Measurement of Weak Phase γ from $B^{\pm} \to D[K_S\pi^+\pi^-]K^{\pm}$ at Belle

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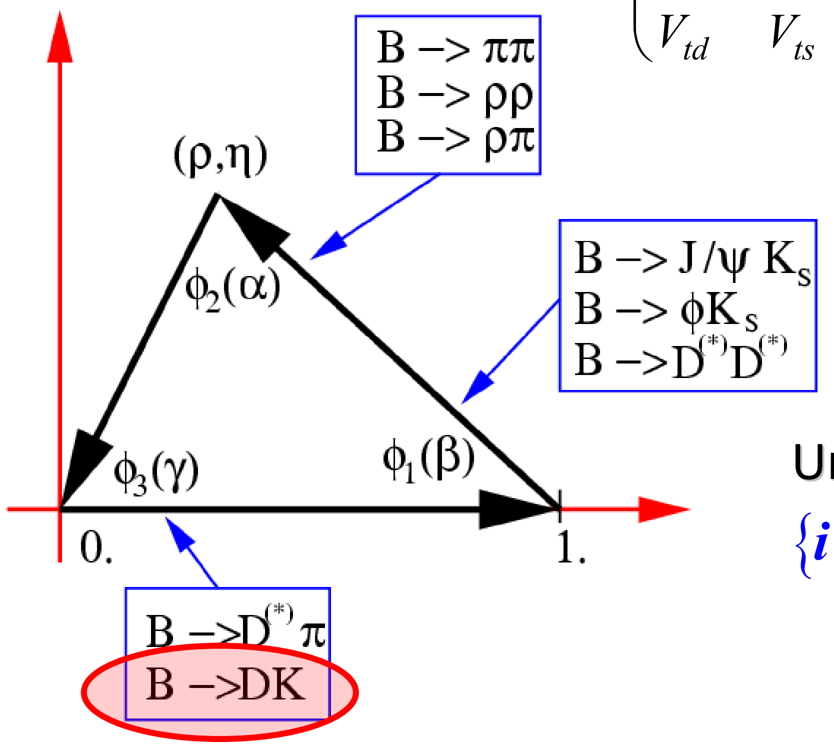
- Analysis
- Results
- Perspectives
- Summary

Introduction: Unitarity triangle

$$V=\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb}\end{pmatrix} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



$\lambda = 0.2235 \pm 0.0033$
$A = 0.81 \pm 0.08$
$|\rho - i\eta| = 0.36 \pm 0.09$
$|1-\rho-i\eta| = 0.79 \pm 0.19$

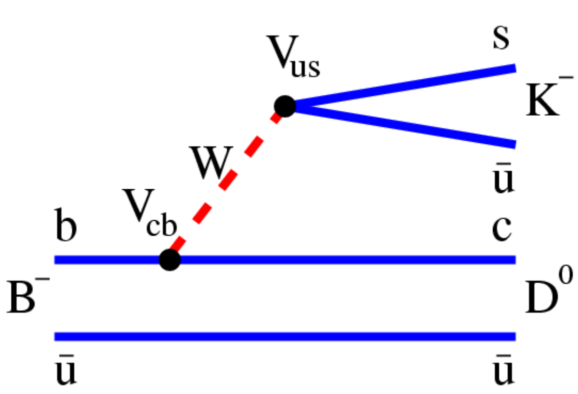
Unitarity requirement:

$$\{i=1, k=3\}: V^*_{ub}V_{ud} + V^*_{cb}V_{cd} + V^*_{tb}V_{td} = 0$$

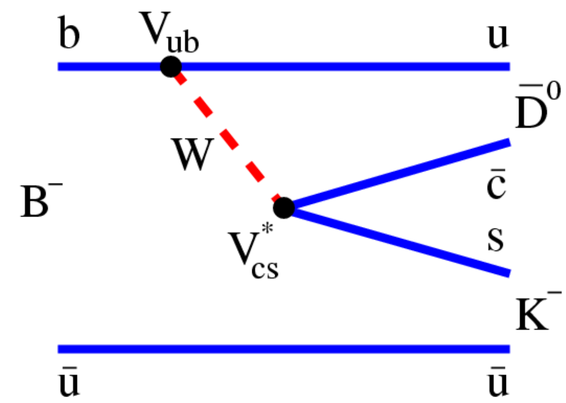
$$\Rightarrow \frac{V^*_{ub}V_{ud}}{V^*_{cb}V_{cd}} + 1 + \frac{V^*_{tb}V_{td}}{V^*_{cb}V_{cd}} = 0$$

β is measured with a high accuracy (~1°) at B-factories.
α and γ are more challenging angles to measure.
Measurement of all the angles needed to test SM.

Introduction: $B^+ \to D^0 K^+$ Decay



$$\mathbf{A}_1 \sim V_{cb} V_{us}^* \sim A\lambda^3$$



$$\mathbf{A}_2 \sim V_{ub} V_{cs}^* \sim A\lambda^3(\rho - i\eta) \sim e^{-i\phi_3}$$

Several methods to measure weak angle ϕ_3 utilizing B->DK decays have been proposed:

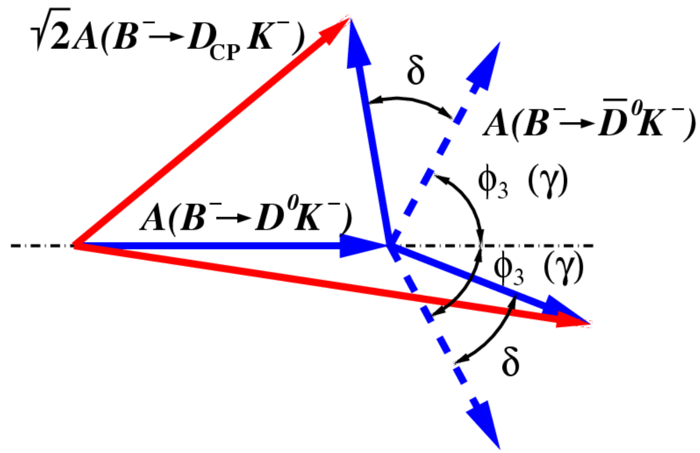
- **GLW** ($D^0 \to K^+K^-$, $\pi^+\pi^-$ ... [CP eigenstates])
  M. Gronau and D. London, PLB 253, 483 (1991);
  M. Gronau and D. Wyler, PLB 265, 172 (1991).
- **ADS** ($D^0 \to K^-\pi^+$, $K^-\pi^+\pi^0$ ...)
  D. Atwood, I. Dunietz and A. Soni, PRL 78, 3357 (1997);
  PRD 63, 036005 (2001).
- **GGSZ** ($D^0 \to K_S\pi^+\pi^-$ ... [common multi-body modes])
  A. Giri, Yu. Grossman, A. Soffer, J. Zupan, PRD 68, 054018 (2003).

Introduction: GLW Method

D-meson decays to CP eigenstates (D_{CP}).

CP-even : $D_1 \to K^+K^-, \pi^+\pi^-$

CP-odd : $D_2 \to K_S\pi^0, K_S\omega, K_S\varphi, K_S\eta$...



Observables:

- CP-Asymmetries:

$$\mathcal{A}_{1,2} = \frac{Br(B^- \to D_{1,2}K^-) - Br(B^+ \to D_{1,2}K^+)}{Br(B^- \to D_{1,2}K^-) + Br(B^+ \to D_{1,2}K^+)} = \frac{2r_B \sin\delta' \sin\gamma}{1 + r_B^2 + 2r_B \cos\delta' \cos\gamma}$$

$$\delta' = \begin{cases} \delta & \text{for } D_1 \\ \delta + \pi & \text{for } D_2 \end{cases} \quad \Rightarrow \quad \mathcal{A}_{1,2} \text{ of different signs}$$

- Double Ratios:

$$\mathcal{R}_{1,2} = \frac{Br(B \to D_{1,2}K)/Br(B \to D_{1,2}\pi)}{Br(B \to D^0K)/Br(B \to D^0\pi)} = 1 + r_B^2 + 2r_B \cos\delta' \cos\gamma$$

4 equations (3 independent: $\mathcal{A}_1\mathcal{R}_1 = -\mathcal{A}_2\mathcal{R}_2$), 3 unknowns (r_B, δ, γ)

Introduction: GLW Method

The relevant (total) branching fractions are of the order of few x 10^{-6}

⇨ larger statistics is required to perform a measurement (especially $\mathcal{A}$)

	Belle (253 fb^{-1}) hep-ex/0601032		BaBar (211 fb^{-1}) hep-ex/0512067	
	$\mathcal{R}$	$\mathcal{A}$	$\mathcal{R}$	$\mathcal{A}$
$B \to D_1 K$	1.13±0.16±0.05	0.06±0.14±0.05	0.90±0.12±0.04	0.35±0.13±0.04
$B \to D_2 K$	1.17±0.14±0.14	-0.12±0.14±0.05	0.86±0.10±0.05	-0.06±0.13±0.03

GLW analysis alone does not constrain γ significantly yet, but

- can be combined with other measurements
- provide information on r_B

$$r_B \equiv \left| \frac{A(B^- \to \overline{D}^0 K^-)}{A(B^- \to D^0 K^-)} \right| \approx \frac{\left|V_{ub}^* V_{cs}\right|}{\left|V_{cb}^* V_{us}\right|} \times [\text{color supp}]$$

Introduction: r_B

$$r_B \equiv \left| \frac{A(B^- \to \overline{D}^0 K^-)}{A(B^- \to D^0 K^-)} \right| \approx \frac{|V_{ub}^* V_{cs}|}{|V_{cb}^* V_{us}|} \times [\text{color supp}]$$

≈ 0.38

Naïve estimation of the color suppression factor gives 1/3

Factorization and experimental information on the b->c transition give [color supp.] ≈ 0.22 ⇨ $r_B \approx 0.09$.

Recent measurements of the color suppressed $B \to D^{(*)}h^0$ suggest that color suppression might be not that effective and r_B could be as large as 0.2.

$$\boxed{r_B \sim 0.1 - 0.2}$$

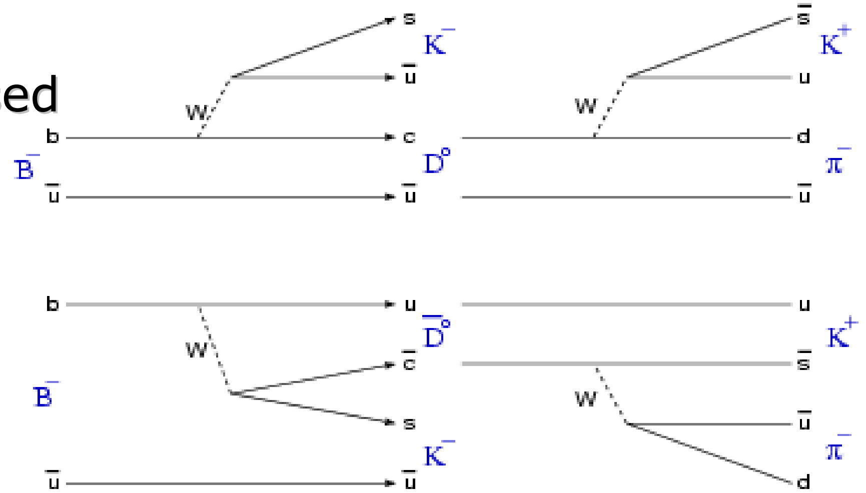
Introduction: ADS method

$B^- \to D^0 K^-$ - color allowed

$D^0 \to K^+\pi^-$ - doubly Cabibbo-suppressed

$B^- \to \bar{D}^0 K^-$ - color suppressed

$\bar{D}^0 \to K^+\pi^-$ - Cabibbo-allowed



Observables:

- Ratios:

$$\mathcal{R}^{\pm}_{K\pi} \equiv \frac{\Gamma([K^{\mp}\pi^{\pm}]_D K^{\pm})}{\Gamma([K^{\pm}\pi^{\mp}]_D K^{\pm})} = r_B^2 + r_D^2 + 2 r_B r_D \cos(\pm\gamma + \delta)$$

$$r_B \equiv \left| \frac{A(B^- \to \bar{D}^0 K^-)}{A(B^- \to D^0 K^-)} \right| \quad r_D \equiv \left| \frac{A(D^0 \to K^+\pi^-)}{A(D^0 \to K^-\pi^+)} \right|$$

2 equations, 3 unknowns

With one more D decay channel added:

⇨ 4 equations, 4 unknowns

Introduction: ADS Method

BaBar (211 fb^{-1}) hep-ex/0504047, PRD **72**, 032004

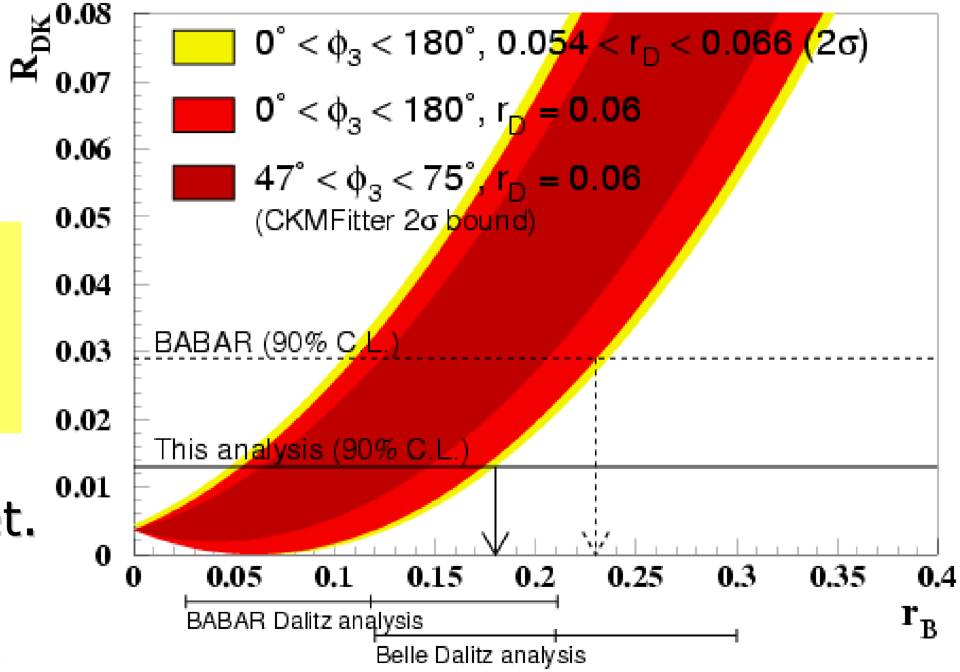
$$\mathcal{R}_{DK} = 13^{+11}_{-9} \times 10^{-3}$$

r_B<0.23 (90% CL)

Belle (357 fb^{-1}) hep-ex/0508048

$$\mathcal{R}_{DK} = (0.0^{+8.4}_{-7.9} \pm 1.0) \times 10^{-3}$$

Using r_D=0.060±0.003,
for maximum mixing (φ_3=0, δ=180°):
r_B<0.18 (90% CL)



Suppressed channel is not visible yet.

