Testing Lorentz and CPT Invariance Using MINOS

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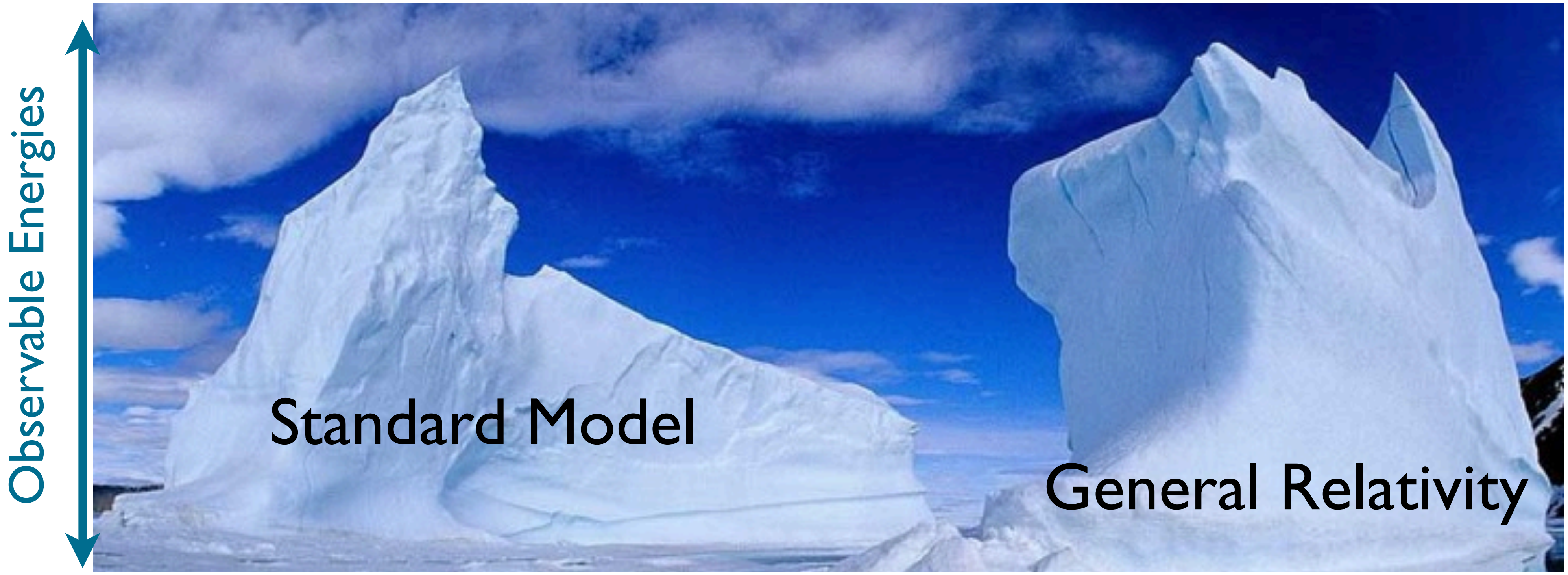
Outline

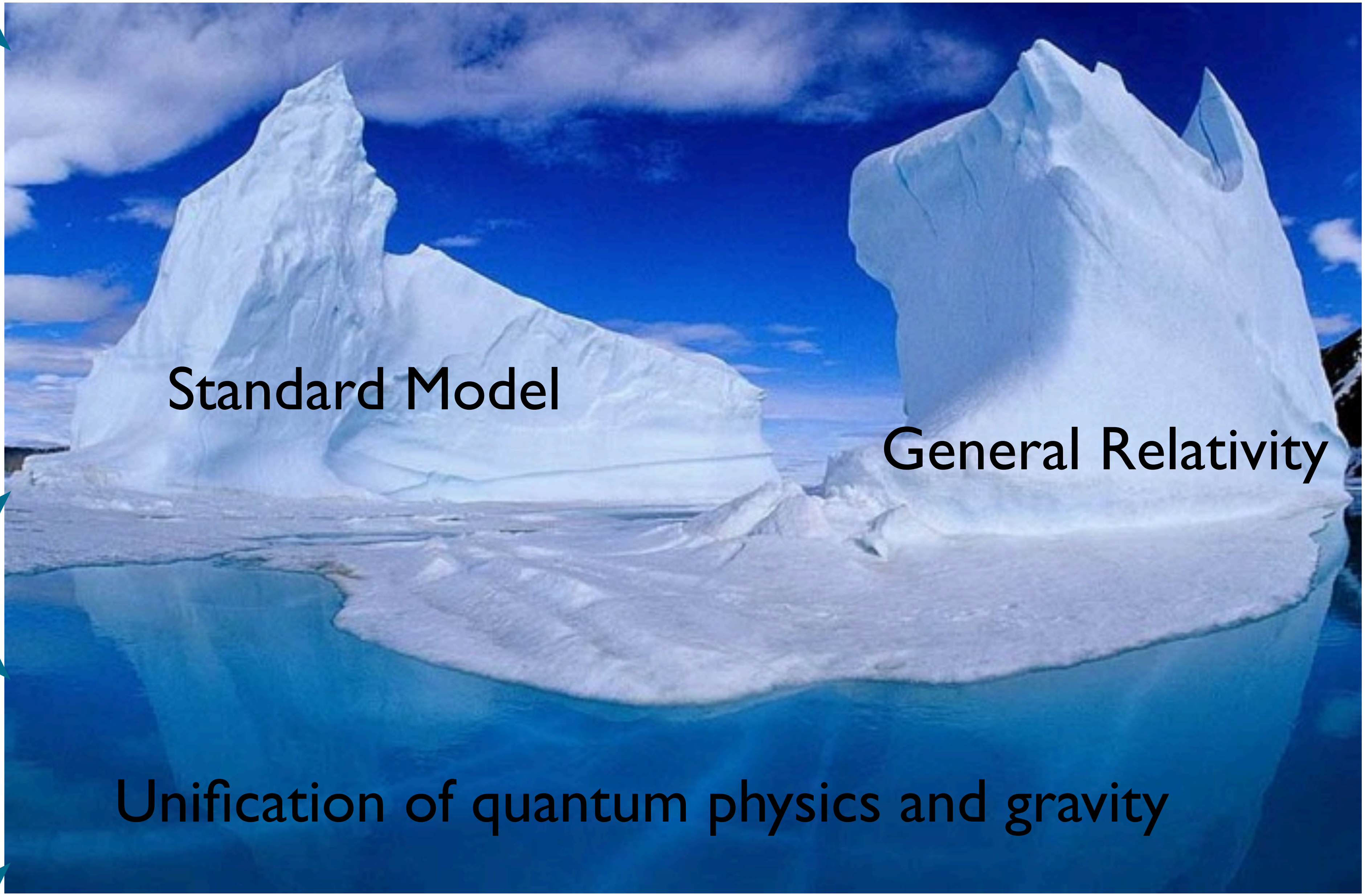
- A framework for Lorentz and CPT violation, the Standard Model Extension
- Introduction to NuMI and MINOS
- Search for sidereal variations in MINOS - the indication of Lorentz and CPT violation
- Conclusions

How to link Standard Model and General Relativity?



How to link Standard Model and General Relativity?

Observable Energies



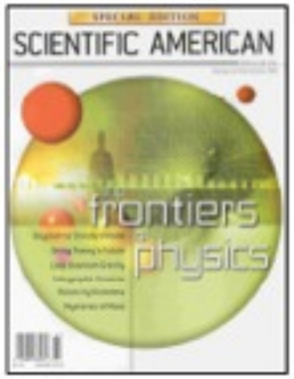
Planck Scale, $m_P = 10^{19}$ GeV

Growing Interest in Lorentz Violation

- Lorentz symmetry is a basic tenant of both the Standard Model and General Relativity - something that fundamental should be tested

- Last 20 years has seen a lot of interest in possibility that Lorentz symmetry can be broken

- Several candidate theories to explain quantum gravity involve breaking Lorentz symmetry:
    - String theory
    - Loop-quantum gravity
    - Non-commutative field theories



How to Look for Lorentz Violation

- Find a general framework that
  - accounts for all possible Lorentz violating effects
  - can be applied to analyze any experiment
- Of course, there are certain constraints:
  - Physical phenomena should be independent of coordinates
  - Any LV must be small because SM and GR are so successful at describing our observations
  - Can only use known forces and particles
- Standard Model Extension (SME) is such a framework

SME = SM + GR + ∑(everything that satisfies the constraints)

Colladay & Kostelecky, PRD 1997
Colladay & Kostelecky, PRD 1998
Kostelecky, PRD 2004

Key Features of the SME

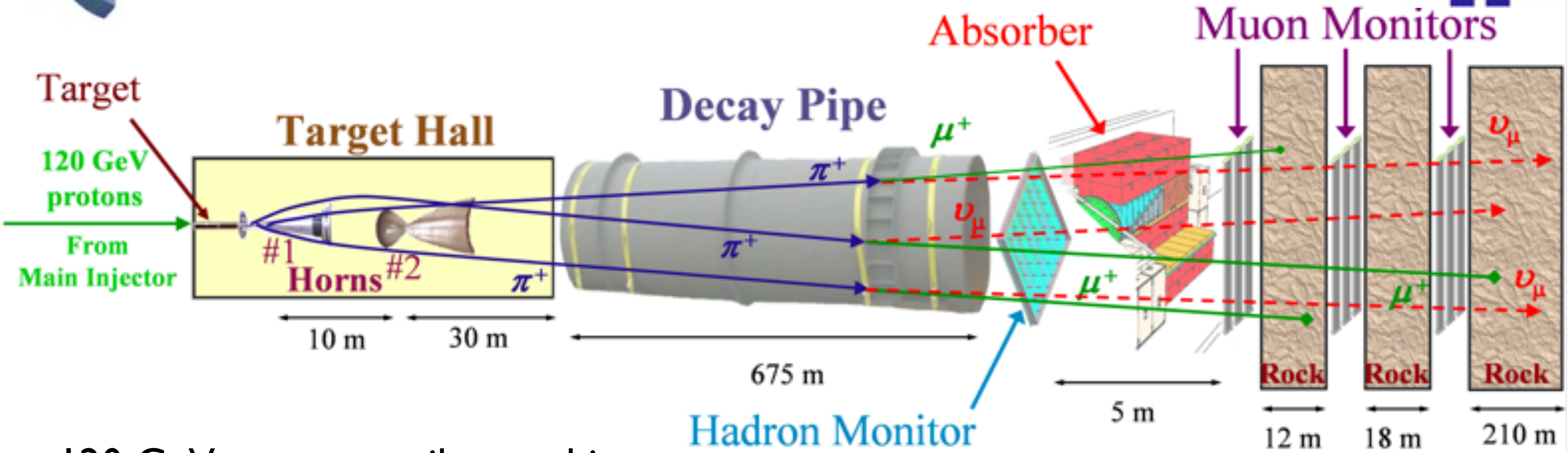
- Structure of the Standard Model is preserved
- Based on quantum field theory
- General framework for studying possible violation of Lorentz symmetry
- CPT violation is included
- Experimental results from different disciplines can be related in a physically meaningful way
- Tells what effects to look for in a given experiment
    - neutrino oscillations
    - oscillations and decays of K, B, D mesons
    - particle-antiparticle comparisons
    - CMB polarization
    - spectroscopy of hydrogen and anti-hydrogen
    - etc

More than 1000 papers published on the SME - see Data Tables for Lorentz and CPT Violation, Kostelecky & Russell, hep-ph/0801.0287v4

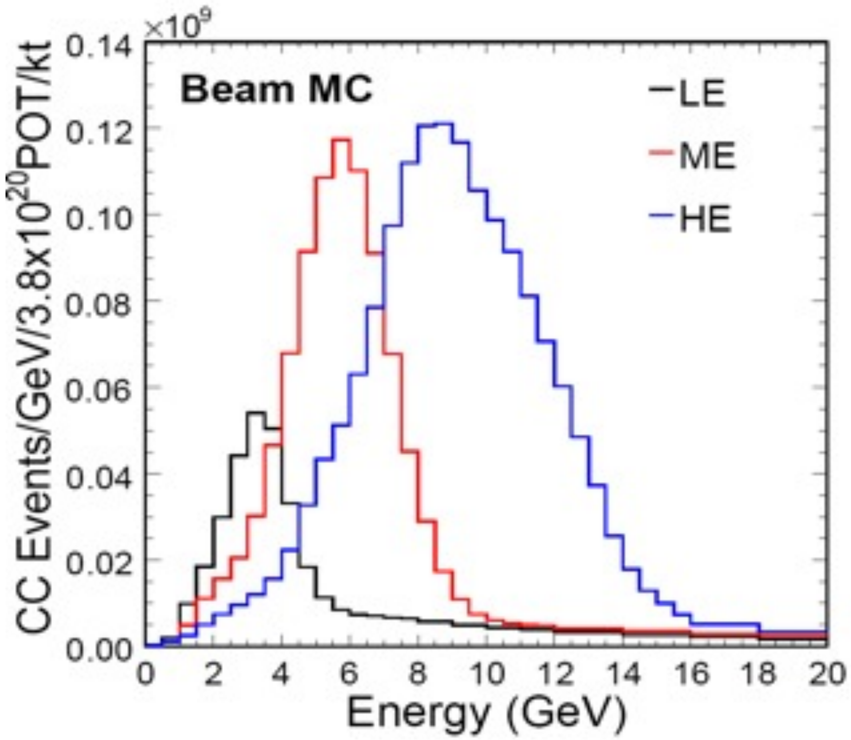
Thanks to Jorge Diaz (Indiana University) for supplying some of this introductory material

NuMI and MINOS

Neutrinos at the Main Injector Beam



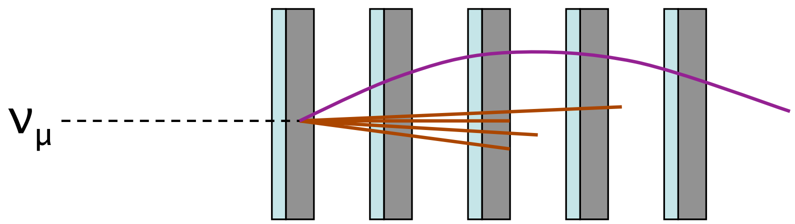
- 120 GeV protons strike graphite target
- Magnetic horns focus produced pions and kaons, pions and kaons decay into muons and neutrinos
- Target position adjusts to change beam energy
- 10 μs spills as fast as once every 2 seconds
- ~3 x 10^{20} POT/year - thanks to AD for delivering excellent beam



