QCD and Collider Physics
Lecture II:Vector boson production

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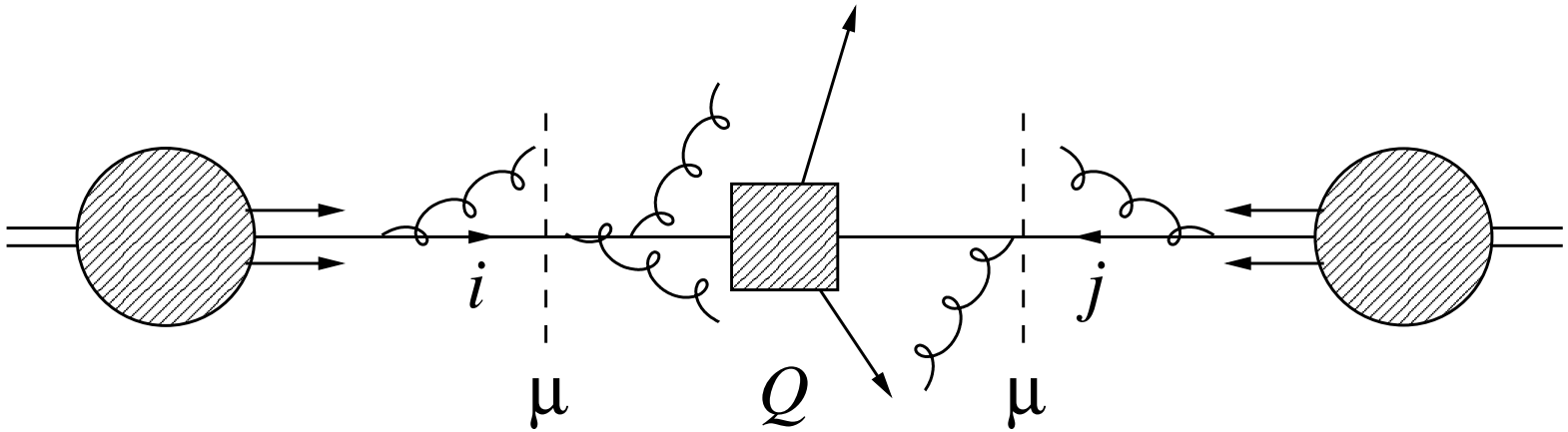
Slides available from http://theory.fnal.gov/people/ellis/3Cycle/

Hadron-hadron processes

- In hard hadron-hadron scattering, constituent partons from each incoming hadron interact at short distance (large momentum transfer Q^2).



