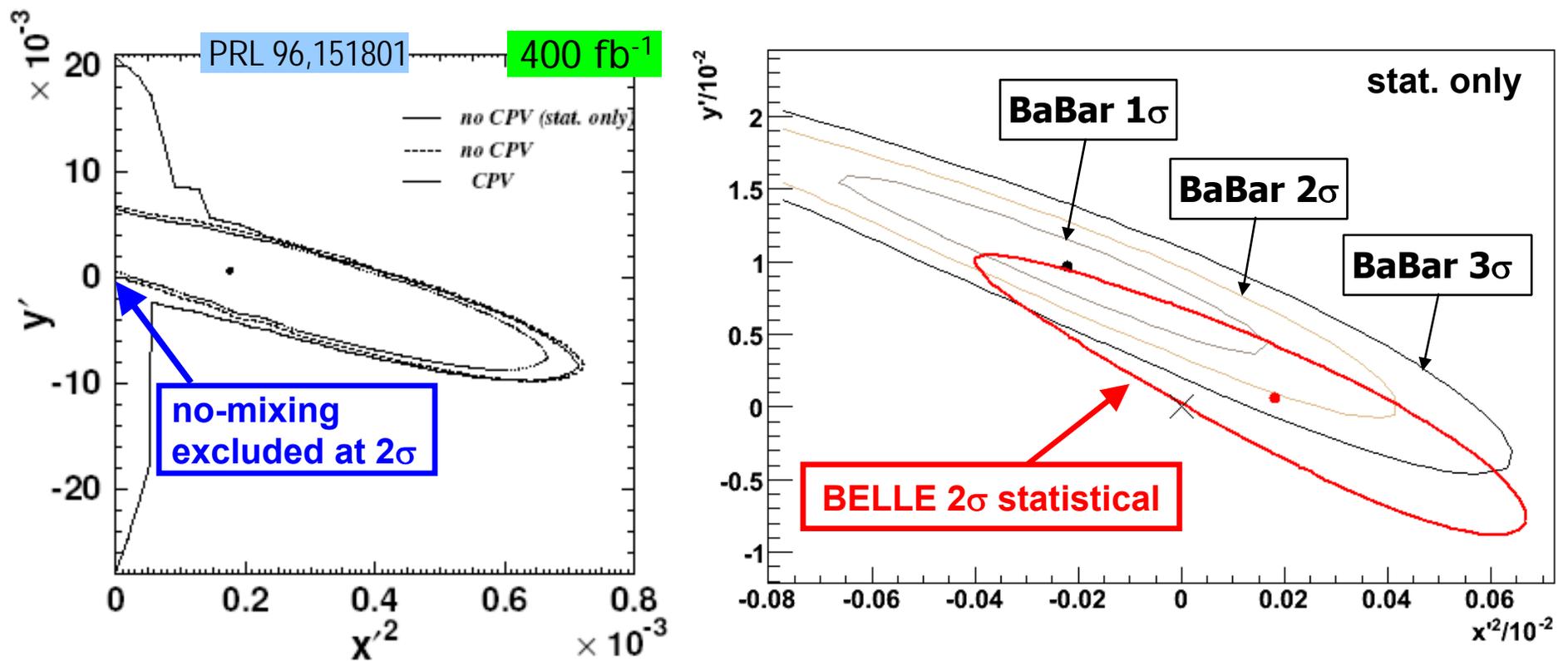
R_D changes between no-mixing and mixing fits.

Fit type	Parameter	Fit Results ($/10^{-3}$)
No CP viol. or mixing	R_D	$3.53 \pm 0.08 \pm 0.04$
No CP violation	R_D	$3.03 \pm 0.16 \pm 0.10$
	x'^2	$-0.22 \pm 0.30 \pm 0.21$
	y'	$9.7 \pm 4.4 \pm 3.1$
CP violation allowed	R_D	$3.03 \pm 0.16 \pm 0.10$
	A_D	$-21 \pm 52 \pm 15$
	x'^{2+}	$-0.24 \pm 0.43 \pm 0.30$
	y'^+	$9.8 \pm 6.4 \pm 4.5$
	x'^{2-}	$-0.20 \pm 0.41 \pm 0.29$
	y'^-	$9.6 \pm 6.1 \pm 4.3$



BELLE 2006 $K\pi$ result

Results consistent within 2σ





Summary

No evidence seen for CP violation

Evidence for mixing at 3.9σ (stat.+syst.)

$$y' = [9.7 \pm 4.4 \text{ (stat.)} \pm 3.1 \text{ (syst.)}] \times 10^{-3}$$

$$x'^2 = [-0.22 \pm 0.30 \text{ (stat.)} \pm 0.219 \text{ (syst.)}] \times 10^{-3}$$

$$R_D = [0.303 \pm 0.016 \text{ (stat.)} \pm 0.010 \text{ (syst.)}] \%$$

Submitted to PRL (hep-ex/0703020)

Results are consistent with other mixing analyses

BABAR $K\pi$, 2003: $(-56 < y' < 39) \times 10^{-3}$, $x' < 11 \times 10^{-3}$ (95% CL)