Like in GLW analyses, no significant constraint on γ, but upper limit on r_B

Introduction: GGSZ Method

Use multi-body final state, common for D^0 and $\bar{D}^0$: $K_S\pi^+\pi^-$, $K_SK^+K^-$, $\pi^0\pi^+\pi^-$ ...

Interference between $B^+\to\bar{D}^0K^+$ and $B^+\to D^0K^+$ amplitudes translates into interference between $\bar{D}^0\to h_1h_2h_3$ and $D^0\to h_1h_2h_3$ decays to be measured on the D Dalitz plot.

Statistical sensitivity of the method depends on the dynamics of the particular 3-body D-meson decay mode:
(For $|M|^2=Const$ there is no sensitivity)

Large variation of the strong phase over the Dalitz plot is an essential ingredient. This is provided by the presence of intermediate resonances.

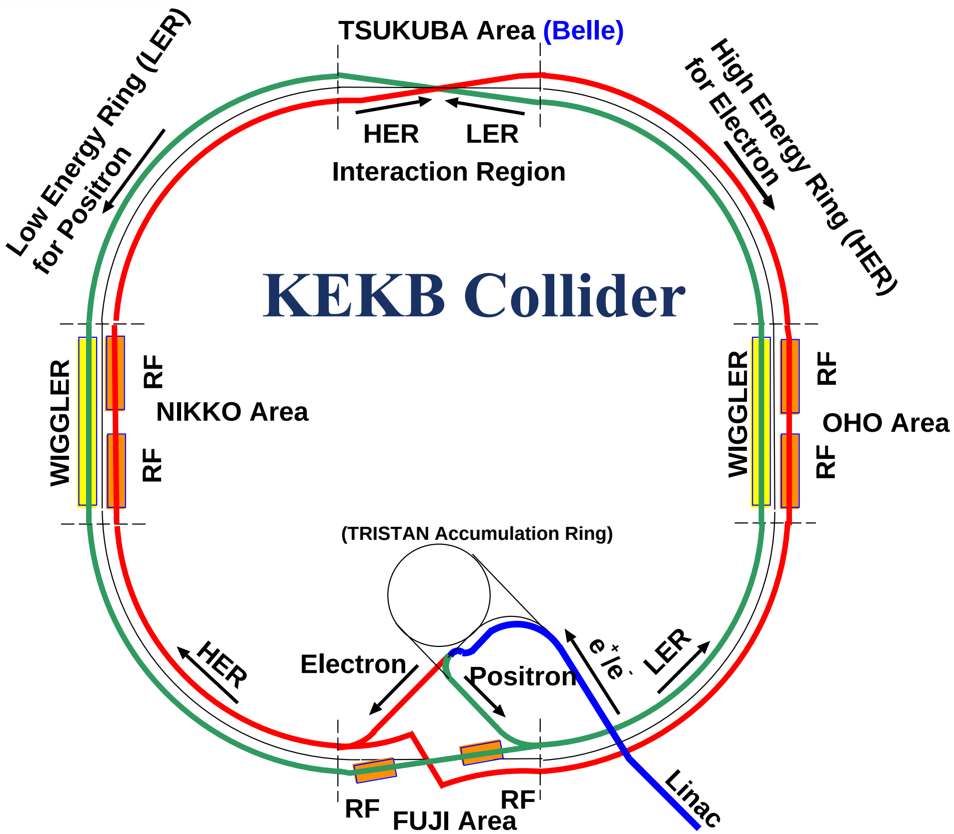
Flavor specific $D^0\to h_1h_2h_3$ amplitude can be independently determined from high statistic sample of flavor tagged $D^{*-}\to D^0\pi^-$, $D^0\to h_1h_2h_3$ decays ($\Rightarrow$ model uncertainty)

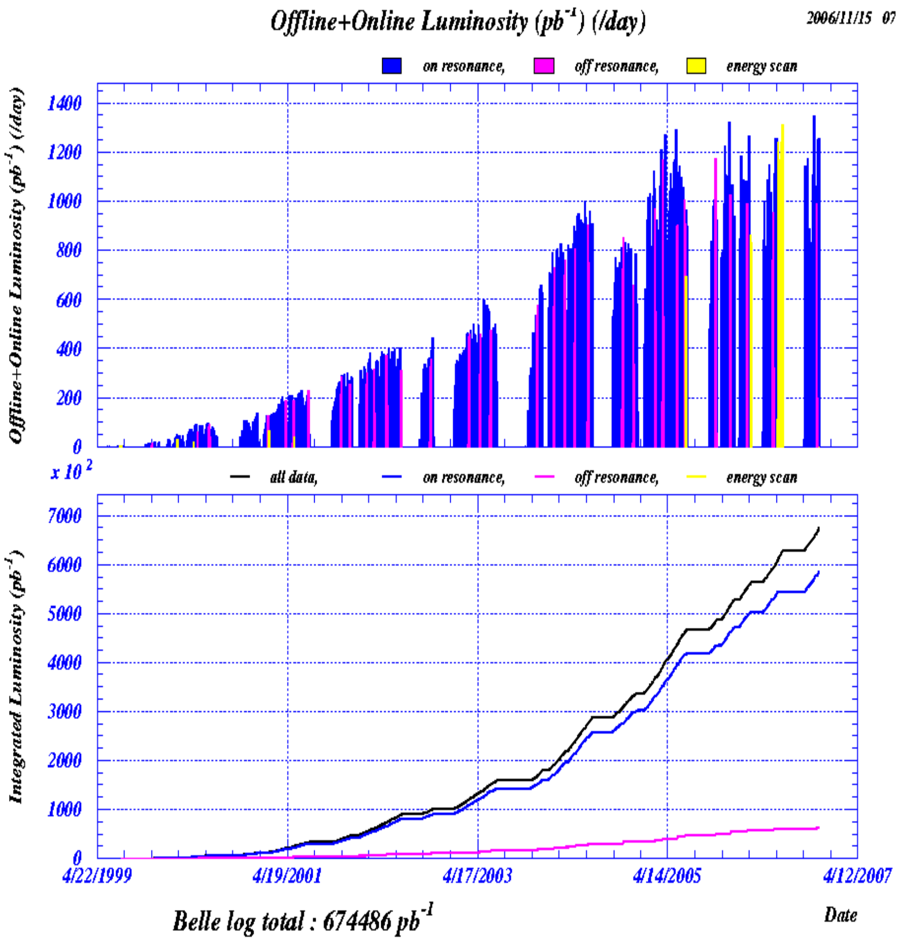
Dalitz distribution of $D\to h_1h_2h_3$ from $B^\pm\to DK^\pm$ decays is fit with only three free parameters (r,δ,φ_3)

Introduction: Historical Remark

- Talk by A. Bondar, at the dedicated Belle Dalitz analysis meeting, Sep. 24-26, 2002, Novosibirsk.
- First arXiv preprint by A.Giri, Yu.Grossman, A.Soffer, J.Zupan, hep-ph/0303187.
- First results by Belle (152M BBbar pairs): Lepton-Photon'03 Aug. 11-16, 2003, Batavia, hep-ex/0308043.
- First results by BaBar (227M BBbar pairs): ICHEP'04 August 16-22, 2004, Bejing, hep-ex/0408088.