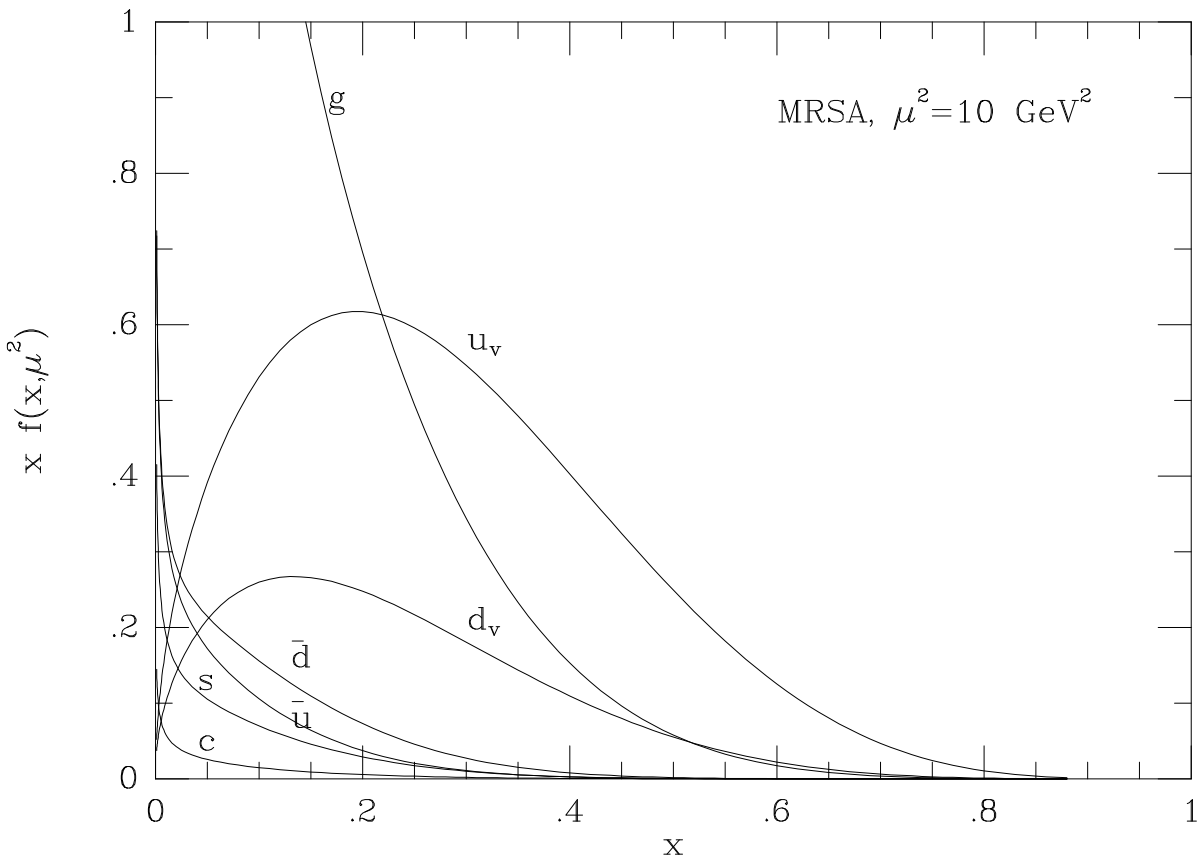
- For hadron momenta P_1, P_2 ($S = 2P_1 \cdot P_2$), form of cross section is

$$\begin{aligned}\sigma(S) &= \sum_{i,j} \int dx_1 dx_2 D_i(x_1, \mu^2) D_j(x_2, \mu^2) \\ &\times \quad \hat{\sigma}_{ij}(\hat{s} = x_1 x_2 S, \alpha_s(\mu^2), Q^2/\mu^2)\end{aligned}$$

where μ^2 is factorization scale and $\hat{\sigma}_{ij}$ is subprocess cross section for parton types i, j.

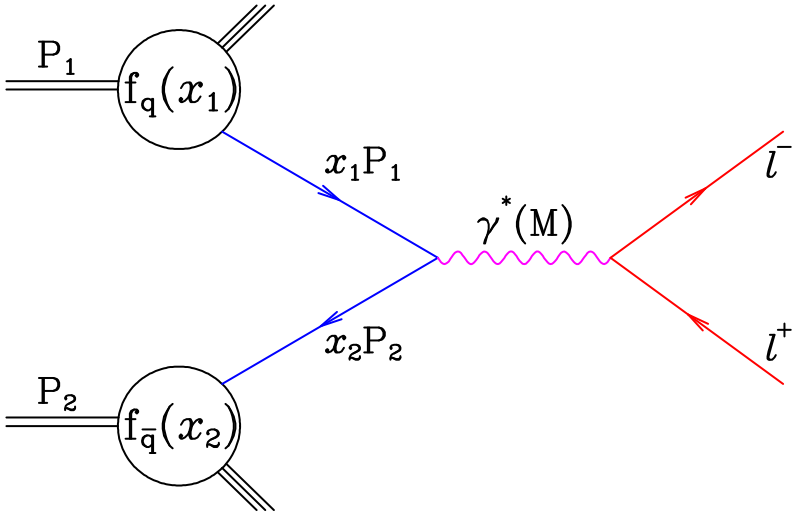
- Notice that factorization scale is in principle arbitrary: affects only what we call part of subprocess or part of initial-state evolution (parton shower).
- Unlike e^+e^- or ep, we may have interaction between spectator partons, leading to *soft underlying event* and/or *multiple hard scattering*.