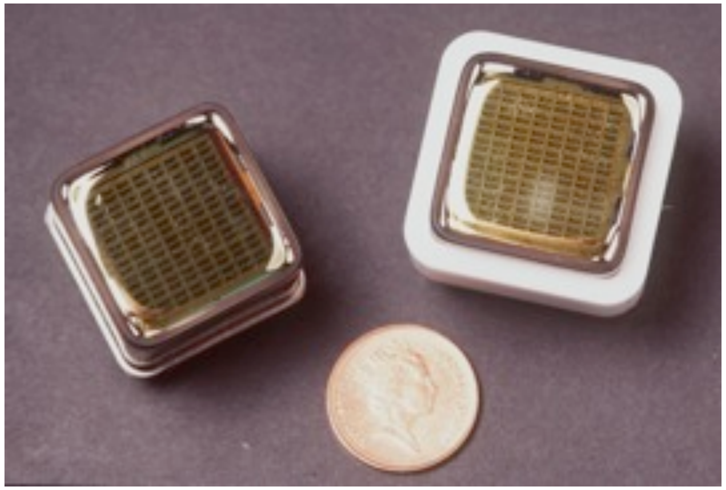
MINOS Overview

- Main Injector Neutrino Oscillation Search is a long baseline neutrino oscillation experiment
- Measure the neutrinos on site with the near detector
- Measure them again using far detector 734 km away in Soudan Mine and compare the two to get oscillation parameters
- MINOS main goal is to make a precision measurement of Δm^2_{32}
- Also search for sterile neutrinos, ν_e appearance in the beam, and measure $\bar{\nu}_\mu$ oscillation parameters
- Other analyses done opportunistically

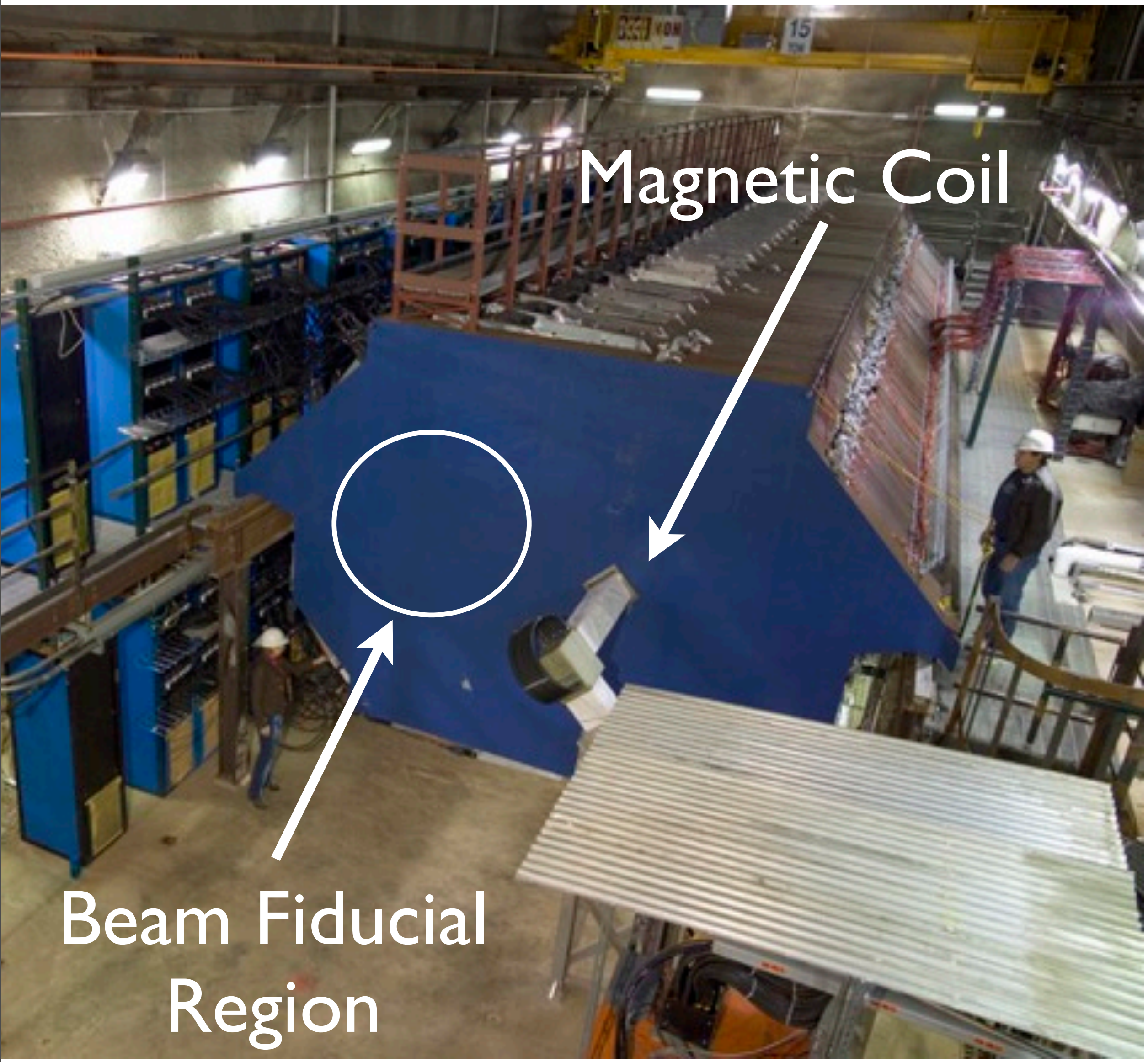
MINOS Detectors



- Detectors are alternating layers of steel and scintillator
- Strips of scintillator collected in aluminum cases and mounted to steel absorber
- Wavelength-shifting (green) fiber used to collect scintillation light
- Steel magnetized to $\langle B \rangle = 1.3$ T

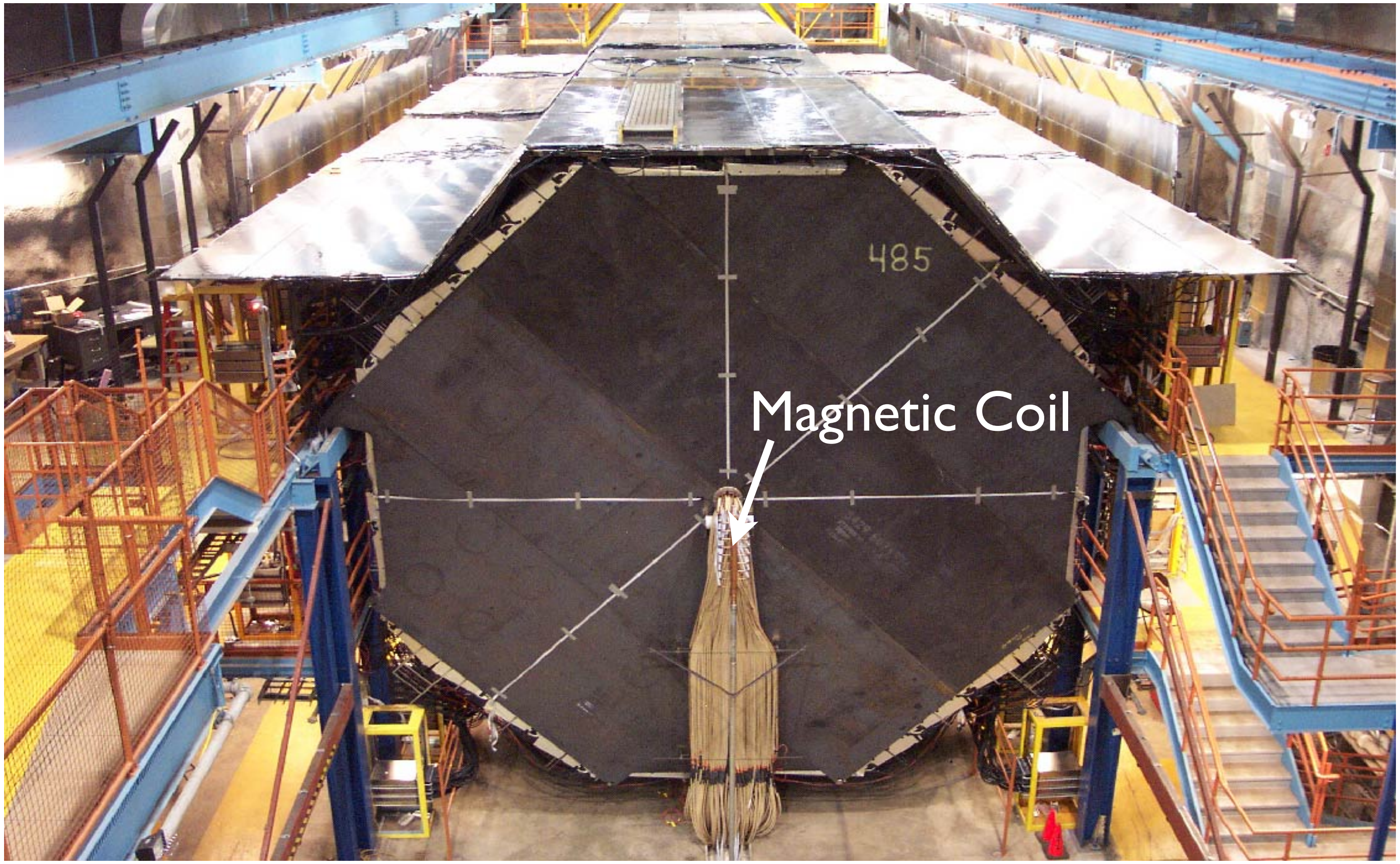


Near Detector



- 980 tons
- 4.8 m x 3.8 m squashed octagon
- Front end electronics designed for fast readout to handle high instantaneous neutrino rates
- Up to 20 neutrino related events in ND for every spill

Far Detector



- 5.4 kT, 8 m octagon, 484 instrumented planes, 735 km from target
- 2 super-modules
- Front end electronics capable of good timing resolution
- 1 - 3 neutrino events/day

The MINOS Collaboration

Argonne - Athens - Brookhaven - Caltech - Cambridge - Campinas - Fermilab - Goias - Harvard - Holy Cross - IIT - Indiana - Iowa State - Minnesota - Minnesota-Duluth - Otterbein - Oxford - Pittsburgh - Rutherford - Sao Paulo - South Carolina - Stanford - Sussex - Texas A&M - Texas - Tufts - University College London - Warsaw - William & Mary - Wisconsin

29 institutions, 120 scientists, funded by DOE, NSF, STFC

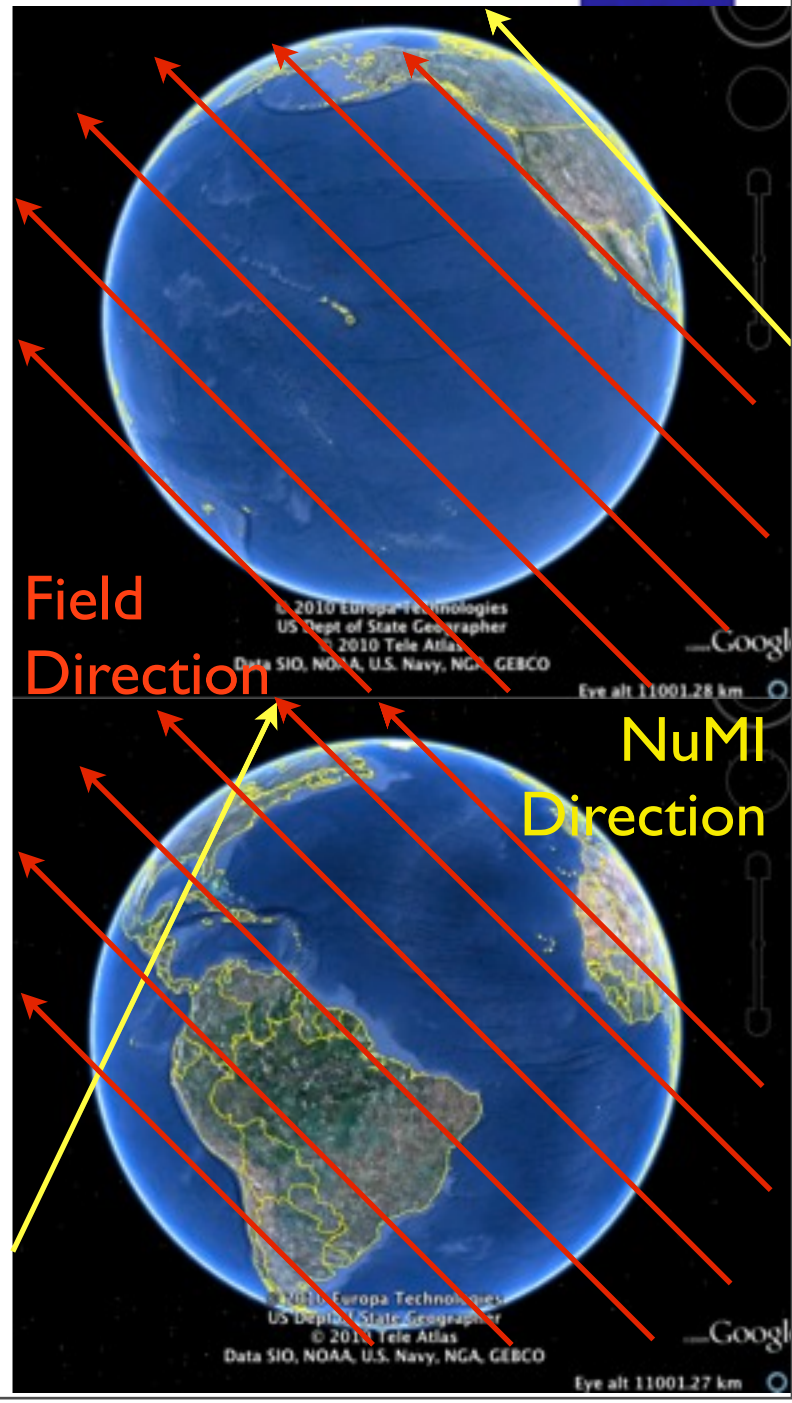
SME and Neutrinos

- CPT and Lorentz Violation could provide a link to Planck scale physics
- Any Lorentz violating signals are suppressed by ratio of the electro-weak and Planck scales

