Physics in extra dimensions: lecture #3

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Lecture 1: Field theory in compact dimensions.
Gauge bosons in the bulk and their collider signatures.

Lecture 2: One universal extra dimension.
Discrete symmetries and cascade decays at colliders.

Lecture 3: **Two universal extra dimensions.**

Lecture 4: Particles in a warped extra dimension.

Try to predict physics at the TeV scale by addressing some of the "problems" of the standard model.

Fermion and scalar gauge charges in the standard model:

	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$
quark doublet: $q_L^i = (u_L^i, d_L^i)$	3	2	$1/3$
right-handed up-type quark: u_R^i	3	1	$4/3$
right-handed down-type quark: d_R^i	3	1	$-2/3$
lepton doublet: $l_L^i = (\nu_L^i, e_L^i)$	1	2	-1
right-handed charged lepton": e_R^i	1	1	-2
Higgs doublet: H	1	2	$+1$

$i = 1, 2, 3$ *labels the fermion generations.*

- Fermion content looks baroque ...

- **Proton stability:**

$$\frac{c}{M^2}QQQL \quad \Rightarrow \quad p \to e^+\pi^0\,, \ldots$$

  For $M \sim 1$ TeV: $c < 10^{-26}$.

- **Quark and lepton mass pattern** *(most explanations run into conflict with flavor-changing neutral currents).*

- **Why $M_Z \ll M_{\mathrm{Planck}}$?**

- ...

Fundamental symmetries

gauge	*spacetime*	*global*	*discrete*
$SU(3) \times SU(2) \times U(1)$;	$SO(3,1)$;	$U(1)_B$;	CPT

Fermions:

$$\left.\begin{array}{rl} q_L: & (3,2,\ \ +1/6) \\ u_R: & (3,1,\ \ +2/3) \\ d_R: & (3,1,\ \ -1/3) \\ l_L: & (1,2,\ \ -1/2) \\ e_R: & (1,1,\ \ -1) \end{array}\right\} \times 3$$

Fundamental symmetries

gauge	*spacetime*	*global*	*discrete*
$SU(3) \times SU(2) \times U(1)$;	$SO(3,1)$;	$U(1)_B$;	CPT

$$SU(5) \subset SO(10)$$

Fermions:

$$\left.\begin{array}{ll} q_L : & (3, 2, \ \ +1/6) \\ u_R : & (3, 1, \ \ +2/3) \\ d_R : & (3, 1, \ \ -1/3) \\ l_L : & (1, 2, \ \ -1/2) \\ e_R : & (1, 1, \ \ -1) \end{array}\right\} \times 3 = (10 + \bar{5}) \times 3 \subset 16 \times 3$$

Fundamental symmetries

gauge *spacetime* *global* *discrete*

$SU(3) \times SU(2) \times U(1)$; $SO(3,1)$; $U(1)_B$; CPT

$SO(5,1)$

6D Lorentz symmetry

Fermions:

$$\left.\begin{array}{rl} q_L: & (3,2,\ \ +1/6) \\ u_R: & (3,1,\ \ +2/3) \\ d_R: & (3,1,\ \ -1/3) \\ l_L: & (1,2,\ \ -1/2) \\ e_R: & (1,1,\ \ -1) \end{array}\right\} \times 3$$

required by global $SU(2)_W$ anomaly cancellation in 6D

Six-Dimensional Field Theory

6D is special...

Fermions in $D = 4 + n$ dimensions: chiral representations of the Lorentz group $SO(3+n,1)$ exist only for even n.

Properties of chirality in $D = 6$ mod 4 are different than in $D = 4$ mod 4.

A chiral fermion in $D = 6$ has 4 degrees of freedom:

6D chirality: $+$ or $-$

4D chirality: L or R

$Q_+ = Q_{+_L} + Q_{+_R}$ or $Q_- = Q_{-_L} + Q_{-_R}$

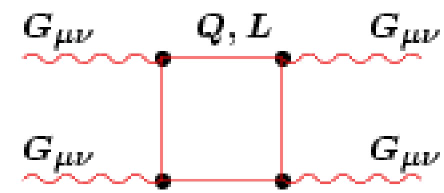
Six-Dimensional Standard Model

$n_\pm \equiv$ **# of spin–1/2 fields with chirality** $\pm$

Local gravitational anomaly in 6D $\propto n_+ - n_-$

$\Rightarrow$ **one right-handed neutrino per generation**

(or else Lorentz symmetry is lost)



Two possible chirality assignments for 6D quarks and leptons:

$$\mathcal{Q}_+ \, , \, \mathcal{U}_- \, , \, \mathcal{D}_- \, , \begin{cases} \mathcal{L}_+ \, , \; \mathcal{E}_- \, , \; \mathcal{N}_- \\ \qquad \text{or} \\ \mathcal{L}_- \, , \; \mathcal{E}_+ \, , \; \mathcal{N}_+ \end{cases}$$

Global $SU(2)_W$ anomaly

In 6D there are 12 classes of $SU(2)_W$ gauge transformations.

Anomaly cancellation condition:

(Bershadsky, Vafa, 1997)

$$n(2_+) - n(2_-) = 0 \bmod 6$$

One generation: $Q_+ \Rightarrow n_Q(2_+) = 3$

$$L_\pm \Rightarrow n_L(2_\pm) = 1$$

Need more than one generation of quarks and leptons:

$$(3 \pm 1)\, n_{\text{gen}} = 0 \bmod 6 \quad \Rightarrow \quad \underline{n_{\text{gen}} = 3 \bmod 3}$$

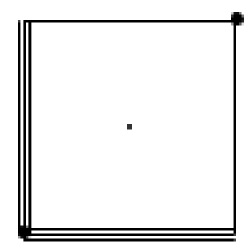
Two Universal Extra Dimensions

hep-ph/0601186, hep-ph/0703231

(G. Burdman, E. Ponton, KC Kong, R. Mahbubani, ...)

All Standard Model particles propagate in $D = 6$ dimensions.

Two dimensions are compactified on a square.