Parton Distributions



- Parton distributions as extracted from Deep Inelastic scattering at low Q^2.

$q\bar{q} \to \mu^+\mu^-$



In the naive parton model, the cross section σ_{AB} for producing such a pair in the collision of beam A and target B is obtained by simply weighting the subprocess cross section $\hat{\sigma}$ for $q\bar{q} \to {}^{+-}$ with the parton distribution functions $f_q(x)$ and $f_{\bar{q}}(x)$ (extracted from deep inelastic scattering), and summing over all quark-antiquark combinations in the beam and target:

$$\sigma_{AB} = \sum_q \int dx_1 dx_2 \; f_q(x_1) f_{\bar{q}}(x_2) \; \hat{\sigma}_{q\bar{q}\to l^+l^-} \; . \tag{1}$$

Thus we wish to calculate the cross section for the process

$$q(p_1) + \bar{q}(p_2) \to \mu^-(p_3) + \mu^+(p_4)$$

Phase space

$$PS^{(2)} = \int \frac{d^4p_3}{(2\pi)^3}\frac{d^4p_4}{(2\pi)^3}\,\delta(p_3^2)\,\delta(p_4^2)\,(2\pi)^4\delta^4(p_1+p_2+p_3+p_4)$$

In the centre of mass system

$$\begin{aligned}
p_1 &= \frac{\sqrt{s}}{2}(1,0,0,+1)\\
p_2 &= \frac{\sqrt{s}}{2}(1,0,0,-1)\\
p_3 &= \frac{\sqrt{s}}{2}(1,0,+\sin\theta^*,+\cos\theta^*)\\
p_4 &= \frac{\sqrt{s}}{2}(1,0,-\sin\theta^*,-\cos\theta^*)
\end{aligned}$$

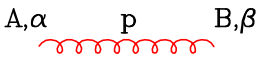
Hence we have that

$$\begin{aligned}
s &= (p_1+p_2)^2\\
t &= (p_1-p_3)^2 = -\frac{s}{2}(1-\cos\theta^*)\\
u &= (p_1-p_4)^2 = -\frac{s}{2}(1+\cos\theta^*)
\end{aligned}$$

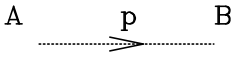
Performing p_4 integral

$$
\begin{aligned}
PS^{(2)} &= \frac{1}{4\pi^2}\int d^4p_3\ \delta(p_3^2)\ \delta((p_1+p_2-p_3)^2) \\
&= \frac{1}{4\pi^2}\int \frac{d^3p_3}{2E_3}\ \delta((s-2\sqrt{s}E_3)) \\
&= \frac{1}{16\pi^2\sqrt{s}}\int |\vec{p}_3|dE_3\ d\cos\theta^*\ d\phi\ \delta((\frac{\sqrt{s}}{2}-E_3)) \\
&= \frac{1}{16\pi}\int_{-1}^{+1} d\cos\theta^*
\end{aligned}
$$