Apparatus: KEKB





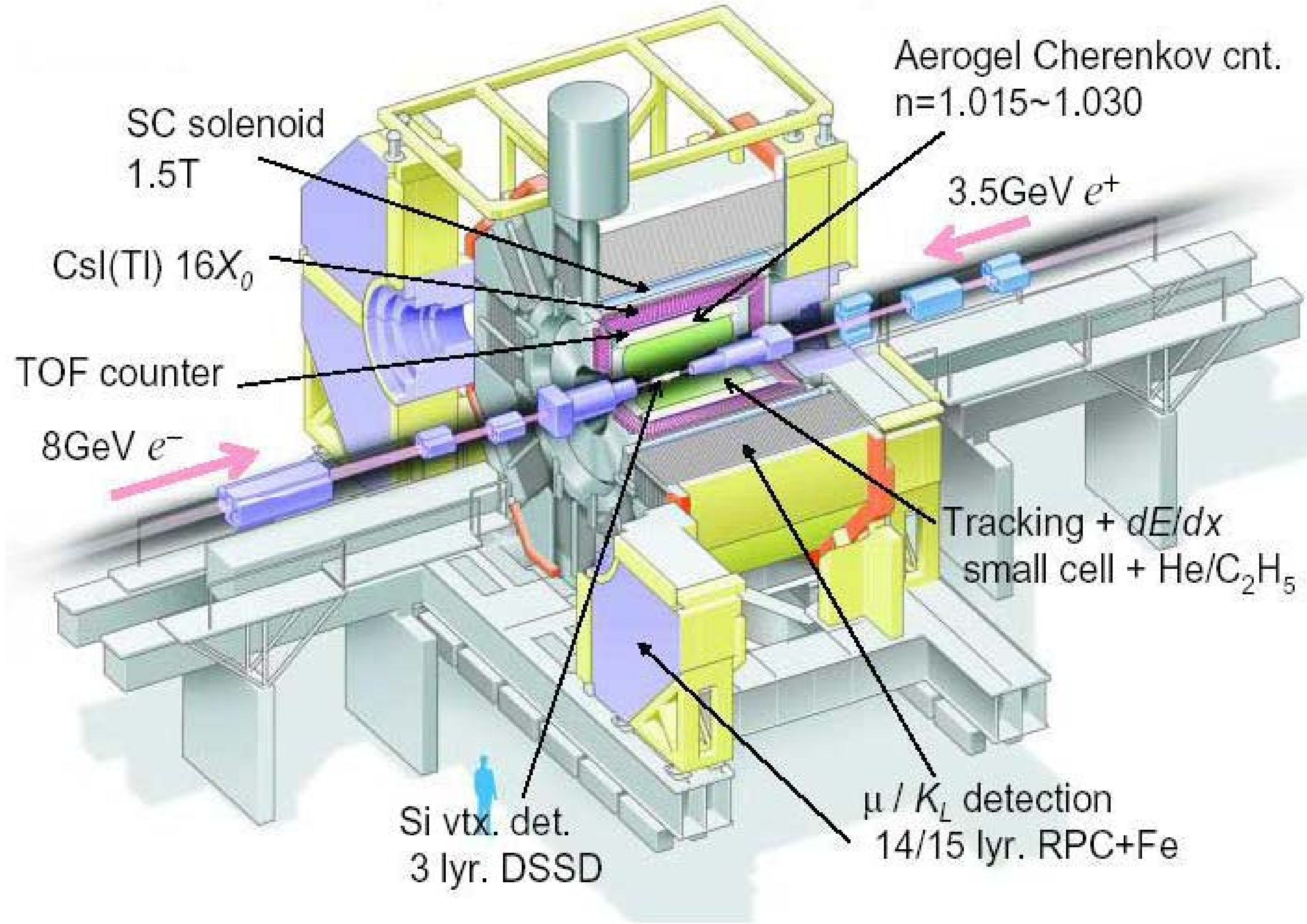
3.5 GeV e^+ & 8 GeV e^- beams

3 km circumference, 11 mrad crossing angle

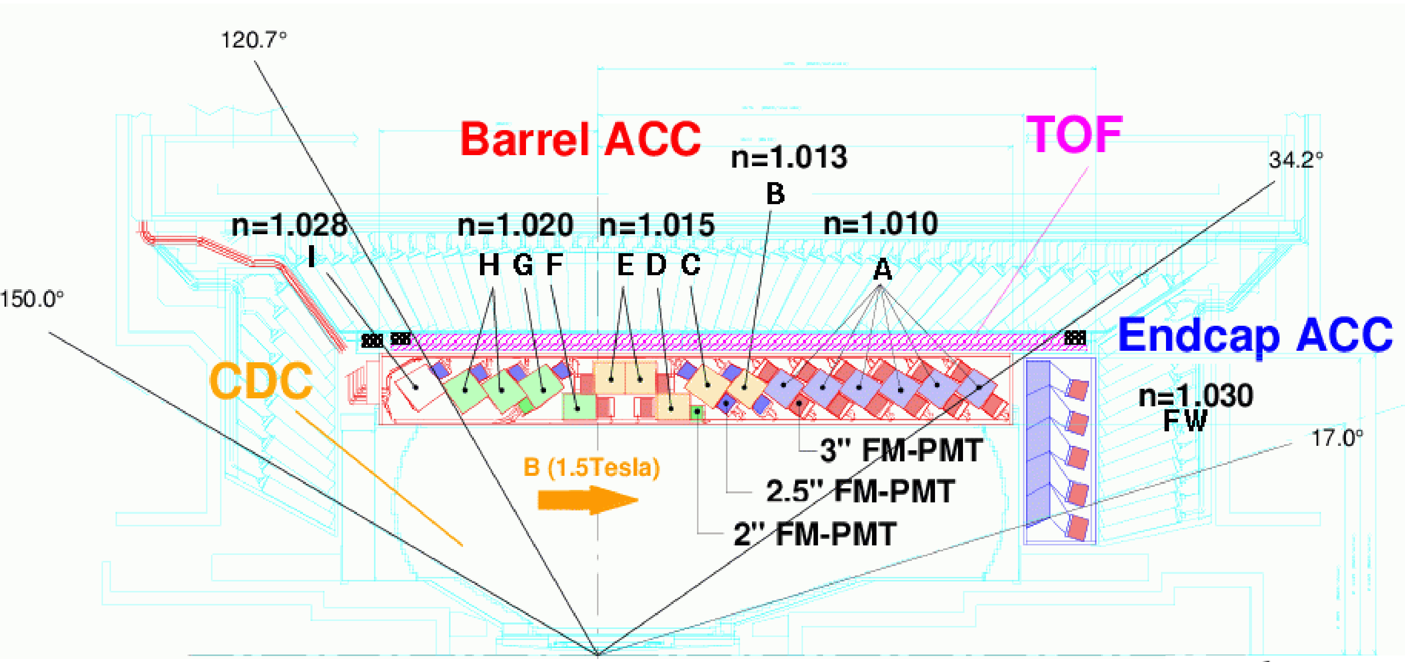
L = 1.66 x 10^{34} $cm^{-2}s^{-1}$ (world record)

$$\int L dt = 675\ fb^{-1}\ @\ \Upsilon(4S) + \text{off}(\sim 10\%)$$

Apparatus: Belle

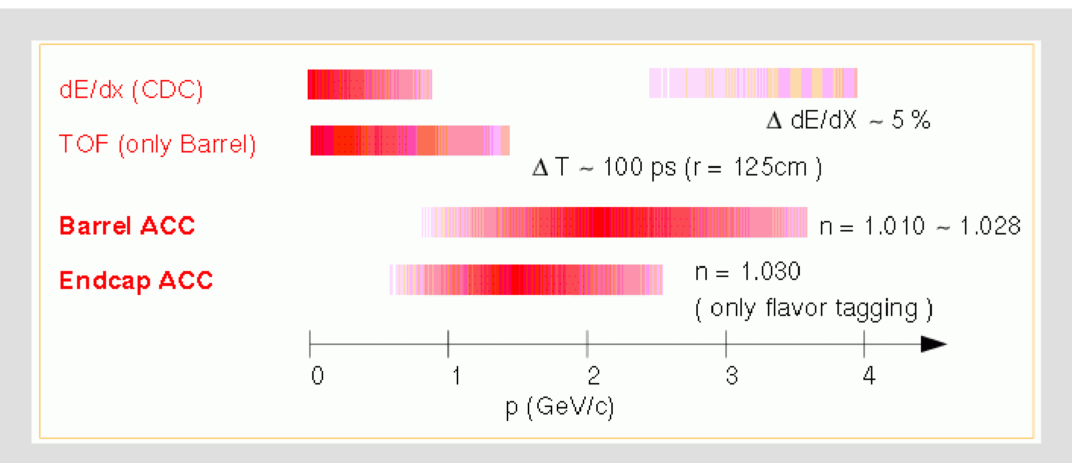


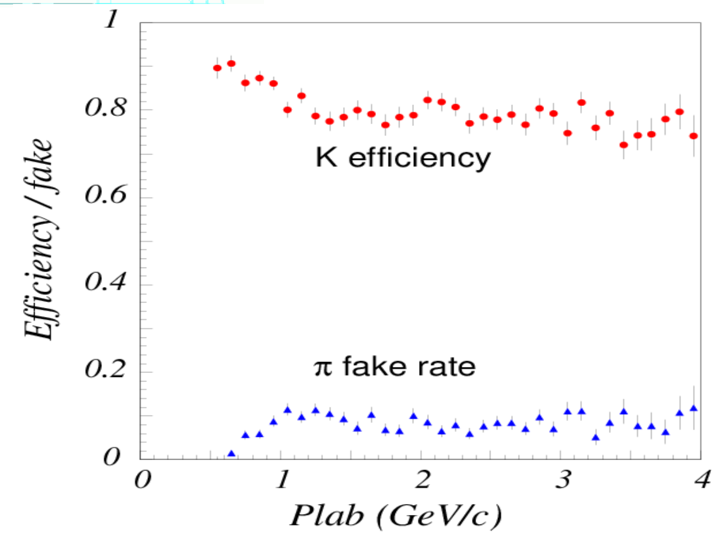
Apparatus: Particle Identification



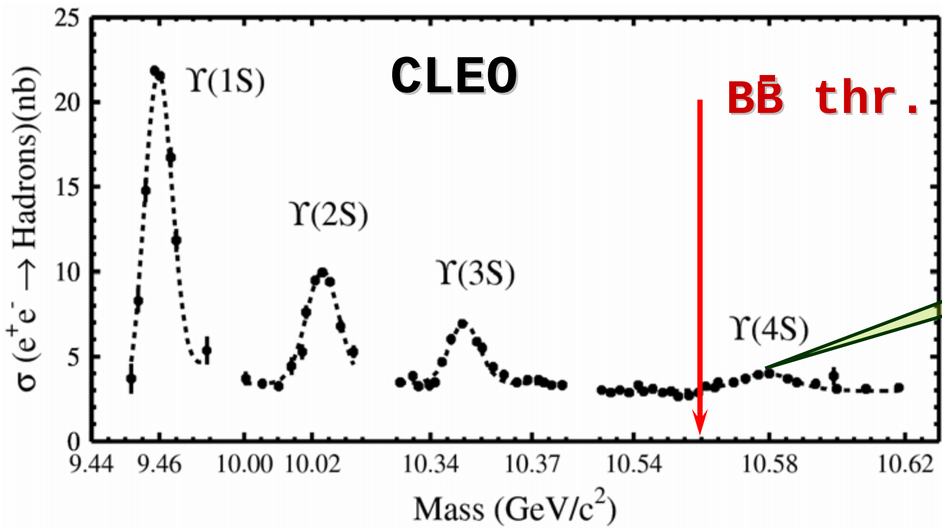
p/K/π separation is based on Likelihood ratio:

$$LR(K) = \frac{L(K)}{L(K)+L(\pi)}$$





Analysis: Kinematical Variables



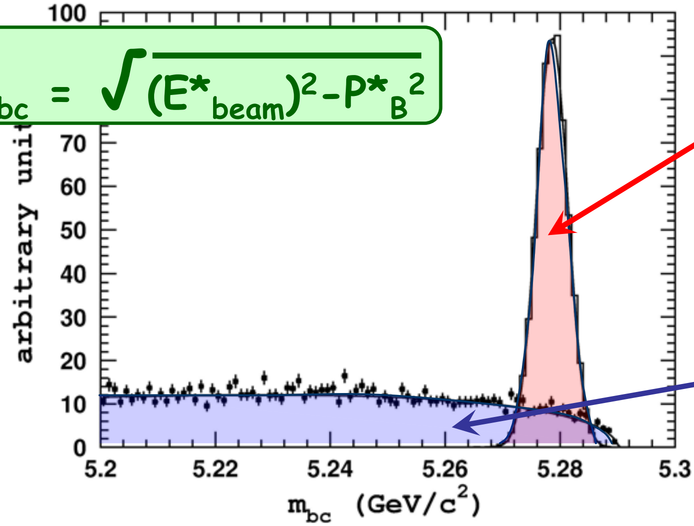
KEKB runs here

Only 20 MeV above $B\bar{B}$ threshold
==> no energy for extra particles

Identification of the B signal

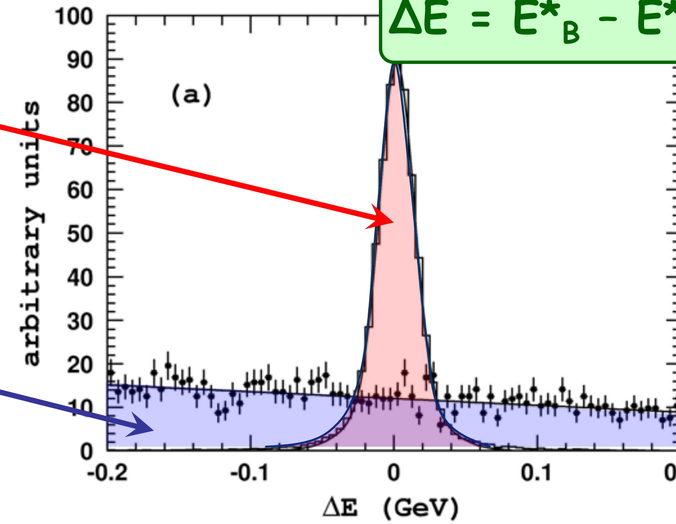
$$M_{bc} = \sqrt{(E^*_{beam})^2 - P^*_B{}^2}$$

$$\Delta E = E^*_B - E^*_{beam}$$

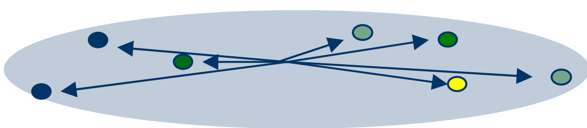


B signal

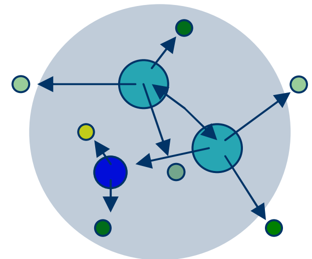
Background

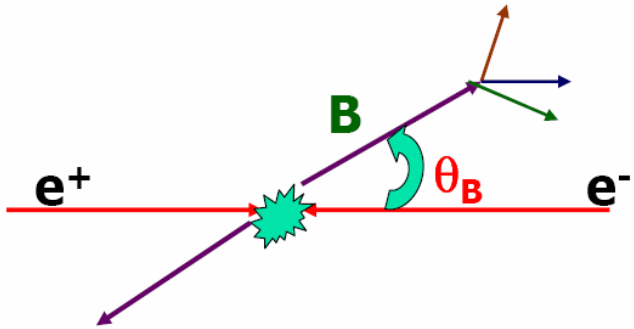


Jetty $e^+e^- \to q\bar{q}$ continuum event



Spherical $B\bar{B}$ event

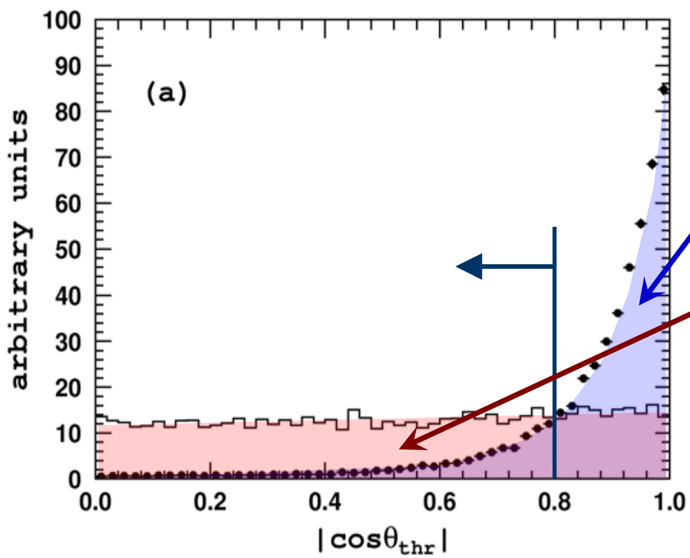




Angle between the two thrust axis:
- B candidate
- rest of the event

$$F = \Sigma \alpha_j x_j$$

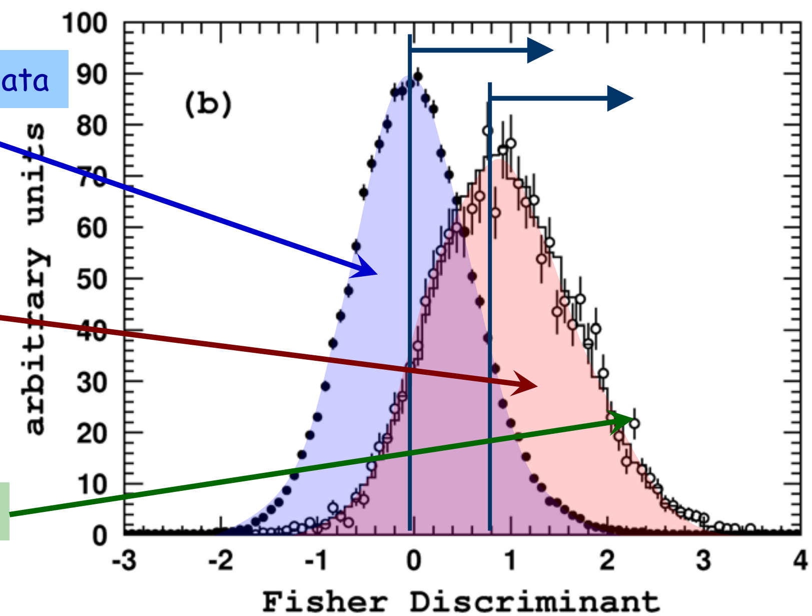
$$T = \max_{\vec{n}} \frac{\Sigma_i \, |(\vec{p}_i \vec{n})|}{\Sigma_i \, |p_i|}$$