$$\frac{m_W}{m_P} \sim \frac{10^2 GeV}{10^{19} GeV} \sim 10^{-17}$$

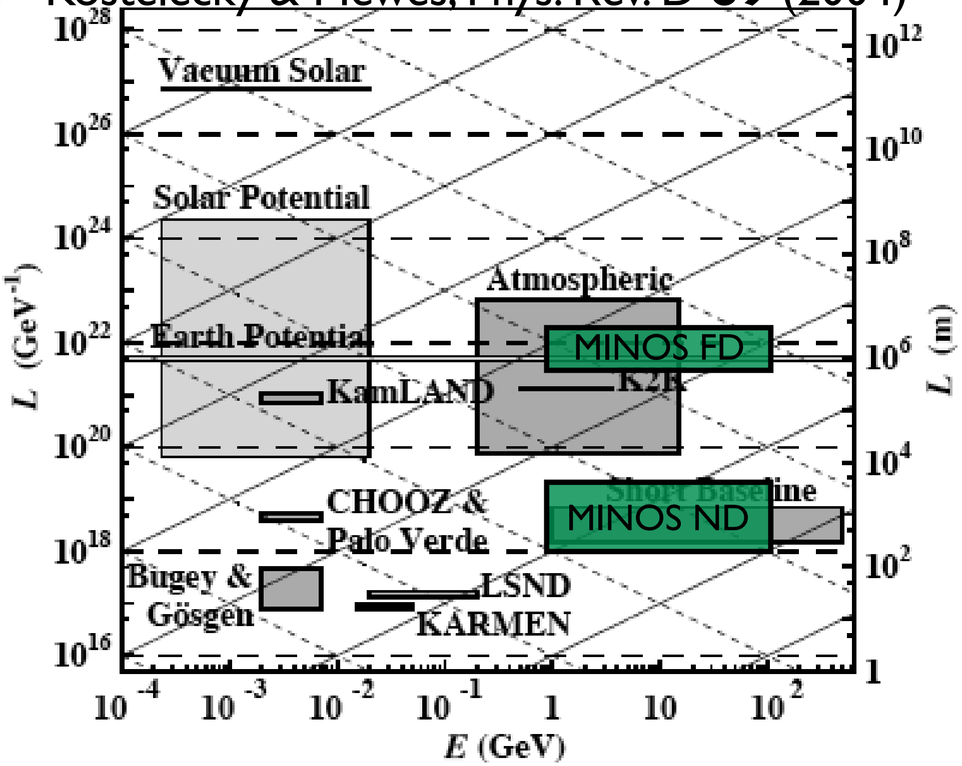
- According to SME, interactions neutrinos have with the field depends on the neutrino energy and direction of travel with respect to the field
- Terrestrial neutrino beams would show variations on the scale of a sidereal day

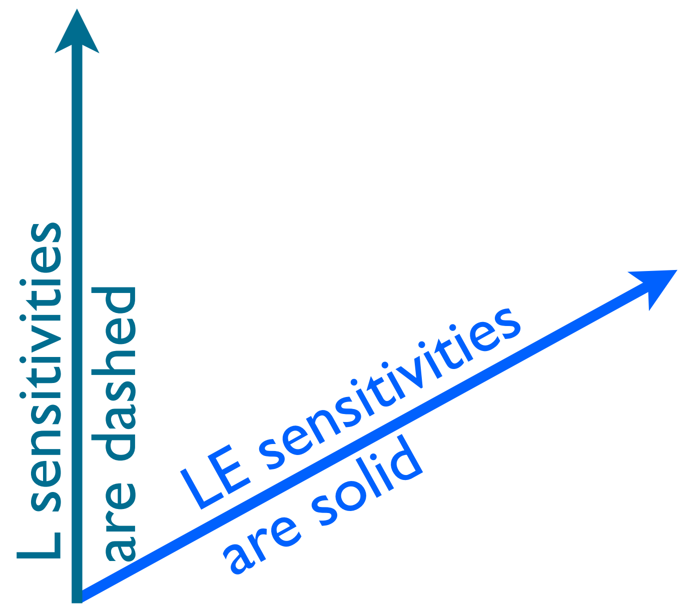
$$\omega_\oplus = 2\pi/(23^h 56^m 04.0982^s)$$



SME and MINOS

Kostelecky & Mewes, Phys. Rev. D **69** (2004)





- Plot shows ranges in L,E space where different experiments are sensitive to CPT and Lorentz violating terms

- MINOS can explore 2 large regions of parameter space
  - ND baseline ~ 750 m (PRL 101:151601 2008)
  - FD baseline ~ 735×10^3 m (PRL 105:151601 2010)
  - Peak E_ν ~ 3 GeV, 0.5 < E_ν < 120 GeV

Near Detector Analysis

Short Baseline Oscillations in SME

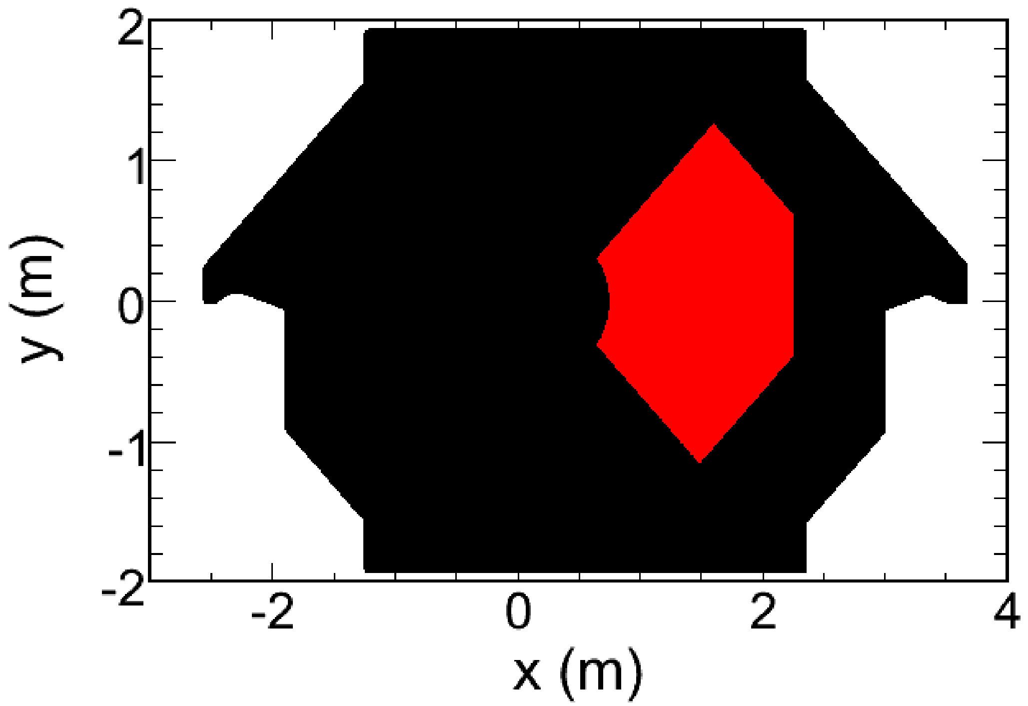
$$P_{\nu_\mu \to \nu_x} \simeq L^2[(\mathcal{C})_{\mu x} + (\mathcal{A}_c)_{\mu x}\cos(\omega_\oplus T_\oplus) + (\mathcal{A}_s)_{\mu x}\sin(\omega_\oplus T_\oplus) + (\mathcal{B}_c)_{\mu x}\cos(2\omega_\oplus T_\oplus) + (\mathcal{B}_s)_{\mu x}\sin(2\omega_\oplus T_\oplus)]^2, \quad (1)$$

- Probability of oscillation for short baselines in SME given above
- C, A, and B terms are combinations of Lorentz violating coefficients from SME, also contain directional information
- Oscillations are between ν_μ and all other flavors
- 4 harmonics in which to search for the oscillations
- Probability goes as L^2 and $(LE)^2$
- Neutrino mass does not play a role in these transitions

Selecting Events in the Near Detector

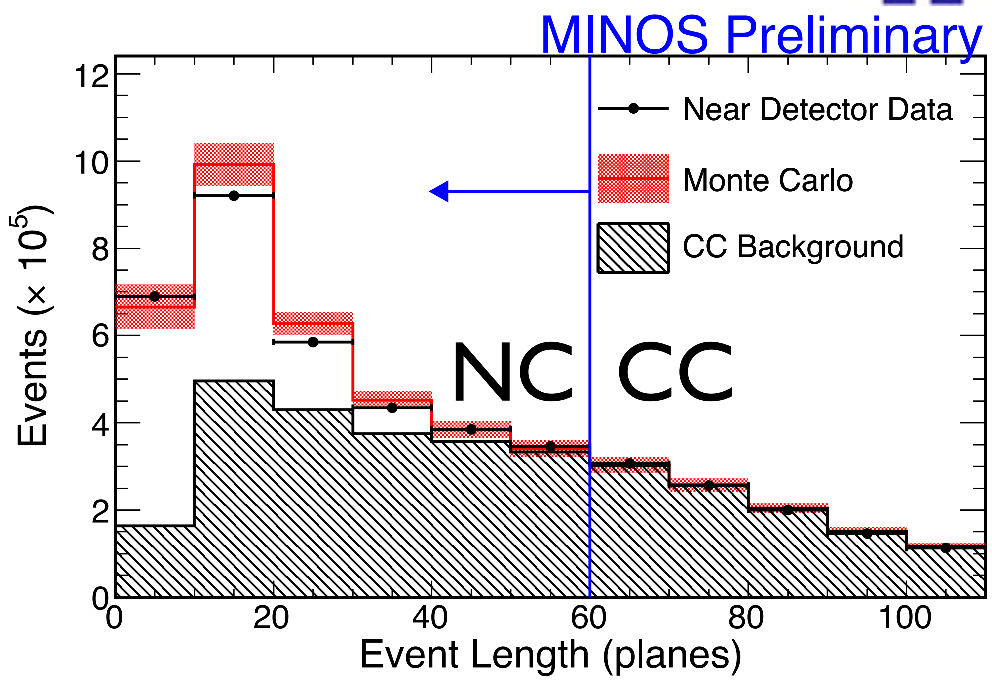
	CC Events	POT	Run Dates
Run I	1.82×10^6	1.25×10^{20}	May05 – Feb06
Run II	1.62×10^6	1.14×10^{20}	Sept06 – Mar07

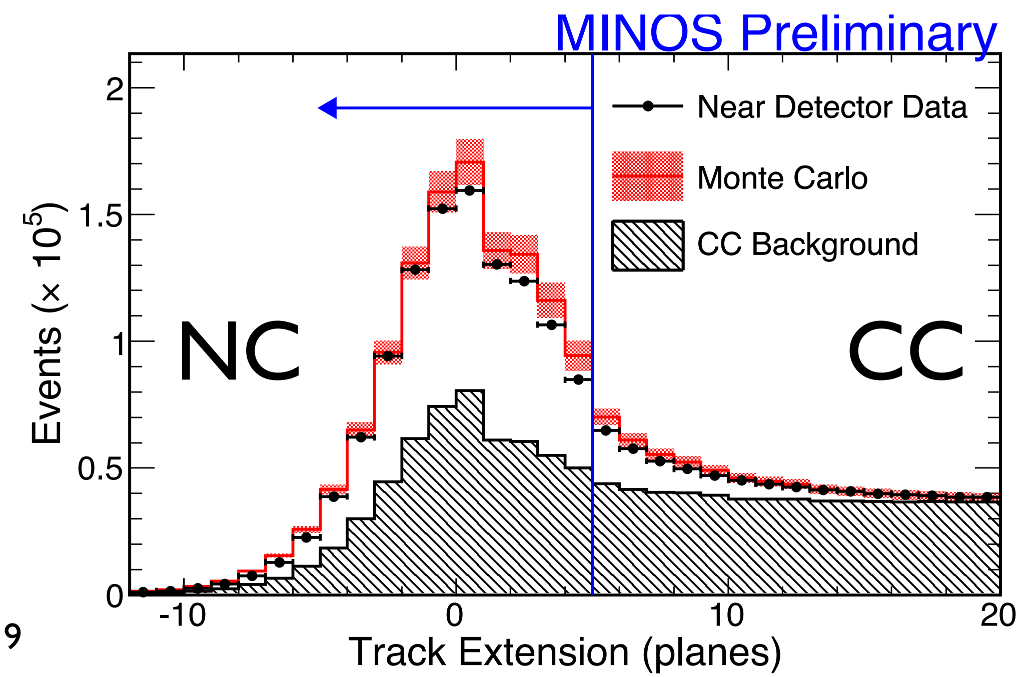
- Basic checks on beam quality and detector performance made
- Remaining events selected if vertex is within fiducial volume
  - Vertex must be >50 cm from edge of partial plane
  - Vertex between planes 30 and 80
  - Ensures containment of any hadronic shower
- MINOS recorded millions of neutrino interactions in the ND during first 2 run periods