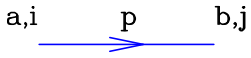
Feynman rules for QCD



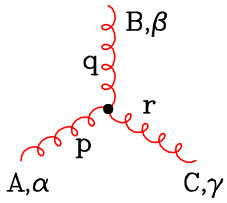
$$\delta^{AB}\left[-g^{\alpha\beta}+(1-\lambda)\frac{p^{\alpha}p^{\beta}}{p^{2}+i\epsilon}\right]\frac{i}{p^{2}+i\epsilon}$$



$$\delta^{AB}\frac{i}{(p^{2}+i\epsilon)}$$

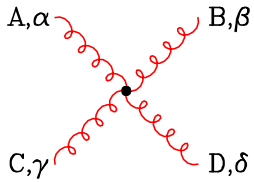


$$\delta^{ab}\frac{i}{(\not{p}-m+i\epsilon)_{ji}}$$

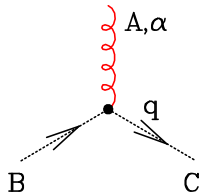


$$-g\ f^{ABC}[(p-q)^{\gamma}g^{\alpha\beta}+(q-r)^{\alpha}g^{\beta\gamma}+(r-p)^{\beta}g^{\gamma\alpha}]$$

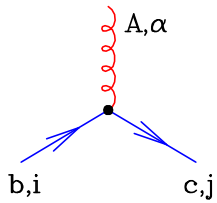
(all momenta incoming)



$$-ig^{2}\ f^{XAC}f^{XBD}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$
$$-ig^{2}\ f^{XAD}f^{XBC}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}]$$
$$-ig^{2}\ f^{XAB}f^{XCD}\ [g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$

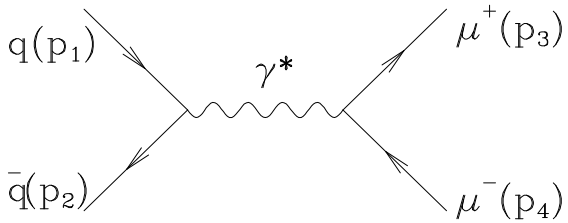


$$g\ f^{ABC}q^{\alpha}$$



$$-ig\ (t^{A})_{cb}\ (\gamma^{\alpha})_{ji}$$

Matrix element



$$(-ie)^2 Q_q \; \bar{v}(-p_2)\gamma^\mu u(p_1) \; \frac{-ig_{\mu\nu}}{s} \; \bar{u}(p_3)\gamma^\nu v(-p_4)$$

Squaring and summing (averaging) over final (initial) spins we get

$$\begin{aligned}
\overline{\sum}_{\text{spins}} |M|^2 &= \overline{\sum}_{\text{spins}} \frac{e^4 Q_q^2}{s^2} \bar{u}(p_1)\gamma^\alpha v(-p_2) \; \bar{v}(-p_2)\gamma^\mu u(p_1) \\
&\quad \bar{v}(-p_4)\gamma^\alpha u(p_3) \; \bar{u}(p_3)\gamma^\mu v(-p_4) \\
&= \frac{1}{4} \frac{e^4 Q_q^2}{s^2} \text{Tr}[\gamma^\alpha \not{p}_2 \gamma^\mu \not{p}_1] \; \text{Tr}[\gamma^\alpha \not{p}_3 \gamma^\mu \not{p}_4]
\end{aligned}$$

Matrix element (continued)

Now using the result

$$\mathrm{Tr}[\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\phi] = 4\Big[g^{\alpha\beta}g^{\delta\phi} + g^{\alpha\phi}g^{\beta\gamma} - g^{\alpha\delta}g^{\beta\phi}\Big]$$

we obtain

$$\mathrm{Tr}[\gamma^\alpha \not{p}_2 \gamma^\mu \not{p}_1]\ \mathrm{Tr}[\gamma^\alpha \not{p}_3 \gamma^\mu \not{p}_4] = +32(p_1.p_3\ p_2.p_4 + p_1.p_4\ p_2.p_3)$$

so that

$$\overline{\sum}_{\mathrm{spins}}|M|^2 = 2e^4 Q_q^2 \frac{t^2+u^2}{s^2} = e^4 Q_q^2 (1+\cos^2\theta^*)$$