OFF Resonance Data

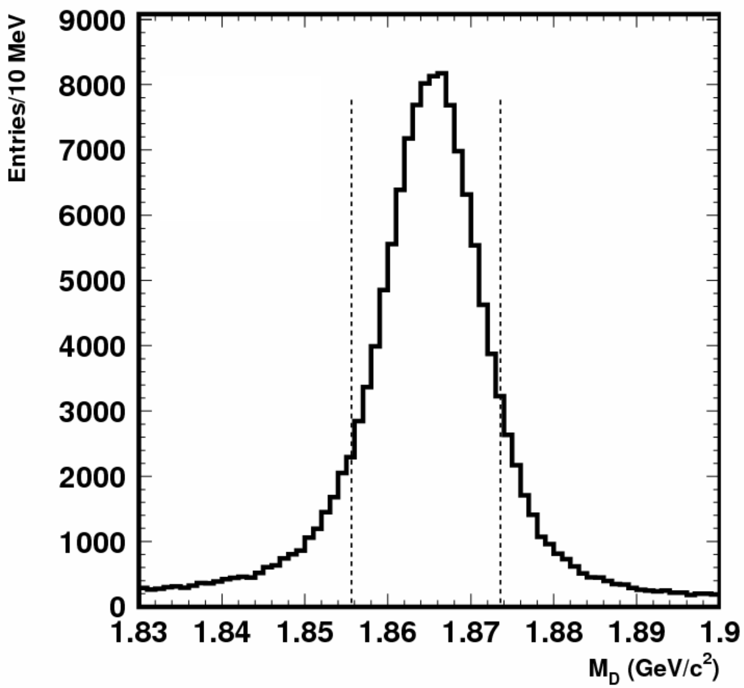
Signal MC

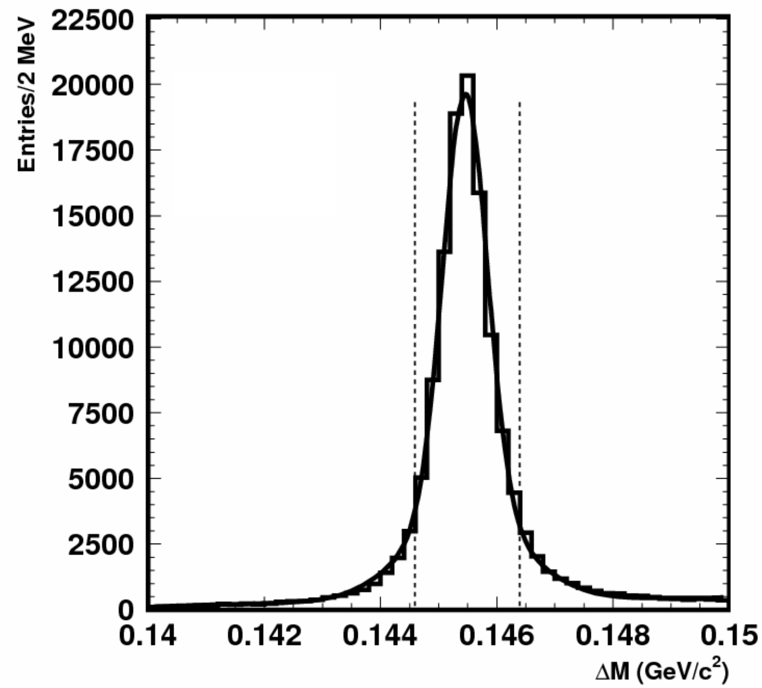
$B^- \to D^0\pi^-$ Data



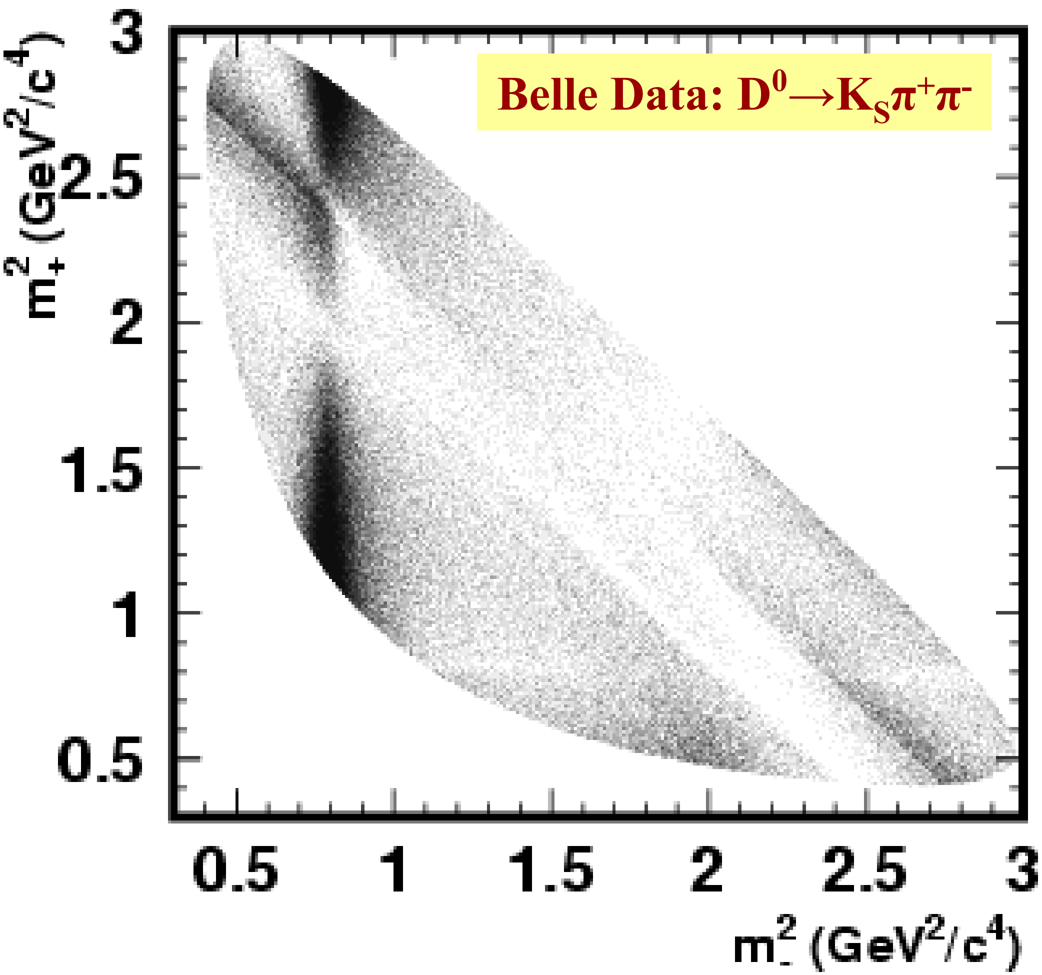
Analysis: $D^0 \to K_s\pi^+\pi^-$ Decay

Reconstruct $D^{*\pm} \to D[K_S\pi^+\pi^-]\pi_s^\pm$ from continuum $e^+e^- \to cc$ production.
Charge of the soft pion π_s from D^* tags the neutral D meson flavor.





With a 357fb^{-1} data sample we reconstruct **262K** $D^{*\pm} \to D\pi^\pm \to [K_S\pi^+\pi^-]\pi^\pm$ signal events with a background fraction of 3.2%



Construct a model (a coherent sum of quasi-two-body amplitudes plus a non-resonant term) to fit the Dalitz plot.

Non-unique choice of the model results in a model-dependent uncertainty in the γ measurement.

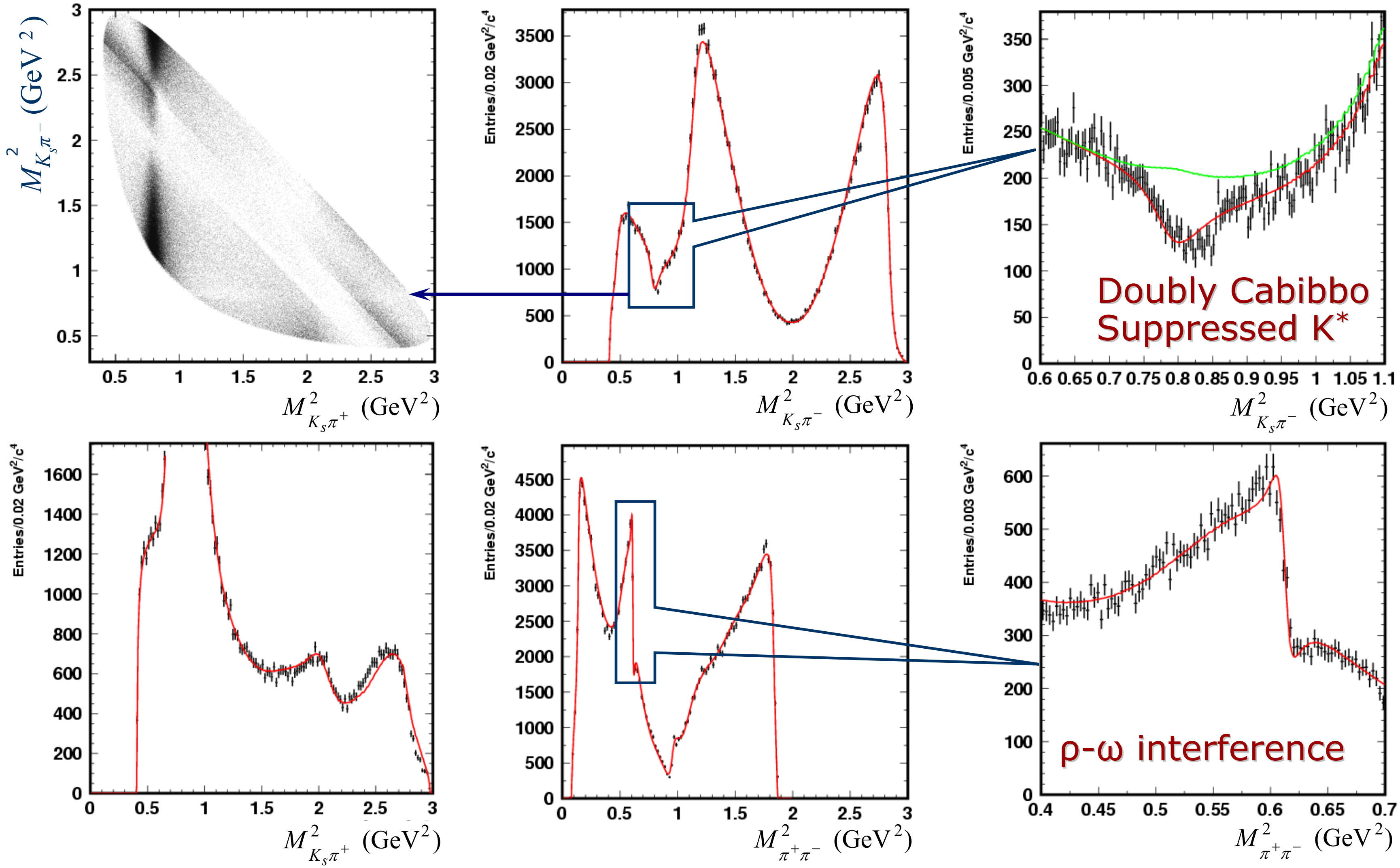
Analysis: $D^0 \to K_S\pi^+\pi^-$ Decay

Parameterization of the D^0 amplitude (isobar model)