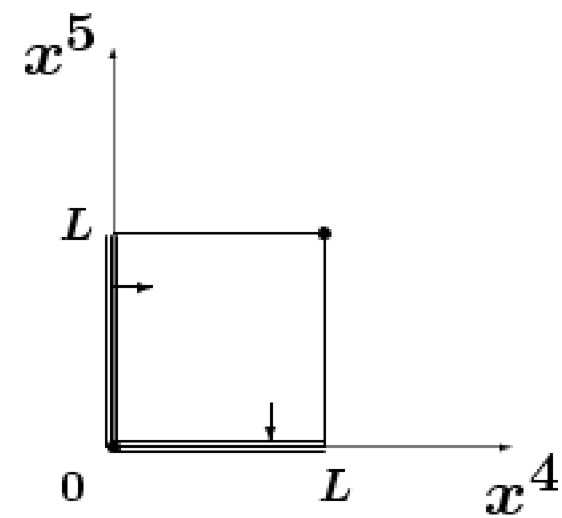
Kaluza-Klein particles are states of definite momenta along the two compact dimensions, labelled by two integers (j, k). Tree-level masses: $\sqrt{j^2 + k^2}/R$

Momentum conservation $\rightarrow$ KK-parity given by $j + k$

$\Rightarrow$ (1,0) particles are produced only in pairs at colliders

Chiral boundary conditions on a square

Identify pairs of adjacent sides:



$$\mathcal{L}(x^\mu, y, 0) = \mathcal{L}(x^\mu, 0, y)$$

$$\mathcal{L}(x^\mu, y, L) = \mathcal{L}(x^\mu, L, y)$$

$\Phi(y, 0) = e^{i\theta}\Phi(0, y), \ \ldots$

$\Rightarrow \ \theta = n\pi/2$

$$\partial_5\Phi|_{(x^4,x^5)=(y,0)} = -e^{in\pi/2}\,\partial_4\Phi|_{(x^4,x^5)=(0,y)}$$

Complete sets of functions satisfying the boundary conditions:

$$f_{0,2}^{(j,k)}(x^4,x^5) = \frac{1}{1+\delta_{j,0}}\left[\cos\left(\frac{jx^4+kx^5}{R}\right) \pm \cos\left(\frac{kx^4-jx^5}{R}\right)\right]$$

$$f_{1,3}^{(j,k)}(x^4,x^5) = i\sin\left(\frac{jx^4+kx^5}{R}\right) \mp \sin\left(\frac{kx^4-jx^5}{R}\right)$$

Spectrum of KK modes:

(j,k)	(1,0)	(1,1)	(2,0)	$(2,1)$ $(1,2)$	(2,2)	(3,0)	$(3,1)$ $(1,3)$
$M_{j,k}R$	1	$\sqrt{2}$	2	$\sqrt{5}$	$2\sqrt{2}$	3	$\sqrt{10}$

KK decomposition of the gauge fields:

$$A_\mu(x^\nu, x^4, x^5) = \frac{1}{L}\left[A_\mu^{(0,0)}(x^\nu) + \sum_{j\geq 1}\sum_{k\geq 0} f_0^{(j,k)}(x^4, x^5) A_\mu^{(j,k)}(x^\nu) \right]$$

$$A_4 + iA_5 \equiv A_+(x^\nu, x^4, x^5) = -\frac{1}{L}\sum_{j\geq 1}\sum_{k\geq 0} f_3^{(j,k)}(x^4, x^5) A_+^{(j,k)}(x^\nu)$$

$$A_4 - iA_5 \equiv A_-(x^\nu, x^4, x^5) = \frac{1}{L}\sum_{j\geq 1}\sum_{k\geq 0} f_1^{(j,k)}(x^4, x^5) A_-^{(j,k)}(x^\nu)$$

Physical degrees of freedom:

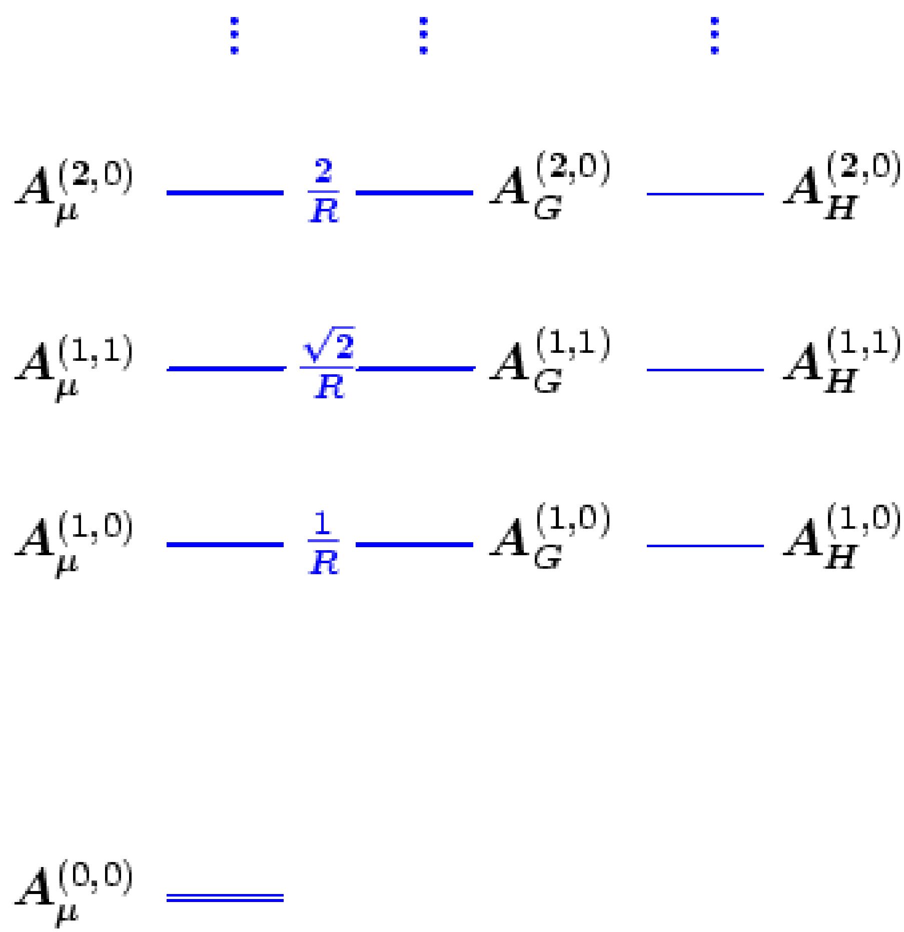
$$A_\pm^{(j,k)} = \frac{j + ik}{\sqrt{j^2 + k^2}}\left(A_H^{(j,k)} \mp i A_G^{(j,k)} \right)$$

$A_G^{(j,k)}$ is the longitudinal polarization of $A_\mu^{(j,k)}$

$A_H^{(j,k)}$ is a real scalar field ("spinless adjoint")

Kaluza-Klein spectrum of gauge bosons

$A_G^{(j,k)}(x^\nu)$ becomes the longitudinal degree of freedom of the spin-1 KK mode $A_\mu^{(j,k)}(x^\nu)$.