Putting it all together

$$
\begin{aligned}
d\sigma &= \frac{1}{N}\frac{1}{2s}\,\frac{1}{16\pi} d\cos\theta^*\; e^4 Q_f^2 (1+\cos^2\theta^*) \\
&= \frac{1}{N}\frac{\pi\alpha^2 Q_f^2}{2s}\int_{-1}^{+1} d\cos\theta^*\;(1+\cos^2\theta^*)
\end{aligned}
$$

$$
\sigma = \frac{1}{N}\frac{4\pi\alpha^2 Q_f^2}{3s}
$$

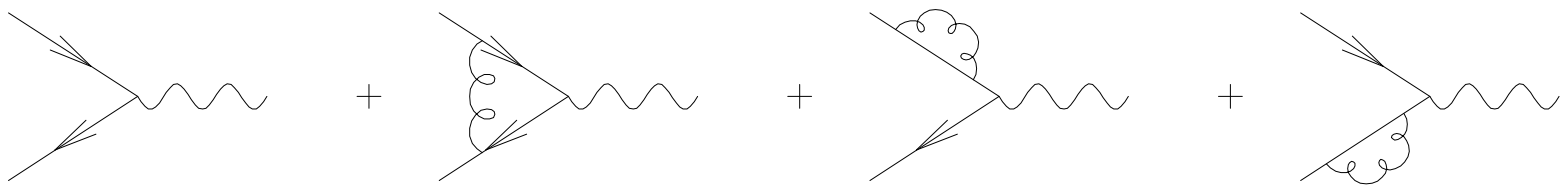
Note that we have added a factor of $1/N$ where N is the number of colors (A red quark can only annihilate with an anti-red quark)

Lepton pair production

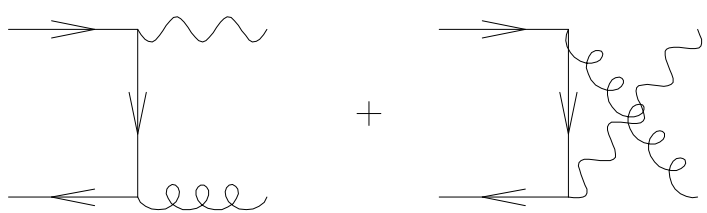
- Inverse of $e^+e^- \to q\bar{q}$ is Drell-Yan process. At $\mathcal{O}(\alpha_s^0)$, mass distribution of lepton pair is given by

$$\frac{d\hat{\sigma}}{dM^2}(q\bar{q} \to \gamma^* \to l^+l^-) = \frac{4\pi\alpha^2}{3\hat{s}}\frac{1}{3}Q_q^2\,\delta(M^2 - \hat{s})$$

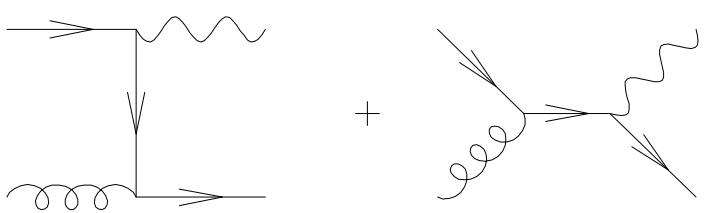
  - ⋆ Factor of $1/3 = 1/N$ instead of $3 = N$ because of *average* over colours of incoming q.



(a)



(b)

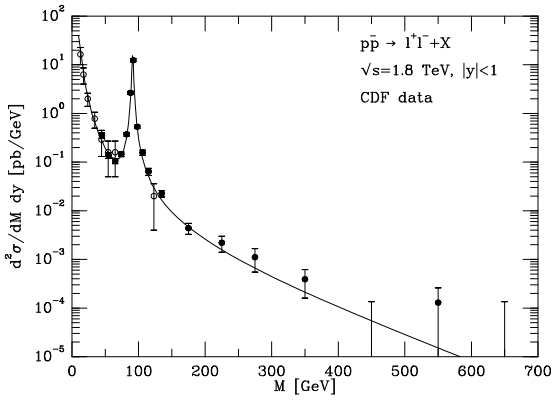


(c)

- In higher orders *vertex corrections* (a) have $M^2 = \hat{s}$, *gluon emission* (b) and *QCD Compton* (c) diagrams give $M^2 < \hat{s}$.
- Rapidity of lepton pair in overall c.m. frame is

$$y \equiv \frac{1}{2} \ln \left(\frac{p^0 + p_3}{p^0 - p_3} \right) = \frac{1}{2} \ln \left(\frac{x_1}{x_2} \right)$$

where $p^\mu = p_1^\mu + p_2^\mu$.