BELLE $K\pi$, 2006: $(-28 < y' < 21) \times 10^{-3}$, $x' < 3.6 \times 10^{-3}$ (95% CL)

BELLE $K_s\pi\pi$ 2007 (hep-ex/0703036):

$$x = (0.80 \pm 0.29 \pm 0.17) \%, \quad y = (0.33 \pm 0.24 \pm 0.15) \%$$

3.2σ evidence for mixing

Lifetime difference analyses:

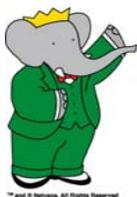
$$\text{BELLE, 2003: } y_{CP} = (11.5 \pm 6.9 \pm 3.8) \times 10^{-3}$$

$$\text{BABAR, 2003: } y_{CP} = (9 \pm 4 \pm 5) \times 10^{-3}$$

Backup slides

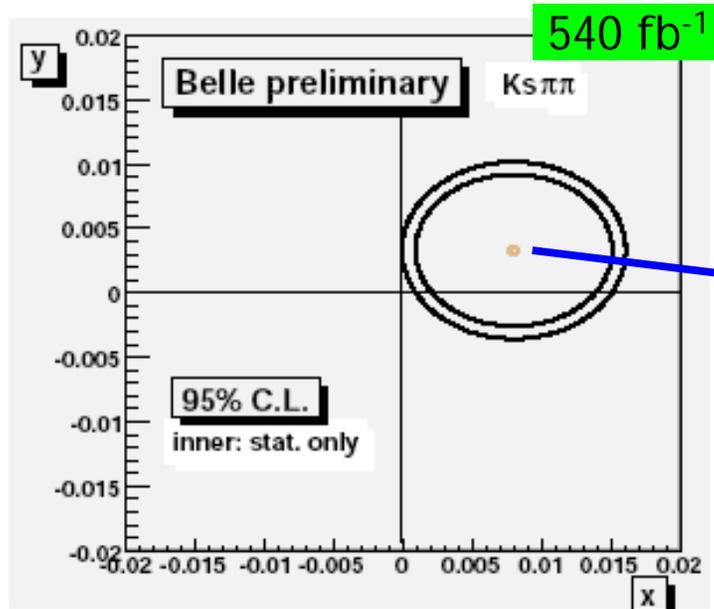


**Massachusetts
Institute of
Technology**



BELLE $K_S \pi \pi$ (Moriond EW 2007)

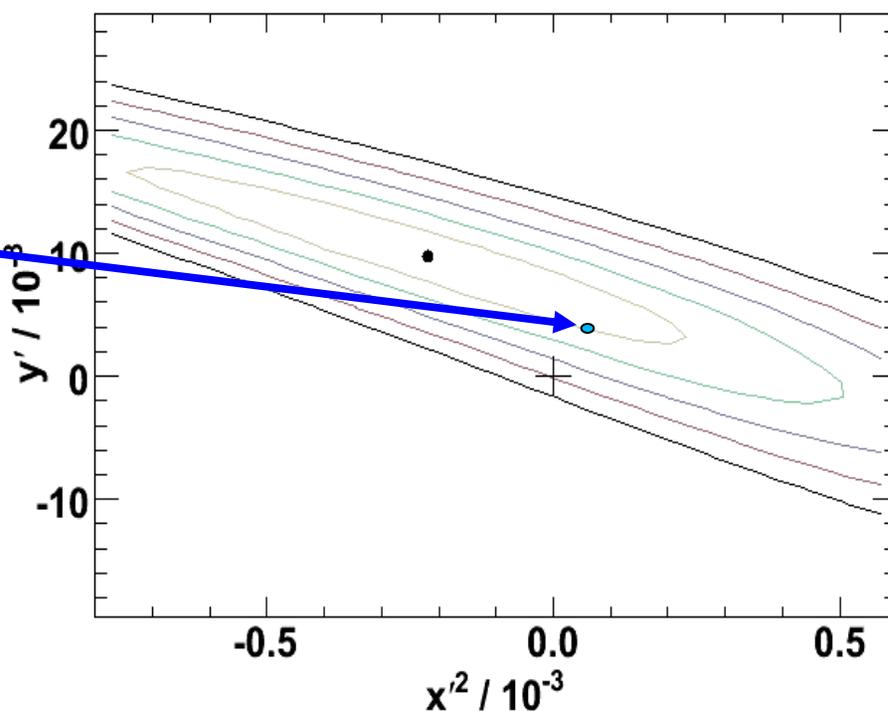
Dalitz analysis of $D^0 \rightarrow K_S \pi \pi$