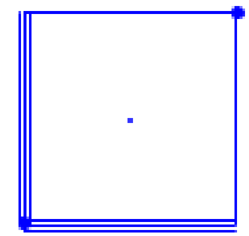


Kaluza-Klein spectrum of quarks and leptons

$(t_L^{(2,0)}, b_L^{(2,0)})$	$\frac{2}{R}$	$(T_R^{(2,0)}, B_R^{(2,0)})$	$T_L^{(2,0)}$	$\frac{2}{R}$	$t_R^{(2,0)}$
$(t_L^{(1,1)}, b_L^{(1,1)})$	$\frac{\sqrt{2}}{R}$	$(T_R^{(1,1)}, B_R^{(1,1)})$	$T_L^{(1,1)}$	$\frac{\sqrt{2}}{R}$	$t_R^{(1,1)}$
$(t_L^{(1,0)}, b_L^{(1,0)})$	$\frac{1}{R}$	$(T_R^{(1,0)}, B_R^{(1,0)})$	$T_L^{(1,0)}$	$\frac{1}{R}$	$t_R^{(1,0)}$
(t_L, b_L)					t_R

Symmetries of the “Chiral Square”

1. Kaluza-Klein parity



Reflections about the center of the square $(L/2, L/2)$,

$$(x^4, x^5) \mapsto (L - x^4, L - x^5)$$

$\Rightarrow$ invariance under Z_2 transformation

$$\Phi^{(j,k)}(x^\mu) \mapsto (-1)^{j+k}\,\Phi^{(j,k)}(x^\mu)$$

2. 6D Lorentz symmetry is broken by compactification,

but not completely: $SO(5,1) \rightarrow SO(3,1) \times Z_8$

Field theory on the chiral square has $Z_8 \times Z_2$ symmetry

Fermion	Z_8 charge (conserved mod 4)	zero-mode
$\mathcal{Q}_{+L}$	$-1/2$	$q_L = (u_L, d_L)$
$\mathcal{U}_{-R}$, $\mathcal{D}_{-R}$	$-1/2$	u_R, d_R
$\mathcal{Q}_{+R}$, $\mathcal{U}_{-L}$, $\mathcal{D}_{-L}$	$+1/2$	—
$\mathcal{L}_{\pm L}$	$\mp 1/2$	$l_L = (\nu_L, e_L)$
$\mathcal{E}_{\mp R}$, $\mathcal{N}_{\mp R}$	$\mp 1/2$	ν_R, e_R
$\mathcal{L}_{\pm R}$, $\mathcal{E}_{\mp L}$, $\mathcal{N}_{\mp L}$	$\pm 1/2$	—

Z_8 imposes an exact selection rule on the low-energy 4D Lagrangian:

$$3\Delta B + \Delta L = 0 \bmod 8$$

Consequences:

- no Majorana ν masses
- neutrino-less double-beta decays are forbidden ($\Delta B = 0$, $\Delta L = 2$)
- no neutron–anti-neutron oscillations ($\Delta B = 2$, $\Delta L = 0$)
- long proton lifetime because the QQQL operator is forbidden ($\Delta B = 1$ transitions induced by very high-dimension operators)

Perturbativity breaks down at a scale $M_s \approx 5/R$
$\Rightarrow$ *6D standard model is an effective theory valid up to* M_s.

Most general operators invariant under Z_8 and $SU(3)_c \times SU(2)_W \times U(1)_Y$ gauge transformations are expected to be present at $M_s \geq 2.5$ TeV.

Proton Decays

$\mathcal{L}_+$ chirality assignment: $\mathcal{O}_{17} = \frac{C_{17}}{M_s^{11}} \left(\overline{\mathcal{L}}_+ \mathcal{D}_-\right)^3 \tilde{\mathcal{H}}$

B-violating processes: $p \to e^- \pi^+ \pi^+ \nu\nu$ **and** $n \to e^- \pi^+ \nu\nu$

$$\tau_p \approx \frac{10^{35}\mathrm{yr}}{C_{17}^2} \left[\frac{(4\pi)^{-7} 10^{-4}}{\Phi_5 F(\pi\pi)}\right] \left[\frac{1/R}{0.5\ \mathrm{TeV}}\right]^{12} \left[\frac{RM_s}{5}\right]^{22}$$

$\tau_n / \tau_p \approx 10^{-3}$ **(larger phase-space)**

Look for nucleon decays into 3 leptons + pions.

Implications for neutrinos:

(Appelquist, Dobrescu, Ponton, Yee: hep-ph/0201131)

- **3 ν_R's required for gravitational anomaly cancellation**
- **Neutrinos are of the Dirac type due to 6D Lorentz invariance**

- **Smallness of neutrino masses due to small wave-function overlap with the ν_R's (assuming a nonuniversal 7th dimension)**

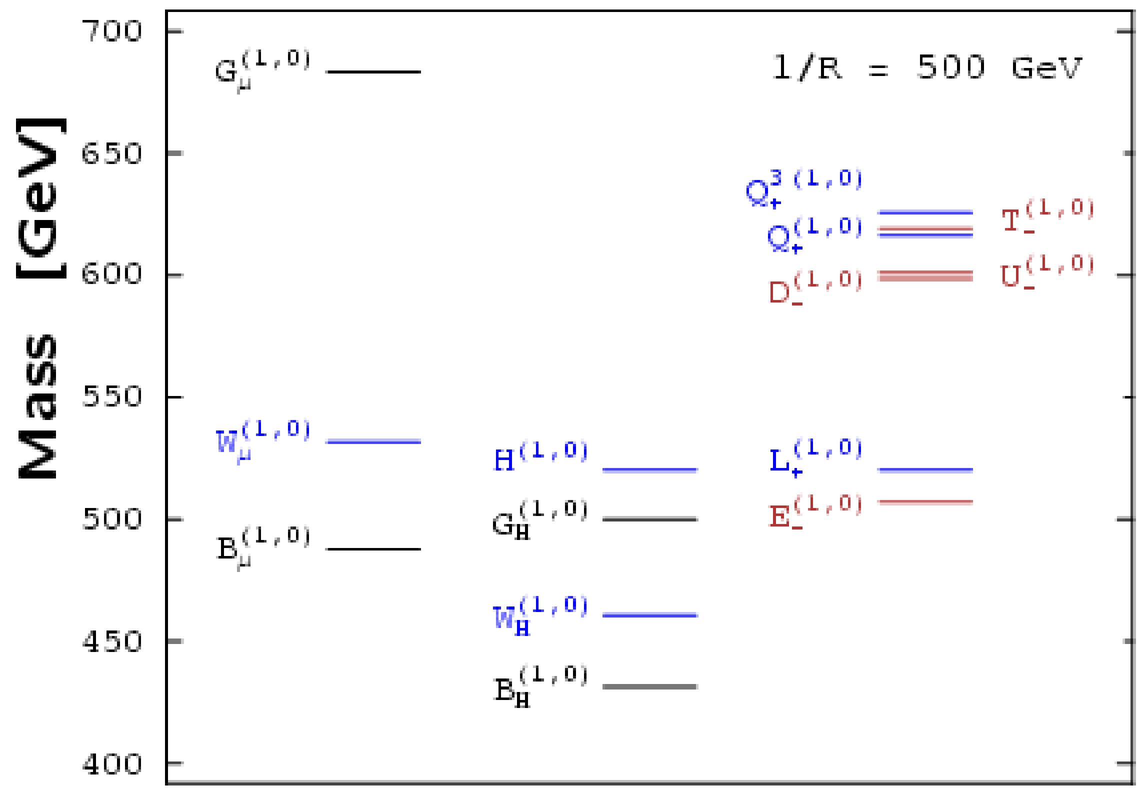
  Typical prediction:
  large θ_{13} & large-mixing-angle solution for solar ν's.

