Fermilab
Wine&Cheese seminar

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CKM matrix and CP violation

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Outline

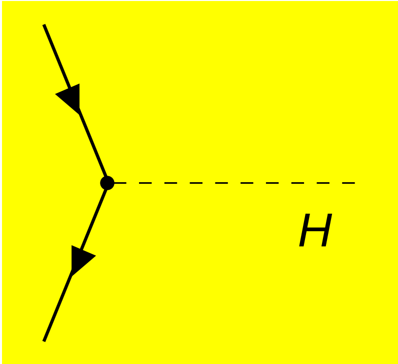
1. Flavor in the Standard Model
2. CKM elements from tree-level decays
3. CKM elements from FCNC processes
4. CP violation in $b \longrightarrow s$ penguin decays
5. Summary

1. Flavor in the Standard Model

Origin of the Cabibbo-Kobayashi-Maskawa matrix:

Yukawa couplings of the Higgs field:

$$y_{ij}\overline{f}_i f_j (v+H)$$



$\Rightarrow$ quark mass matrix: $m_{ij} = y_{ij} v$

diagonalization $\Rightarrow$ fermion masses and CKM matrix V.

$V \neq 1$

$\Rightarrow$ couplings of the W-Bosons to quarks of different generations, flavor physics

y_{ij}, V complex $\Rightarrow$ one physical CP violating phase

The CKM matrix... ...expanded in $\lambda \simeq 0.22$:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3\left(1+\frac{\lambda^2}{2}\right)(\overline{\rho}-i\overline{\eta}) \\ -\lambda - iA^2\lambda^5\overline{\eta} & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\overline{\rho}-i\overline{\eta}) & -A\lambda^2 - iA\lambda^4\overline{\eta} & 1 \end{pmatrix}$$

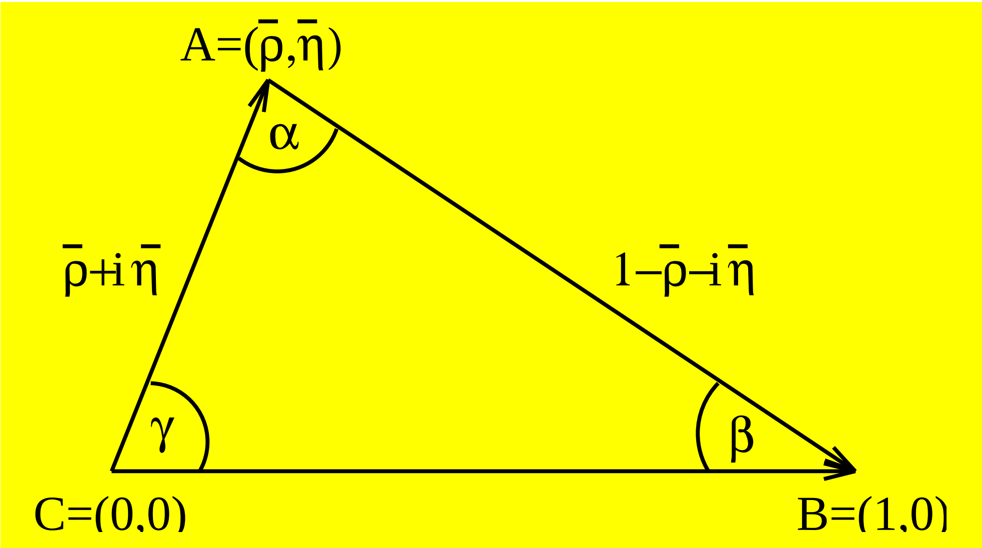
with the Wolfenstein parameters λ, A, $\overline{\rho}$, $\overline{\eta}$

CP violation $\Leftrightarrow \overline{\eta} \neq 0$

Unitarity triangle:

Exact definition:

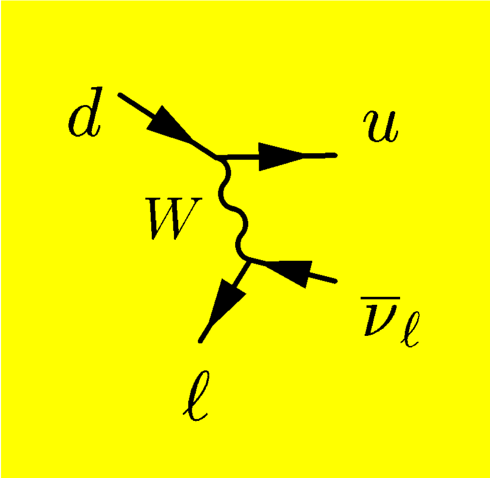
$$\overline{\rho} + i\overline{\eta} = -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} = \left|\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right| e^{i\gamma}$$



2. CKM elements from tree-level decays

V_{ud}

V_{ud} is determined from semileptonic $d \to u \ell^- \overline{\nu}_\ell$ decays through lifetime measurements:



- superallowed $(0^+ \to 0^+)$ nuclear β decay,
- $n \to p\,\ell\,\overline{\nu}_\ell(\gamma)$ or
- $\pi^- \to \pi^0\,\ell\,\overline{\nu}_\ell(\gamma)$.

All methods involve the hadronic form factor of the vector current:

$$\langle\, f\, |\overline{u}\gamma_\mu d|\, i\,\rangle$$

$n \to p\,\ell\,\overline{\nu}_\ell(\gamma)$ further involves the form factor of the axial vector current:

$$\langle\, f\, |\overline{u}\gamma_\mu\gamma_5 d|\, i\,\rangle$$

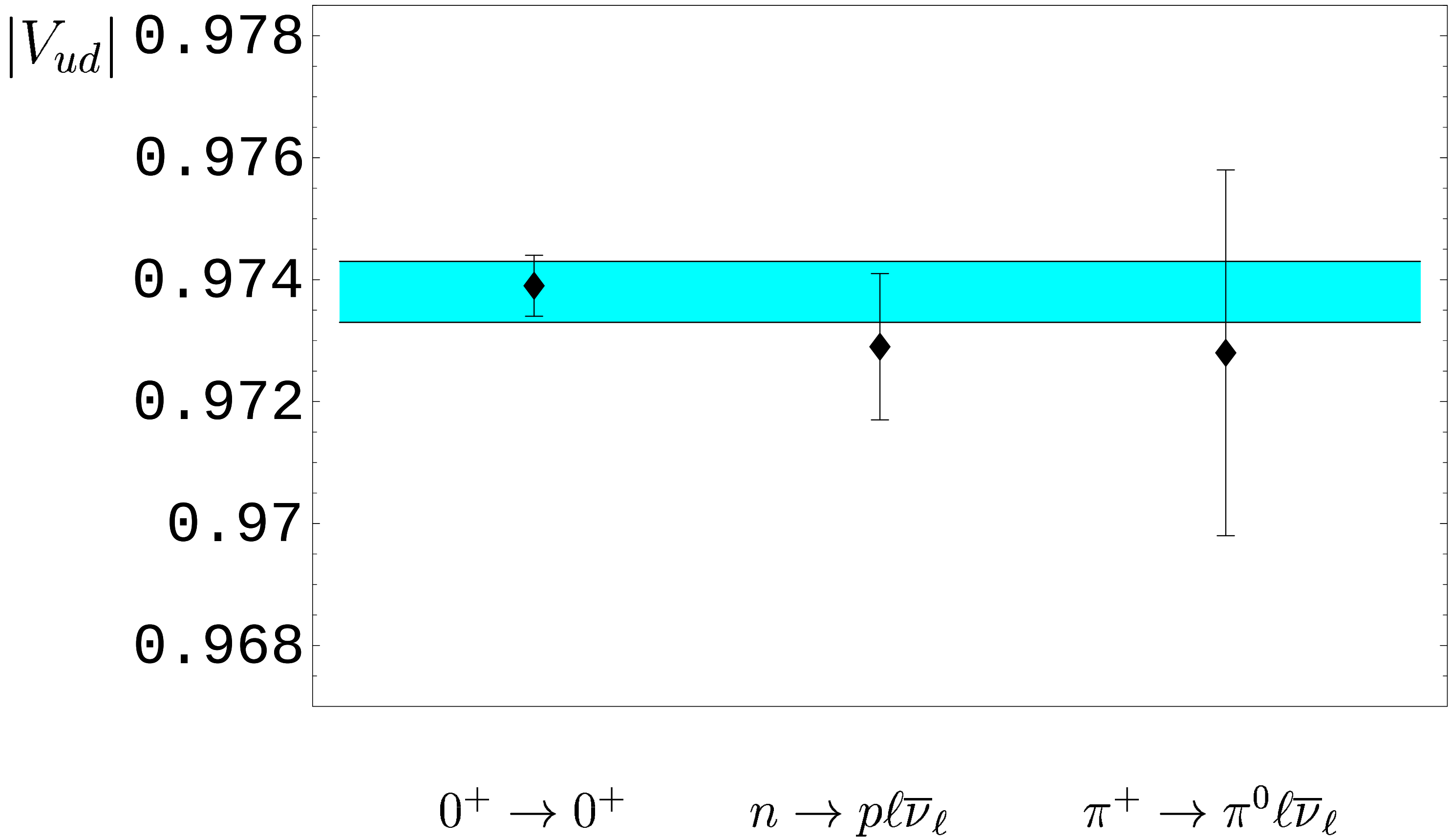
Here $(i, f) = (0^+, 0^+), (n, p)$ or $(\pi^\pm, \pi^0)$.

Isospin symmetry $\Rightarrow$ normalization of the vector form factor is fixed at $p_i = p_f$.

Ademollo-Gatto theorem: Corrections are $\mathcal{O}\left(\frac{(m_d - m_u)^2}{\Lambda_{\text{had}}^2}\right)$, i.e. second order in the symmetry breaking parameter.

The normalization of the axial form factor is not fixed, but the corresponding parameter G_A can be fixed from asymmetries in the Dalitz plot.

method	theory drawback	theoretical cleanliness	experimental precision in V_{ud}
$0^+ \to 0^+$	nuclear effects	$*$	$***$
$n \to p\,\ell\,\overline{\nu}_\ell(\gamma)$	depends on G_A	$**$	$**$ (but contradictory)
$\pi^- \to \pi^0\,\ell\,\overline{\nu}_\ell(\gamma)$		$***$	$*$



courtesy of Vincenzo Cirigliano

Average: $V_{ud} = 0.9738 \pm 0.0005$

Theoretical progress: calculation of the leading (two-loop) electroweak $\mathcal{O}(\alpha^2)$ corrections to $n \to p\,\ell\,\overline{\nu}_\ell(\gamma)$ by Czarnecki, Marciano, Sirlin 2004.

V_{us}

V_{us} from Kaon decays:

Chiral Perturbation Theory (χPT): The pseudoscalar mesons $\pi, K, \ldots$ are Goldstone bosons of a dynamically broken chiral symmetry of the QCD lagrangian.

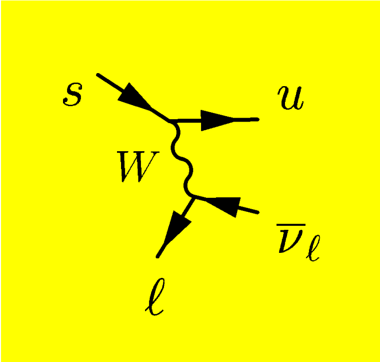
$\Rightarrow$ systematic expansion in $\frac{p}{\Lambda_{\text{had}}}, \frac{M}{\Lambda_{\text{had}}}, \frac{m_\ell}{\Lambda_{\text{had}}}$ and the electroweak coupling e.

p and M denote meson momenta and masses.