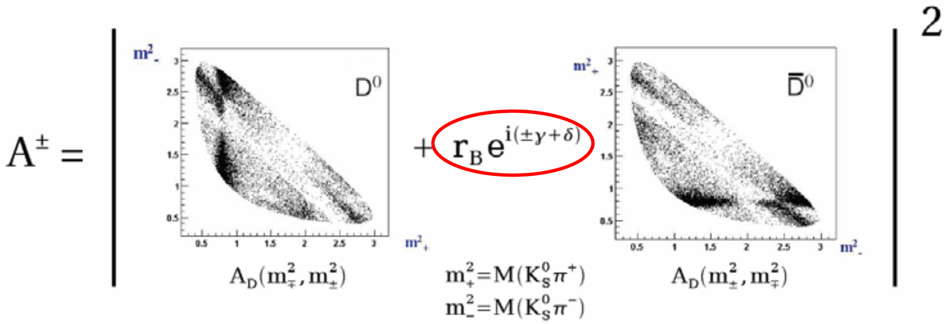
Intermediate state	Amplitude	Phase, °	Fit fraction
$K_S\,\sigma_1$ (M=520±15 MeV, Γ=466±31 MeV)	1.43±0.07	212±4	9.8%
$K_S\,\rho(770)$	1 (fixed)	0 (fixed)	21.6%
$K_S\,\omega$	0.0314±0.0008	110.8±1.6	0.4%
$K_S\,f_0(980)$	0.365±0.006	201.9±1.9	4.9%
$K_S\,\sigma_2$ (M=1059±6 MeV, Γ=59±10 MeV)	0.23±0.02	237±11	0.6%
$K_S\,f_2(1270)$	1.32±0.04	348±2	1.5%
$K_S\,f_0(1370)$	1.44±0.10	82±6	1.1%
$K_S\,\rho(1450)$	0.66±0.07	9±8	0.4%
$K^*(892)^+\pi^-$	1.644±0.010	132.1±0.5	61.2%
$K^*(892)^-\pi^+$	0.144±0.004	320.3±1.5	0.55%
$K^*(1410)^+\pi^-$	0.61±0.06	113±4	0.05%
$K^*(1410)^-\pi^+$	0.45±0.04	254±5	0.14%
$K^*_0(1430)^+\pi^-$	2.15±0.04	353.6±1.2	7.4%
$K^*_0(1430)^-\pi^+$	0.47±0.04	88±4	0.43%
$K^*_2(1430)^+\pi^-$	0.88±0.03	318.7±1.9	2.2%
$K^*_2(1430)^-\pi^+$	0.25±0.02	265±6	0.09%
$K^*(1680)^+\pi^-$	1.39±0.27	103±12	0.36%
$K^*(1680)^-\pi^+$	1.2±0.2	118±11	0.11%
Nonresonant	3.0±0.3	164±5	9.7%

Analysis: $D^0 \to K_s\pi^+\pi^-$ Decay

$D^* \to D^0\pi^+ \to [K_S\pi^+\pi^-]\pi^+$



Analysis: Sensitivity to γ

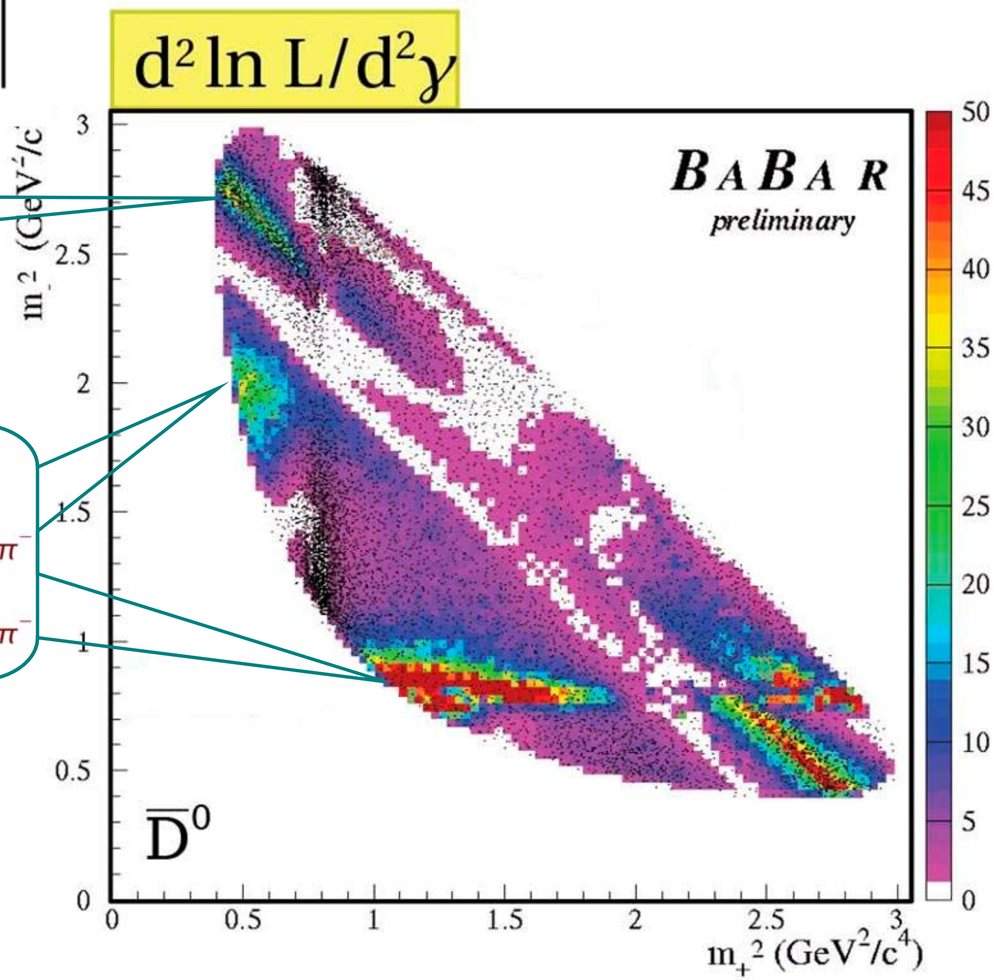


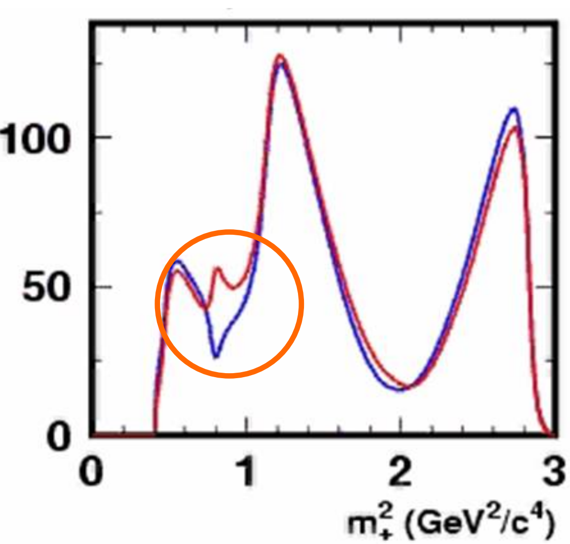
Sensitivity to γ (MC)

$\gamma=75°$, $\delta=180°$, $r_B=0.125$

GLW like
Interference of
$B^- \to D^0 K^-$, $D^0 \to K_S^0 \rho^0$
with
$B^- \to \overline{D}^0 K^-$, $\overline{D}^0 \to K_S^0 \rho^0$

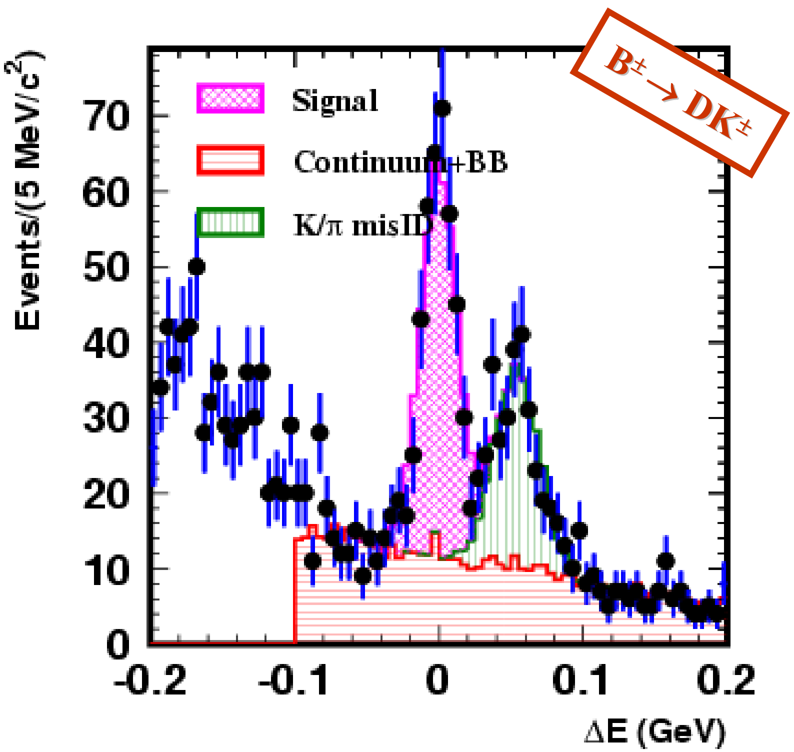
ADS like
Interference of
$B^- \to D^0 K^-$, $D^0 \to K^{*+} \pi^-$
with
$B^- \to \overline{D}^0 K^-$, $\overline{D}^0 \to K^{*+} \pi^-$



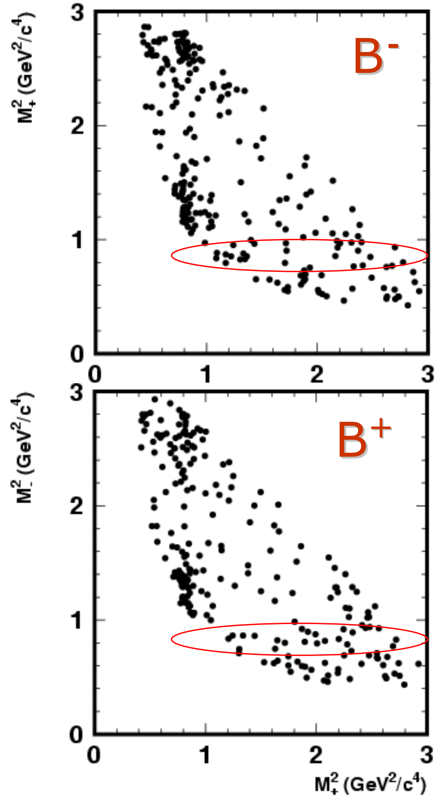


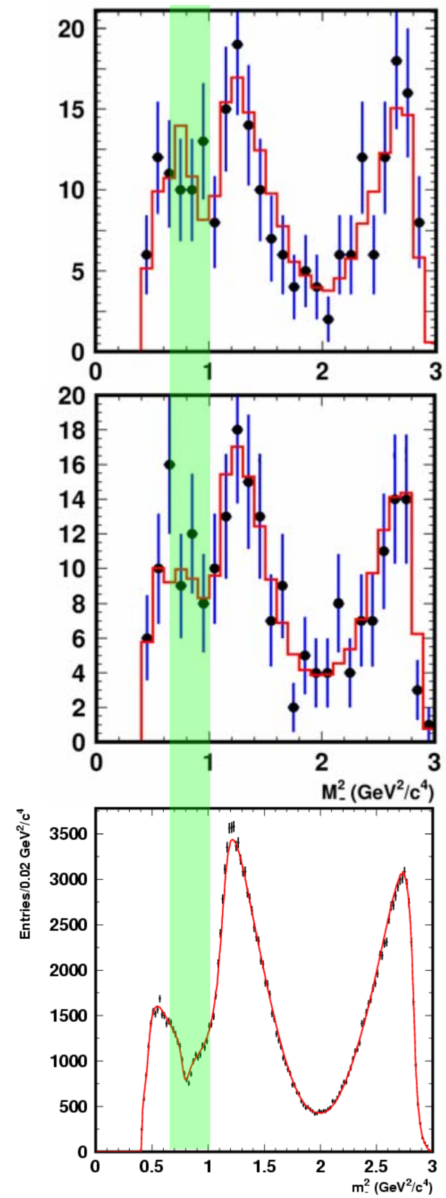
Results: B±->DK± Signal

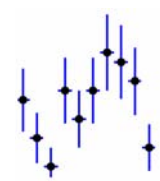
Belle results (357 fb^{-1})