$$x = 0.80 \pm 0.29 \pm 0.17 \%$$

$$y = 0.33 \pm 0.24 \pm 0.15 \%$$

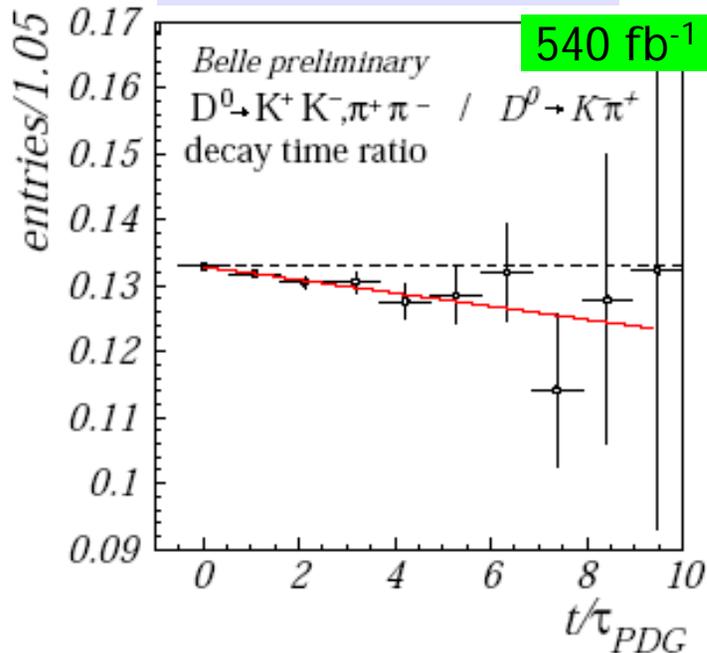
Compare assuming $\delta=0$:
($x'=x$, $y'=y$)





BELLE lifetime ratio (Moriond EW 2007)

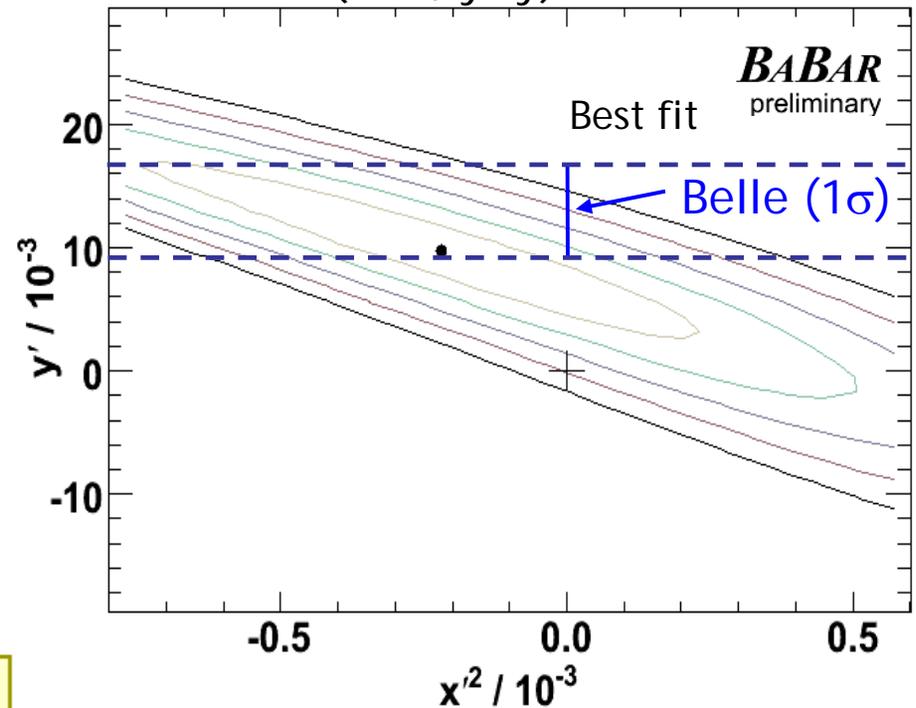
Lifetime ratio in $D^0 \rightarrow KK/\pi\pi$ to $K\pi$



$$y_{CP} = 1.31 \pm 0.32 \pm 0.25 \%$$

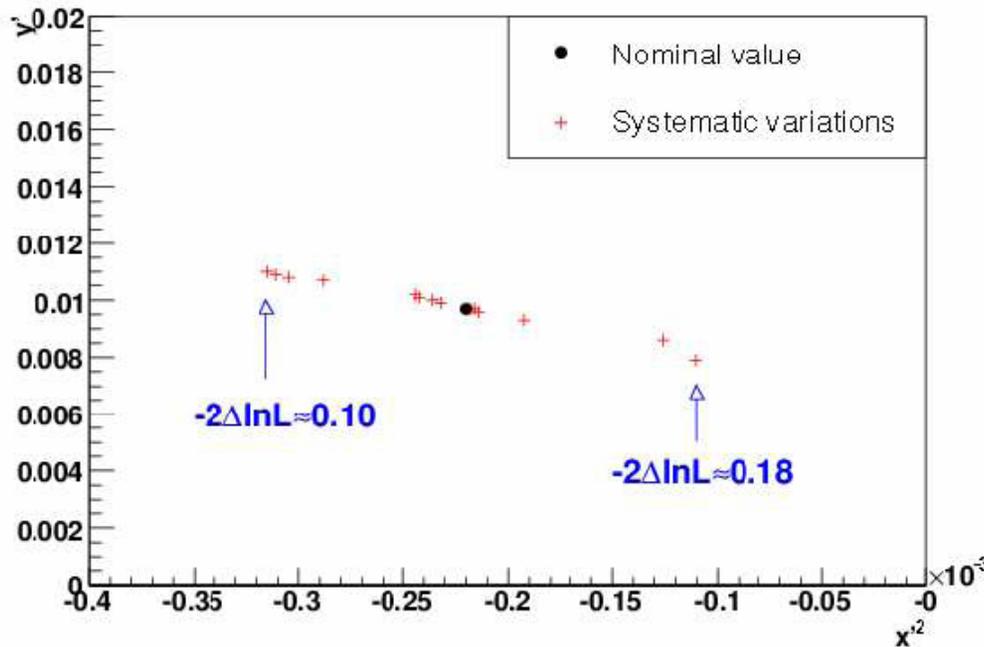
> 3 σ above zero
 (4.1 σ stat. only)