$K\ell 3$ decays:

$K^0 \to \pi^- \ell^+ \nu_\ell$, $K^0 \to \pi^- \mu^+ \nu_\ell$

$K^+ \to \pi^0 \ell^+ \nu_\ell$, $K^+ \to \pi^0 \mu^+ \nu_\ell$



Dependence on key quantities:

$$\Gamma(K \to \pi \ell^+ \nu_\ell) \;\propto\; V_{us}^2 \left| f_+^{K^0\pi^-}(0) \right|^2 \left[1 \,+\, 2\Delta_{SU(2)}^{K} \,+\, 2\Delta_{\rm em}^{K\ell} \right]$$

with $\quad \langle\, \pi^-(p_\pi)\, | \overline{s} \gamma^\mu u |\, K^0(p_K)\, \rangle \;=\; f_+^{K^0\pi^-}(0)(p_K^\mu + p_\pi^\mu) + \mathcal{O}(p_K - p_\pi)$

$$\Delta_{SU(2)}^{K^+} \;=\; \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1, \qquad \Delta_{SU(2)}^{K^0} \;=\; 0$$

$\Delta_{\rm em}^{K\ell}$: QED corrections

and $f_+^{K^0\pi^-}(0) = 1 + \mathcal{O}\left(\frac{(m_s - m_d)^2}{\Lambda_{\rm had}^2} \right)$

Substantial theory (χPT) improvements within recent years:

Key references:

$\Delta_{\mathrm{em}}^{K\ell}$: Cirigliano, Knecht, Neufeld, Rupertsberger, Talavera 2002

Cirigliano, Neufeld, Pichl 2004

Andre 2004

Moussallam, Descotes 2005

significant effect of $\mathcal{O}(e^2 p^2)$ corrections on differential distributions, must be included in Monte Carlo simulations.

$f_+^{K^0\pi^-}(0) - 1$: Leutwyler, Roos 1984

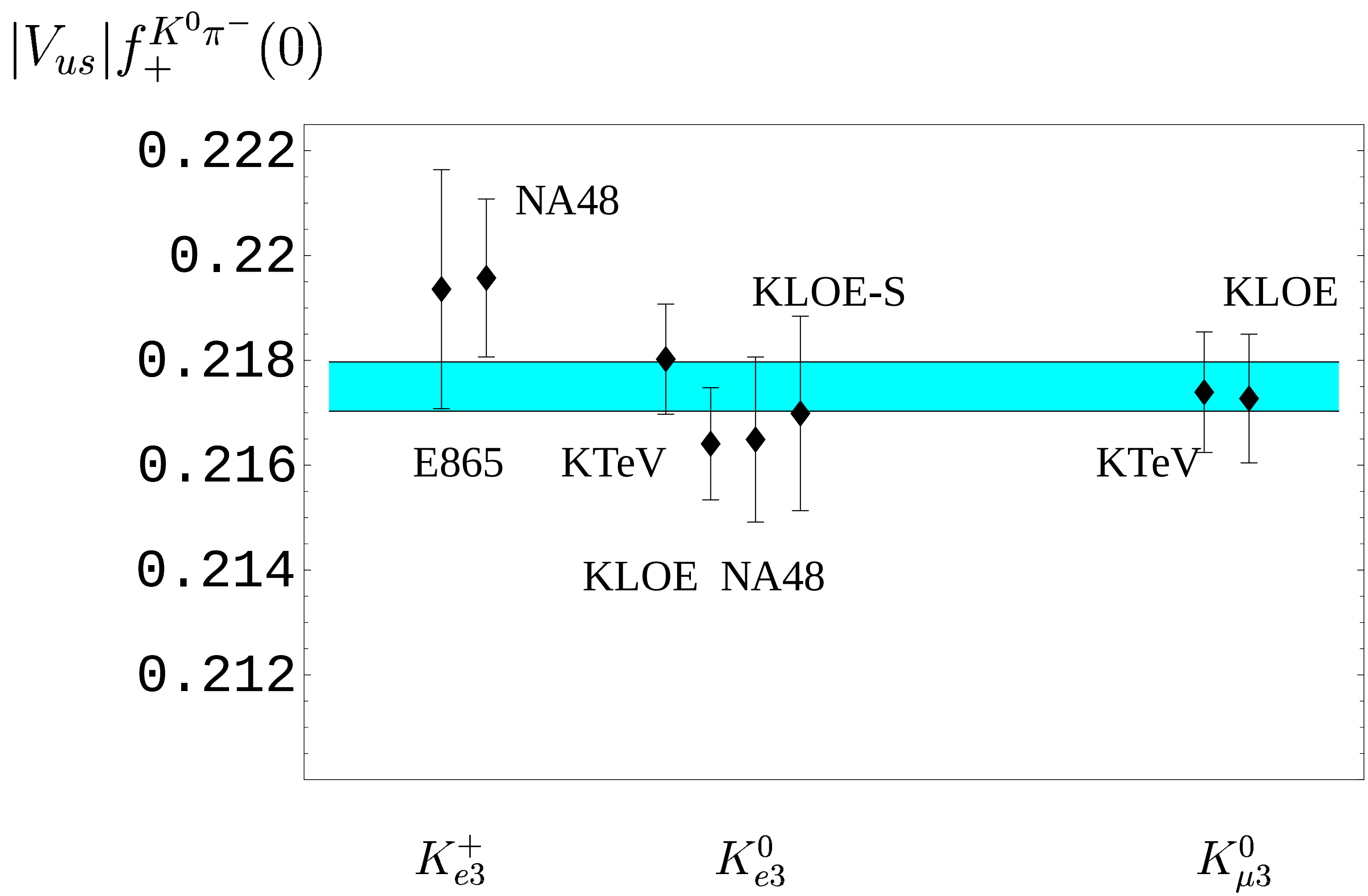
Post, Schilcher 2002

Bijnens, Talavera 2003

Jamin, Oller, Pich 2004

Becirevic et al. (SPQcdR)2004 (lattice!)

Cirigiliano et al. 2005



courtesy of Vincenzo Cirigliano

$f_+^{K^0\pi^-} V_{us} = 0.2175 \pm 0.0008$, theory: $f_+^{K^0\pi^-} = 0.972 \pm 0.012$

$\Rightarrow \quad V_{us} = 0.2238 \pm 0.0029$ from $K\ell 3$

$K\mu 2$ decay: $K^+ \to \mu^+ \nu_\mu(\gamma)$

$$\frac{\Gamma(K^+ \to \mu^+ \nu_\mu(\gamma))}{\Gamma(\pi^+ \to \mu^+ \nu_\mu(\gamma))} = \frac{V_{us}^2}{V_{ud}^2} \frac{F_K^2}{F_\pi^2} \frac{M_K^2 - m_\mu^2}{M_\pi^2 - m_\mu^2} \left[1 \; - \; \frac{\alpha}{\pi} (C_\pi - C_K) \right]$$

QED corrections: $\quad C_\pi - C_K = 3.0 \pm 1.5$ Marciano 2004

Lattice: $\quad \dfrac{F_K}{F_\pi} = 1.210 \pm 0.004 \pm 0.013$ MILC 2004

$$\Rightarrow \quad V_{us} = 0.2223 \pm 0.0026 \qquad \text{from } K\mu 2$$

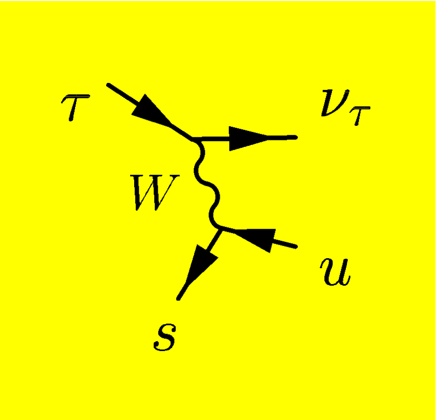
...astonishingly precise, it starts to constrain new physics (charged Higgs).

Marciano 2004

V_{us} from τ decay:

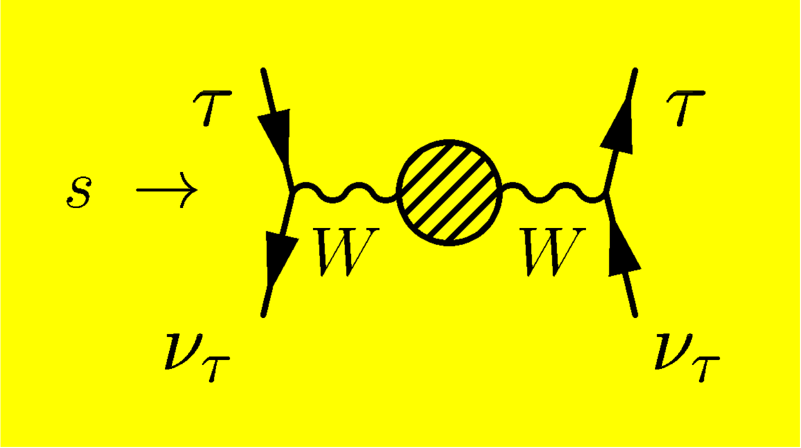
$$R_{\tau s} = \frac{\Gamma^{\Delta S=1}(\tau \to \text{hadrons } \nu_\tau(\gamma))}{\Gamma(\tau \to e\overline{\nu}_e\nu_\tau(\gamma))} \propto V_{us}^2$$

$$R_{\tau d} = \frac{\Gamma^{\Delta S=0}(\tau \to \text{hadrons } \nu_\tau(\gamma))}{\Gamma(\tau \to e\overline{\nu}_e\nu_\tau(\gamma))} \propto V_{ud}^2$$



Here S is the strangeness. $R_{\tau s,d}$ can be related to the QCD current-current correlators Π_S^T and Π_S^L (with $z = s/M_\tau^2$):

$$R_{\tau s,d} = 12\pi \int_0^1 d\,z(1-z)^2 \left[(1+2z)\,\mathrm{Im}\,\Pi_{s,d}^T(z) + \mathrm{Im}\,\Pi_{s,d}^L(z)\right]$$



$\Pi_{s,d}^{T,L}$ can be computed through the operator product expansion (OPE). The leading term is massless perturbative QCD, subleading operators are m_s^2 and $m_s\langle\overline{q}q\rangle$.

Compute the flavor SU(3) breaking difference:

$$\delta R_\tau \;\equiv\; \frac{R_{\tau d}}{V_{ud}^2} - \frac{R_{\tau s}}{V_{us}^2}$$

Pich, Prades; ALEPH (1998)

With $R_{\tau d} = 3.469 \pm 0.014$ and $R_{\tau s} = 0.1694 \pm 0.0049$ from experiment find:

$$V_{us} \;=\; \sqrt{\frac{R_{\tau s}}{R_{\tau d}/|V_{ud}|^2 \;-\; \delta R_\tau}}$$

theory: $\delta R_\tau \;=\; 0.218 \,\pm\, 0.026$

$$= 0.2219 \pm 0.0033_{\rm exp} \pm 0.0009_{\rm th} \;=\; 0.2219 \pm 0.0034,$$

Gámiz, Jamin, Pich, Prades, Schwab 2003/04

Dominant source of uncertainty in δR_τ is from m_s. δR_τ is small!

Near future: reduce the uncertainty with τ-data from BABAR and BELLE.