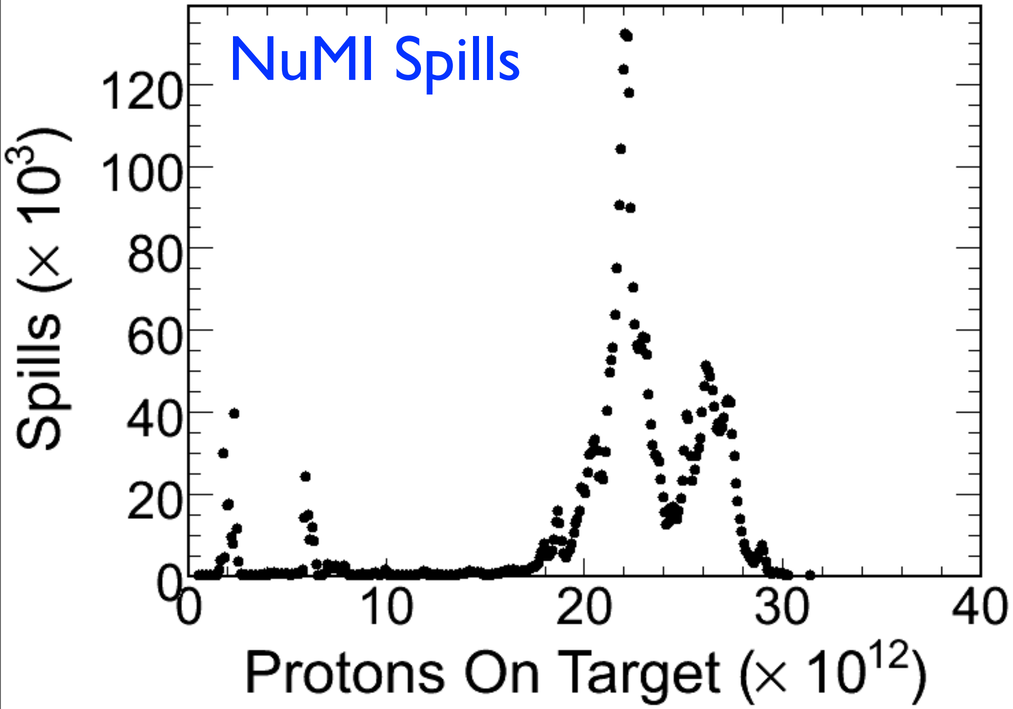
Selecting CC-like Events

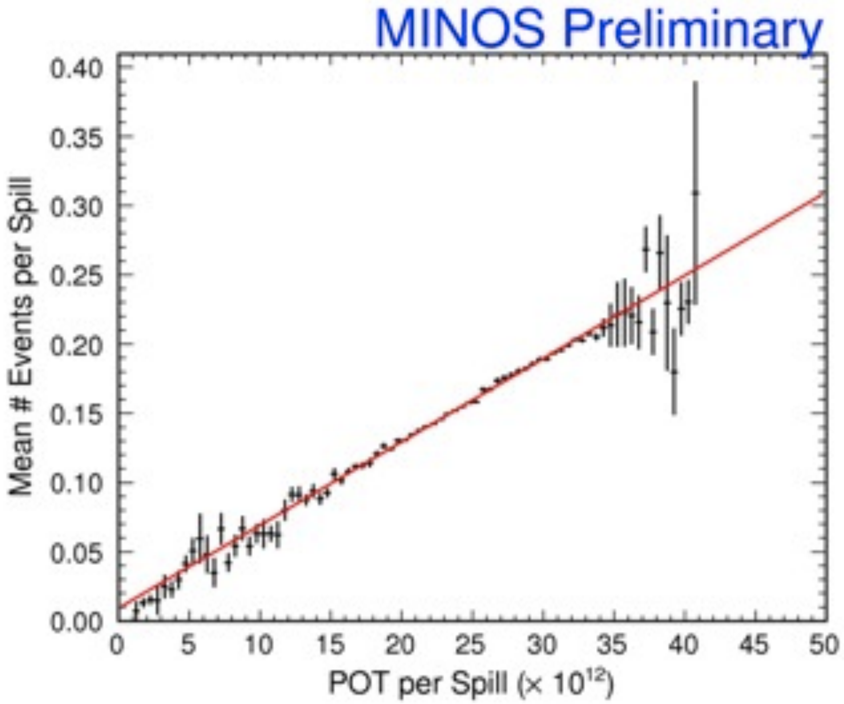
- Need charged-current (CC) interactions to look for disappearance
- Took advantage of work done by the sterile neutrino analysis to distinguish CC-like events from NC-like events
- Neutral current (NC) events span fewer planes than CC events
- Expect tracks in CC events, large showers and no tracks in NC events
- Apply 3 criteria to distinguish events
  - Events crossing > 60 planes → CC
  - Remaining events without a track → NC
  - Remaining events with track extension > 5→ CC





Search for Sidereal Variations

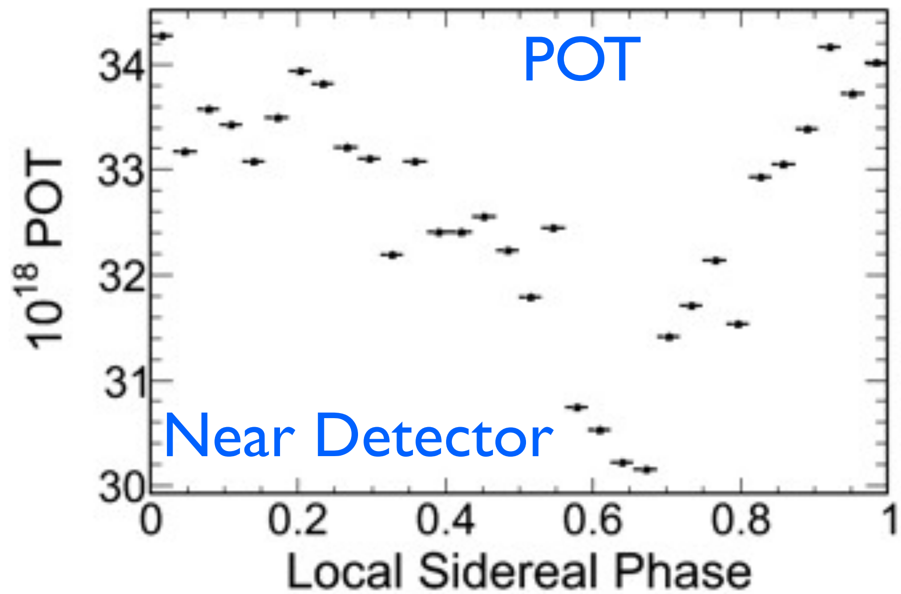


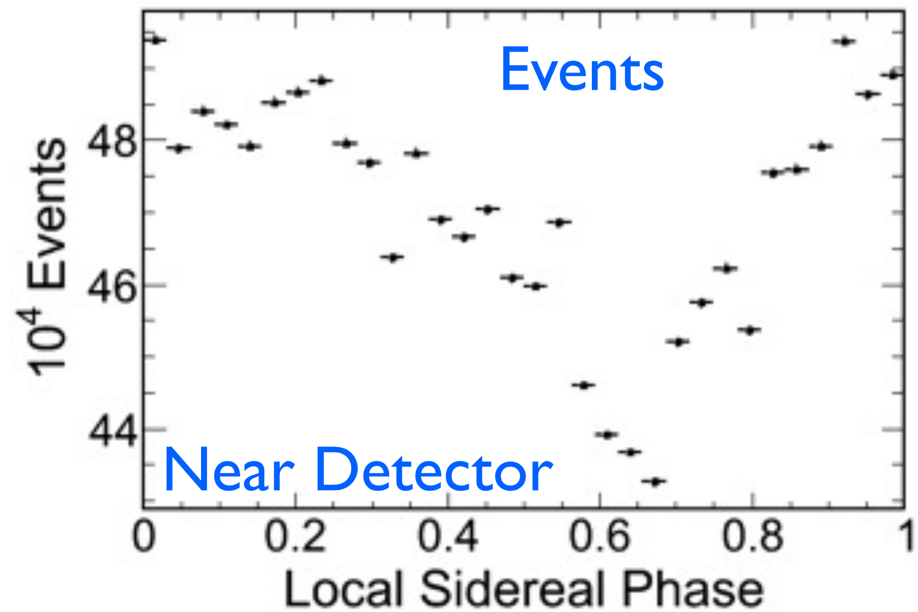


- Typical values of protons on target /spill for May 2005 through February 2006 are shown in top plot
- Variations in the number of protons delivered per spill could introduce fake signals
- Thankfully, number of neutrinos produced scales with POT so we can normalize out the variations

Analysis Strategy

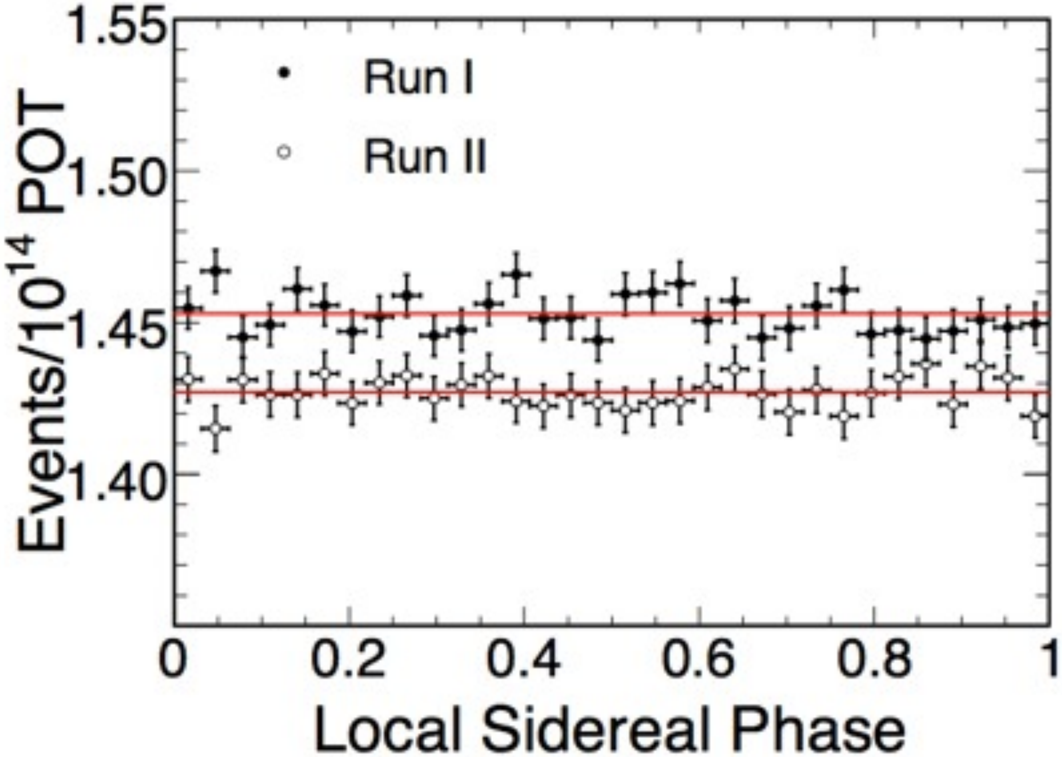
- Plot number of events and POT as a function of local sidereal phase (LSP)
- Number of POT delivered varies throughout the diurnal day and lack of a complete yearly cycle introduces the variation seen
- Use Events/POT as the normalized quantity in which to search for variations
- Perform Fast Fourier Transform (FFT) of the rate vs LSP to search for power in relevant harmonics
- Set number of sidereal bins to retain appropriate powers and provide sufficient resolution





Near Detector Results

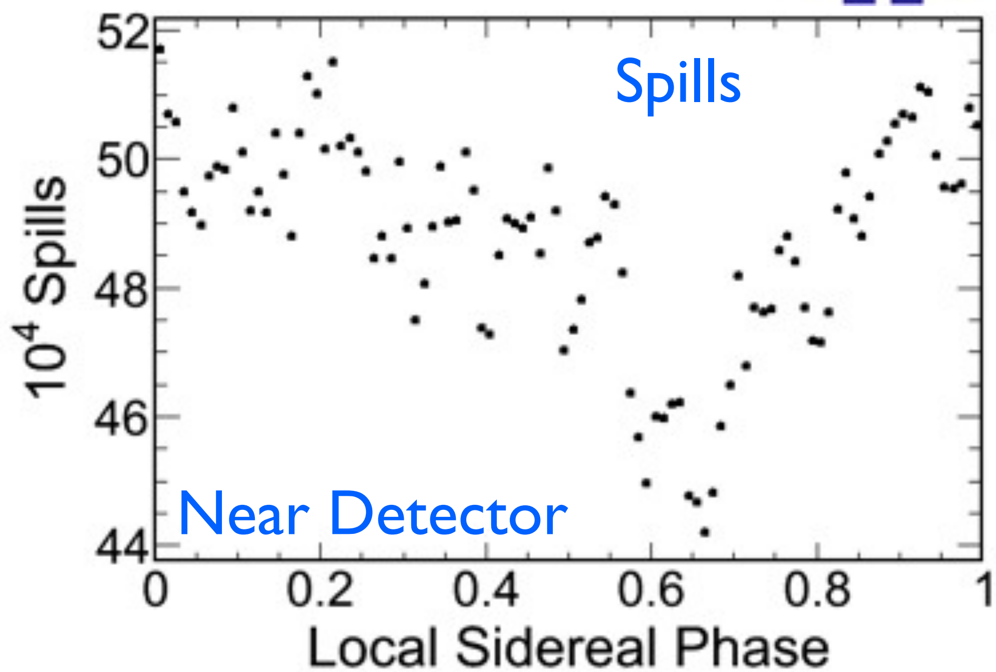
- Event rates for 2 run periods used in ND analysis are shown, differences are due to target degradation
- Combine data sets by weighting each according to the mean event rate
- Rates appear to be well described by constant values
- However have to determine how significant the returned FFT powers are

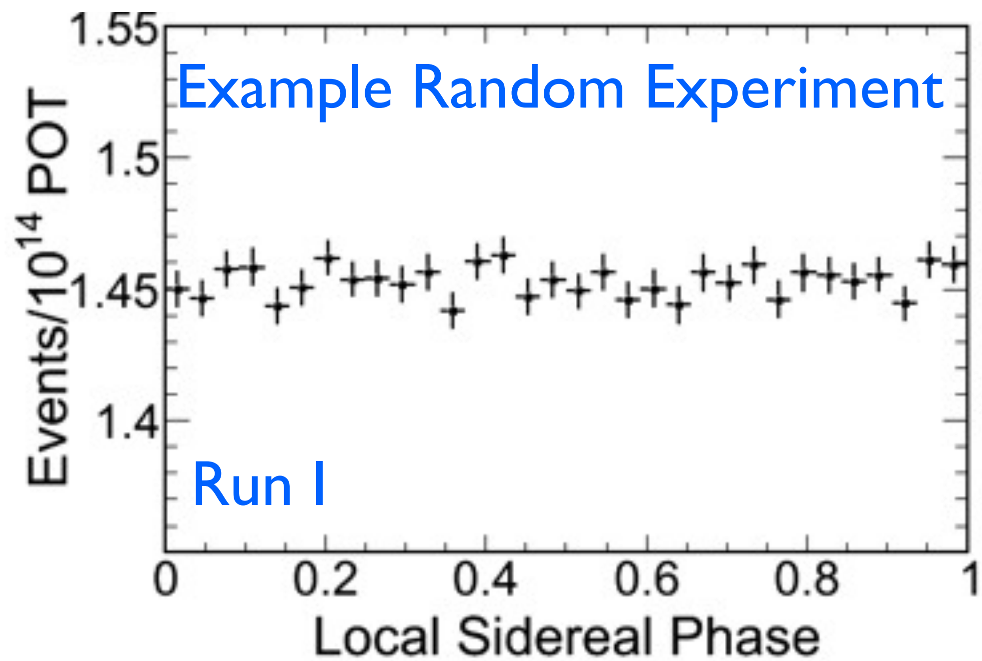


$\cos()$	$\bar{p}(\mathrm{FFT})$	$\sin()$	$\bar{p}(\mathrm{FFT})$
$(\omega_{\oplus}T_{\oplus})$	-0.002	$(\omega_{\oplus}T_{\oplus})$	0.024
$(2\omega_{\oplus}T_{\oplus})$	0.011	$(2\omega_{\oplus}T_{\oplus})$	0.011
$(3\omega_{\oplus}T_{\oplus})$	-0.006	$(3\omega_{\oplus}T_{\oplus})$	-0.004
$(4\omega_{\oplus}T_{\oplus})$	-0.016	$(4\omega_{\oplus}T_{\oplus})$	0.023