Compare assuming $\delta_{K\pi} = 0$:
 ($x'=x, y'=y$)



Including Systematics in Contours

Systematic variations produce new mixing parameters sets
 - tend to scatter along correlation axis:



Included in contours as follows:
 - for each variation calculate change in likelihood between new and old point in old likelihood

$$m_i^2 = (-2\Delta\ln L) / 2.3 \quad \leftarrow 1\sigma \text{ in } 2D$$

- Scale likelihood with $\frac{1}{1 + \sum m_i^2}$
- Should correspond to scaling the statistical uncertainty up

CPV systematics use same scale factor plus tiny correction for charge asymmetry in efficiency

Is this a correct or approximately correct thing to do?



More on systematics

Accounting for systematic errors in contours

■ Sources

- variations in functional form of signal and background terms
- variations in the parameters
- variations in proper time, proper time error and D^* overlap removal criteria

■ (x'^2, y) contours:

- for each variation, compute $s_i^2 = 2 [\ln \mathcal{L}_0 - \ln \mathcal{L}_i] / 2.3$
where \mathcal{L}_0 is the maximum likelihood from the standard fit and \mathcal{L}_i is the likelihood from the standard fit with (x'_i, y_i) fixed to the values obtained from the fit with the i^{th} variation
 - PDF variations: $\sum s_i^2 = .06$
 - selection criteria: $\sum s_i^2 = .18$
 - total: $\sum s_i^2 = .24$
- divide change in $-2 \log \mathcal{L}$ by the factor $f = 1 + \sum s_i^2 = 1.24$
to account for systematic errors

"Final" Systematics

fit:	y' ($\times 10^{-2}$)	$\delta y'/\sigma$	R_M ($\times 10^{-4}$)	$\delta R_M/\sigma$	m^2
default fit:	0.97 ± 0.44	-	-0.63 ± 1.07	-	-
No offset in core resolution:	1.10 ± 0.44	$+0.30\sigma$	-0.97 ± 1.06	-0.33σ	0.045
offset in all resolution Gaussians:	0.97 ± 0.44	-0.01σ	-0.61 ± 1.07	$+0.02\sigma$	0.000
Proper time error distributions from sidebands, not sPlot:					
widest core Gaussian without per-event errors:	0.96 ± 0.44	-0.02σ	-0.61 ± 1.07	$+0.02\sigma$	0.001
Fix scale factor $s_1 = 1$:	0.93 ± 0.44	-0.09σ	-0.53 ± 1.08	$+0.10\sigma$	0.004
Fix D^0 lifetime to PDG value:	0.97 ± 0.44	-0.00σ	-0.62 ± 1.07	$+0.01\sigma$	0.001
Change Category 3 Model:	0.96 ± 0.44	-0.05σ	-0.61 ± 1.07	$+0.02\sigma$	0.003
Cat. 4 t from low sideband:	0.85 ± 0.43	-0.28σ	-0.46 ± 1.06	$+0.16\sigma$	0.060
Cat. 4 t from high sideband:	1.01 ± 0.44	$+0.08\sigma$	-0.65 ± 1.07	-0.02σ	0.011
Vary $(m_{K_S}, \Delta m)$ fit model:	1.00 ± 0.44	$+0.06\sigma$	-0.68 ± 1.07	-0.05σ	0.002
Vary $(m_{K_S}, \Delta m)$ parameters:	1.02 ± 0.44	$+0.10\sigma$	-0.70 ± 1.06	-0.07σ	0.007
$(-1 < t < 3.5)$ ps:	0.86 ± 0.44	-0.26σ	-0.26 ± 1.10	$+0.34\sigma$	0.061
$(-5 < t < 10)$ ps:	1.08 ± 0.44	$+0.24\sigma$	-0.94 ± 1.05	-0.30σ	0.039
$(\delta_1 < 0.4)$ ps:	1.07 ± 0.45	$+0.23\sigma$	-0.87 ± 1.07	-0.22σ	0.023
$(\delta_1 < 0.6)$ ps:	0.79 ± 0.43	-0.41σ	-0.27 ± 1.07	$+0.34\sigma$	0.077
Keep all overlapping candidates	0.99 ± 0.44	$+0.05\sigma$	-0.67 ± 1.06	-0.04σ	0.002
Remove all overlapping candidates	1.09 ± 0.45	$+0.27\sigma$	-0.95 ± 1.07	-0.31σ	0.042
Total variation:		0.71σ		0.70σ	0.306