331±17 B± signal events



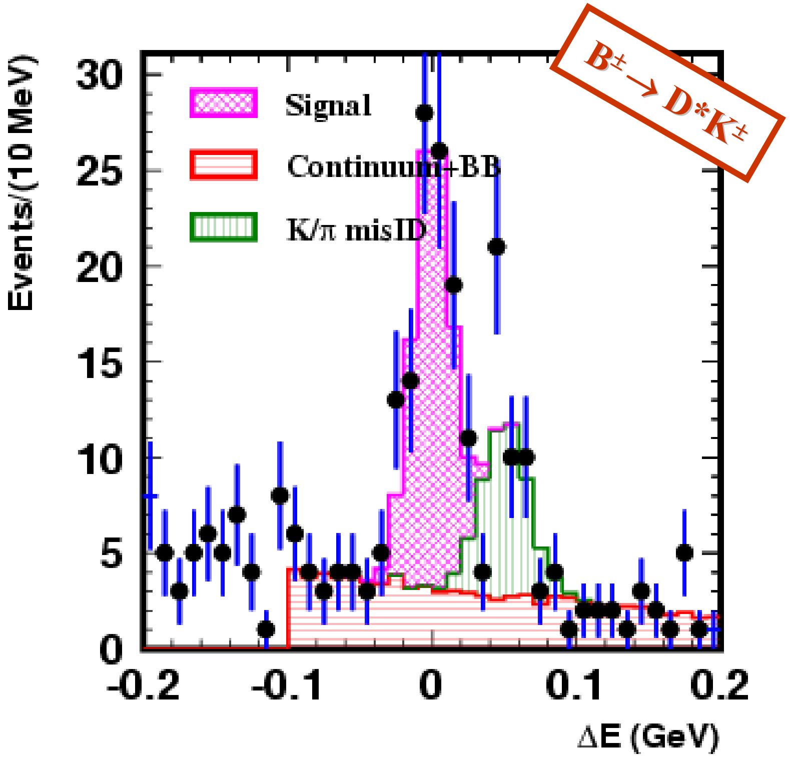


 - Data

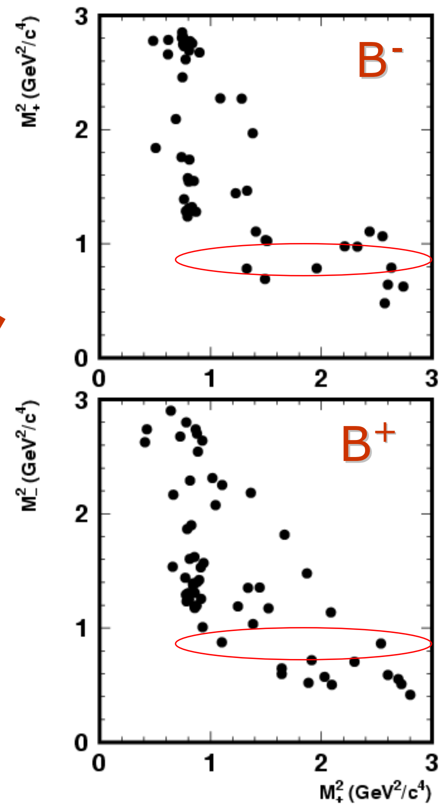
 - Fit

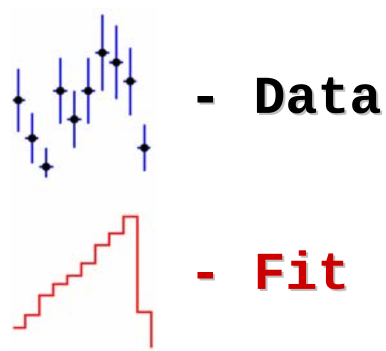
Results: $B^\pm \to D^* K^\pm$ Signal

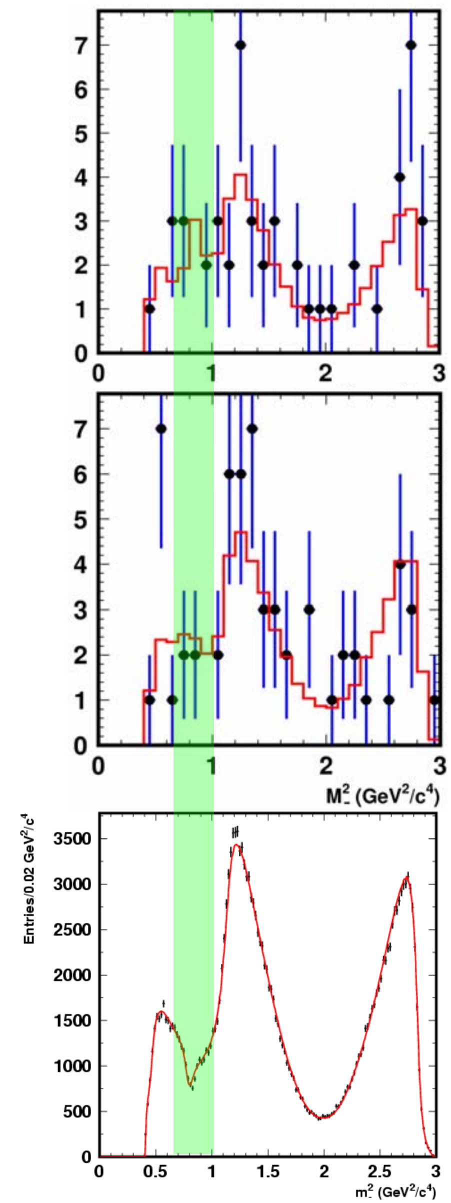
Belle results (357 fb^{-1})



81±8 B± signal events

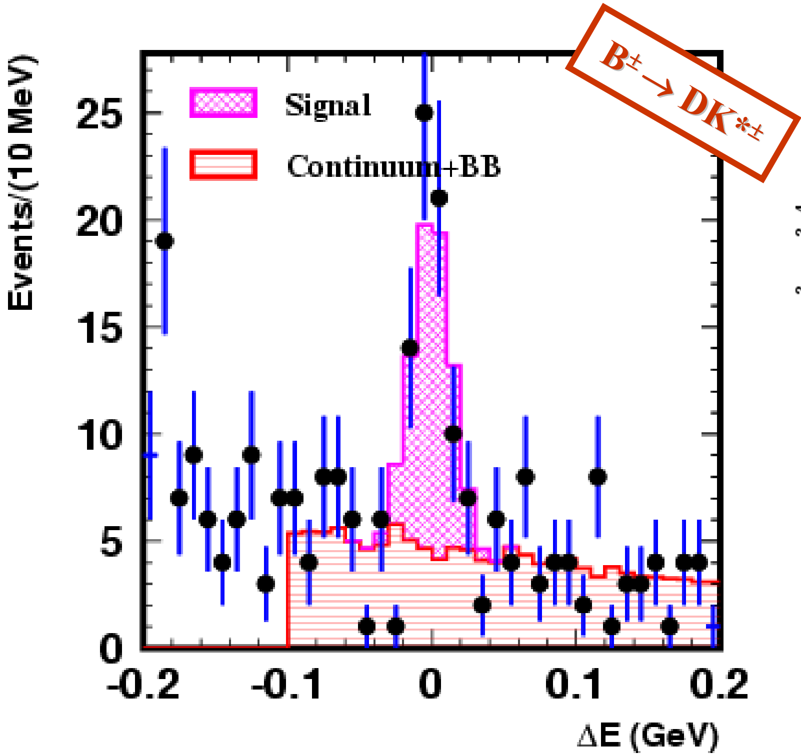




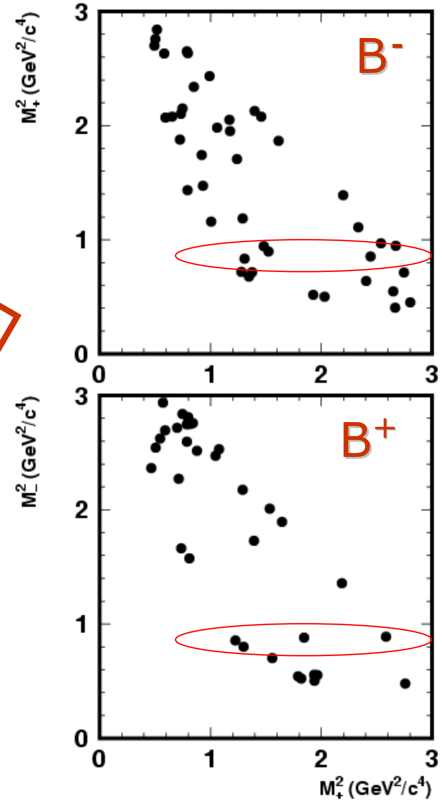


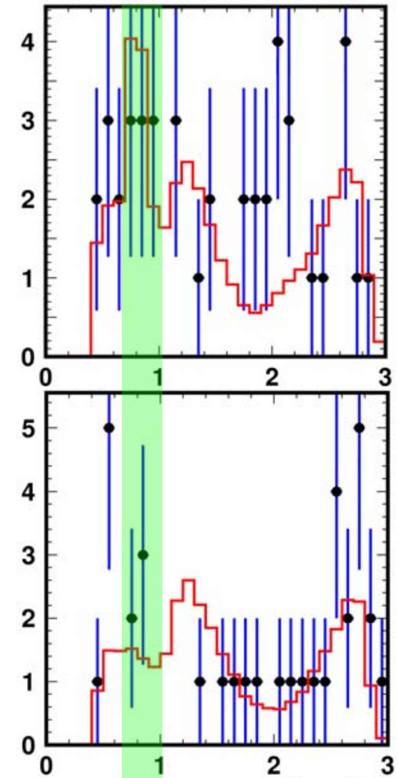
Results: $B^{\pm} \to DK^{*\pm}$ Signal

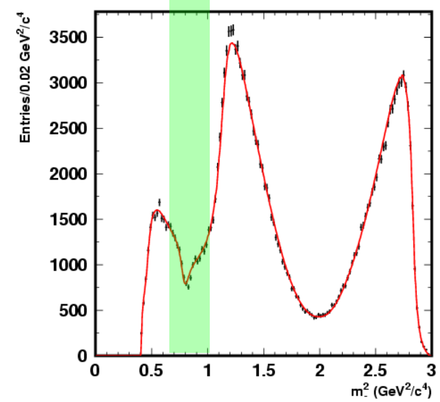
Belle results (357 fb^{-1})

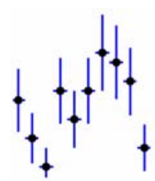


54±8 B± signal events







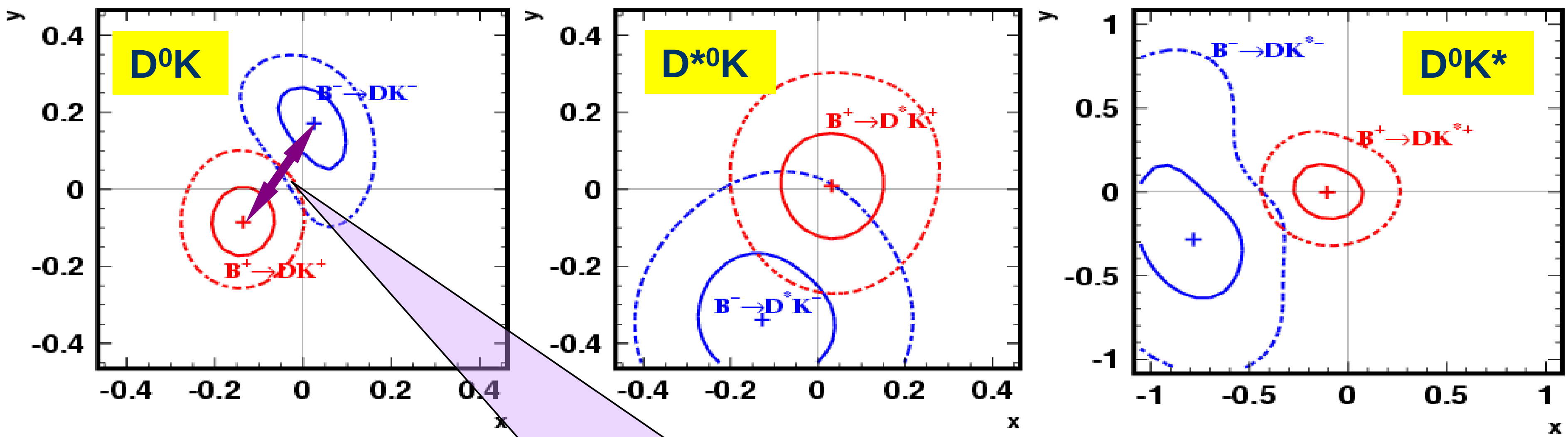
 - Data

 - Fit

Results: x-y plane

Fit B^+ and B^- samples separately

$$x_\pm = r_\pm \cos(\pm\phi_3 + \delta) \qquad y_\pm = r_\pm \sin(\pm\phi_3 + \delta)$$



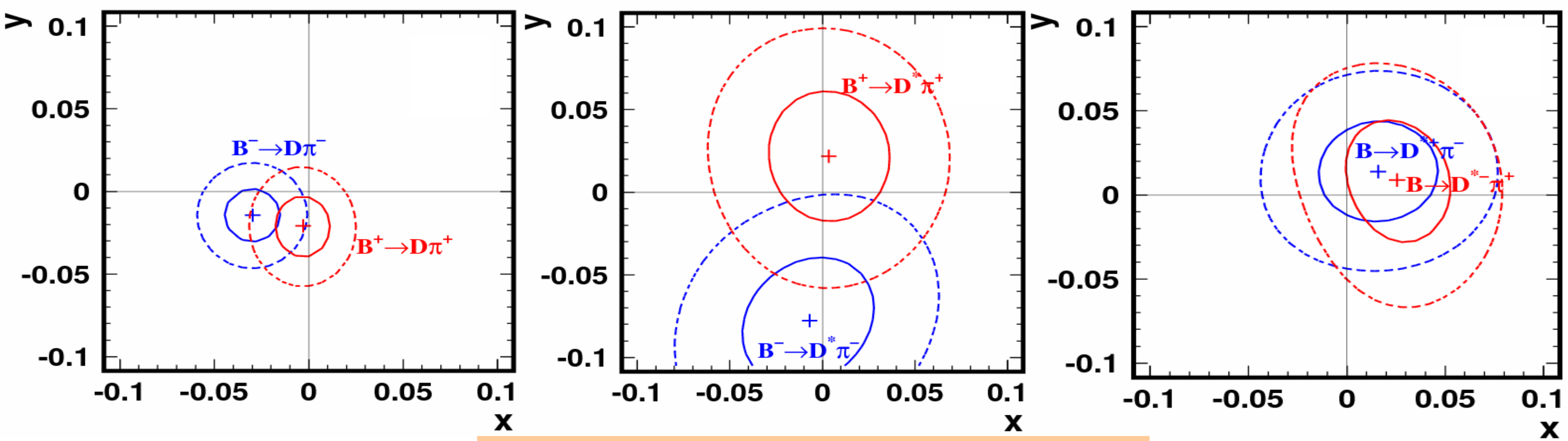
$$d = \sqrt{(x_- - x_+)^2 + (y_- - y_+)^2} = 2r_B|\sin\gamma|$$