Determining Significance of FFT Powers

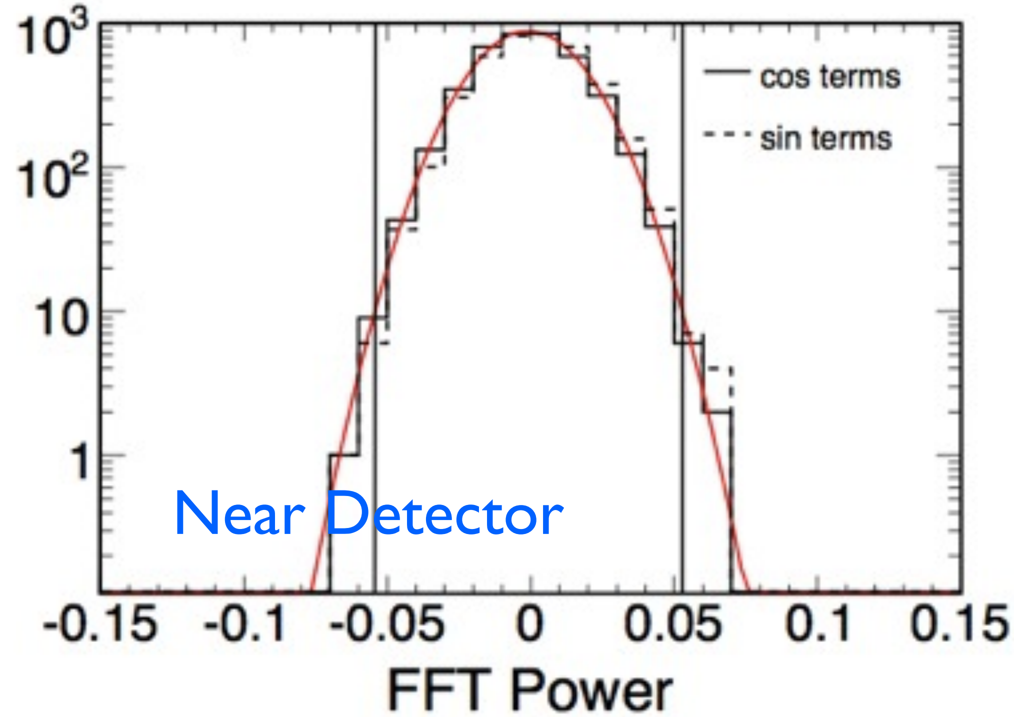
- Used the data to produce set of 1000 random experiments without signal
- For every spill, pull a random LSP out of the distribution for all spills
- Place each event in the spill into a histogram and POT for the spill into a second histogram
- Divide the event and POT histograms to produce the random experiment
- Randomization of spill times removes any possible sidereal variation from the events





Determining Significance of FFT Powers

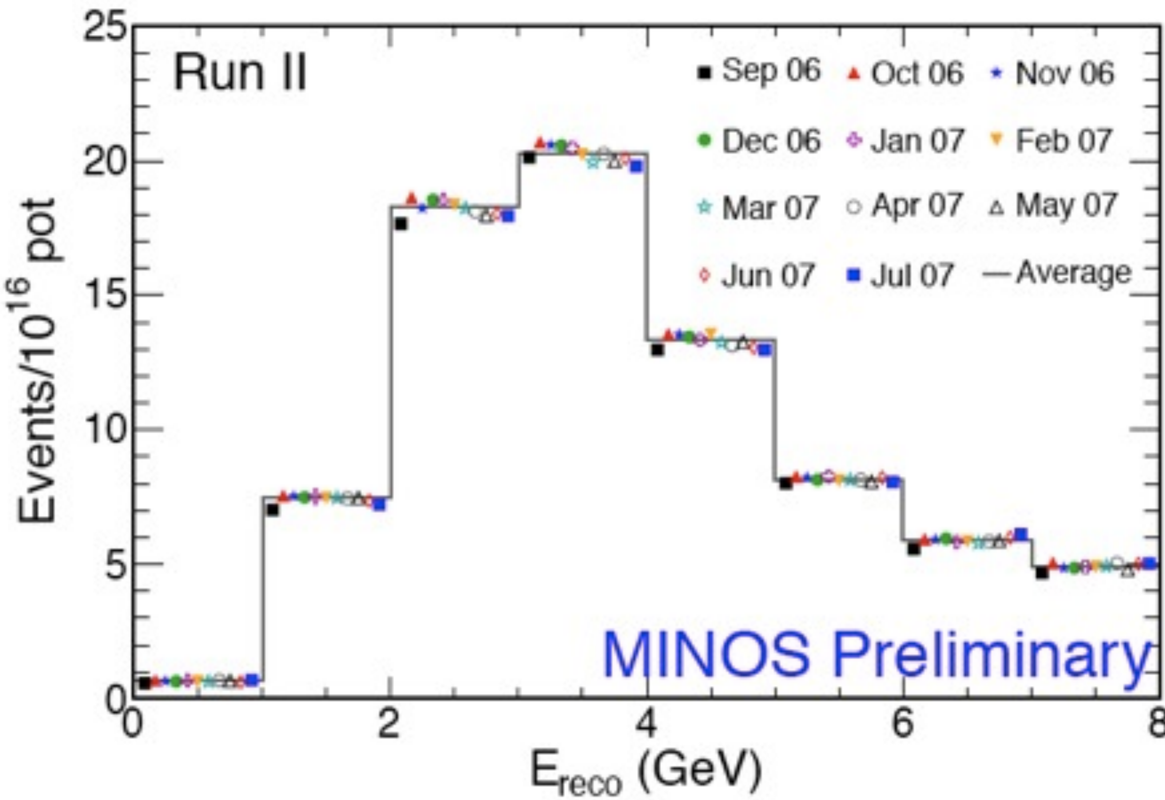
- Determine the FFT powers for each relevant harmonic and place into a distribution
- That distribution allows determination of the probability that a measured power is from a data set without sidereal variations
- $\mathcal{P}_{\mathcal{F}}$ is the probability of drawing a value from the FFT power distribution at least as large as the data value
- No data power is more than 1.3σ from the mean, implying no sidereal variations



cos ()	$\bar{p}$(FFT)	$\mathcal{P}_F$	sin ()	$\bar{p}$(FFT)	$\mathcal{P}_F$
$(\omega_\oplus T_\oplus)$	-0.002	0.91	$(\omega_\oplus T_\oplus)$	0.024	0.18
$(2\omega_\oplus T_\oplus)$	0.011	0.54	$(2\omega_\oplus T_\oplus)$	0.011	0.54
$(3\omega_\oplus T_\oplus)$	-0.006	0.74	$(3\omega_\oplus T_\oplus)$	-0.004	0.83
$(4\omega_\oplus T_\oplus)$	-0.016	0.37	$(4\omega_\oplus T_\oplus)$	0.023	0.20

Systematic Uncertainties

- Look for effects that could mask a sidereal signal
- Target degradation during this period caused event rates to decrease 5% over a period of six months; no impact because rate of decrease was large compared to size of sidereal bins
- No day/night difference in rates > 0.1%
- No modulation in CC/NC ratio as a function of sidereal time, indicates no effects associated with neutrino production in the beam



Limiting Size of SME Coefficients

- In the absence of sidereal variations in the event rate, we set limits on the size of the coefficients
- Treat each coefficient individually by setting all coefficients to 0, increase the size of the desired coefficient from 0 until a signal is detected
- Randomly assign a LSP value to each simulated spill using the data distribution of LSP values
- Set the survival probability of each neutrino based on its energy, baseline, and sidereal phase and put the weighted neutrino into one histogram
- Put POT corresponding to the spill into a second histogram with same phase
- Perform FFT on the experiment and record the largest power, repeat process 200 times to get the average largest power
- Increase size of desired coefficient and repeat, limit is where coefficient magnitude is large enough to produce a detectable signal

Near Detector Limits on SME Coefficients

a_L^X	3.0×10^{-20}	a_L^Y	3.0×10^{-20}
c_L^{TX}	0.9×10^{-22}	c_L^{TY}	0.9×10^{-22}
c_L^{XX}	5.6×10^{-21}	c_L^{YY}	5.5×10^{-21}
c_L^{XY}	2.7×10^{-21}	c_L^{YZ}	1.2×10^{-21}
c_L^{XZ}	1.3×10^{-21}	–	–

PRL 101:151601 2008

- Limit is value where the average size of at least one harmonic power was more than 3σ from the mean of the harmonic power distribution
- These values are all 3-4 orders of magnitude lower than previous measurements by LSND
- Compare scale of results to suppression of electroweak scale of SM to Planck scale, $m_W/m_P \sim 10^{-17}$

Interesting Side Effects

- Indian University issued a press release about this work because it involved an IU theorist (Kostelecky), an IU experimentalist (Mufson), and two IU alums (Mewes and Rebel)
- Press release went viral
- Picked up by Der Spiegel, Voice of America: Russia, Fermilab Today, etc
- Surreal experience to see your work described on popular websites in languages other than english

Einstein's relativity survives neutrino test

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FOR IMMEDIATE RELEASE
Oct. 15, 2008

BLOOMINGTON, Ind. -- Physicists working to disprove "Lorentz invariance" -- Einstein's prediction that matter and massless particles will behave the same no matter how they're turned or how fast they go -- won't get that satisfaction from muon neutrinos, at least for the time being, says a consortium of scientists.

Physicist Stuart Mufson

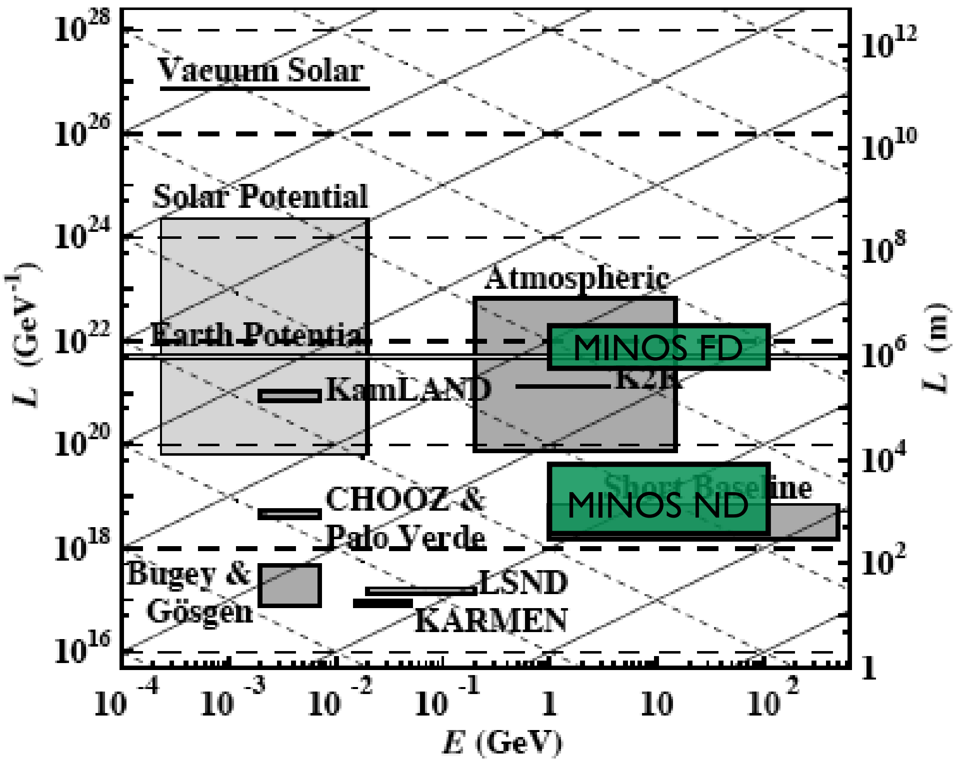
Print-Quality Photo

The test of Lorentz invariance, conducted by MINOS Experiment scientists and reported in the Oct. 10 issue of *Physical Review Letters*, started with a stream of muon neutrinos produced at Fermilab particle accelerator, near Chicago, and ended with a neutrino detector 750 meters away and 103 meters below ground. As the Earth does its daily rotation, the neutrino beam rotates too.

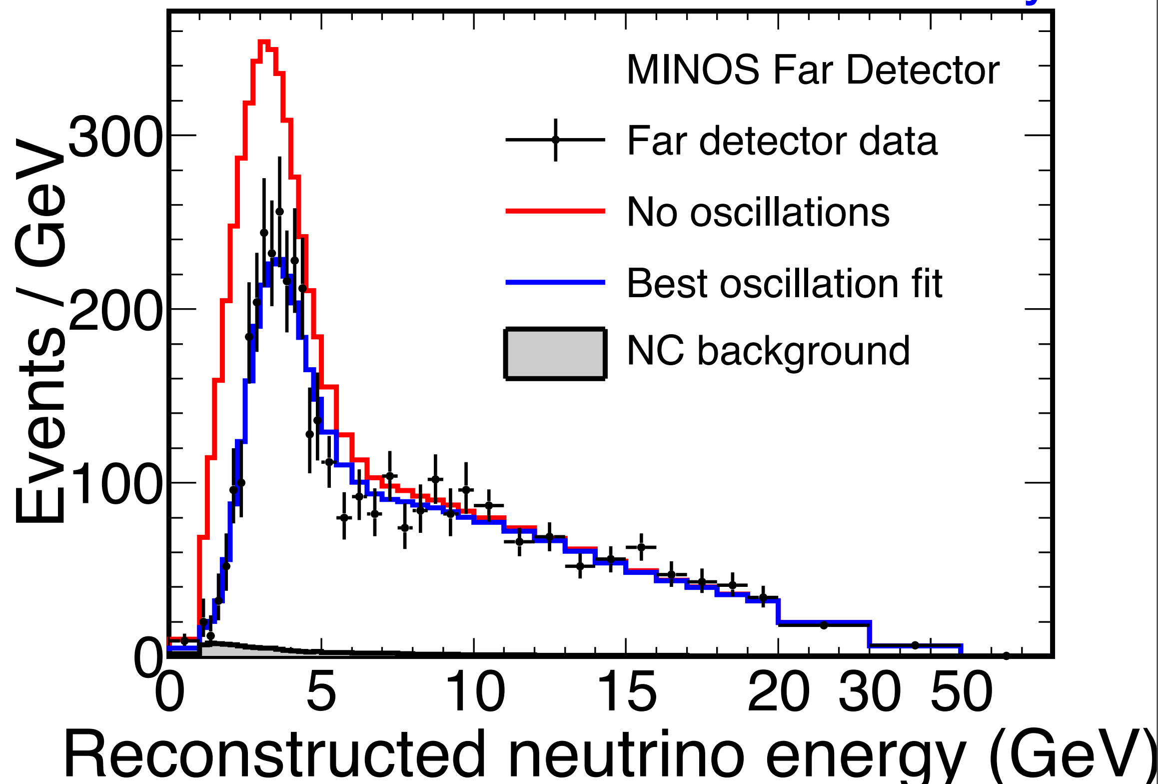
Far Detector Analysis

Increase Baseline, Increase Sensitivity

Kostelecky & Mewes, Phys. Rev. D **69** (2004)