Systematics summary:

systematic source:	R_D	y'	x'^2
PDF:	0.59σ	0.45σ	0.40σ
selection criteria:	0.24σ	0.55σ	0.57σ
Quadrature total:	0.63σ	0.71σ	0.70σ

Validation: fit to generic Monte Carlo

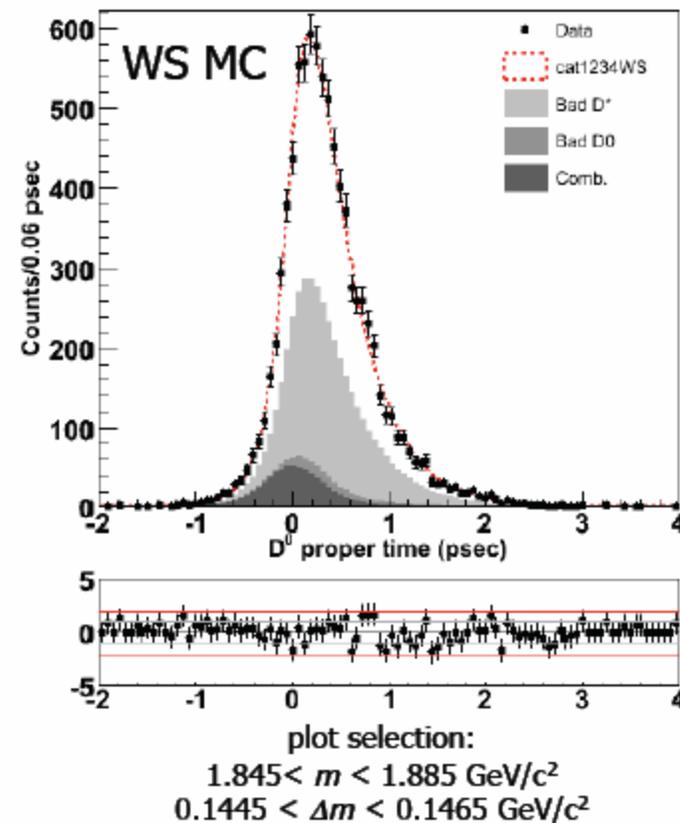
- repeat fitting procedure on R18b generic Monte Carlo sample ($\sim 400 \text{ fb}^{-1}$)
 - WS mixing fit results:

$$y' = (-0.22 \pm 0.30)\%$$

$$\chi^2 = (2 \pm 18) \times 10^{-5}$$

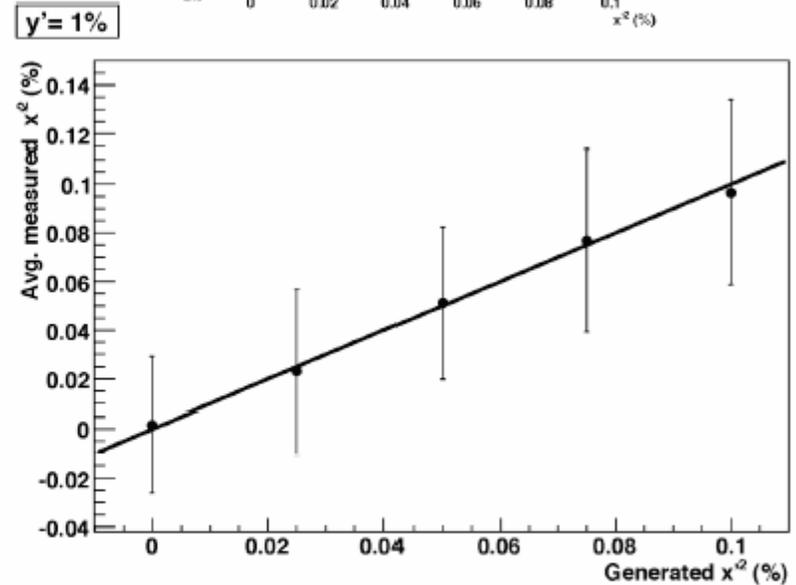
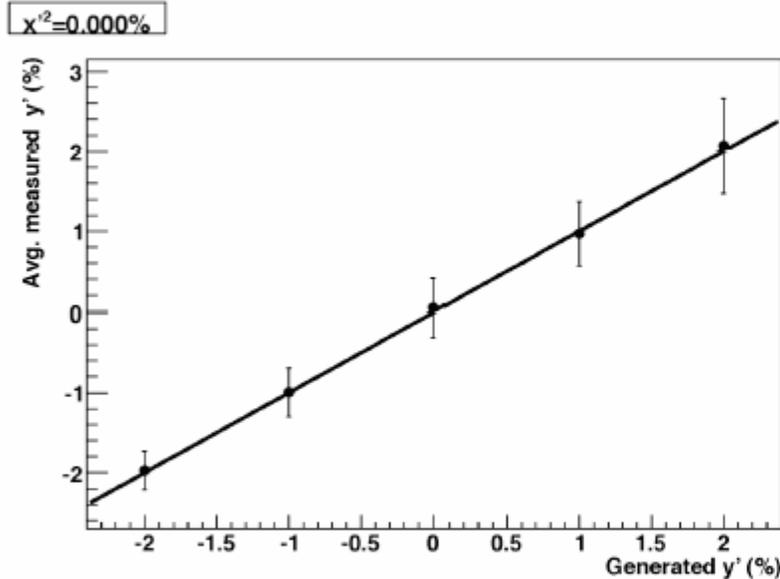
$$R_D = (0.413 \pm 0.014)\%$$