(1,0) modes have a tree-level mass of $1/R$, and KK parity $-$.

One-loop contributions and EWSB split the spectrum

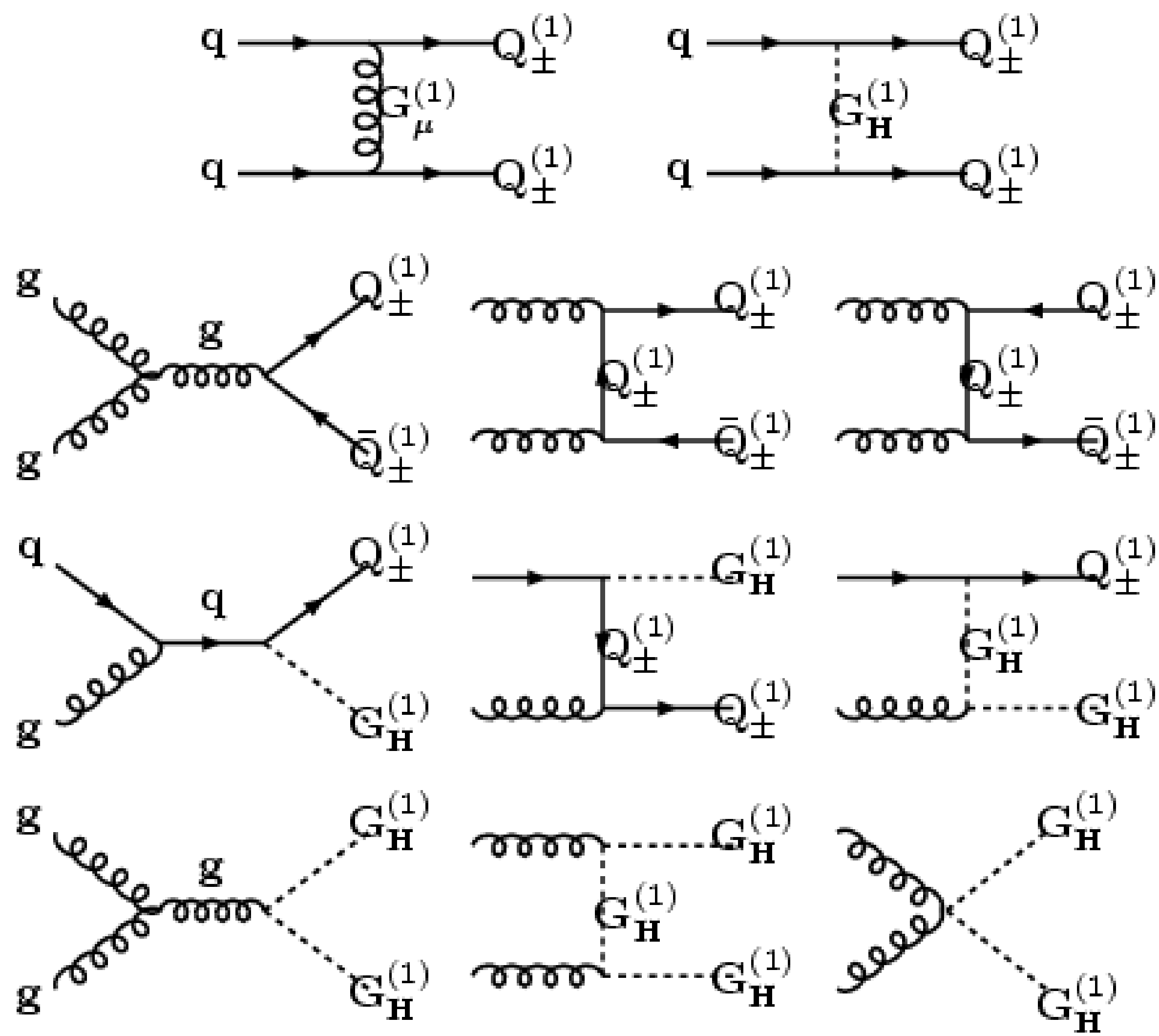
(Cheng, Matchev, Schmaltz, hep-ph/0204342 ; Ponton, Wang, hep-ph/0512304)

Mass spectrum of the (1,0) level:



Homework 3.1: compute the branching fractions of the (1,0) particles.

Production of (1,0) particles at the LHC

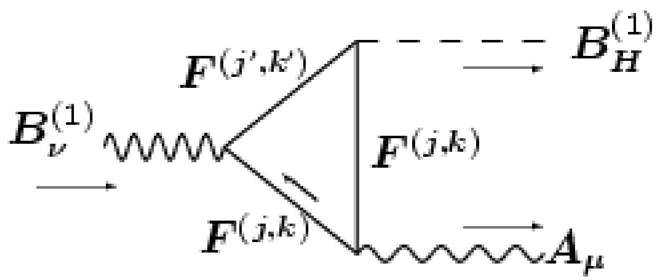


Use CalcHEP to compute cross section for (1,0) pair production.

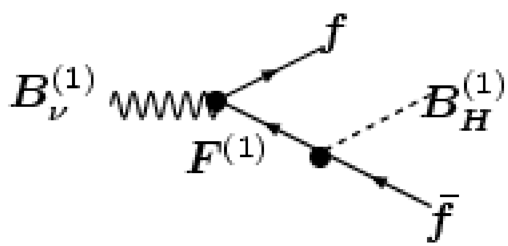
http://theory.fnal.gov/people/kckong/6d/

At one loop: $$\frac{c}{R^{-1}} B_H^{(1,0)} B_{\mu\nu}^{(1,0)} \tilde{F}^{\mu\nu}$$

Competition between 1-loop induced 2-body decays and tree-level 3-body decays of the (1,0) bosons.