- $W^\pm$ boson production is similar, except sensitive to different parton distributions, e.g.

$$u\bar{d} \to W^+ \to l^+\nu_l$$

Parton luminosity

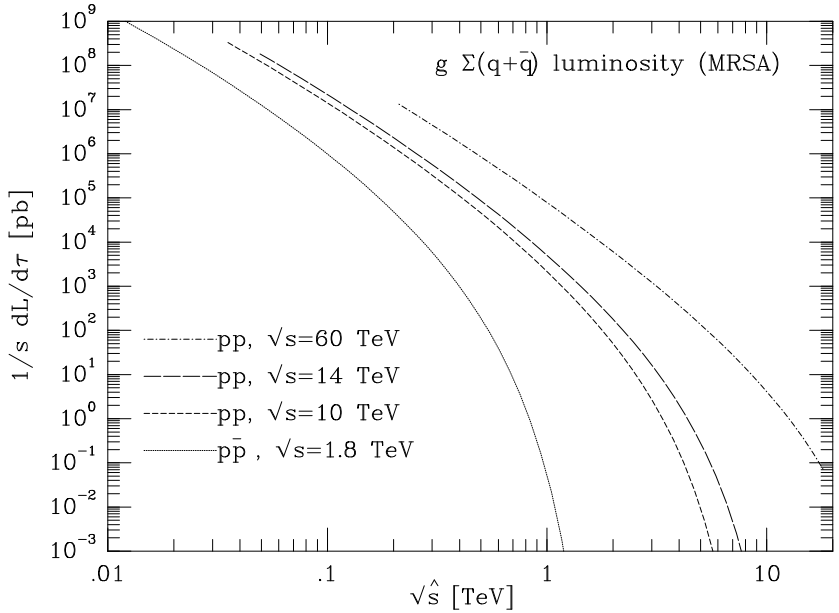
- parton luminosity is determined by the parton distribution functions, $f_i(x_1, \mu^2)$ and $f_j(x_2, \mu^2)$.
- the available centre-of-mass energy-squared of the parton-parton collision, $\hat{s}$, is less than the overall hadron-hadron collision energy, s, by a factor of $x_1 x_2 \equiv \tau$.
- Define differential parton luminosities

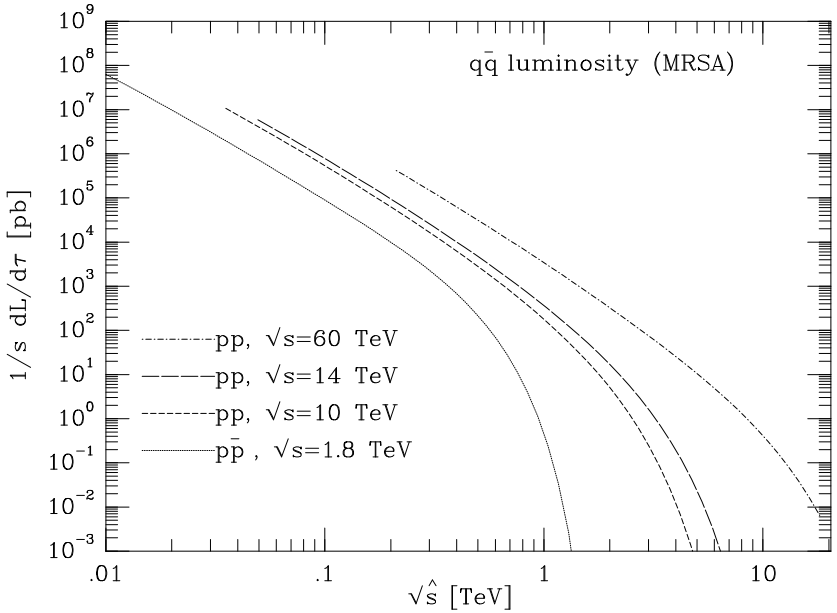
$$\tau \frac{dL_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}} \int_0^1 dx_1 dx_2$$
$$\times \Big[\big(x_1 f_i(x_1, \mu^2)\; x_2 f_j(x_2, \mu^2) \big) + \big(1 \leftrightarrow 2\big) \Big] \delta(\tau - x_1 x_2).$$