- MC generated without mixing
- No mixing is observed
- R_D consistent with dialed value



Validation: Toy studies

- test for unbiasedness:



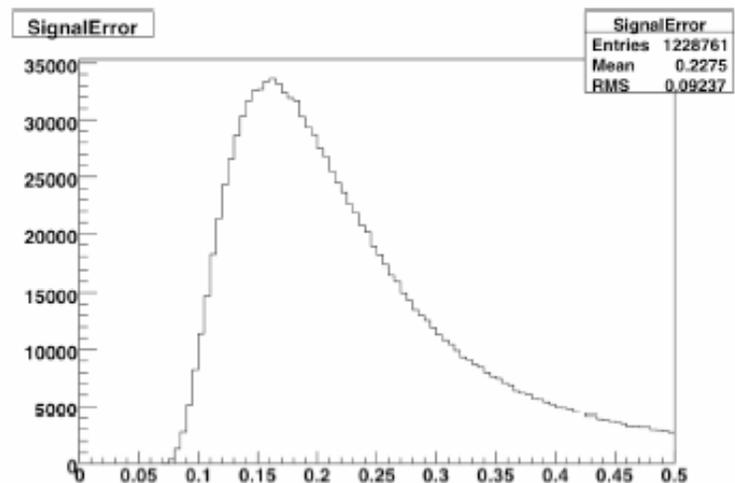
100	100	100	100	100
1	48	100	100	100
100	75	51	93	100
100	100	100	100	100

average fitted value of mixing parameter versus generated value.
 Error bars: RMS of fitted values: expected parameter errors
 Straight line has unit slope, 0 intercept.

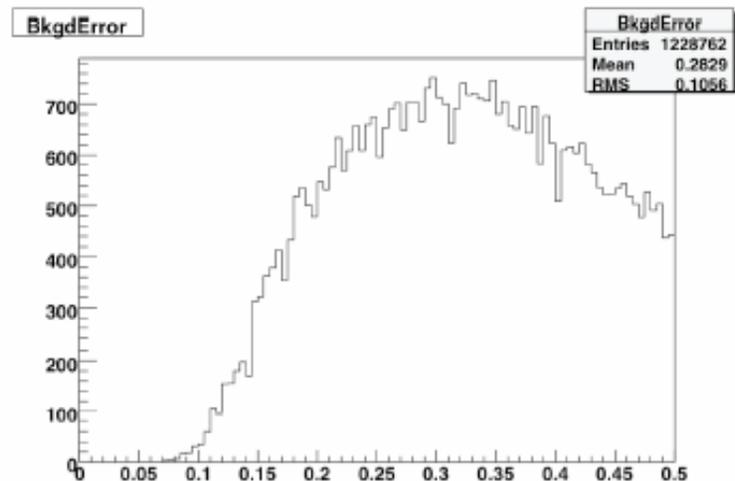
Results indicate no bias in estimating mixing parameters

R18b data decay time error distributions

category 1-3 DecayTimeError sPlot:



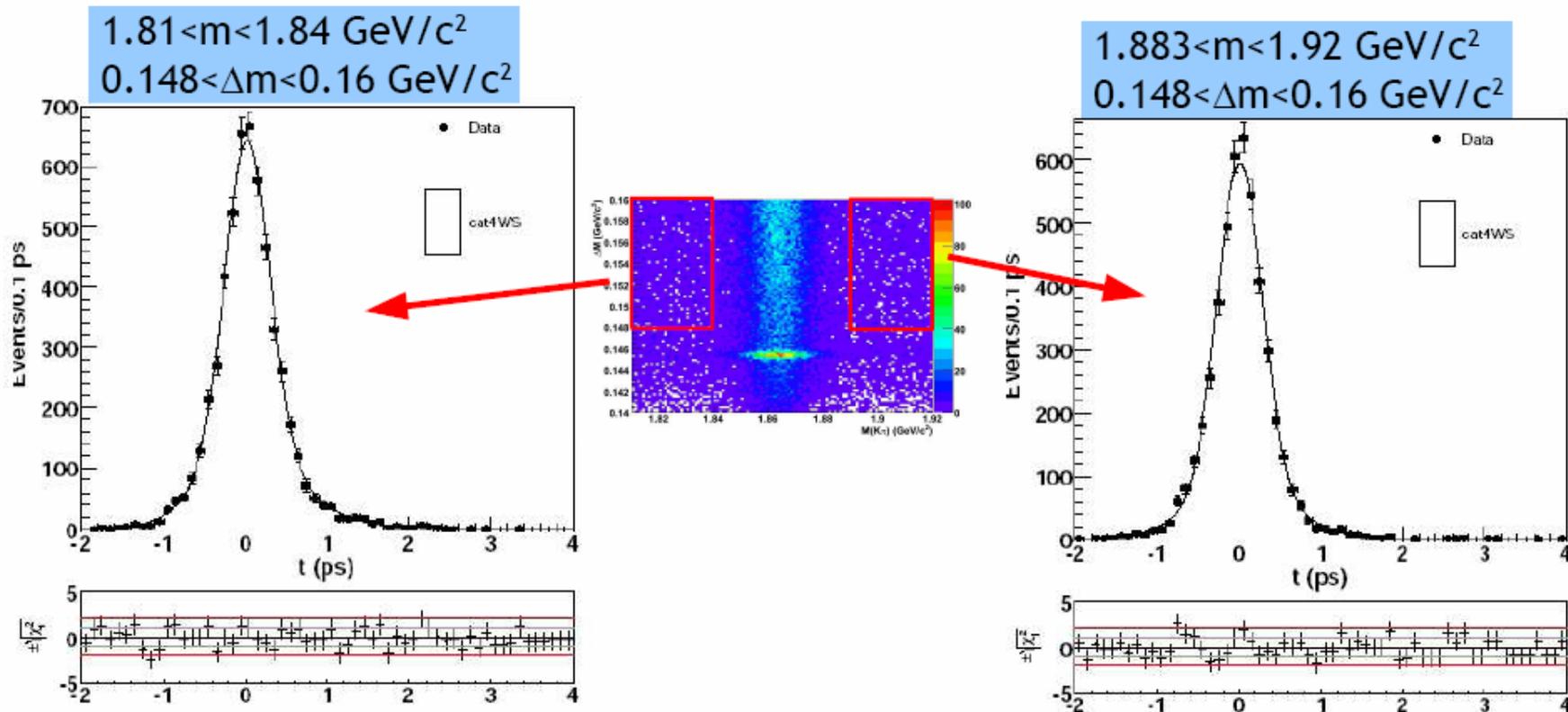
category 4 DecayTimeError sPlot:



Proper Time from Sidebands

Assigning systematic

Instead of fitting proper time for background in full fit, fix it to fits in pure background sidebands:





Time-dependent decay rate

The time-dependent decay rate of an initially-pure D^0 or \bar{D}^0 can be written

$$\begin{aligned}|D^0(t)\rangle &= g_+(t)|D^0\rangle - (q/p)g_-(t)|\bar{D}^0\rangle \\ |\bar{D}^0(t)\rangle &= g_+(t)|\bar{D}^0\rangle - (p/q)g_-(t)|D^0\rangle\end{aligned}$$

where $g_{\pm}(t) = \frac{1}{2}e^{-iMt - \frac{1}{2}\Gamma t} \left(e^{-\frac{i}{2}\Delta Mt - \frac{1}{4}\Delta\Gamma t} \pm e^{+\frac{i}{2}\Delta Mt + \frac{1}{4}\Delta\Gamma t} \right)$

This yields the time-dependent decay rate

$$\begin{aligned}\frac{d\Gamma}{dt} [|D^0(t)\rangle \rightarrow f] &\propto e^{-\Gamma t} \times \\ &[(|A_f|^2 + |(q/p)\bar{A}_f|^2) \cosh(y\Gamma t) + (|A_f|^2 - |(q/p)\bar{A}_f|^2) \cos(x\Gamma t) \\ &+ 2\text{Re}((q/p)A_f^*\bar{A}_f) \sinh(y\Gamma t) - 2\text{Im}((q/p)A_f^*\bar{A}_f) \sin(x\Gamma t)]\end{aligned}$$



Time-dependent decay rate (1)

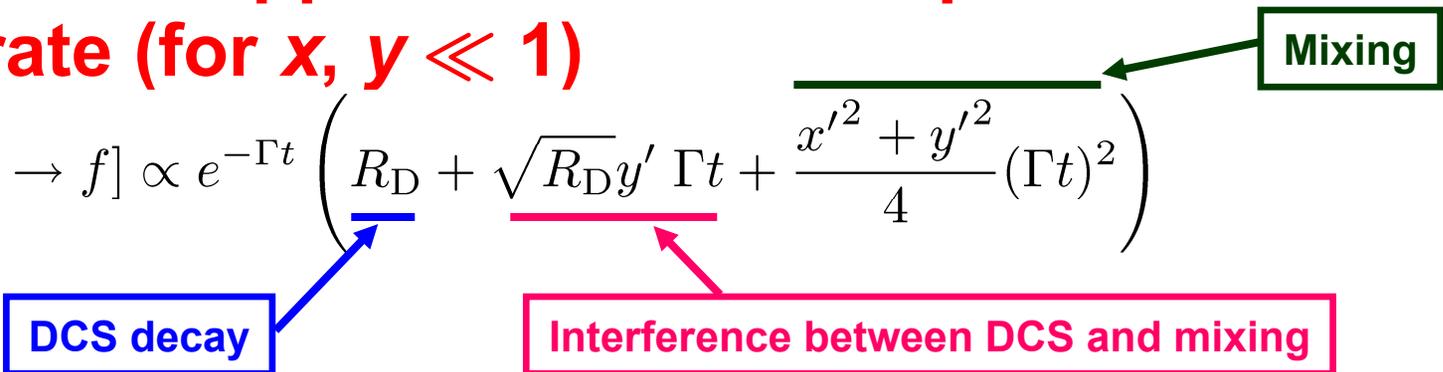
Solving the Hamiltonian for the time-dependence of the D_1, D_2 eigenstates yields