MINOS Preliminary



- FD provides increased sensitivity for LV according to the SME because the effect goes as the baseline, L, or the product of the baseline and neutrino energy, LE
- For FD analysis, must account for oscillations in addition to LV
- Reference is Diaz, Kostelecky and Mewes, Phys. Rev. D **80** (2009)

Long Baseline Oscillations in SME

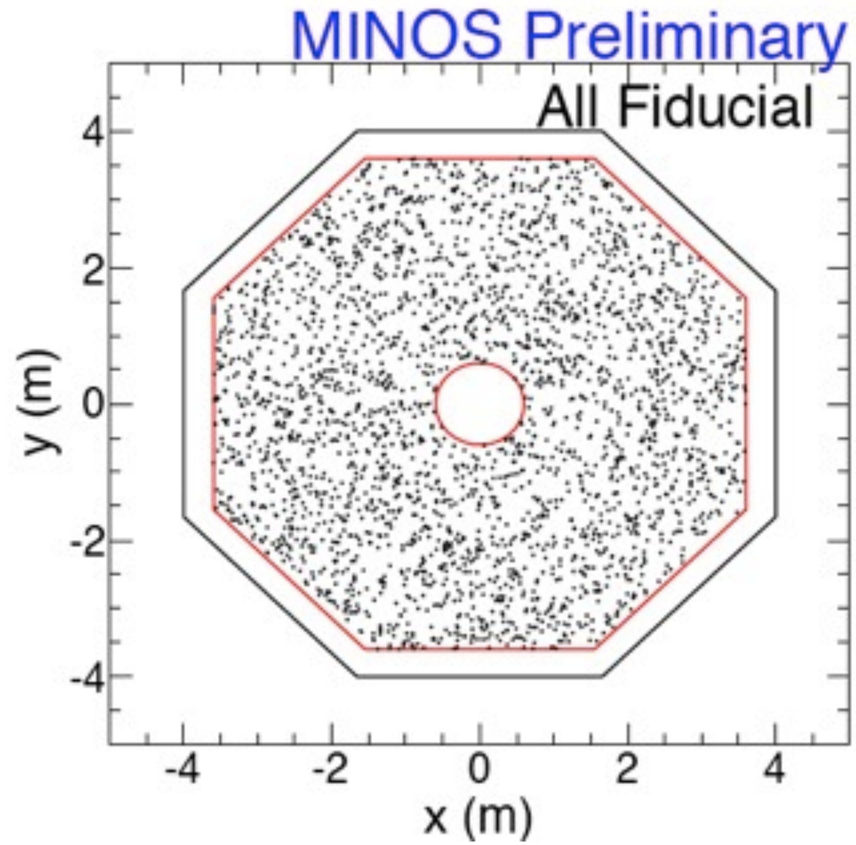
$$P_{\nu_\mu \to \nu_\tau} \simeq \sin^2(2\theta_{23}) \sin^2(1.267 \Delta m^2_{32} L/E_\nu) + P^{(1)}_{\mu\tau}$$

$$\begin{aligned} P^{(1)}_{\mu\tau} &= 2L\Big\{ (P^{(1)}_{\mathcal{C}})_{\mu\tau} \\ &+ (P^{(1)}_{\mathcal{A}_s})_{\mu\tau} \sin\omega_\oplus T_\oplus + (P^{(1)}_{\mathcal{A}_c})_{\mu\tau} \cos\omega_\oplus T_\oplus \\ &+ (P^{(1)}_{\mathcal{B}_s})_{\mu\tau} \sin 2\omega_\oplus T_\oplus + (P^{(1)}_{\mathcal{B}_c})_{\mu\tau} \cos 2\omega_\oplus T_\oplus \Big\} \end{aligned}$$

- At long baselines the LV effects become perturbations to mass oscillations
- Probability of oscillation for long baselines in SME given above
- Assume 2 flavor oscillations, maximal mixing
- Probability goes as L and LE

Data Set

- Used data from first 3 run periods, total of 7.06 x 10^{20} POT
- Standard beam quality criteria applied
- Fiducial cuts required event vertices to be
  - 60 cm from coil center, 40 cm from outside edges
  - > 3 planes (20 cm) from start of SM, > 20 planes (120 cm) from end
- Removed cosmic rays and detector noise with << 1% background remaining

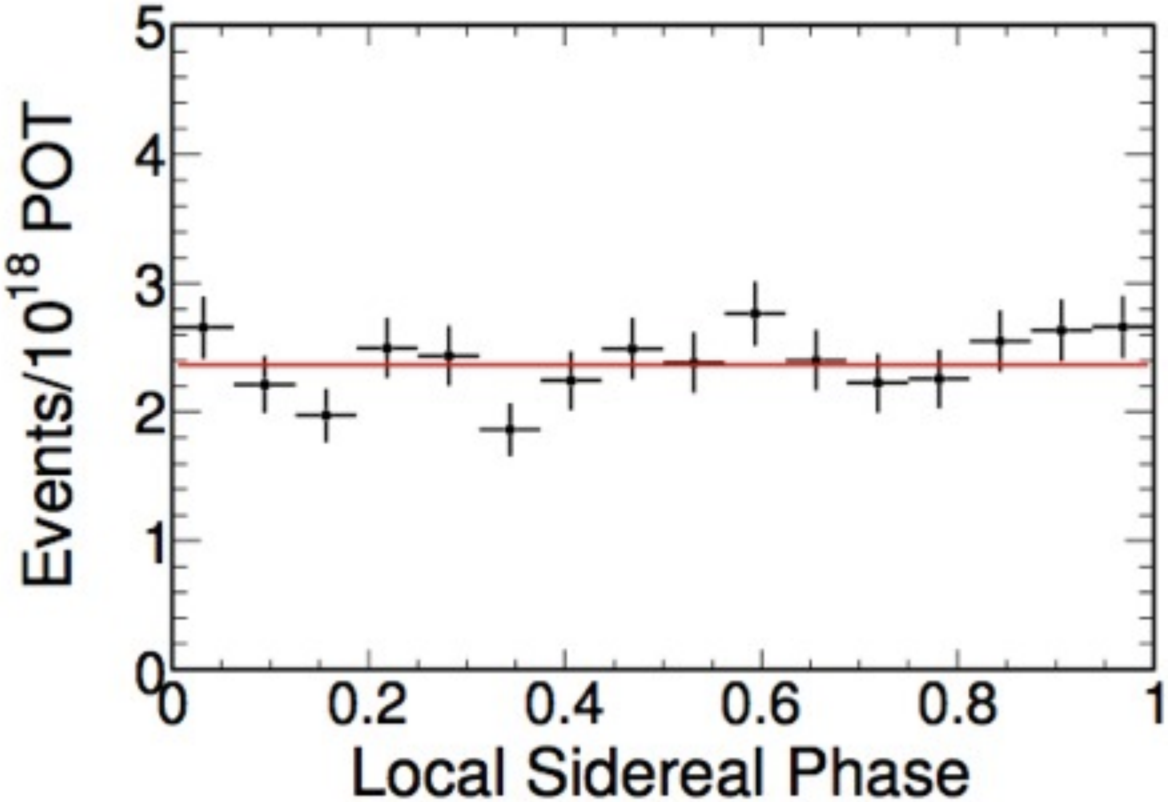


	Run Dates	POT	CC Events
Run I	May05 – Feb06	1.24×10^{20}	281
Run II	Sep06 – Jul07	1.94×10^{20}	453
Run III	Sep07 – Jun09	3.88×10^{20}	954

Blind Analysis

- Used a blinding scheme to prevent biases from influencing the analysis
- Procedures were determined using only the Run I and Run II data, less than 50% of the total data set
- No changes to analysis procedures were allowed after the box was opened

FD Data



- Analysis uses the same approach as with the ND analysis, but use 16 sidereal bins as we only need to look at first and second harmonics
- Event rate for combined data from 3 run periods used in FD analysis are shown, low rate at FD means target degradation is not observable there
- The data are well described by a single rate

Search for Variations

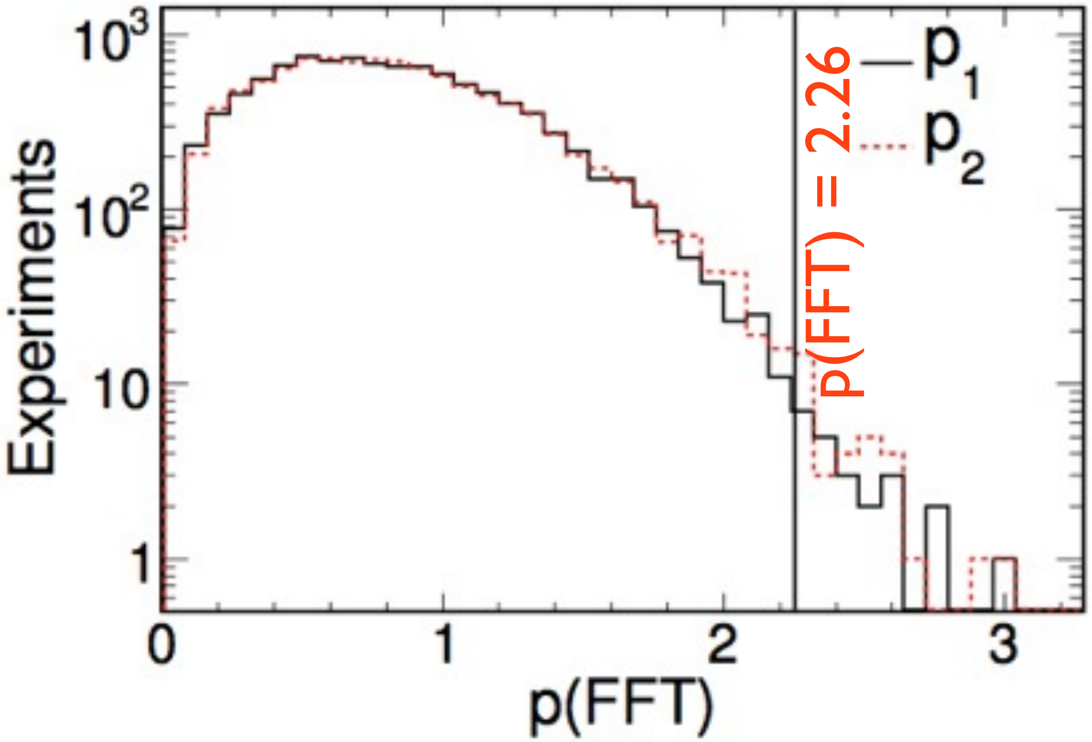
- Use data to define size of FFT power that signals a detection
- Make 10k experiments
  - Distribution of LSP for actual spills used to randomize spills in each experiment
  - LSP of each event randomized based on actual spill LSP distribution as well
- Perform FFT on each experiment
- Take quadratic sum of sine and cosine components of each harmonic

$$p_1 = \sqrt{S_1^2 + C_1^2},$$

$$p_2 = \sqrt{S_2^2 + C_2^2}.$$

- Define the detection threshold as the value of p_1 and p_2 that is greater than that found for 99.7% of the experiments

Search for Variations



Data Powers

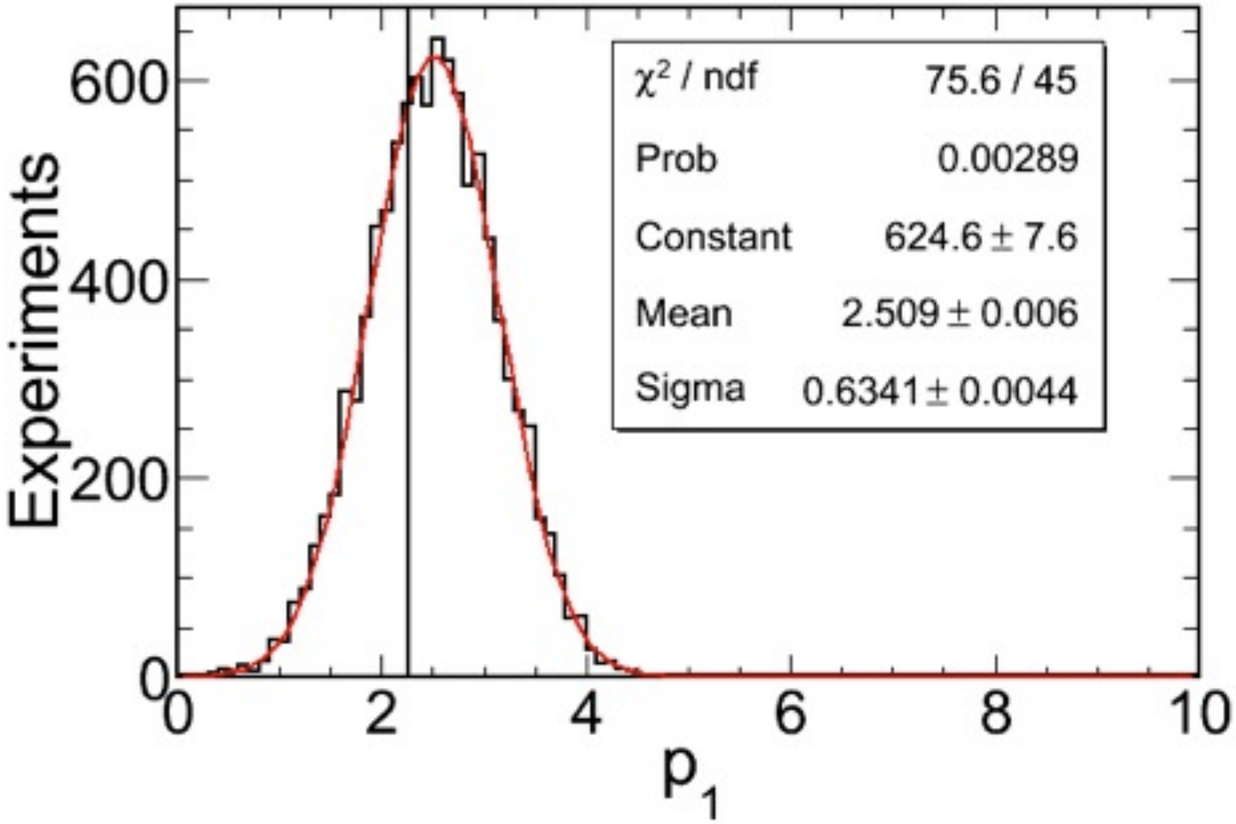
Harmonic	$p(FFT)$	$\mathcal{P}_{\mathcal{F}}$
p_1	1.09	0.26
p_2	1.13	0.24

$$p_1 = \sqrt{S_1^2 + C_1^2},$$

$$p_2 = \sqrt{S_2^2 + C_2^2}.$$

- Distribution of powers for each harmonic shown
- Set the threshold for a detection to be a power > 99.7% of the powers for the random experiments, p = 2.26
- Table shows data powers, probability, $\mathcal{P}_{\mathcal{F}}$, of getting larger value for each harmonic also shown