  case of identical partons.
- We now assume that $\hat{\sigma}$ depends only on $\hat{s}$.

$$\sigma(s) = \sum_{\{ij\}} \int_{\tau_0}^1 \frac{d\tau}{\tau} \left[\frac{1}{s} \frac{dL_{ij}}{d\tau} \right] \left[\hat{s} \hat{\sigma}_{ij} \right],$$

Other luminosities





Factorization of the cross section

- Why does the factorization property hold and when it should fail?
- For a heuristic argument Consider the simplest hard process involving two hadrons

$$H_1(P_1) + H_2(P_2) \to V + X.$$

- Do the partons in hadron H_1, through the influence of their colour fields, change the distribution of partons in hadron H_2 before the vector boson is produced? Soft gluons which are emitted long before the collision are potentially troublesome.
- A simple model from classical electrodynamics. The vector potential due to an electromagnetic current density J is given by

$$A^\mu(t, \vec{x}) = \int dt' d\vec{x}' \; \frac{J^\mu(t', \vec{x}')}{|\vec{x} - \vec{x}'|} \; \delta(t' + |\vec{x} - \vec{x}'| - t) \; ,$$

where the delta function provides the retarded behaviour required by causality.

Consider a particle with charge e travelling in the positive z direction with constant velocity β. The non-zero components of the current density are

$$\begin{aligned} J^t(t', \vec{x}') &= e\delta(\vec{x}' - \vec{r}(t')) \ , \\ J^z(t', \vec{x}') &= e\beta\delta(\vec{x}' - \vec{r}(t')), \ \ \vec{r}(t') = \beta t' \hat{z}, \end{aligned}$$

$\hat{z}$ is a unit vector in the z direction. At an observation point (the supposed position of hadron H_2) described by coordinates x, y and z, the vector potential (after performing the integrations using the current density given above) is

$$\begin{aligned} A^t(t, \vec{x}) &= \frac{e\gamma}{\sqrt{[x^2 + y^2 + \gamma^2(\beta t - z)^2]}} \\ A^x(t, \vec{x}) &= 0 \\ A^y(t, \vec{x}) &= 0 \\ A^z(t, \vec{x}) &= \frac{e\gamma\beta}{\sqrt{[x^2 + y^2 + \gamma^2(\beta t - z)^2]}}, \end{aligned}$$

where $\gamma^2 = 1/(1 - \beta^2)$. Target hadron H_2 is at rest near the origin, so that $\gamma \approx s/m^2$.