V_{ud} and V_{us} summary

$$V_{us} = 0.2238 \pm 0.0029 \qquad \text{from } K\ell 3$$

$$V_{us} = 0.2223 \pm 0.0026 \qquad \text{from } K\mu 2$$

$$V_{us} = 0.2219 \pm 0.0034 \qquad \text{from } \tau \to \text{hadrons } \nu_\tau$$

Average:

$$V_{us} = 0.2227 \pm 0.0017$$

$$V_{ud} = 0.9738 \pm 0.0005$$

Unitarity check:

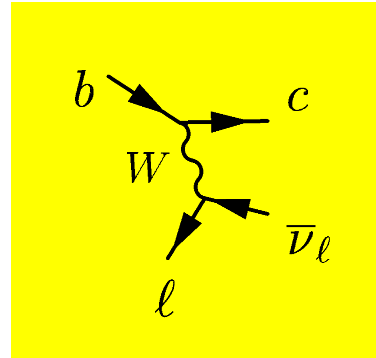
$$V_{us}^2 + V_{ud}^2 + |V_{ub}|^2 - 1 \simeq V_{us}^2 + V_{ud}^2 - 1 = -0.0021 \pm 0.0012$$

The Cabibbo matrix is unitary at the 1.8σ level,
just as at Lepton-Photon 2003:

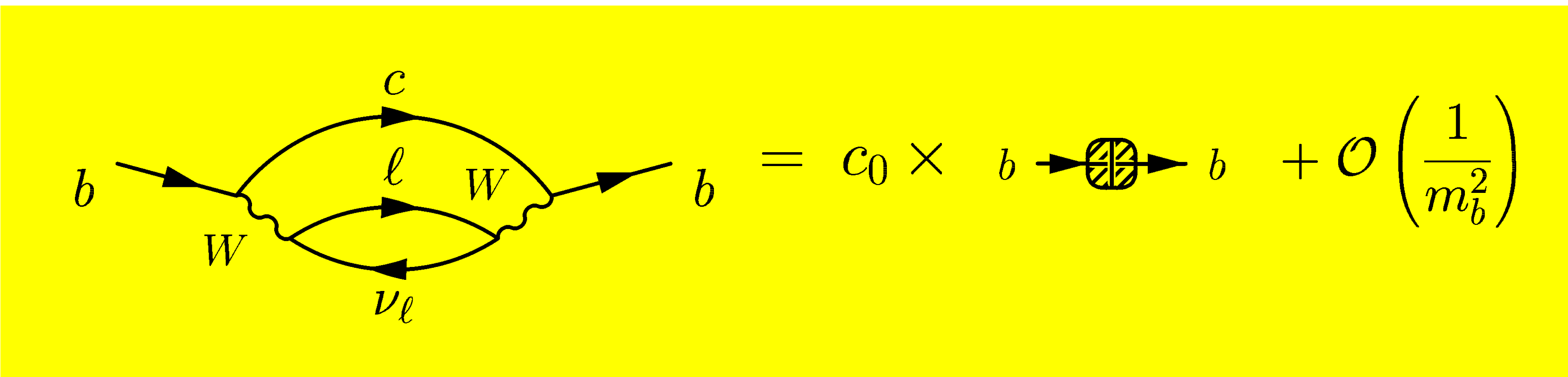
$$V_{us}^2 + V_{ud}^2 - 1 = -0.0031 \pm 0.0017$$

V_{cb}

Here: Determinations of V_{cb} from
inclusive semileptonic B decays.



In B decay exploit that $m_b \gg \Lambda_{QCD}$. Apply the operator product expansion (OPE) to inclusive decay rates, schematically:



c_0 multiplies the leading dimension-3 operator $\bar{b}b$. It can be calculated in perturbative QCD.

OPE: expansion in Λ_{QCD}/m_b and $\alpha_s(m_b)$

First non-perturbative corrections appear at order Λ_{QCD}^2/m_b^2 and involve

$$-\langle\, B\, |\bar{b} D_\perp^2 b|\, B\,\rangle \,\propto\, \mu_\pi^2, \qquad\qquad \langle\, B\, |\bar{b} i \sigma_{\mu\nu} G^{\mu\nu} b|\, B\,\rangle \,\propto\, \mu_G^2.$$

$$\uparrow \qquad\qquad\qquad\qquad \uparrow$$

kinetic operator — chromomagnetic operator

μ_G^2 can be determined from spectroscopy.

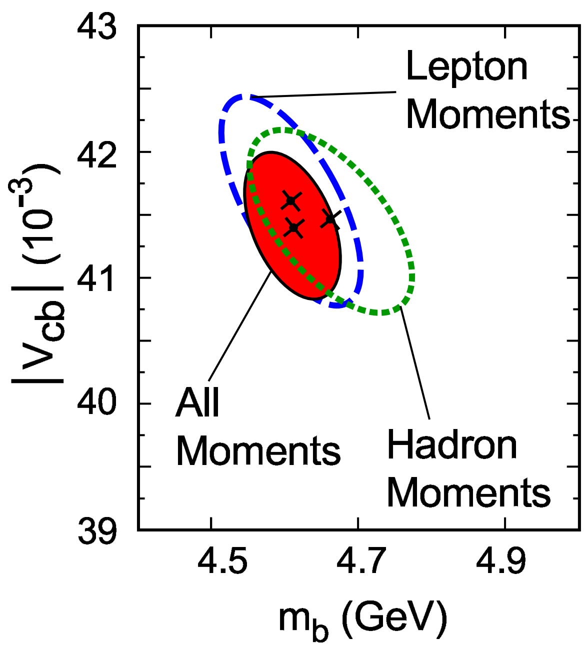
$\Rightarrow$ To order Λ_{QCD}^2/m_b^2 only involve m_b, m_c and μ_π^2.

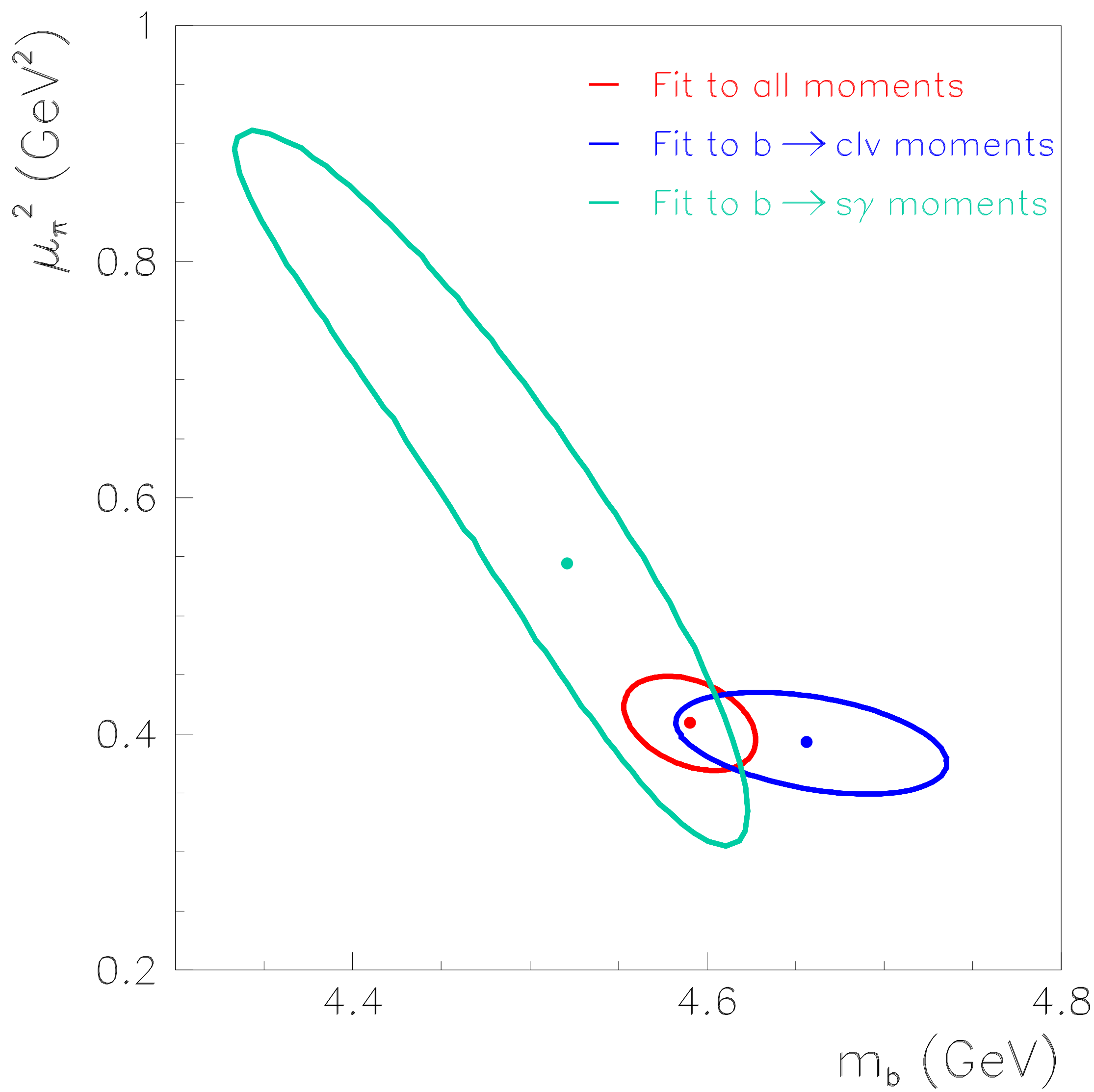
The OPE can further be applied to certain spectral moments of the $B \to X \ell \overline{\nu}_\ell$ decay, the distributions of the hadron invariant mass M_X and of the lepton energy and the same parameters govern different inclusive decays.

State of the art: Fits to order Λ_{QCD}^3/m_b^3 (involving 7 parameters)

Gambino, Uraltsev; Bauer, Ligeti, Luke, Manohar, Trott

Here $m_b = m_b(1\,\text{GeV})$ is defined in the kinetic scheme.