$$\begin{aligned}
 |D^0(t)\rangle &= g_+(t)|D^0\rangle - (q/p)g_-(t)|\bar{D}^0\rangle \\
 |\bar{D}^0(t)\rangle &= g_+(t)|\bar{D}^0\rangle - (p/q)g_-(t)|D^0\rangle
 \end{aligned}$$

where $g_{\pm}(t) = \frac{1}{2}e^{-iMt - \frac{1}{2}\Gamma t} \left(e^{-\frac{i}{2}\Delta Mt - \frac{1}{4}\Delta\Gamma t} \pm e^{+\frac{i}{2}\Delta Mt + \frac{1}{4}\Delta\Gamma t} \right)$

This yields the approximate time-dependent decay rate (for $x, y \ll 1$)

$$\frac{d\Gamma}{dt} [|D^0(t)\rangle \rightarrow f] \propto e^{-\Gamma t} \left(R_D + \sqrt{R_D} y' \Gamma t + \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right)$$





Systematics: decay time resolution

Decay-time resolution

Sum of 3 Gaussians

Narrowest has a non-zero mean of 3.6 fsec

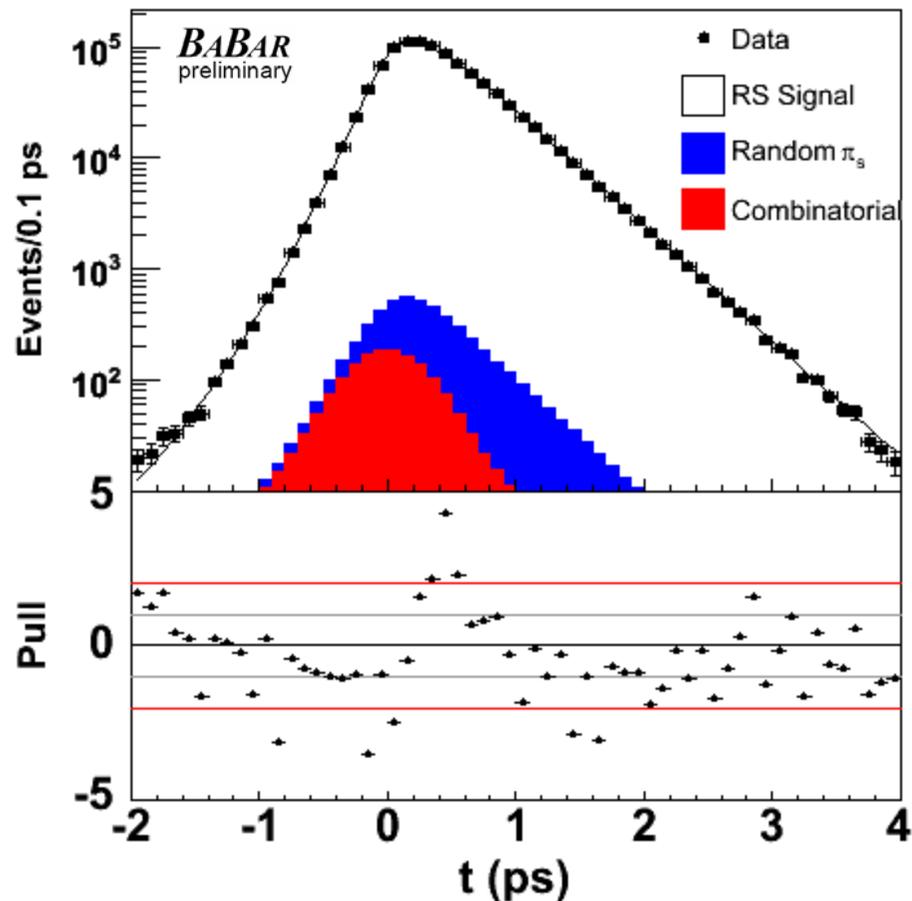
Most likely due to alignment issues.

Also seen in other analyses.

Check by setting offset to zero and refitting for mixing parameters.

χ^2 changes by -0.3σ

y' changes by $+0.3\sigma$



RS decay time fit with zero offset.