Minimum Detectable Signal



- Understanding minimum detectable signal size helps to determine if a systematic effect could impact the analysis
- Injected known signal into random experiments to determine minimum signal size where at least 68% of experiments are above 2.26
- Minimum signal amplitude is 9%

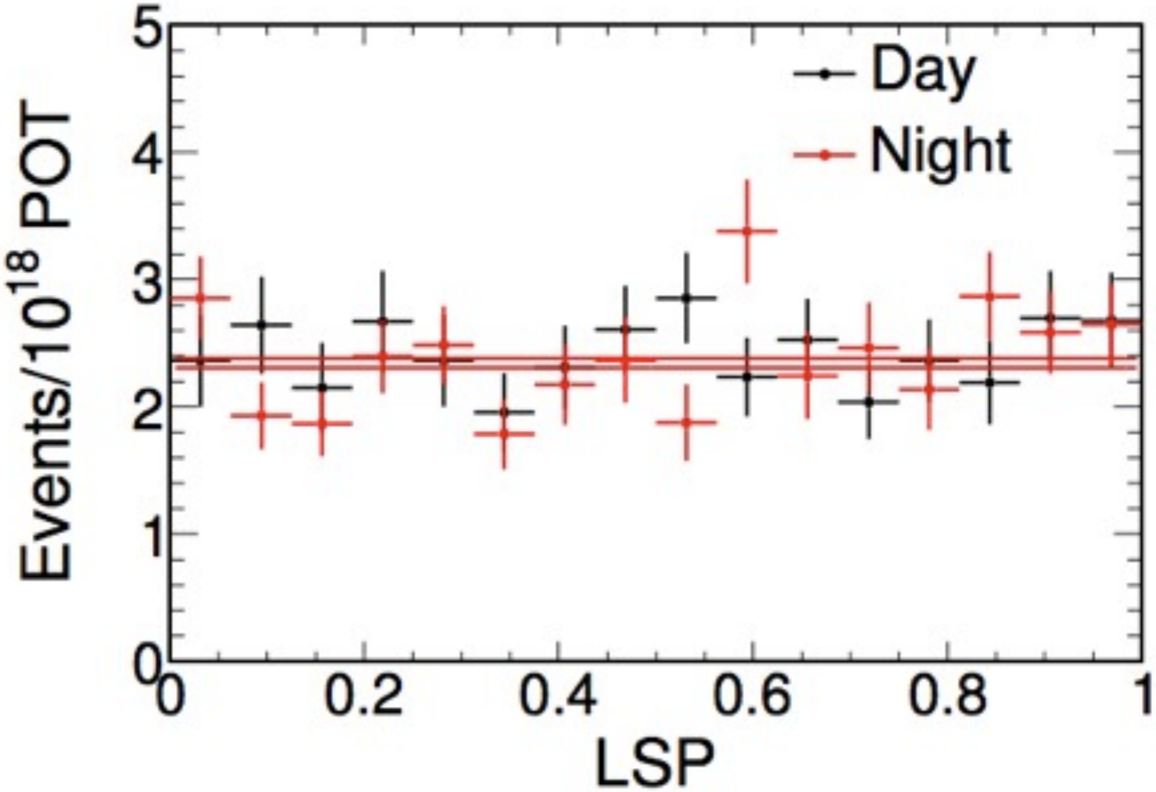
Systematic Uncertainties - Target Degradation and POT Counting

- Introduced linear decrease in neutrino yield/POT into data at level of 5% for every 6 months
- Repeated FFT analysis of resulting rate histogram - no change in size of powers found compared to the data because rate of change is large compared to LSP bin size
- Tested linear increase at same level, no change in powers found
- POT counting uncertainty is ±1%
- Could introduce a signal if the counting were coherently high or low for only certain phases
- Uncertainty in POT counting well below size necessary to cause false signal

Systematic Uncertainties - Choice of Zero Point in Phase

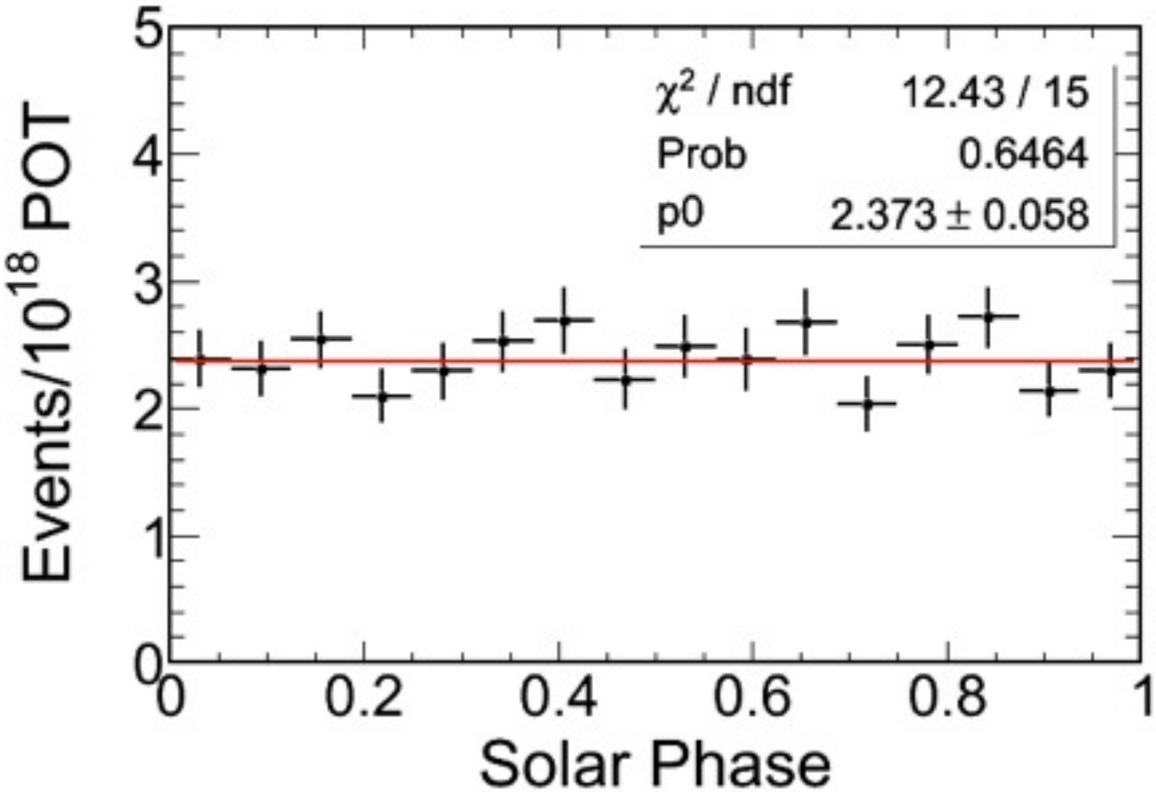
- Choice of zero point shifts power between sine and cosine terms in harmonics
- Tested sensitivity to the choice by injecting a 20% signal in the first harmonic into 10^4 random experiments
- Compared power from that trial with several others where the zero point was chosen randomly
- Average power for all trials was 5.12 ± 0.02
- Use of quadratic sum of powers removes the zero point choice as a potential systematic uncertainty

Systematic Uncertainties - Day/Night



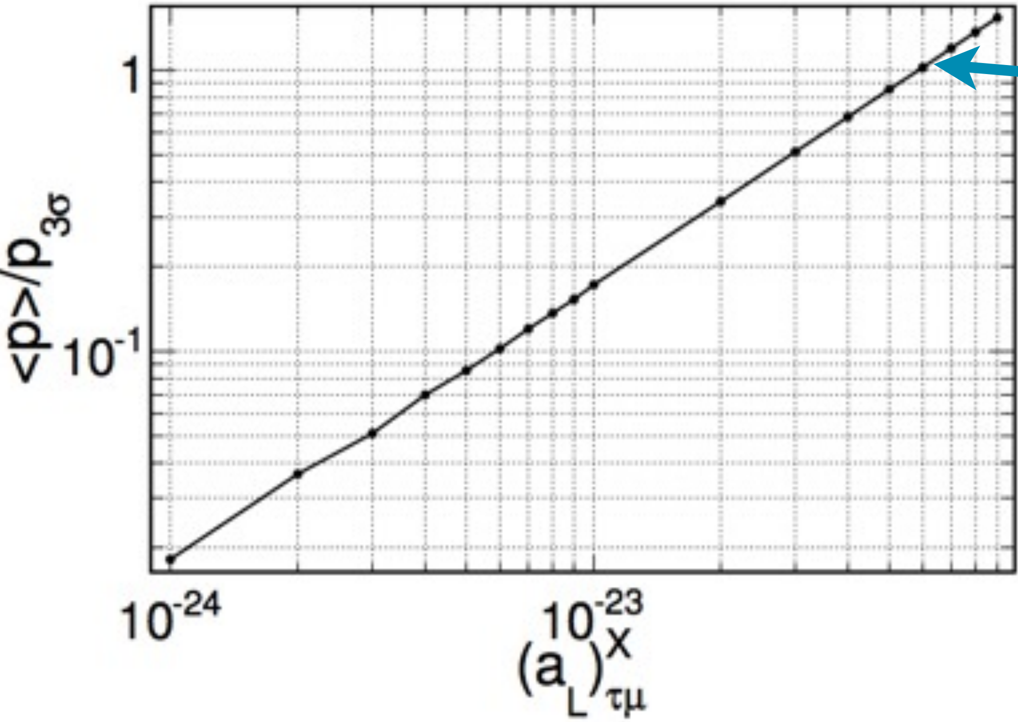
- Day defined as 6:00 to 18:00, night is remaining 12 hours
- Variations in POT delivered coupled with incomplete annual coverage could introduce false signal
- Average rates consistent to within < 0.1 sigma of both samples
- No Day/Night differences

Faux Sidereal Modulation



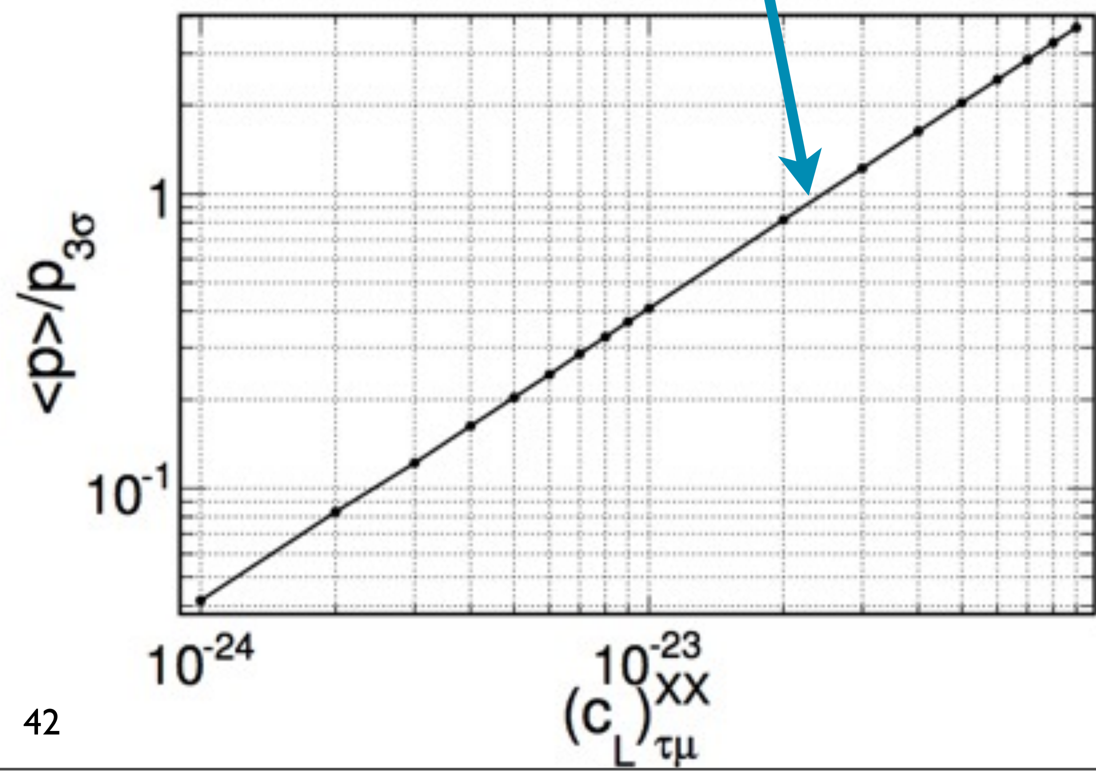
- A solar diurnal modulation beating with yearly modulation can cause a faux sidereal modulation
- Solar modulation from day/night differences is 0.03, yearly modulation from above plot is 0.03
- Product of solar and yearly modulation sets the scale of the faux sidereal signal, 10^{-3}, well below detection threshold

Limiting Size of SME Coefficients



Limit is where the average power is greater than the detection threshold

- Nearly identical method for finding limits as with the near detector analysis
- Set the survival probability of each neutrino based on its baseline, energy, sidereal phase and neutrino oscillation parameters



Low Statistics Cross-Check

- The method for determining limits just described used the high statistics Monte Carlo scaled to the same exposure as the data
- It is possible that statistical fluctuations could alter the size of the limits determined
- Simulated 750 experiments using the total number of events in the data and the limits determined from the previous method to ask the question, "At what confidence level do these limits exclude the results from our data?"
- Used distributions of p_1 and p_2 from all simulated experiments to determine the confidence interval at which the measured values of p_1 and p_2 from the data are excluded
- Exclusion is greater than 99.7% for all coefficients