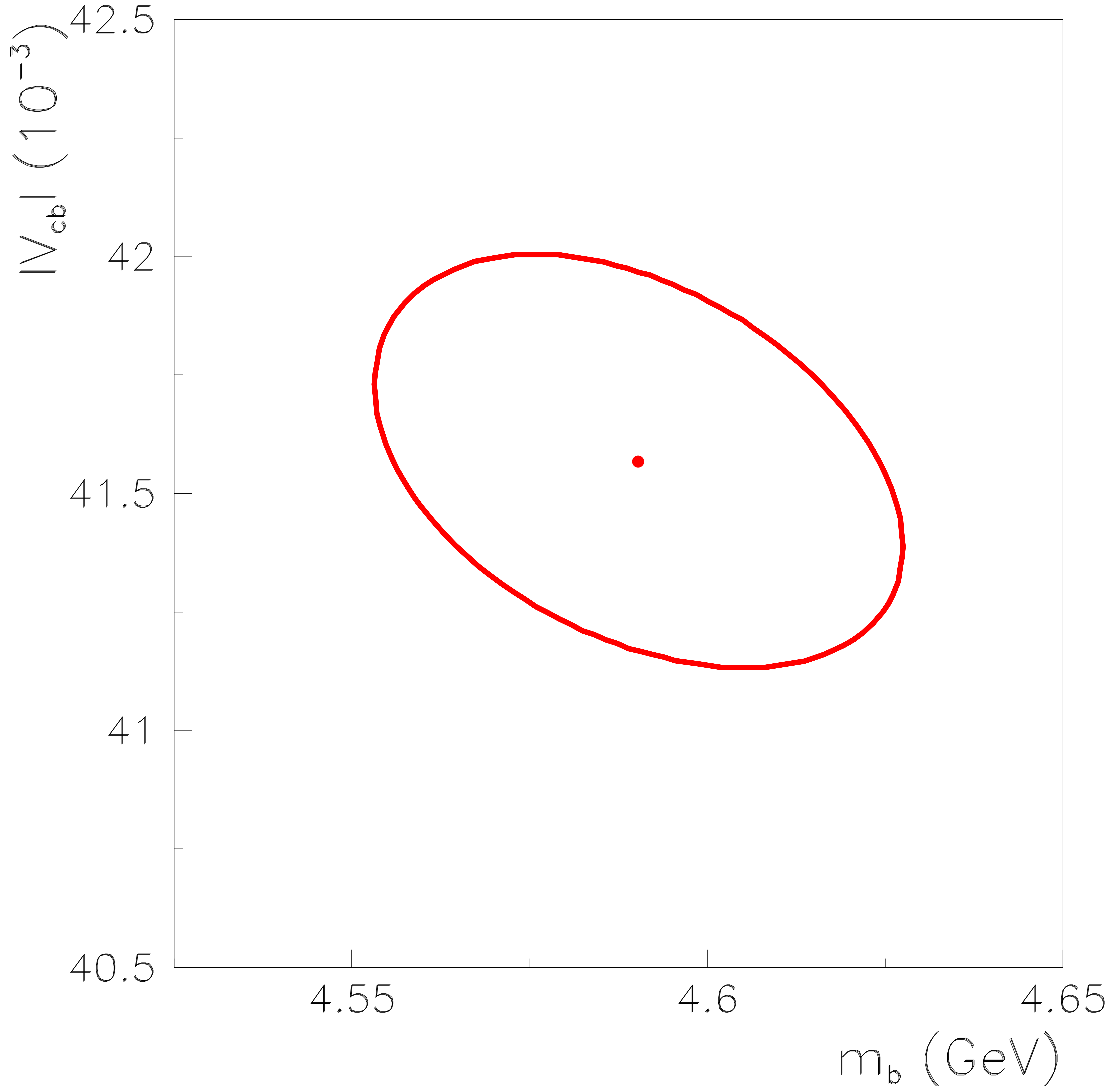




BaBar

courtesy of Oliver Buchmüller and Henning Flächer

Global fit to hadron and lepton moments in $B \to X \ell \nu_\ell$ and photon energy moments in $B \to X_s \gamma$ from BaBar, BELLE, CDF, CLEO, DELPHI:



fit and plot courtesy of Oliver Buchmüller and Henning Flächer

V_{cb}^{incl} summary

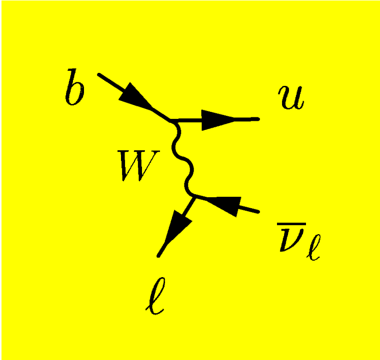
Result:

$$V_{cb} = 41.6 \pm 0.3_{\mathrm{exp}} \pm 0.3_{\mathrm{OPE\,moments}} \pm 0.3_{\mathrm{OPE}\,\Gamma_{\mathrm{sl}}}$$
$$= (41.6 \pm 0.5) \cdot 10^{-3}$$

from inclusive $B \to X \ell \nu_\ell$

compared to CKM 2005: new BaBar photon energy moment included

$|V_{ub}|$



First: Determinations of $|V_{ub}|$ from
inclusive $B \to X_u \ell \overline{\nu}_\ell$ decays

Problem: Huge background from $b \to c\ell\overline{\nu}_\ell$.
$\Rightarrow$ employ cuts on judiciously chosen combinations of
M_X, E_X and E_ℓ.
$\Rightarrow$ M_X is typically too low for OPE

Still some components of the hadron momentum $\vec{P}_X$ are large. The description of inclusive B decays in this region involves the non-perturbative shape function S, which is a parton distribution function of the B meson.

At leading order in $1/m_b$: The same S governs the photon spectrum in $B \to X_s\gamma$ and differential decay rates in $B \to X_u \ell \overline{\nu}_\ell$.
$\Rightarrow$ Extract S from $B \to X_s\gamma$ for use in $B \to X_u \ell \overline{\nu}_\ell$.

Goal: Push total uncertainty in $|V_{ub}|$ below 10%:

Theoretical progress:

1. Factorization formula at $\mathcal{O}(\alpha_s)$:

$$d\Gamma \propto H \int_0^{P_+} d\omega \, J\left(m_b(P_+ - \omega)\right) \, S(\omega)$$

$$\nearrow \qquad\qquad\qquad\qquad \uparrow \qquad\qquad\qquad\qquad \nwarrow$$

hard QCD	jet function	shape function
$\mu \sim m_b$	$\mu \sim M_X \sim m_b \Lambda_{QCD}$	$\mu \sim \Lambda_{QCD}$

where $M_X \sim P_+ \equiv E_X + |\vec{P}_X| \ll P_- \equiv E_X - |\vec{P}_X| \leq m_b$.

Bauer, Manohar; Bosch, Lange, Neubert, Paz

$\Rightarrow$ radiative QCD corrections properly included

2. Factorization of subleading ($\mathcal{O}(1/m_b)$) shape functions $s_i(\omega)$:

Unrelated in $B \to X_s \gamma$ and $B \to X_u \ell \overline{\nu}_\ell$, but their moments are known in terms of OPE parameters like μ_π^2.

Lee, Stewart; Bosch, Neubert, Paz; Beneke, Campanario, Mannel, Pecjak 2004

3. Event generator for $B \to X_u \ell \overline{\nu}_\ell$:
uses full theoretical information on $B \to X_s \gamma$ and $B \to X_u \ell \overline{\nu}_\ell$ shape functions, interpolates to the OPE region

Lange, Neubert, Paz 2005

LNP promotes to cut on the variable P_+, which is related to the photon energy in $B \to X_s \gamma$ as $P_+ = M_B - 2E_\gamma$ and allows for the most efficient use of the extracted $S(\omega)$.

Mannel, Recksiegel 1999

$|V_{ub}|^{\mathrm{incl}}$ summary

Improved Wolfenstein parameters:

$$V_{cb} = A\lambda^2$$
$$|V_{ub}| = A\lambda^3 (1+\lambda^2) \sqrt{\overline{\rho}^2 + \overline{\eta}^2} + \mathcal{O}(\lambda^7)$$

$$\Rightarrow \quad \sqrt{\overline{\rho}^2 + \overline{\eta}^2} = \frac{|V_{ub}|}{V_{cb}} \frac{1-\lambda^2+\mathcal{O}(\lambda^4)}{\lambda},$$

which is a circle in the $(\overline{\rho}, \overline{\eta})$ plane. With $V_{cb} = 0.0416 \pm 0.0005$ and

$$|V_{ub}| = (4.39 \pm 0.20_{\mathrm{exp}} \pm 0.27_{m_b,\mathrm{th}}) \cdot 10^{-3} = (4.39 \pm 0.34) \cdot 10^{-3}$$

from inclusive semileptonic decays one finds

$$\sqrt{\overline{\rho}^2 + \overline{\eta}^2} = 0.45 \pm 0.04$$

$|V_{ub}|$ from $B \to \pi \ell^+ \nu_\ell$

The determination of $|V_{ub}|$ from the exclusive decay $B \to \pi \ell^+ \nu_\ell$ involves the form factor $F(B \to \pi)$:

$$d\Gamma(B \to \pi \ell^+ \nu_\ell) \propto [F(B \to \pi)]^2 \, |V_{ub}|^2$$

$F(B \to \pi)$ is a hadronic quantity $\Rightarrow$ need non-perturbative QCD.

NEW! Lattice QCD has left the **dark age** of **quenched** calculations and has entered the era of 2+1 dynamical staggered fermions.

The Fermilab/HPQCD/MILC Lattice collaboration has computed $F(B \to \pi)$. The result for $|V_{ub}|$ reads

$$\begin{aligned} |V_{ub}|^{\text{excl}} &= (3.48 \pm 0.29_{\text{stat}} \pm 0.38_{\text{syst}} \pm 0.47_{\text{exp}}) \cdot 10^{-3} \\ &= (3.48 \pm 0.67) \cdot 10^{-3} \end{aligned}$$

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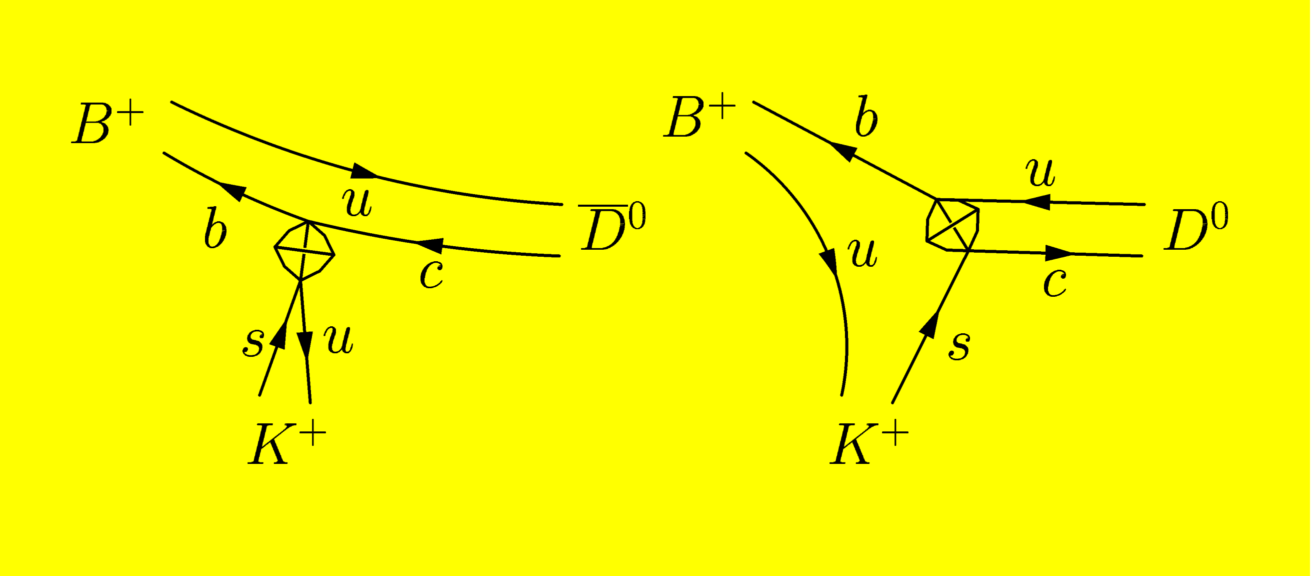
FNAL
PRIME

$\arg V_{ub}$

$\gamma = \arg V_{ub}^*$ from $B \to D^0 X$:

Basic idea: Use interference of the two tree amplitudes $b \to c\overline{u}q$ and $b \to u\overline{c}q$ (with $q = d$ or $q = s$) to get γ from $B \to \overset{(}{\overline{D^0}}{}^{)} X$ branching fractions.