$$Br\left(B_\mu^{(1,0)} \to B_H^{(1,0)}\gamma\right) \approx 30\%$$

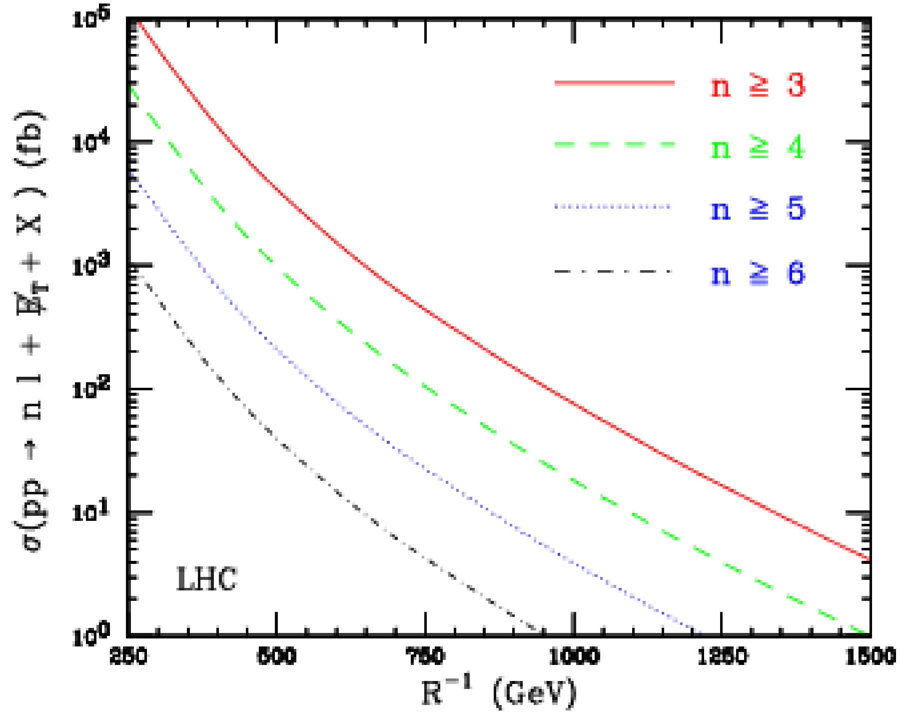


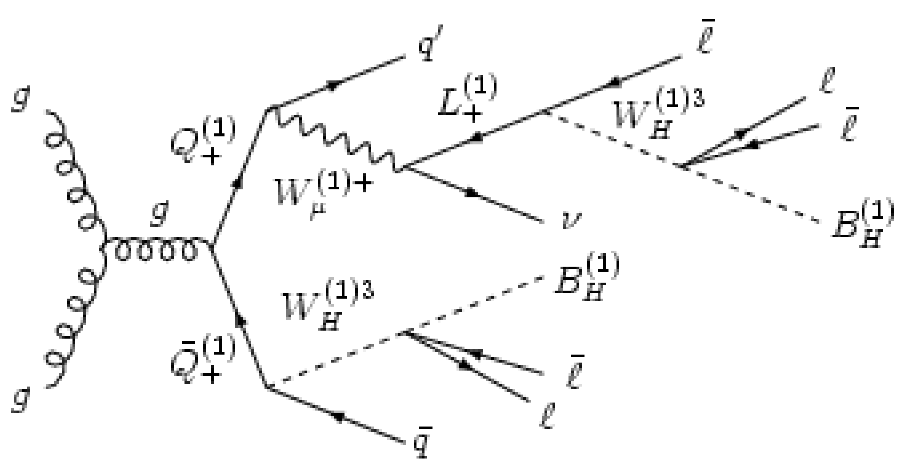
$$Br\left(B_\mu^{(1,0)} \to B_H^{(1,0)}\ell^+\ell^-\right) \approx 23\%$$

$\to$ Events with leptons, photons and missing $\not{E}_T$.

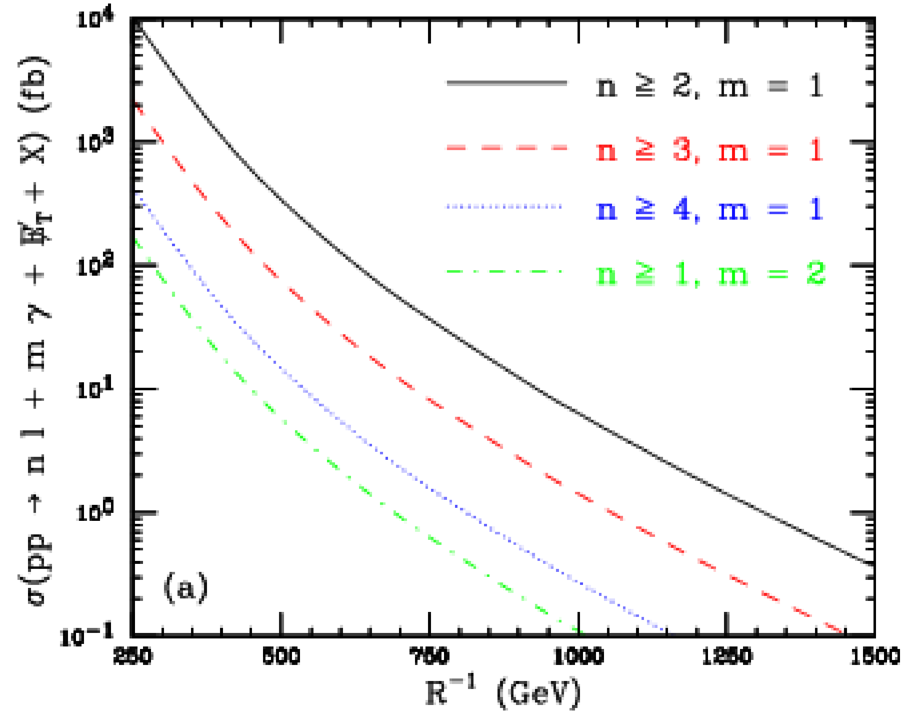
Work with K.C. Kong and Rakhi Mahbubani (hep-ph/0703231).

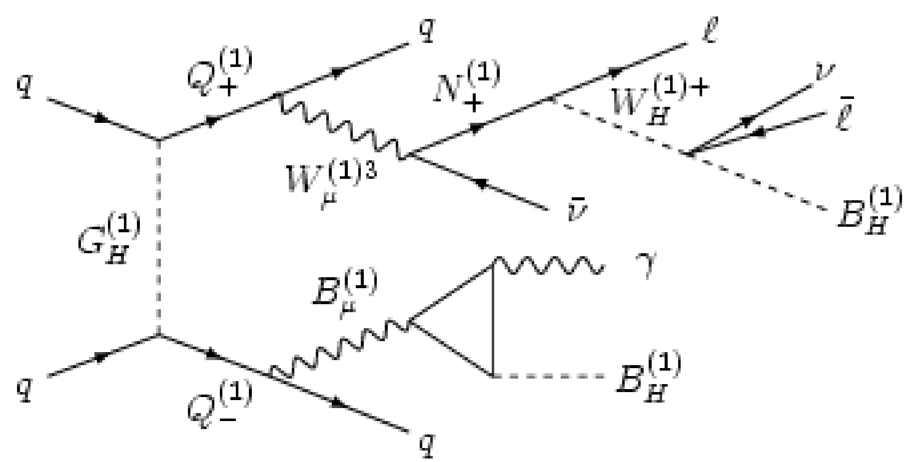
Multi-lepton signal at the LHC:





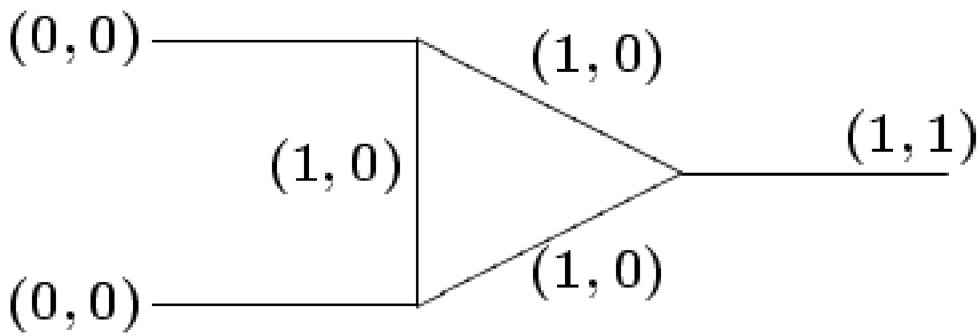
Leptons + photons at the LHC:





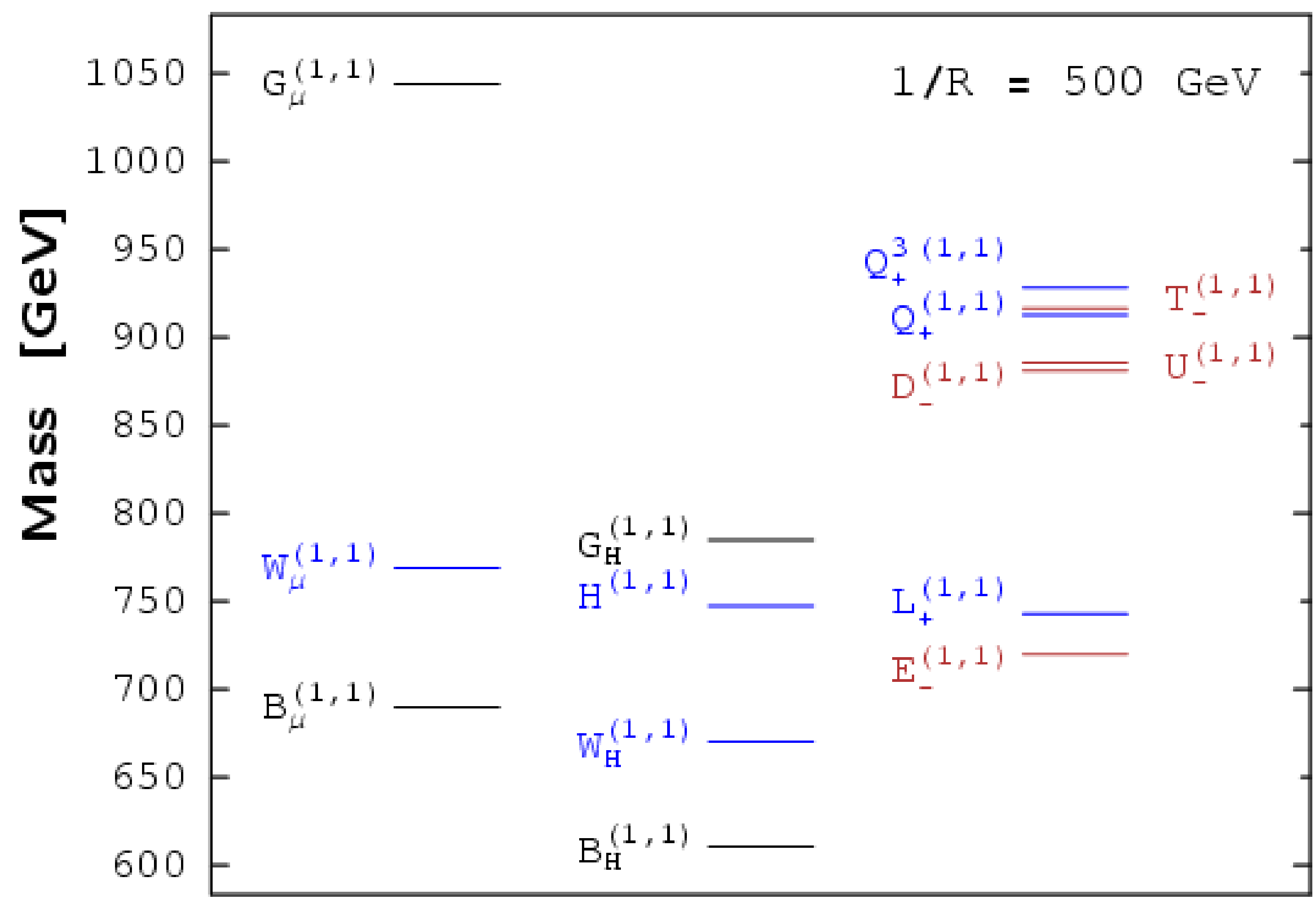
KK parity is conserved: $(-1)^{j+k}$

At colliders: s-channel production of the even-modes at 1-loop



(1,1) modes have a tree-level mass of $\sqrt{2}/R$, and KK parity $+$.

Mass spectrum of the (1,1) level for $1/R = 500$ GeV:



Spinless adjoints interact with the zero-mode fermions only via dimension-5 or higher operators:

$$\frac{g_s \tilde{C}^{qG}_{j,k}}{M_{j,k}} \left(\overline{q}\gamma^{\mu} T^a q\right) \partial_\mu G_H^{(j,k)a}$$

$\tilde{C}^{qG}_{j,k}$ are real dimensionless parameters.

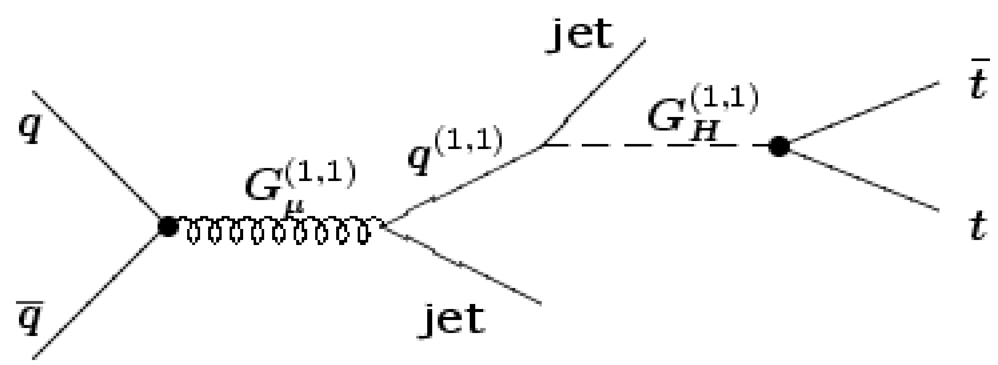
$\Rightarrow$ G_H, W_H and B_H couple to usual quarks and leptons proportional to the fermion mass!