Limits on SME Coefficients

Near Detector

a_L^X	3.0×10^{-20}	a_L^Y	3.0×10^{-20}
c_L^{TX}	0.9×10^{-22}	c_L^{TY}	0.9×10^{-22}
c_L^{XX}	5.6×10^{-21}	c_L^{YY}	5.5×10^{-21}
c_L^{XY}	2.7×10^{-21}	c_L^{YZ}	1.2×10^{-21}
c_L^{XZ}	1.3×10^{-21}	–	–

Far Detector

$(a_L)^X_{\mu\tau}$	5.9×10^{-23}	$(a_L)^Y_{\mu\tau}$	6.1×10^{-23}
$(c_L)^{TX}_{\mu\tau}$	0.5×10^{-23}	$(c_L)^{TY}_{\mu\tau}$	0.5×10^{-23}
$(c_L)^{XX}_{\mu\tau}$	2.4×10^{-23}	$(c_L)^{YY}_{\mu\tau}$	2.4×10^{-23}
$(c_L)^{XY}_{\mu\tau}$	1.2×10^{-23}	$(c_L)^{YZ}_{\mu\tau}$	0.7×10^{-23}
$(c_L)^{XZ}_{\mu\tau}$	0.7×10^{-23}	–	–

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- Limits are for real components of coefficients are shown
- All FD limits are 20-500 times smaller than found with the ND
- Improvement due to increased baseline to far detector

Conclusions

- MINOS has measured several SME coefficients using both the short and long baseline approximations
- The limits in the FD are between 10^{-24} and 10^{-23}, or 6 - 7 orders of magnitude smaller than the ratio of the electroweak scale to the Planck scale $m_W/m_P \sim 10^{-17}$
- The analysis with the FD is the first to treat LV and CPTV as a perturbation to neutrino mass oscillations
- The method for detecting sidereal variations and limiting the SME coefficients has been adopted by IceCube to improve on our limits for a subset of the coefficients shown today

Long-baseline Oscillations in SME

- Assuming 2 flavor oscillations the expressions become

$$(P^{(1)}_{\mathcal{C}})_{\tau\mu} = \frac{1}{2}\sin(\frac{2.534\Delta m^2_{32}L}{E})\{(a_L)^T_{\tau\mu} - \hat{N}^Z(a_L)^Z_{\tau\mu} - \frac{1}{2}(3-\hat{N}^Z\hat{N}^Z)E(c_L)^{TT}_{\tau\mu} + 2\hat{N}^Z E(c_L)^{TZ}_{\tau\mu} + \frac{1}{2}(1-3\hat{N}^Z\hat{N}^Z)E(c_L)^{ZZ}_{\tau\mu}\}. \quad (12)$$

$$(P^{(1)}_{\mathcal{A}_s})_{\tau\mu} = \frac{1}{2}\sin(\frac{2.534\Delta m^2_{32}L}{E})\{\hat{N}^Y(a_L)^X_{\tau\mu} - \hat{N}^X(a_L)^Y_{\tau\mu} - 2\hat{N}^Y E(c_L)^{TX}_{\tau\mu} + 2\hat{N}^X E(c_L)^{TY}_{\tau\mu} + 2\hat{N}^Y\hat{N}^Z E(c_L)^{XZ}_{\tau\mu} - 2\hat{N}^X\hat{N}^Z E(c_L)^{YZ}_{\tau\mu}\}. \quad (13)$$

$$(P^{(1)}_{\mathcal{A}_c})_{\tau\mu} = \frac{1}{2}\sin(\frac{2.534\Delta m^2_{32}L}{E})\{-\hat{N}^X(a_L)^X_{\tau\mu} - \hat{N}^Y(a_L)^Y_{\tau\mu} + 2\hat{N}^X E(c_L)^{TX}_{\tau\mu} + 2\hat{N}^Y E(c_L)^{TY}_{\tau\mu} - 2\hat{N}^X\hat{N}^Z E(c_L)^{XZ}_{\tau\mu} - 2\hat{N}^Y\hat{N}^Z E(c_L)^{YZ}_{\tau\mu}\}. \quad (14)$$

$$(P^{(1)}_{\mathcal{B}_s})_{\tau\mu} = \frac{1}{2}\sin(\frac{2.534\Delta m^2_{32}L}{E})\{\hat{N}^X\hat{N}^Y E((c_L)^{XX}_{\tau\mu} - (c_L)^{YY}_{\tau\mu}) - (\hat{N}^X\hat{N}^X - \hat{N}^Y\hat{N}^Y)E(c_L)^{XY}_{\tau\mu}\}. \quad (15)$$

$$(P^{(1)}_{\mathcal{B}_c})_{\tau\mu} = \frac{1}{2}\sin(\frac{2.534\Delta m^2_{32}L}{E})\{-2\hat{N}^X\hat{N}^Y E(c_L)^{XY}_{\tau\mu} - \frac{1}{2}(\hat{N}^X\hat{N}^X - \hat{N}^Y\hat{N}^Y)E((c_L)^{XX}_{\tau\mu} - (c_L)^{YY}_{\tau\mu})\}. \quad (16)$$

- Mass oscillations come in through the sine factor in front of each equation

Oscillations in SME

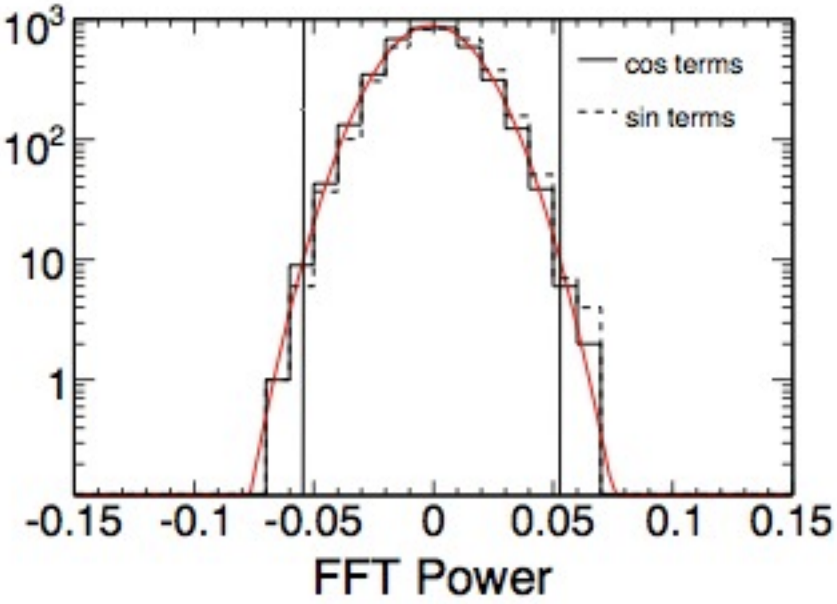
- Factors multiplying the sidereal frequencies are below

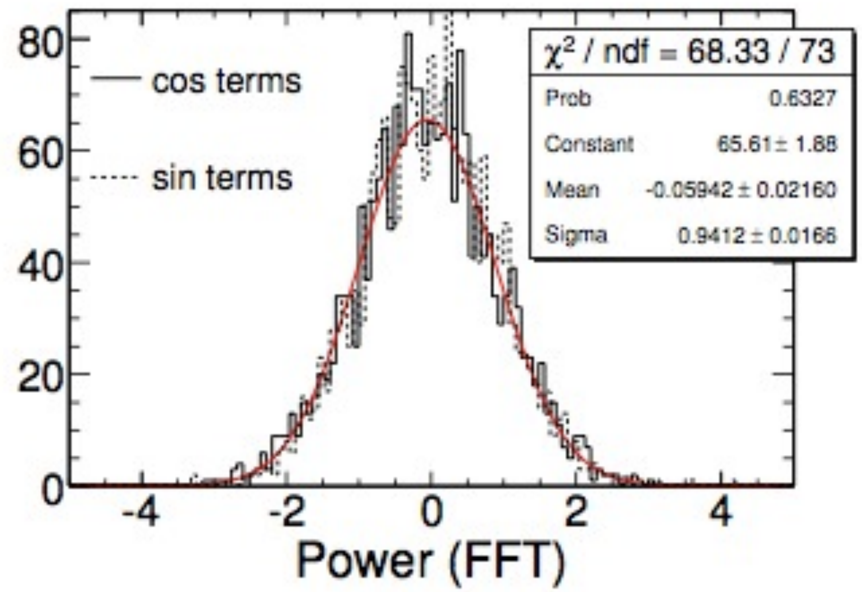
$$
\begin{aligned}
(P^{(1)}_{\mathcal{C}})_{\tau\mu} &= \{(\tilde{a}_L)^T_{\tau\mu} - \hat{N}^Z(\tilde{a}_L)^Z_{\tau\mu} - \frac{1}{2}(3-\hat{N}^Z\hat{N}^Z)E(\tilde{c}_L)^{TT}_{\tau\mu} \\
&+ 2\hat{N}^Z E(\tilde{c}_L)^{TZ}_{\tau\mu} + \frac{1}{2}(1-3\hat{N}^Z\hat{N}^Z)E(\tilde{c}_L)^{ZZ}_{\tau\mu}\}, \\
(P^{(1)}_{\mathcal{A}_s})_{\tau\mu} &= \{\hat{N}^Y(\tilde{a}_L)^X_{\tau\mu} - \hat{N}^X(\tilde{a}_L)^Y_{\tau\mu} - 2\hat{N}^Y E(\tilde{c}_L)^{TX}_{\tau\mu} + 2\hat{N}^X E(\tilde{c}_L)^{TY}_{\tau\mu} \\
&+ 2\hat{N}^Y\hat{N}^Z E(\tilde{c}_L)^{XZ}_{\tau\mu} - 2\hat{N}^X\hat{N}^Z E(\tilde{c}_L)^{YZ}_{\tau\mu}\}, \\
(P^{(1)}_{\mathcal{A}_c})_{\tau\mu} &= \{-\hat{N}^X(\tilde{a}_L)^X_{\tau\mu} - \hat{N}^Y(\tilde{a}_L)^Y_{\tau\mu} + 2\hat{N}^X E(\tilde{c}_L)^{TX}_{\tau\mu} + 2\hat{N}^Y E(\tilde{c}_L)^{TY}_{\tau\mu} \\
&- 2\hat{N}^X\hat{N}^Z E(\tilde{c}_L)^{XZ}_{\tau\mu} - 2\hat{N}^Y\hat{N}^Z E(\tilde{c}_L)^{YZ}_{\tau\mu}\}, \\
(P^{(1)}_{\mathcal{B}_s})_{\tau\mu} &= \{\hat{N}^X\hat{N}^Y E\big((\tilde{c}_L)^{XX}_{\tau\mu} - (\tilde{c}_L)^{YY}_{\tau\mu}\big) \\
&- (\hat{N}^X\hat{N}^X - \hat{N}^Y\hat{N}^Y)E(\tilde{c}_L)^{XY}_{\tau\mu}\}, \\
(P^{(1)}_{\mathcal{B}_c})_{\tau\mu} &= \{-2\hat{N}^X\hat{N}^Y E(\tilde{c}_L)^{XY}_{\tau\mu} \\
&- \frac{1}{2}(\hat{N}^X\hat{N}^X - \hat{N}^Y\hat{N}^Y)E\big((\tilde{c}_L)^{XX}_{\tau\mu} - (\tilde{c}_L)^{YY}_{\tau\mu}\big)\},
\end{aligned}
$$

Directional Dependence

$$
\begin{aligned}
\hat{N}^X &= \cos\chi\sin\theta\cos\phi + \sin\chi\cos\theta, \\
\hat{N}^Y &= \sin\theta\sin\phi, \\
\hat{N}^X &= -\sin\chi\sin\theta\cos\phi + \cos\chi\cos\theta.
\end{aligned}
$$

Why is the Far Detector More Sensitive?





- Statistics decrease by a factor of ~10^{-4}, so why are limits better?
- First checked that the statistics seen at the far detector are reasonable by comparing baseline, fiducial volume, oscillation probability, detection efficiencies and likelihood of a muon crossing each detector; all checks out
- Checked that spread in the powers for the simulated experiments scales according to statistics - expect FD to have broader distribution by square-root of the ratio of number of events in each; expect increase of factor of 72, see increase by factor of 52

Why is the Far Detector More Sensitive?

$$(P_{\mu x})_{ND} \simeq L^2[(\mathcal{C})_{\mu x} + (\mathcal{A}_c)_{\mu x}\cos(\omega_\oplus T_\oplus) + (\mathcal{A}_s)_{\mu x}\sin(\omega_\oplus T_\oplus) + (\mathcal{B}_c)_{\mu x}\cos(2\omega_\oplus T_\oplus) + (\mathcal{B}_s)_{\mu x}\sin(2\omega_\oplus T_\oplus)]^2.$$

$$P^{(1)}_{\mu\tau} = 2L\Big\{(P^{(1)}_{\mathcal{C}})_{\tau\mu} + (P^{(1)}_{\mathcal{A}_s})_{\tau\mu}\sin\omega_\oplus T_\oplus + (P^{(1)}_{\mathcal{A}_c})_{\tau\mu}\cos\omega_\oplus T_\oplus + (P^{(1)}_{\mathcal{B}_s})_{\tau\mu}\sin 2\omega_\oplus T_\oplus + (P^{(1)}_{\mathcal{B}_c})_{\tau\mu}\cos 2\omega_\oplus T_\oplus\Big\}$$

- The increase in power has to be inherent to the theory
- The two baselines test different limiting cases of the same theory
- Oscillation probability goes as L^2 in the ND and $2L$ in the FD
- For same detection threshold, ratio of coefficients in two cases is given below
- 523 when all the numbers are plugged in; just what we see

$$(a_L)_{FD}/(a_L)_{ND} = \sqrt{3} \times (N_{ND})^{1/4}/(N_{FD})^{1/2} \times (L_{ND}/2L_{FD}) \times [\frac{1}{2}\sin(\frac{2.534\Delta m^2_{32}L_{FD}}{E})]^{-1}$$

Selecting Neutrino Events

- Basic checks on beam quality and detector performance made
- Remaining events selected if vertex is within fiducial volume
  - Vertex must be >50 cm from edge of partial plane
  - Vertex between planes 30 and 80
  - Ensures containment of any hadronic shower
- High event rate can overestimate the number of events
  - Showers split into multiple events
  - Vertex migrates into fiducial volume
- Remove reconstruction failures using
  - Time and spatial separation between events
  - Total strips in event
  - Activity outside fiducial volume