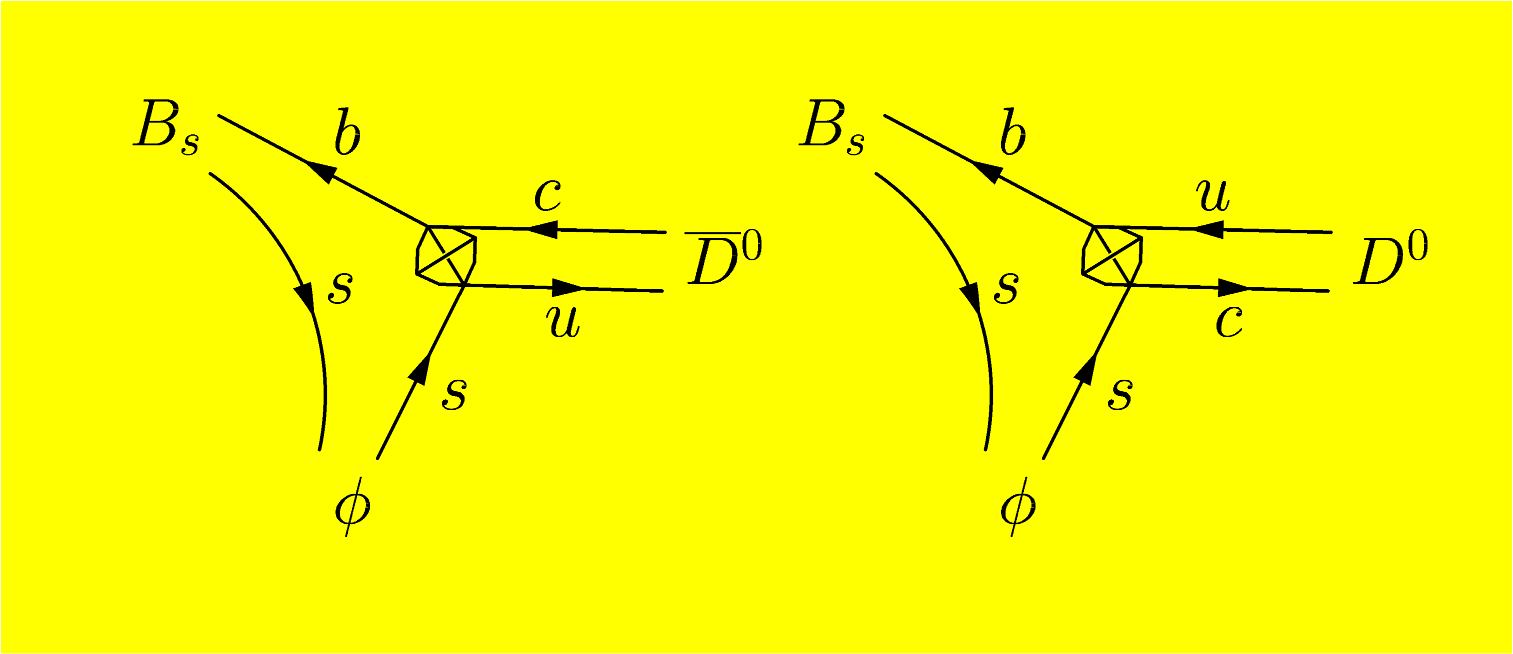
Prototype: Gronau-London-Wyler (GLW) method:



Interference if both $D^0 \to f$ and $\overline{D^0} \to f$ are allowed.

Need four measurements to solve for $|A(b \to c)|$, $|A(b \to u)|$, the strong phase δ and the weak phase γ. E.g. measure the branching fractions of $B^+ \to \overset{(}{\overline{D^0}}{}^{)}[\to K^\pm \pi^\mp] K^+$ and $B^\pm \to \overset{(}{\overline{D^0}}{}^{)}[\to \pi^+\pi^-] K^\pm$.

This works with non-flavor-specific $B \to D^0 X$ decays as well:
E.g. both B_s and $\overline{B}_s$ can decay to $\overset{(-)}{D^0}\phi$:



One can still solve for all hadronic parameters and γ, if at least three pairs of $\overset{(-)}{B}_s \to \overset{(-)}{D^0}[\to f_i]\phi$ and $\overset{(-)}{B}_s \to \overset{(-)}{D^0}[\to \overline{f}_i]\phi$ branching fractions are measured, where $\overline{f}_i = CP f_i$ (and the f_i's are not CP eigenstates).

Gronau, Grossman, Shuhmaher, Soffer, Zupan 2004

Can CDF contribute to γ measurements?

γ from α

Tagged analyses of $B^0 \to \pi\pi$, $B^0 \to \rho\rho$ or $B^0 \to \rho\pi$ decays determine α. (Ignoring discrete ambiguities in extracting α from $\sin(2\alpha)$) one finds:

$$\alpha_{\rm exp}(\rho\rho, \pi\pi, \rho\pi \text{ combined}) = (98.6^{+12.6}_{-8.1})^\circ \qquad \text{CKM Fitter 2005}$$

The loop-induced penguin decay amplitude is eliminated through an isospin analysis. Combine this information with

$$2\beta_{\rm exp} = (43.7 \pm 2.4)^\circ$$

from the measured $a_{\rm CP}^{\rm mix}$ in $b \to c\bar{c}s$ decays to eliminate potential new physics in the $\mathrm{B_d} - \overline{\mathrm{B}}_\mathrm{d}$ mixing phase $2\beta_{\rm exp}$ (again ignoring discrete ambiguities):

$$\gamma = (59.6^{+8.2}_{-12.7})^\circ$$

This is $\gamma = \arg V_{ub}^*$ determined from the tree-level $b \to u\bar{u}d$ amplitude.

$\gamma = \arg V_{ub}^*$ summary

Combining the $B^+ \to D^{0(*)} K^{+(*)}$ results gives

$$\gamma = (63^{+15}_{-12})^\circ \qquad \text{CKM Fitter 2005}$$

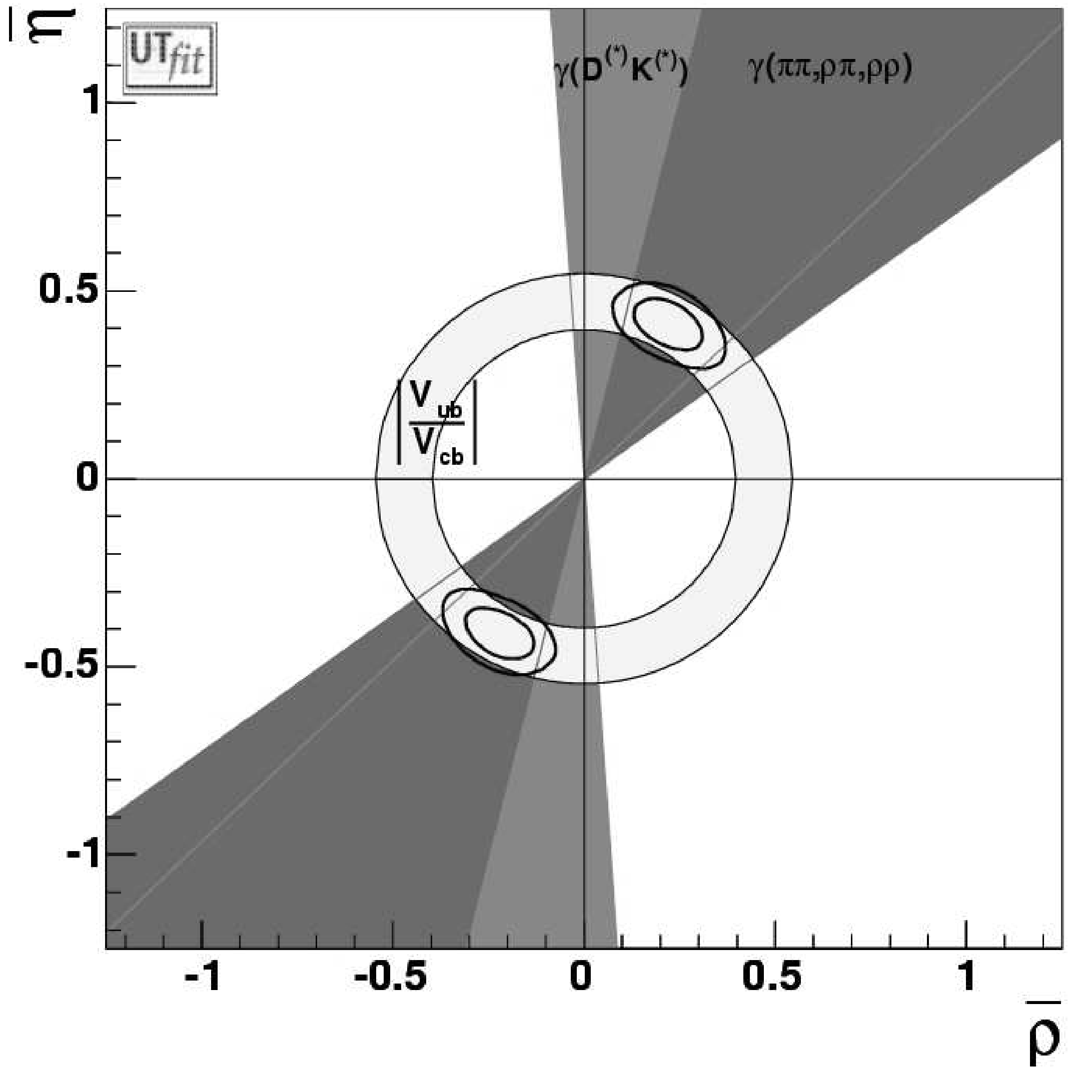
This is $\gamma = \arg V_{ub}^*$ determined from the tree-level $b \to u\bar{c}d$ amplitude.

Analyses extracting $2\beta + \gamma$ from $B_d \to D^0\pi$ give only very weak constraints at present, mainly excluding very large values for γ.

Combining with $\gamma = (59.6^{+8.2}_{-12.7})^\circ$ inferred from α gives (naive average):

$$\gamma = \arg V_{ub}^* = (61^{+7}_{-8})^\circ$$

Unitarity triangle from tree-decays only:



courtesy of Maurizio Pierini

3. CKM elements from FCNC processes

In the Standard Model flavor-changing neutral current (FCNC) processes suffer from several suppression factors:

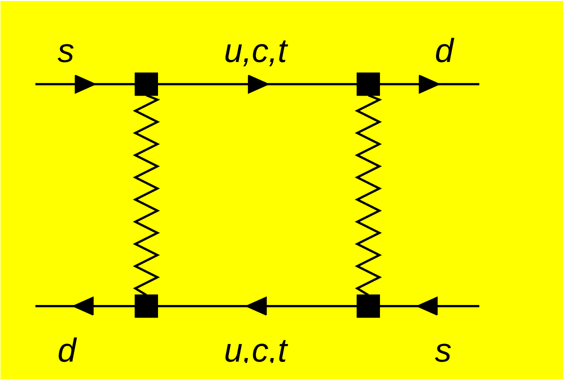
- FCNCs proceed through electroweak loops, no FCNC tree graphs,
- small CKM elements, e.g. $|V_{ts}| = 0.04$, $|V_{td}| = 0.01$,
- GIM suppression, $\propto m_c^2/M_W^2$ in loops with charm,
- helicity suppression in radiative and leptonic decays, e.g. $\propto m_b/M_W$, because FCNCs involve only left-handed fields.