Results: Control Sample

As a cross check of the fitting procedure we perform a fit to several control samples: $B^- \to D^{(*)0}\pi^-$ and $B^0 \to D^{*-}\pi^+$

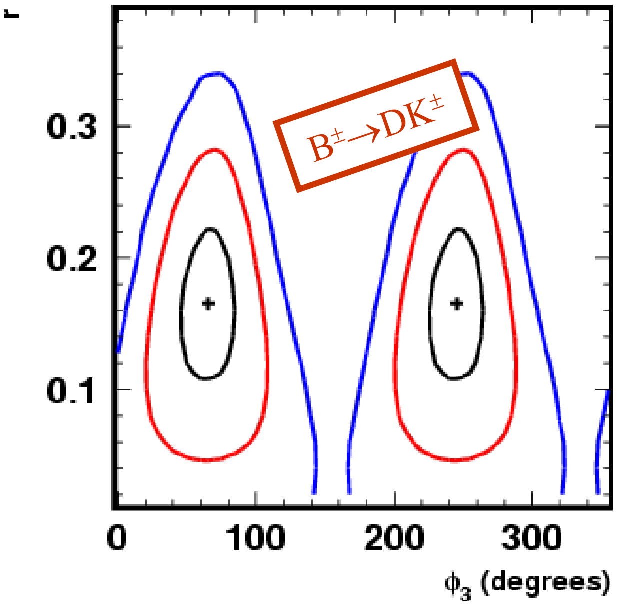
$$|V_{ub}V^*_{cd}|/|V_{cb}V^*_{ud}| \sim 0.02$$
$$r \sim 0.01$$

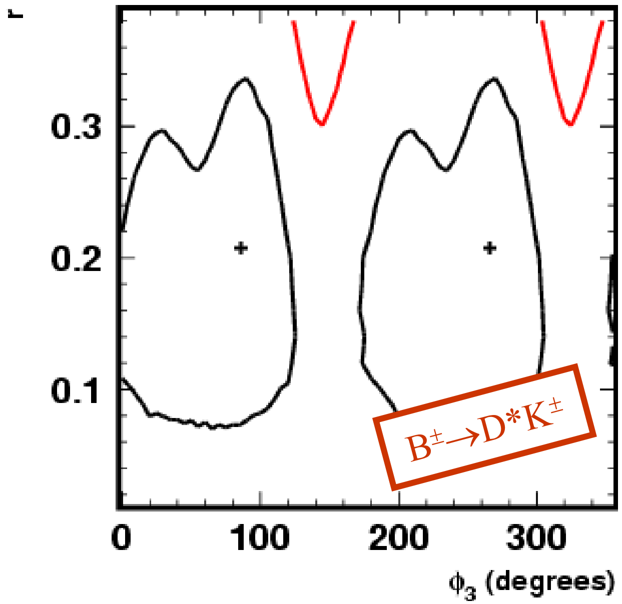
$$r = 0.0$$

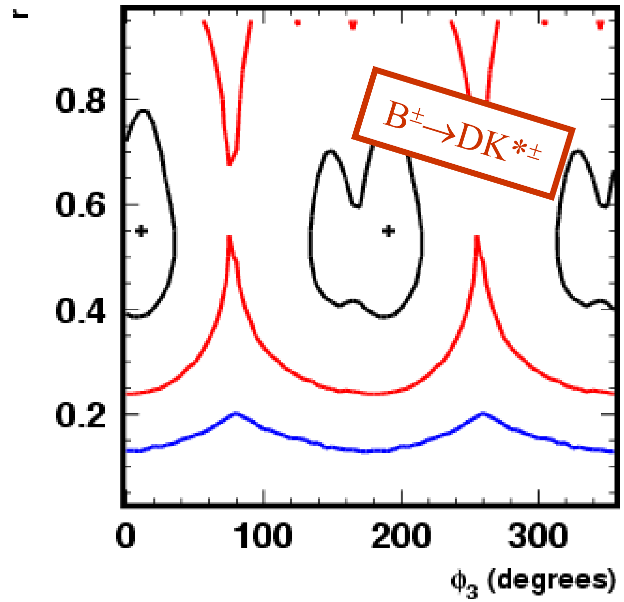


Consistent with expectations

Results: r_B-γ plane







$\gamma = 66^{+19}_{-20}{}^\circ$(stat)

$\gamma = 86^{+37}_{-93}{}^\circ$(stat)

$\gamma = 11^{+23}_{-57}{}^\circ$(stat)

3 modes combined:

CPV significance: 78%

$\gamma = 53^{+15}_{-18} \pm 3\,(\mathrm{syst}) \pm 9\,(\mathrm{model})^\circ$

$8^\circ < \gamma < 111^\circ$ (2σ interval)

DK : $r_B = 0.159^{+0.054}_{-0.050} \pm 0.012(\mathrm{syst}) \pm 0.049(\mathrm{model})$; $\delta = (146^{+19}_{-20})^\circ$
D*K: $r_B = 0.175^{+0.108}_{-0.099} \pm 0.013(\mathrm{syst}) \pm 0.049(\mathrm{model})$; $\delta = (302^{+34}_{-35})^\circ$
DK*: $r_B = 0.564^{+0.216}_{-0.155} \pm 0.041(\mathrm{syst}) \pm 0.084(\mathrm{model})$; $\delta = (243^{+20}_{-23})^\circ$

Results: γ vs. Statistics

$152\times10^6\, B\bar{B}$ [*PRD70*, 072003 (2004)]

$$r_B = 0.26^{+0.10}_{-0.14} \pm 0.03 \pm 0.004$$

$$r_B^* = 0.20^{+0.19}_{-0.17} \pm 0.02 \pm 0.04$$

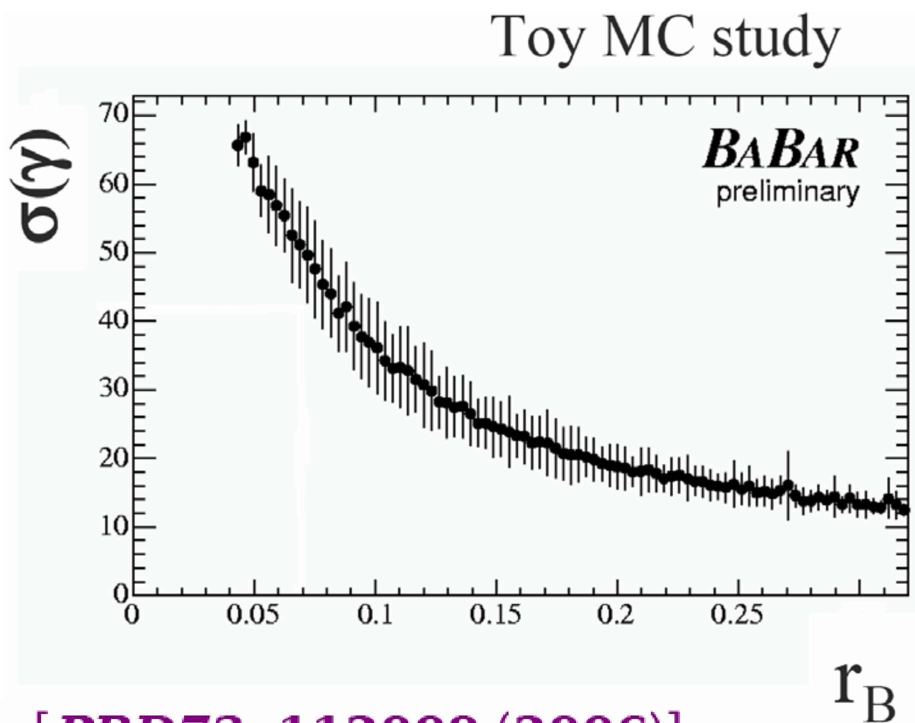
$$\gamma = 77^{\circ}{}^{+17^{\circ}}_{-19^{\circ}}(stat) \pm 13^{\circ}(syst) \pm 11^{\circ}(model)$$

$275\times10^6\, B\bar{B}$ [*BELLE–CONF–0476*]

$$r_B = 0.21 \pm 0.08 \pm 0.03 \pm 0.004$$

$$r_B^* = 0.12^{+0.16}_{-0.11} \pm 0.02 \pm 0.04$$

$$\gamma = 68^{\circ}{}^{+14^{\circ}}_{-15^{\circ}}(stat) \pm 13^{\circ}(syst) \pm 11^{\circ}(model)$$



this analysis: $386\times10^6\, B\bar{B}$ [*PRD73*, 112009 (2006)]

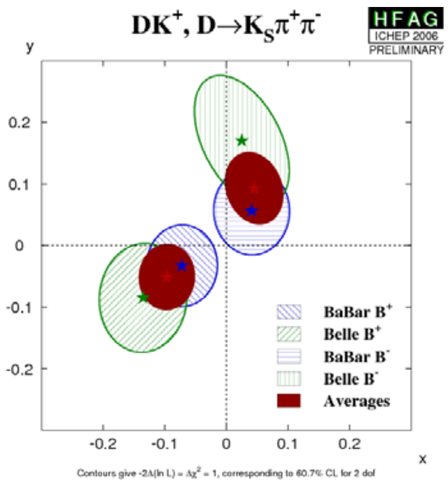
$$r_B = 0.159^{+0.054}_{-0.050} \pm 0.012 \pm 0.049 \qquad r_B^* = 0.175^{+0.108}_{-0.099} \pm 0.013 \pm 0.049$$

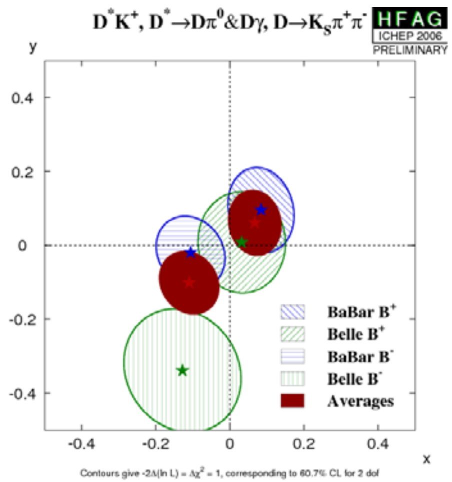
$$\gamma = 53^{\circ}{}^{+15^{\circ}}_{-18^{\circ}}(stat) \pm 3^{\circ}(syst) \pm 9^{\circ}(model)$$

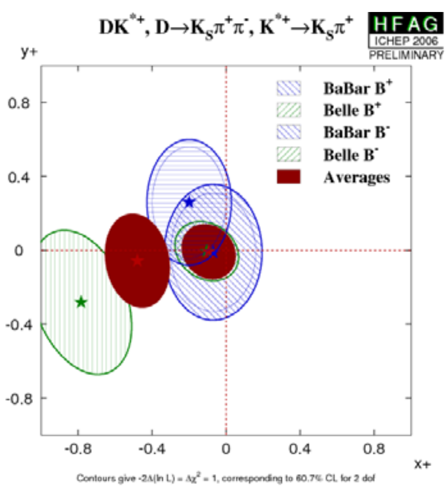
Strong dependence of the stat. error of the (unknown) r_B value
➔ hard to predict evolution of $\delta\gamma$ with statistics

Results: Summary

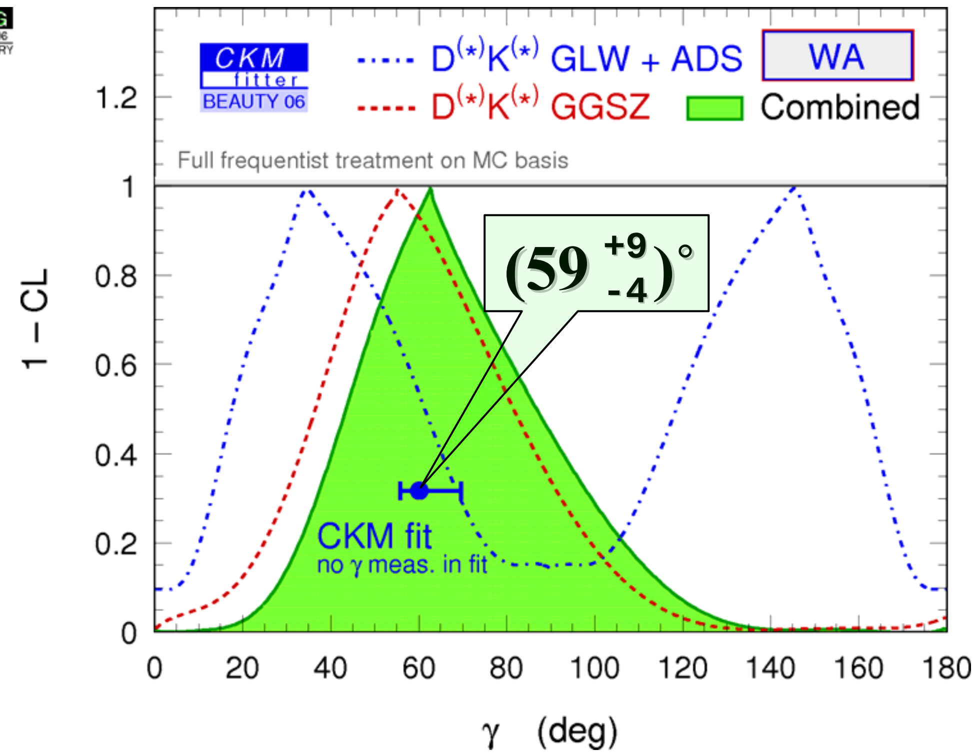
GGSZ: Belle vs. BaBar







ADS+GLW+GGSZ Belle only

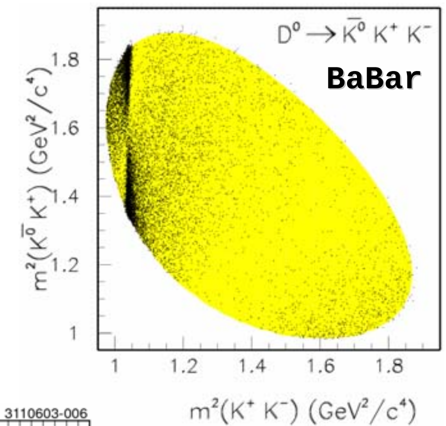


ADS+GLW+GGSZ Belle+BaBar

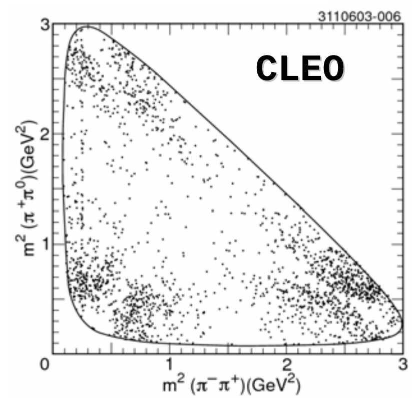
$$\gamma = (62^{+32}_{-28})^\circ$$

Prospects: Other D^0 Modes

- $K_S\pi^+\pi^-$: $\boxed{\text{BF} \approx 2\%}$ (including $K_S \to \pi^+\pi^-$ BF)
  $\equiv BF_0$ as normalization channel