$\Rightarrow$ KK-number violating couplings of the spinless adjoints are large only in the case of the top quark.

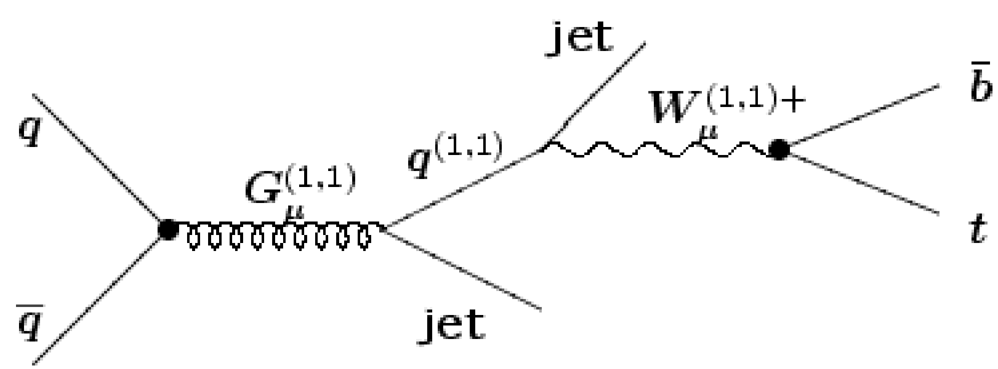
Signals of (1,1) particles at the Tevatron and LHC:

1. s-channel production of a (1,1) gluon of mass $\sim \sqrt{2}/R\,(1+\alpha_s)$

$\rightarrow$ $t\bar{t}$ **resonance + 2 jets ($\sim 50 - 100$ GeV):**



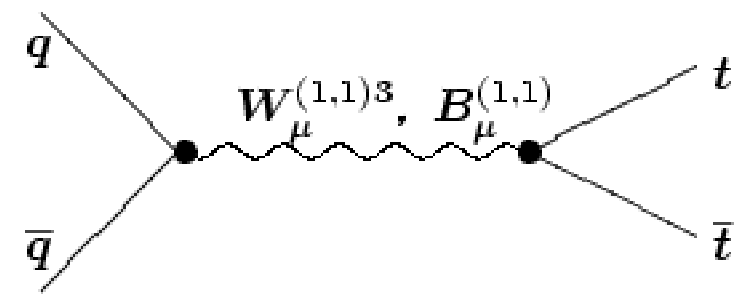
$\rightarrow$ $t\bar{b}$ **resonance + 2 jets ($\sim 50 - 100$ GeV):**



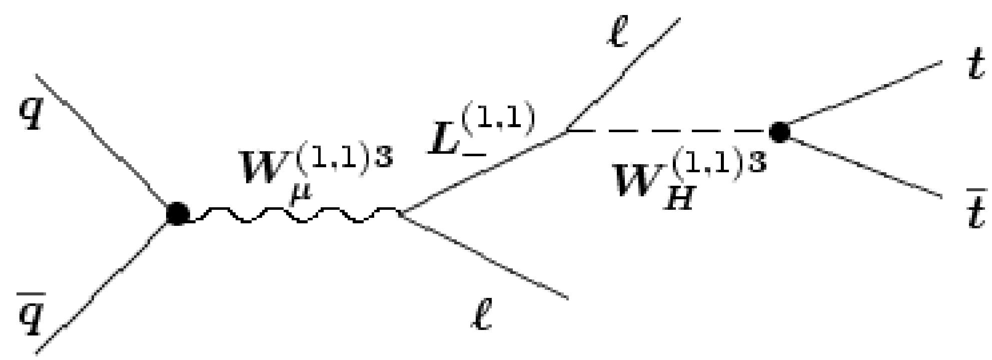
More signals at the Tevatron and LHC:

2. s-channel production of a (1,1) electroweak gauge boson

$\rightarrow$ $t\bar{t}$ resonance:

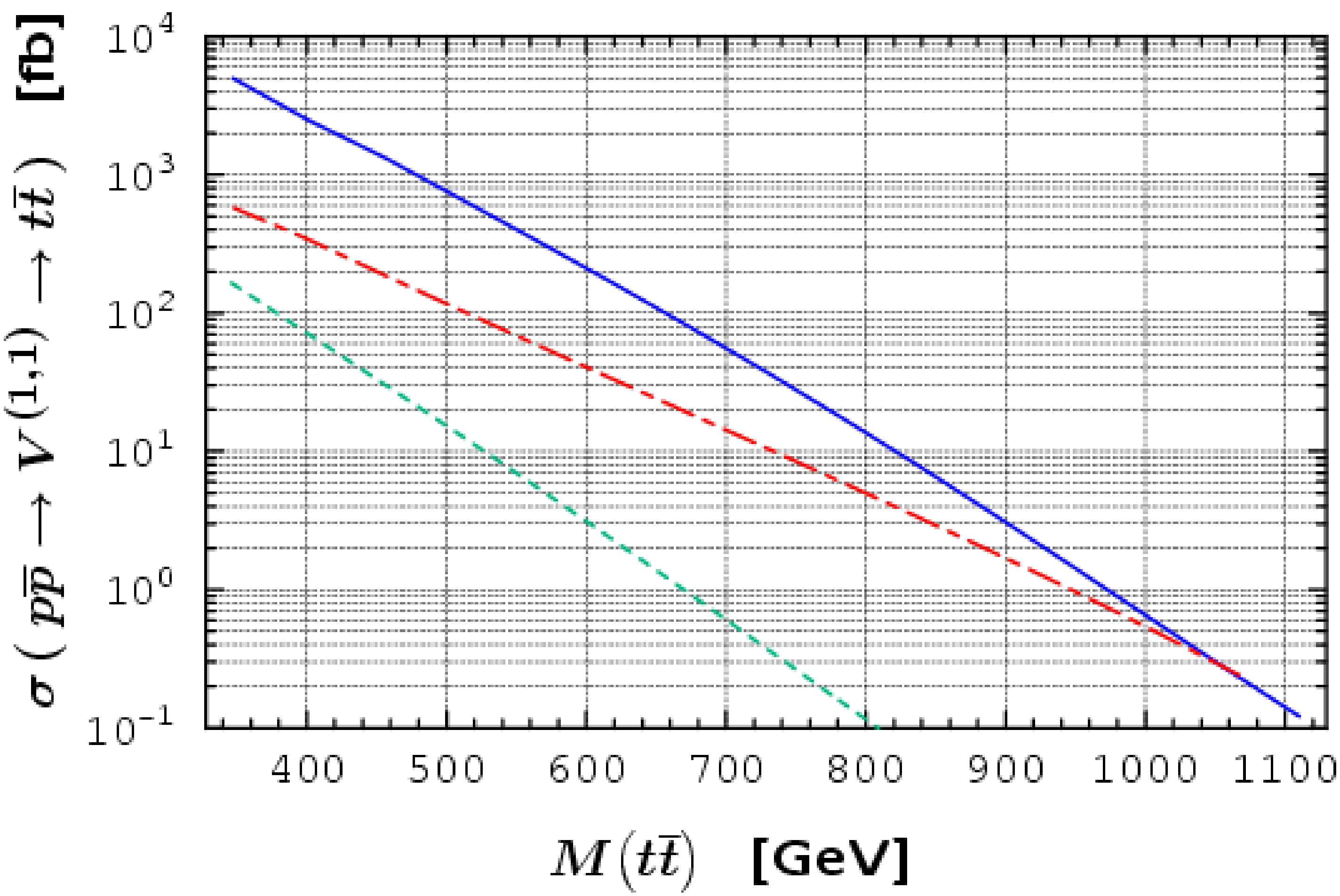


$\rightarrow$ $t\bar{t}$ resonance + 1 lepton ~ 70 GeV + 1 lepton ~ 20 GeV:



Production of $t\bar{t}$ pairs at the Tevatron from mass peaks at:

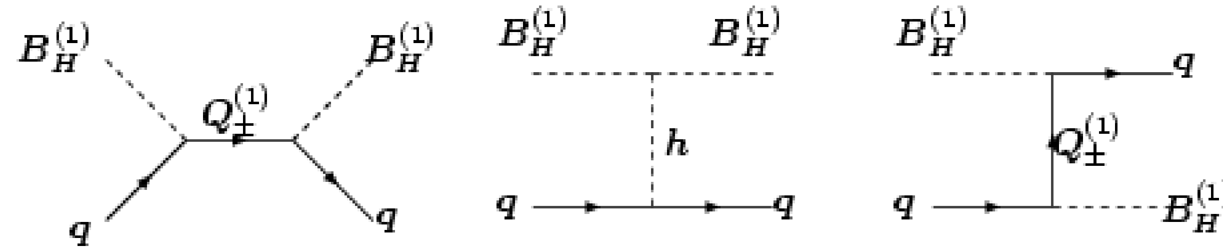
- $G_H^{(1,1)} + W_\mu^{(1,1)3}$ —— $M_{t\bar{t}} \simeq 1.10\,\sqrt{2}/R$
- $W_H^{(1,1)3} + B_\mu^{(1,1)}$ - - - $M_{t\bar{t}} \simeq 0.96\,\sqrt{2}/R$
- $B_H^{(1,1)}$ - - - $M_{t\bar{t}} \simeq 0.87\,\sqrt{2}/R$

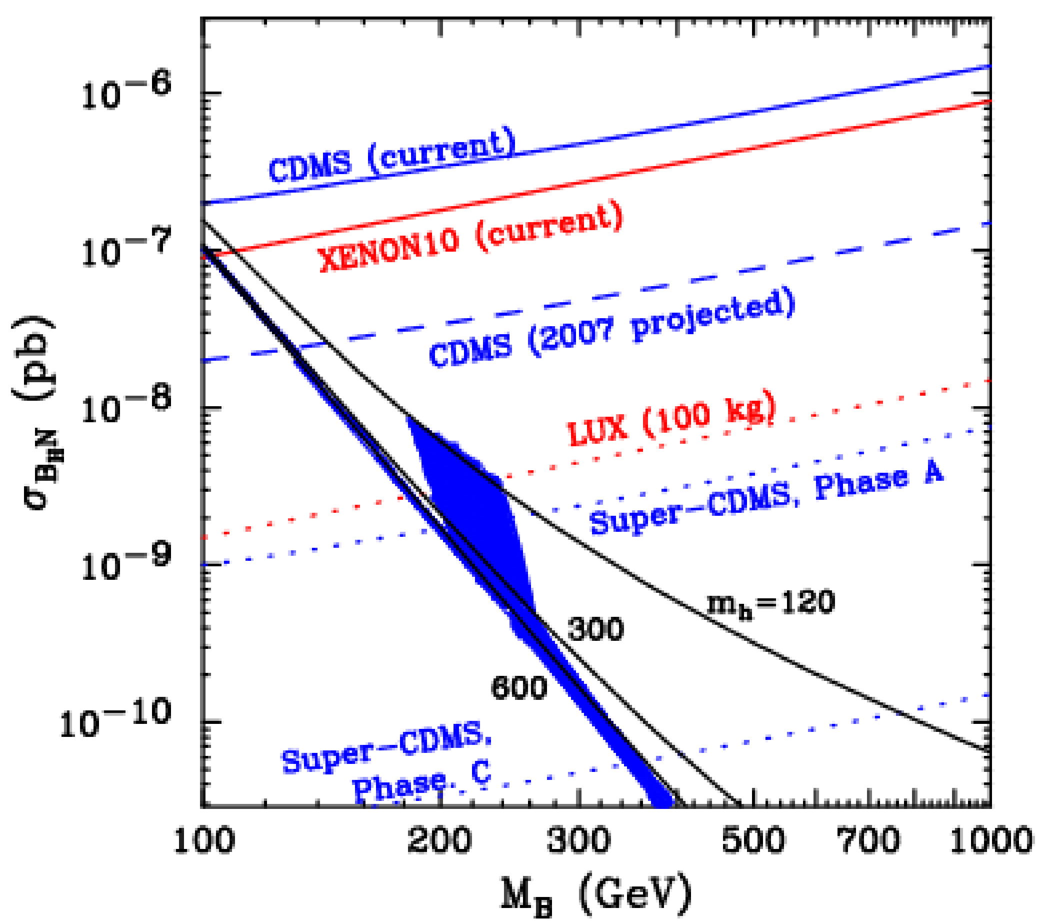


$B_H^{(1,0)}$ ("spinless photon") is a viable dark matter candidate

(hep-ph/0706.3409)

Direct detection:





Conclusions (2 UED)

- 6-Dimensional Standard Model

– 3 generations of quarks and leptons are required for global $SU(2)_W$ anomaly cancellation

– proton is long-lived due to 6D Lorentz invariance

– neutrinos are special

- *At colliders, look for:*
  - $t\bar{t}$ and $t\bar{b}$ resonances
  - many leptons + jets + missing E_T
  - leptons + photons + jets + missing E_T
  - other signatures of Kaluza-Klein modes