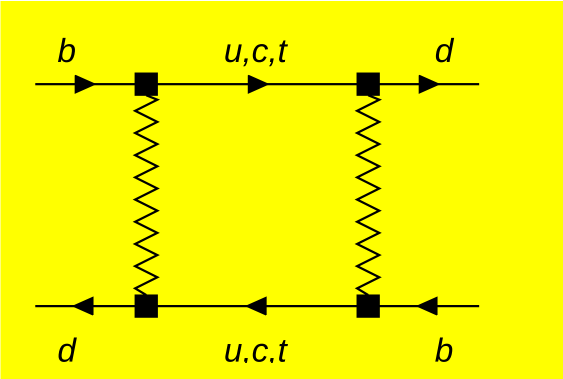
The suppression of FCNC processes is not the consequence of any symmetry of the Standard Model. It results from the particle content of the Standard Model and the accidental smallness of most Yukawa couplings. The suppression is absent in generic extensions of the Standard Model.

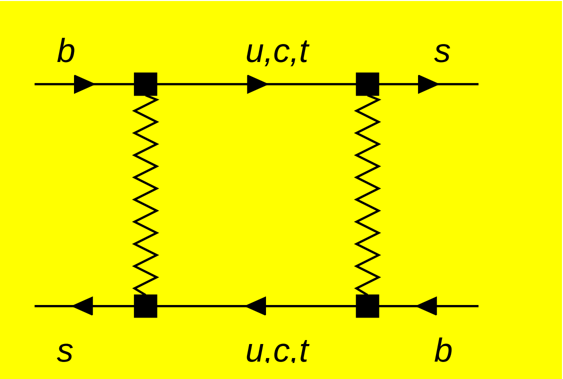
$\Rightarrow$ FCNC processes are extremely sensitive to new physics, probing scales in the range 200 GeV–100 TeV.

Meson-antimeson mixing

$\mathrm{K}-\overline{\mathrm{K}}$ mixing $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ mixing $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing







Observables:

$$\Delta m_{B_d} \propto |V_{td}|^2 \qquad \Delta m_{B_s} \propto |V_{ts}|^2$$

$$\sqrt{(1-\overline{\rho})^2+\overline{\eta}^2}$$

$\cancel{\mathrm{CP}}$:

ϵ_K gives $\mathrm{Im}\,(V_{ts}V_{td}^*)^2$ $\sin(2\beta)$, $\beta \simeq \arg V_{td}^*$ $\sin(2\beta_s)$, $\beta_s \simeq \arg(-V_{ts})$

$$\overline{\eta}[(1-\overline{\rho})+\mathrm{const.}] \qquad \frac{\overline{\eta}}{1-\overline{\rho}} \qquad \overline{\eta}$$

Hadronic matrix elements:

$$\langle\, \overline{K}^0\, |\overline{s}d_{V-A}\overline{s}d_{V-A}|\, K^0\, \rangle \quad \langle\, \overline{B}^0\, |\overline{b}d_{V-A}\overline{b}d_{V-A}|\, B^0\, \rangle \quad \langle\, \overline{B}^0_s\, |\overline{b}s_{V-A}\overline{b}s_{V-A}|\, B^0_s\, \rangle$$

ϵ_K

Hadronic matrix element:

$$\langle \overline{K}^0 | \overline{s} d_{V-A} \overline{s} d_{V-A}(\mu) | K^0 \rangle \propto f_K^2 M_K^2 B_K(\mu),$$

where μ is the energy scale.

New lattice result with 2+1 dynamical staggered fermions:

$$\begin{aligned} B_K^{\overline{\mathrm{MS}}-\mathrm{NDR}}(\mu = 2\,\mathrm{GeV}) &= 0.630 \pm 0.018_{\mathrm{stat}} \pm 0.015_{\mathrm{chiral\ extrapolation}} \\ &\qquad \pm 0.030_{\mathrm{discret.}} \pm 0.130_{\mathrm{perturb.\ matching}} \\ &= 0.630 \pm 0.135 \qquad \text{HPQCD} \end{aligned}$$

Transform to the usually used $\widehat{B}_K$, which is independent of the renormalization scale and scheme:

$$\widehat{B}_K = 0.85 \pm 0.18$$

Perturbative matching: Becher, Gámiz, Melnikov

FNAL
PRIME

Previous results from quenched lattice QCD quoted a similar uncertainty, which included an educated guess of the quenching error. In the new 2+1 flavor unquenched result the estimate of the uncertainty is on a firm footing now. Dominant error from perturbative matching:

$\Rightarrow$ Want lattice-continuum matching at order α_s^2!

V_{td} from $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ mixing

The hadronic matrix element of $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ mixing drops out from the relation between $a_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to J/\psi K_S)$ and

$$\sin(2\beta) \;=\; 0.69 \pm 0.03, \qquad \cos(2\beta) > 0$$
$$\Rightarrow \qquad \arg\left(\pm V_{td}^*\right) \;=\; \beta \;=\; (21.8 \pm 1.2)^\circ,$$

but not from the relation between Δm_{B_d} and $|V_{td}|$: With

$$\langle\, \overline{B}^0 \,|\bar{b}d_{V-A}\bar{b}d_{V-A}|\, B^0 \,\rangle \;=\; M_{B_d}^2 f_{B_d}^2 B_{B_d}$$

one has

$$\Delta m_{B_d} \;\propto\; |V_{td}|^2\, f_{B_d}^2 B_{B_d}.$$

New lattice result for f_{B_d} with 2+1 dynamical staggered fermions from HPQCD:

$$f_{B_d} \; = \; (216 \pm 22)\,\text{MeV}$$

Combine the with the old result $\widehat{B}_{B_d} = 1.27 \pm 0.10$:

$$f_{B_d}\sqrt{\widehat{B}_{B_d}} \; = \; (243 \pm 27)\,\text{MeV}$$

Effect on $|V_{td}|$ (from $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ mixing alone):

$$|V_{td}|^{\text{old}} = 0.00796 \pm 0.00128 \quad \longrightarrow \quad |V_{td}| = 0.00758 \pm 0.00084$$

for $\Delta m_{B_d} = 0.502\text{ps}^{-1}$ and $m_t(m_t) = 163.0\,\text{GeV}$.

$|V_{td}|/|V_{ts}|$ from $\mathrm{B}-\overline{\mathrm{B}}$ mixing

A measurement of the ratio $\Delta m_{B_d}/\Delta m_{B_s}$ will determine $|V_{td}|/|V_{ts}|$ via

$$\left|\frac{V_{td}}{V_{ts}}\right| = \sqrt{\frac{\Delta m_{B_d}}{\Delta m_{B_s}}} \sqrt{\frac{M_{B_s}}{M_{B_d}}}\,\xi$$

with the hadronic quantity

$$\xi = \frac{f_{B_s}\sqrt{\widehat{B}_{B_s}}}{f_{B_d}\sqrt{\widehat{B}_{B_d}}}$$

which equals $\xi = 1$ in the limit of exact SU(3)$_{\mathrm{F}}$.

A critical role for f_{B_s}/f_{B_d} is played by chiral logarithms, which are not correctly reproduced in quenched calculations. Kronfeld, Ryan 2002

New lattice result for f_{B_s}/f_{B_d} with 2+1 dynamical staggered fermions from HPQCD:

$$\frac{f_{B_s}}{f_{B_d}} = 1.20 \pm 0.03$$

Use this to refine

$$\xi = 1.21 \pm 0.03.$$

The lower bound $\Delta m_{B_s} \geq 14.5\,\mathrm{ps}^{-1}$ implies

$$\left|\frac{V_{td}}{V_{ts}}\right| \leq 0.235$$

which constrains one side of the unitarity triangle

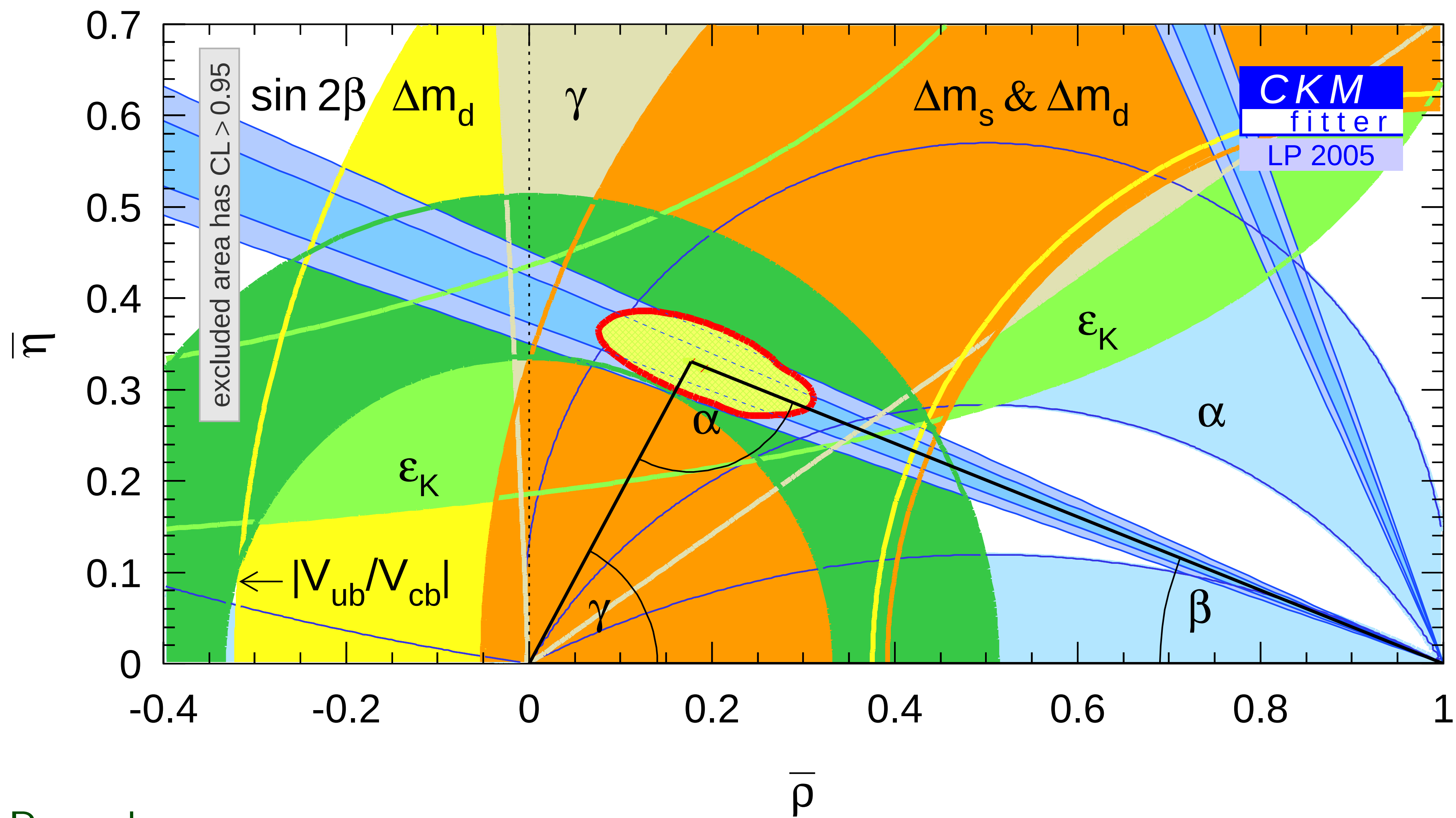
$$R_t \equiv \sqrt{(1-\overline{\rho})^2 + \overline{\eta}^2} = \left|\frac{V_{td}}{V_{ts}\lambda}\right| \leq 1.06$$

Using $V_{ts} \simeq -V_{cb}$ the combined constraint from Δm_{B_d} and Δm_{B_s} is

$$|V_{td}|^{\mathrm{old}} = 0.0085^{+0.0017}_{-0.0019} \quad \longrightarrow |V_{td}| = 0.0083^{+0.0006}_{-0.0020}$$ CKM Fitter 2005

showing the impact of the new unquenched lattice results.