- $K_SK^+K^-$: BF $\approx BF_0/6$
  Dominated by $a(980)^0K_S$, $\varphi(1020)K_S$ and $a(980)^+K^-$
  Sensitivity to γ might be provided by interference between $\varphi(1020)$ and $a(980)^+$



- $\pi^+\pi^-\pi^0$: BF $\approx BF_0/2$
  Dominated by three $\rho\pi$ channels
  ~100% overlapping between D^0 and $\bar{D}^0$ => large sensitivity to γ is expected
  High combinatorial background due to π^0

- Four-Body: $K_S\pi^+\pi^-\pi^0$ (BF $\approx 2\times BF_0$), $K^+K^-\pi^+\pi^-$ (BF $\approx BF_0/10$)
  Complicated resonant substructure => model uncertainty might be large

Model-independent approach

- D^0 flavor specific decay amplitude: $f = |f(m_+^2, m_-^2)|\, e^{i\phi(m_+^2, m_-^2)}$
- D^0-$\bar{D}^0$ interference from $B^+ \to D^0 K^+$:

$$A_{\tilde{D}^0} = |f(m_+^2, m_-^2)|\, e^{i\phi(m_+^2, m_-^2)} + re^{i\theta}\, |f(m_-^2, m_+^2)|\, e^{i\phi(m_-^2, m_+^2)}$$

$$= |f(m_+^2, m_-^2)| + re^{i\theta}\, |f(m_-^2, m_+^2)|\, e^{i[\phi(m_+^2, m_-^2) - \phi(m_-^2, m_+^2)]}$$

$|f|$ is measured directly, $\phi(m_+^2, m_-^2) - \phi(m_-^2, m_+^2)$ is model-dependent

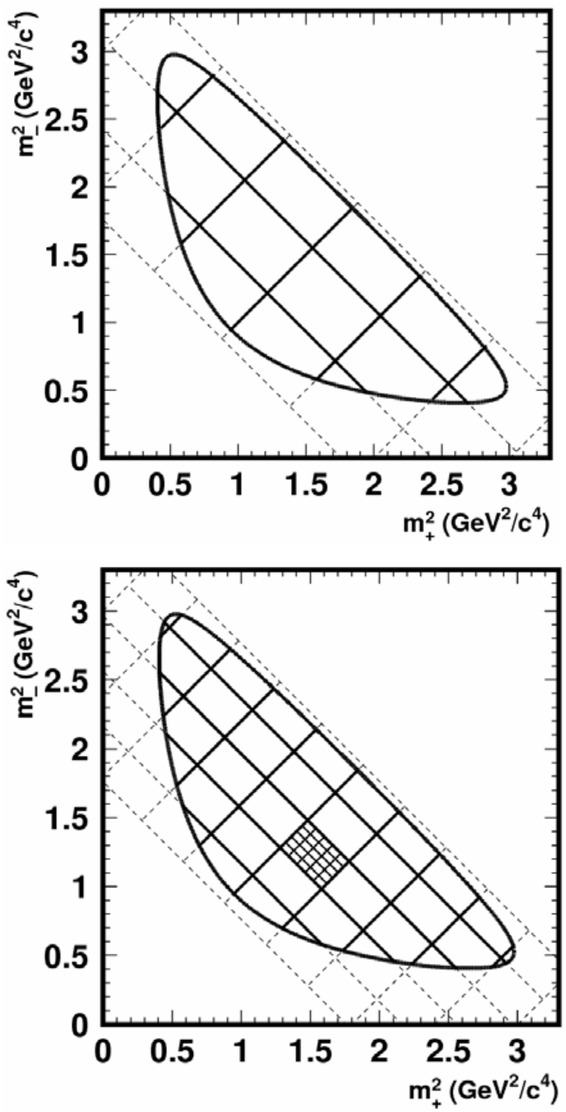
- If CP-tagged D are available (e.g. from $\psi'' \to D^0 \overline{D^0}$, where tag-side D decays into CP-eigenstate) phase difference can also be measured:

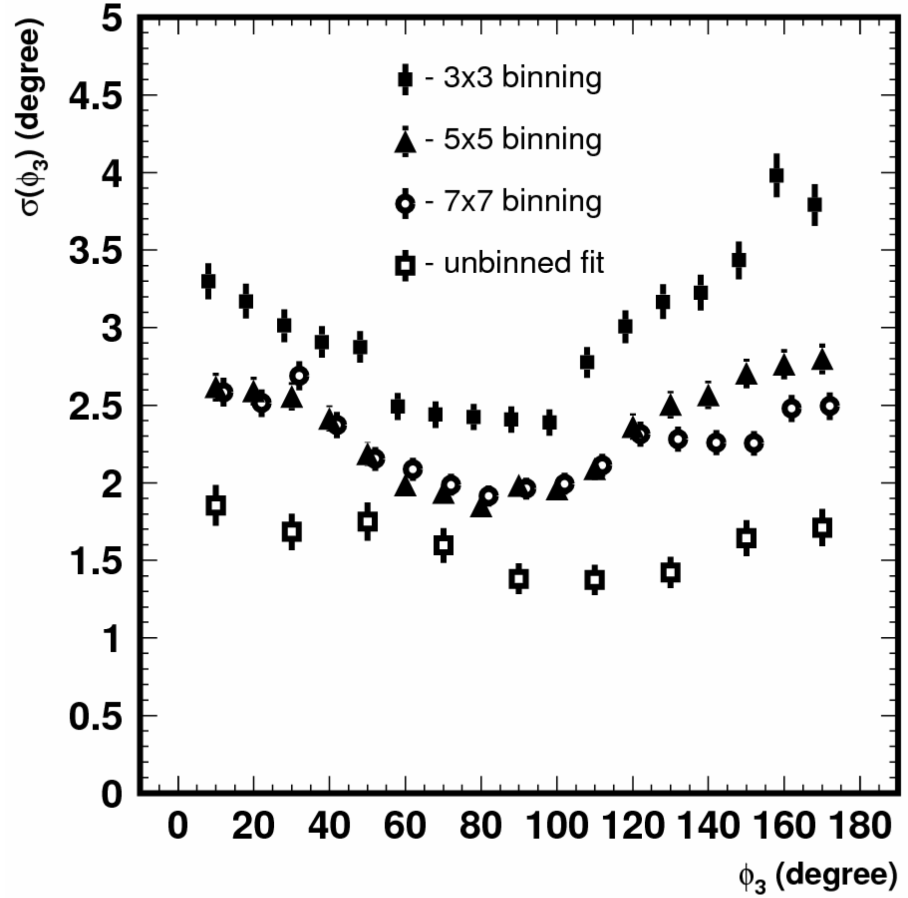
$$A_{CP} = \frac{|f(m_+^2, m_-^2)|\, e^{i\phi(m_+^2, m_-^2)} \pm |f(m_-^2, m_+^2)|\, e^{i\phi(m_-^2, m_+^2)}}{\sqrt{2}}$$

$$= \frac{|f(m_+^2, m_-^2)| \pm |f(m_-^2, m_+^2)|\, e^{i[\phi(m_-^2, m_+^2) - \phi(m_+^2, m_-^2)]}}{\sqrt{2}}$$

Model-independent approach

A.Bondar, A.Poluektov hep-ph/0510246



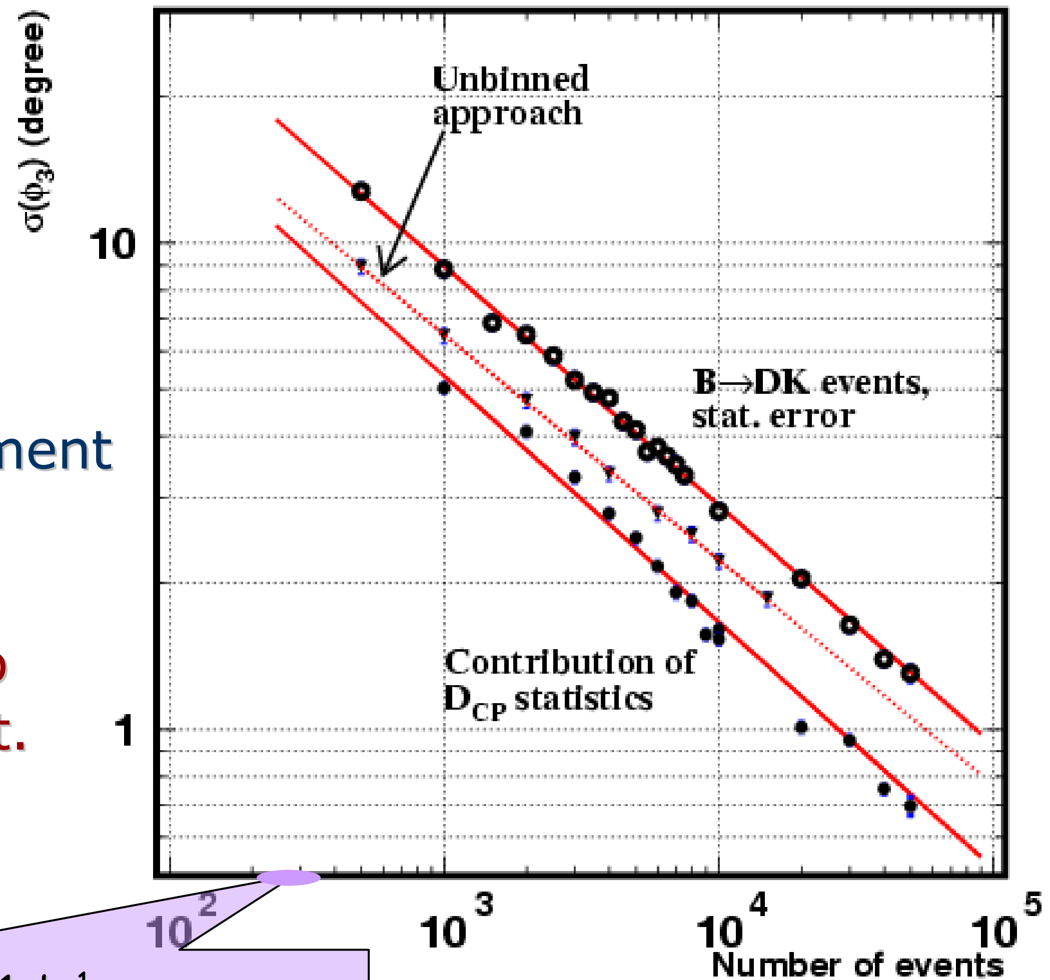


MC studies show that (given sufficient D_{CP} sample) $2\mathrm{x}10^4$ signal $B \to DK$ events and "5x5" DP binning is enough to extract γ with ~3° uncertainty

Model-independent Approach

A.Bondar, A.Poluektov hep-ph/0510246

A.Giri, Yu. Grossman, A. Soffer, J. Zupan, PRD 68, 054018 (2003)



50 ab^{-1} at SuperB factory should be enough for model-independent measurement of γ with accuracy below 2°

~10 fb^{-1} at $\psi(3770)$ needed to accompany this measurement.

Current CLEO **ψ(3770)** statistic is 281pb^{-1}
Expected factor 3 increase by 2008 (hep-ex/0608008)

Summary

- GGSZ method has been proven to be the most powerful one (amongst known today) in providing information on weak phase γ

- Combining all the information from Belle and BaBar direct measurements give

$$\gamma = (62^{+32}_{-28})^\circ$$

in perfect agreement with SM fit: $(59^{+9}_{-4})^\circ$

- Precision of γ depends strongly on (unknown) parameter r_B. By 2008 we expect to have $1ab^{-1}$/experiment. If we are lucky enough, stat. accuracy will be $\sigma_\gamma \sim 10^\circ$