Global fit to the unitarity triangle



Remark:

The first UT fit with theory input computed at NLO in QCD was done in 1995, from ϵ_K, $|V_{ub}|$ and Δm_{B_d} only. Herrlich, UN

$K \to \pi\nu\overline{\nu}$

The rare decays $K^+ \to \pi^+\nu\overline{\nu}$ and $K_L \to \pi^0\nu\overline{\nu}$ provide an excellent opportunity to determine the unitarity triangle from $s \to d$ transitions. With planned dedicated experiments $(\overline{\rho}, \overline{\eta})$ can be determined with a similar precision as today from $b \to d$ and $b \to u$ transitions at the B factories.

$\Rightarrow$ Powerful probe of the CKM mechanism of FCNCs.

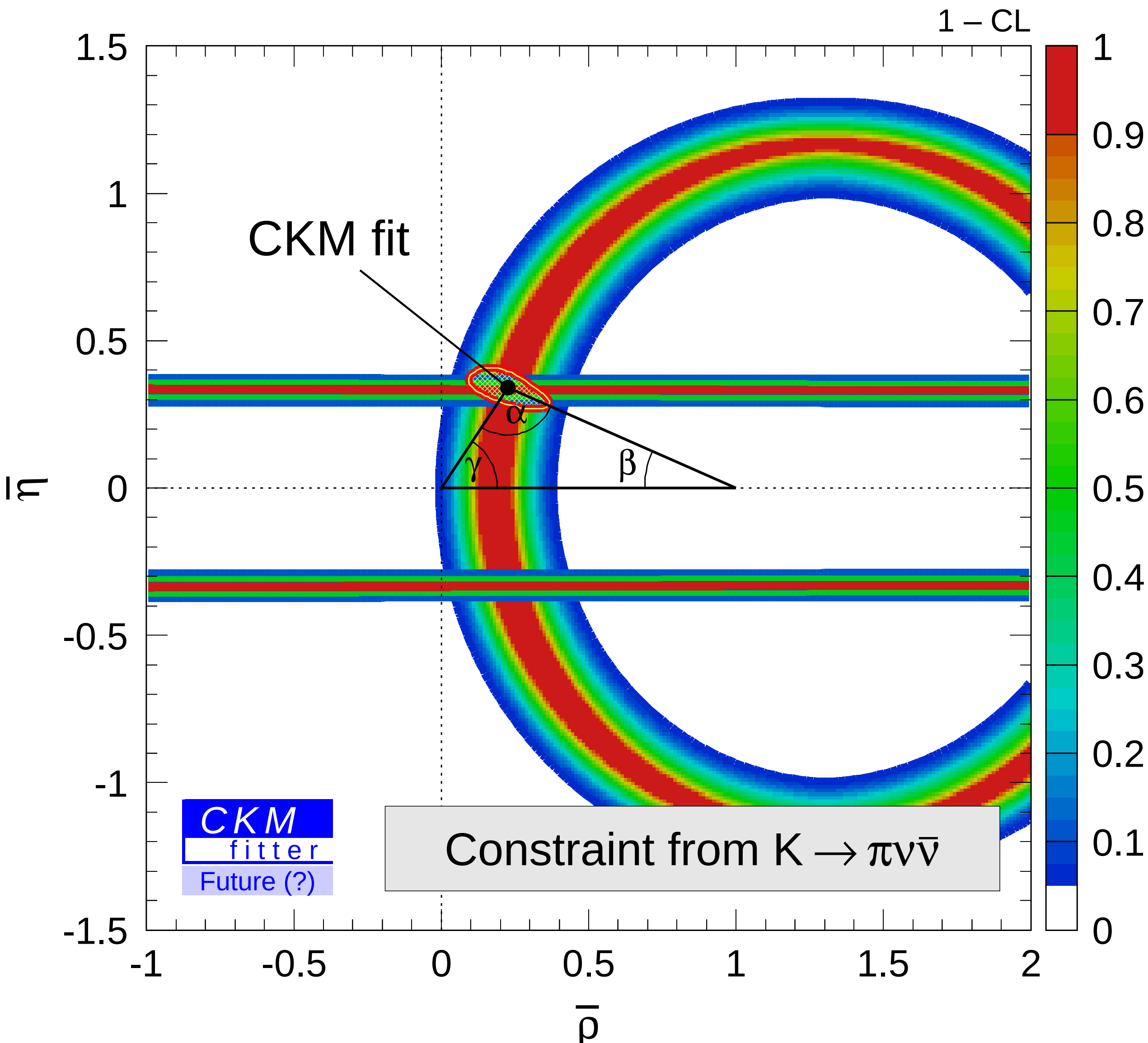
$Br(K_L \to \pi^0\nu\overline{\nu})$ is proportional to $\overline{\eta}^2$, top-dominated, theoretical uncertainty of the next-to-leading order (NLO) prediction below 2%.

$Br(K^+ \to \pi^+\nu\overline{\nu})$ defines an ellipse in the $(\overline{\rho}, \overline{\eta})$ plane, sizeable charm contribution, theoretical accuracy of next-to-leading order (NLO) prediction unsatisfactory

$K \to \pi\nu\overline{\nu}$ dominated by $\overline{s}dZ$-penguin, sensitive to new physics with particle masses of up to $\mathcal{O}(100\,\text{TeV})$.

Buchalla, Buras

Assume a 10% measurement of $Br(K_L \to \pi^0 \nu \overline{\nu})$ and $Br(K^+ \to \pi^+ \nu \overline{\nu})$:



plot courtesy of Andreas Höcker

$$Br(K^+ \to \pi^+ \overline{\nu}\nu) \;\propto$$

$$\left[\mathrm{Im}\,(V_{ts}^* V_{td}) X\left(\frac{m_t^2}{M_W^2}\right)\right]^2 + \left[\mathrm{Re}\,(V_{ts}^* V_{td}) X\left(\frac{m_t^2}{M_W^2}\right) + \mathrm{Re}\,(V_{cs}^* V_{cd}) \lambda^4 \left(P_c + \delta P_{cu}\right)\right]^2$$

$\nearrow$ $\uparrow$ $\uparrow$ $\nearrow$ $\uparrow$

$\propto \overline{\eta}$ top-dependence $\propto 1-\overline{\rho}$ leading charm subleading charm and up

Charm contribution:

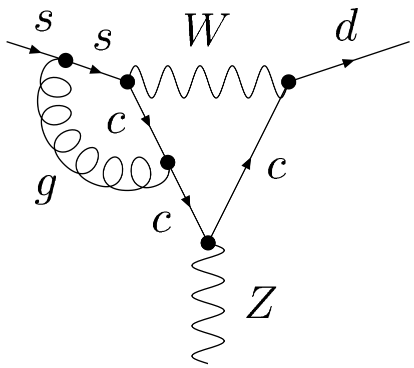
Two expansion parameters: m_K^2/m_c^2 and $\alpha_s(m_c)$.

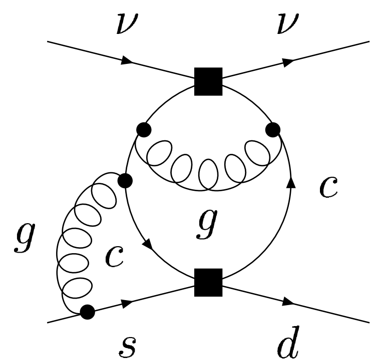
$\mathcal{O}(m_K^2/m_c^2)$ corrections in δP_{cu}:

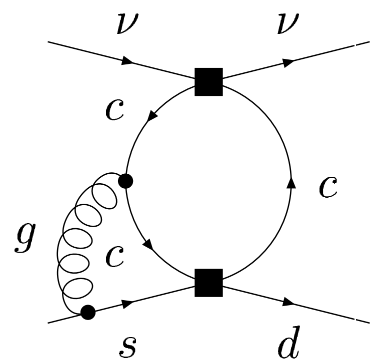
$$\frac{\delta P_{cu}}{P_c} = 0.11 \pm 0.05$$

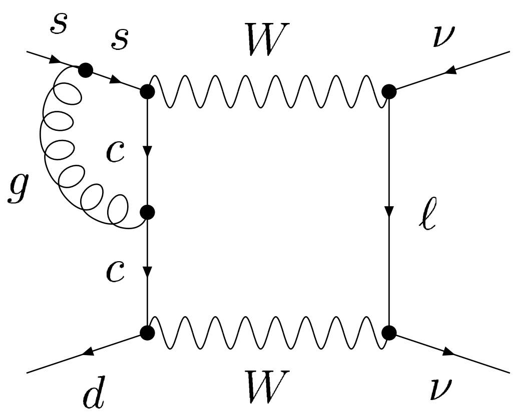
Isidori, Mescia, Smith 2005

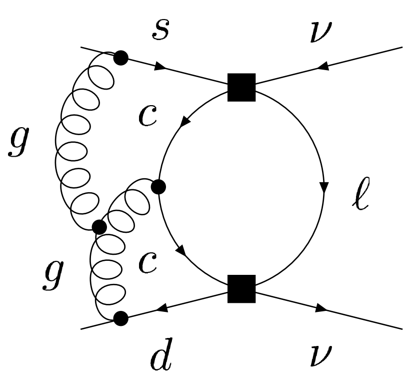
NEW! QCD corrections to P_c at NNLO (three-loop):

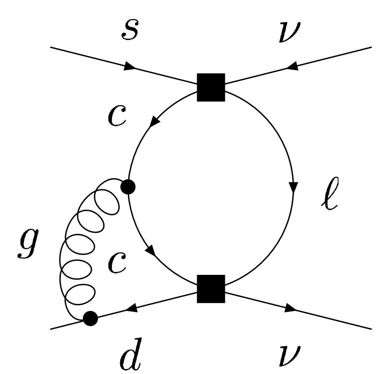












NNLO result:

$$P_c = 0.371 \pm 0.031_{m_c} \pm 0.009_{\alpha_s} \pm 0.009_{\mu_c}$$

charm mass

scale dependence reduced by a factor of 4 from NLO

$$Br(K^+ \to \pi^+ \overline{\nu} \nu) = (8.0 \pm 0.5_{P_c} \pm 0.8_{\text{other}}) \cdot 10^{-11}$$

FNAL PRIME

Buras, Gorbahn, Haisch, UN

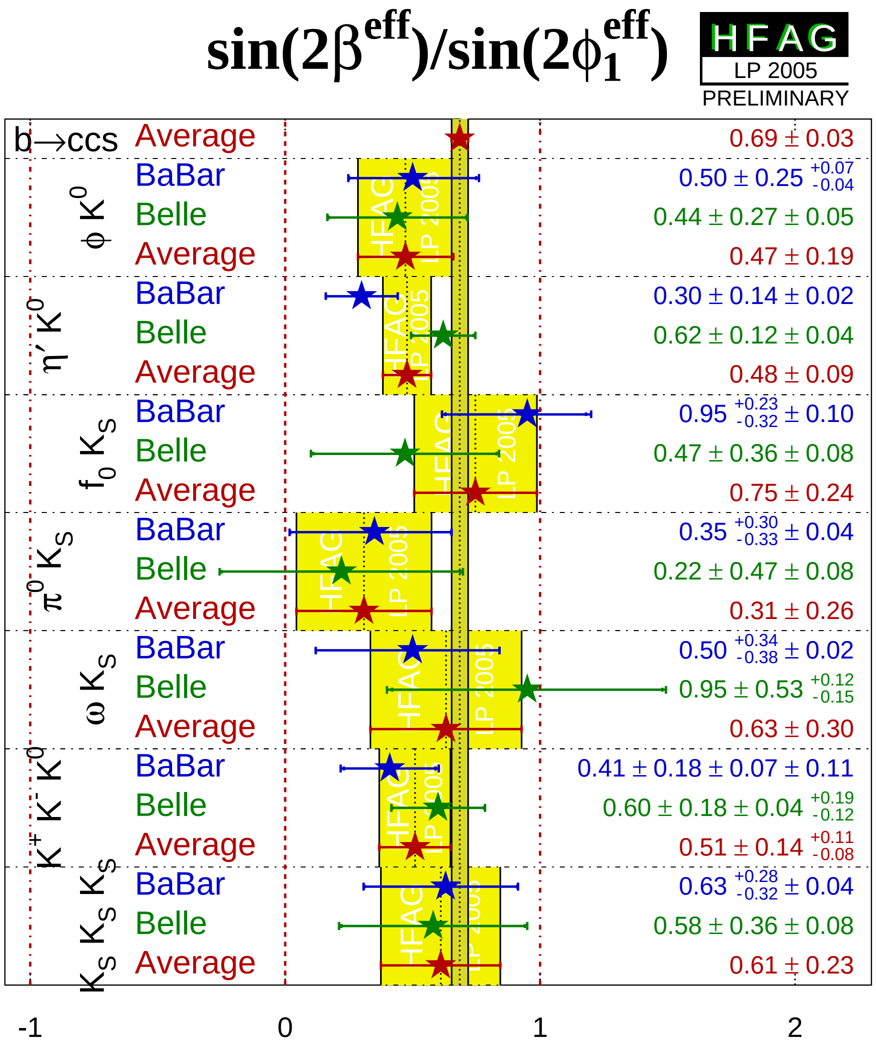
Perturbation theory at the charm scale is in good shape!

Only other NNLO calculation of a flavor-changing decay:

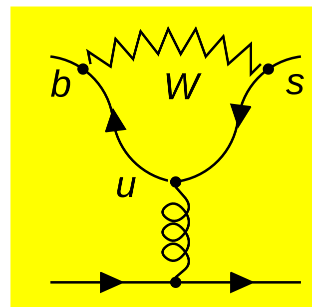
FNAL PRIME

$Br(B \to X_s \ell^+ \ell^-)$ Bobeth, Gambino, Gorbahn, Haisch

4. CP violation in $b \to s$ penguin decays



$b \;\to\; s\bar{q}q$ penguin amplitudes are polluted by a penguin loop with u quark:



$$\propto\; V_{ub}V_{us}^{*}$$
$$\propto\; e^{-i\gamma}$$

$b \to s\bar{u}u$ transitions further have a color-suppressed tree contribution.

CKM suppression with respect to the leading charm penguin loop:

$$\left|\frac{V_{ub}V_{us}}{V_{cb}V_{cs}}\right| \sim 0.025$$

Naive averages:

Winter 2005:

$\sin(\beta)^{\text{eff}} = 0.43 \pm 0.07$,

see right plot:

LP 2005:

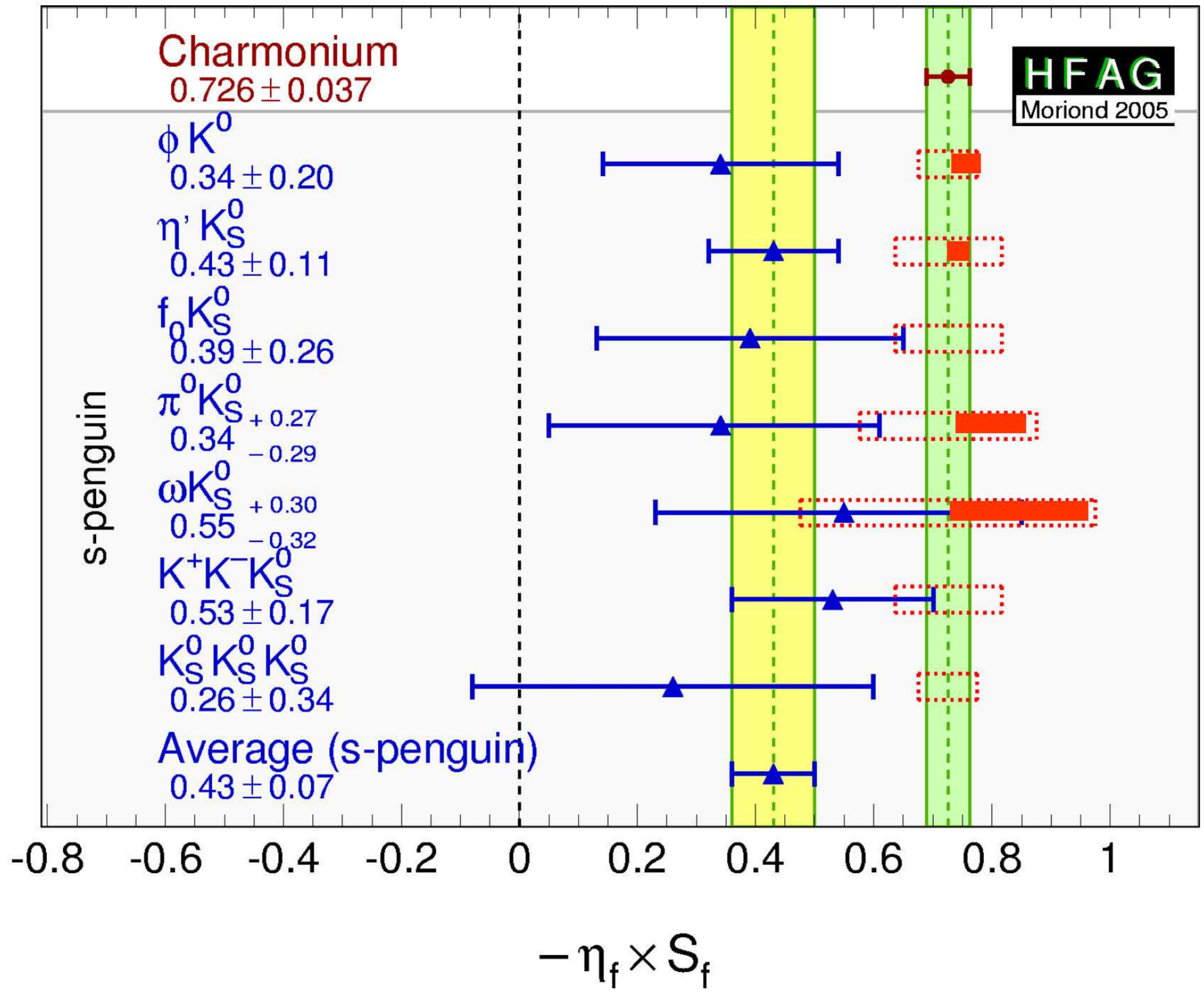
$\sin(\beta)^{\text{eff}} = 0.51 \pm 0.06$,

well below

$\sin(2\beta)^{b\to c\bar{c}s} = 0.69 \pm 0.03$.

QCD Factorization finds small corrections to $\sin(2\beta_{\text{eff}}) - \sin(2\beta)$, which are positive, see red bars:

Beneke 2005



Note: To measure a mixing-induced CP asymmetry (S_f term) in a $b \to s\overline{q}q$ decay of a B_d meson one needs a neutral Kaon in the final state, so that the

$$b(\overline{d}) \to \overline{q}qs(\overline{d}) \qquad \text{and} \qquad \overline{b}(d) \to \overline{q}q\overline{s}(d)$$

decays of B_d and $\overline{B}_d$ can interfere.

In a $\overset{(-)}{B}_s$ decay, however, one has a flavorless final state:

$$b(\overline{s}) \to \overline{q}qs(\overline{s}), \qquad \overline{b}(s) \to \overline{q}q\overline{s}(s)$$

and the needed interference occurs in any final state.

$\Rightarrow$ B_s physics is the El Dorado of $b \to s\overline{q}q$ penguin physics!

CDF does a superb job on charmless B_s branching fractions. More effort on B_s tagging to tackle mixing-induced CP asymmetries will be rewarding!

If there is any new physics in $b \to s\overline{q}q$ penguin decays, most likely $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing will also be affected. To this end the magnitude and phase of the $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing amplitude should be determined. Key measurements are

- the mass difference Δm_{B_s},
- the width difference $\Delta \Gamma_{B_s}$,
- the semileptonic CP asymmetry a_{fs}, which requires to count the positive and negative leptons from $B_s \to D_s^{(*)-} \ell^+ \nu_\ell$ and $\overline{B}_s \to D_s^{(*)+} \ell^- \overline{\nu}_\ell$ decays and
- the mixing-induced CP asymmetry in $B_s \to J/\psi \phi$.

5. Summary

- V_{ud} is known with a precision of $5 \cdot 10^{-4}$, $n \to p\,\ell\,\overline{\nu}_\ell$ and $\pi^+ \to \pi^0\,\overline{\ell}\,\nu_\ell$ are gaining importance.
- V_{us} determinations from $K\ell 3$, $K\mu 2$ and τ decays agree perfectly, V_{us} has a precision of $8 \cdot 10^{-3}$.
- V_{cb} from inclusice $\overline{B} \to X_c \ell \overline{\nu}_\ell$ decays is a success story of the operator product expansion. Many redundant measurements agree, the precision of V_{cb} is 1.2%.
- In inclusice $\overline{B} \to X_u \ell \overline{\nu}_\ell$ decays $\mathcal{O}(\alpha_s)$ corrections are understood and $1/m_b$ corrections are attacked. The current precision in $|V_{ub}|$ is 8%.
- Significant progress in finding $\gamma = \arg V_{ub}^*$ from tree-level decays, which determine γ to 13% accuracy.
- New unquenched 2+1 flavor lattice results from HPQCD for B_K, f_{B_d} and f_{B_d}/f_{B_s} improve the extraction of $(\overline{\rho}, \overline{\eta})$ from ϵ_K, Δm_{B_d} and $\Delta m_{B_d}/\Delta m_{B_s}$.

- The gold-plated modes $K_L \to \pi^0 \nu\overline{\nu}$ and $K^+ \to \pi^+ \nu\overline{\nu}$ will allow to determine $(\overline{\rho}, \overline{\eta})$ from $s \to d$ transitions and provide a powerful CKM test. The precision matches that of the B factories. The needed three-loop calculation of the charm piece in $Br(K^+ \to \pi^+ \nu\overline{\nu})$ is available now.
- Standard QCD dynamics cannot explain sizeable deviations of $\sin(2\beta_{\text{eff}})$ in $b \to s\overline{q}q$ penguin decays from $\sin(2\beta)$. Further subleading QCD effects tend to increase $\sin(2\beta_{\text{eff}})$. This remains a hot field to look for new physics.
- Fermilab should keep its high priority for B_s physics.

Penguins in $b \to s\bar{q}q, \ldots s \to d\nu\bar{\nu}$:

Wake-up call for New Physics?