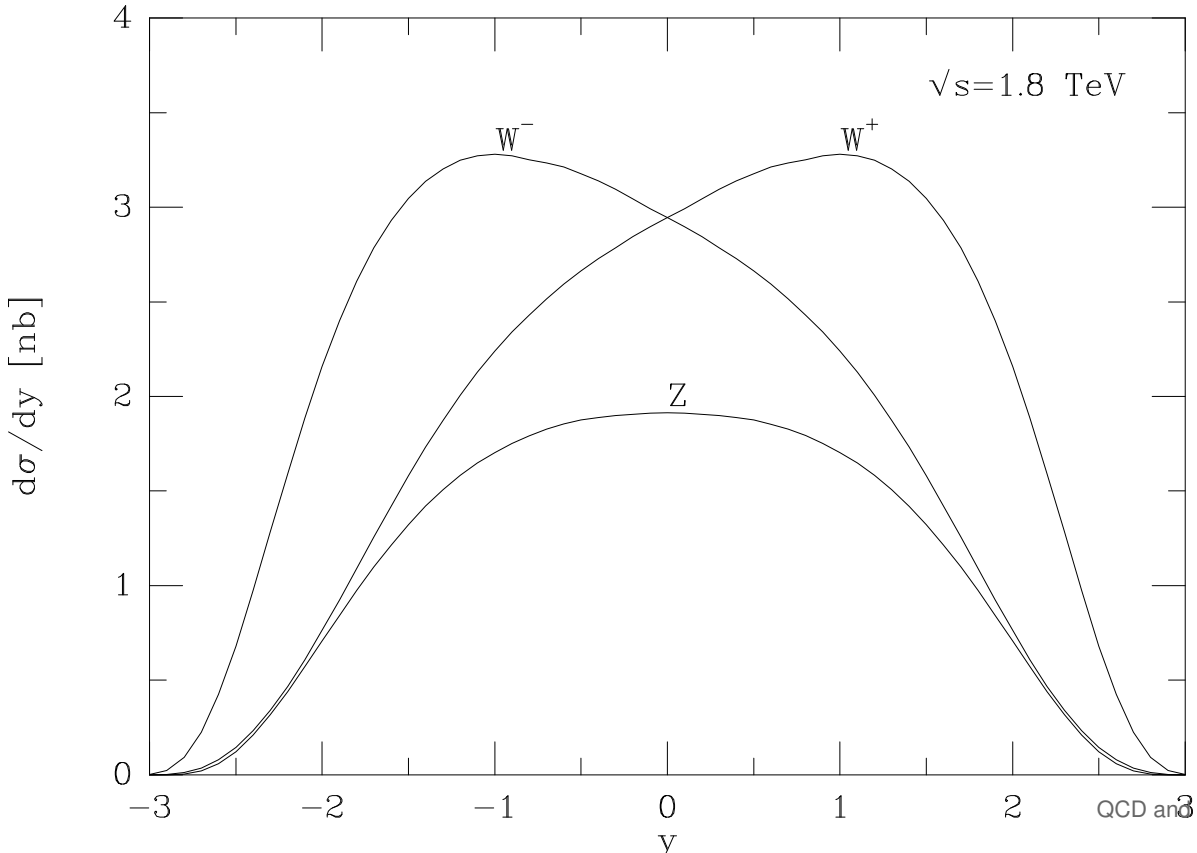
- Note that for large γ and fixed non-zero $(\beta t - z)$ some components of the potential tend to a constant independent of γ, suggesting that there will be non-zero fields which are not in coincidence with the arrival of the particle, even at high energy.
- However at large γ the potential is a pure gauge piece, $A^\mu = \partial^\mu \chi$ where χ is a scalar function
- Covariant formulation using the vector potential A has large fields which have no effect.
- For example, the electric field along the z direction is

$$E^z(t, \vec{x}) = F^{tz} \equiv \frac{\partial A^z}{\partial t} + \frac{\partial A^t}{\partial z} = \frac{e\gamma(\beta t - z)}{[x^2 + y^2 + \gamma^2(\beta t - z)^2]^{\frac{3}{2}}}.$$

The leading terms in γ cancel and the field strengths are of order $1/\gamma^2$ and hence of order m^4/s^2. The model suggests the force experienced by a charge in the hadron H_2, at any fixed time before the arrival of the quark, decreases as m^4/s^2.

W rapidity asymmetry

The W and Z rapidity distributions are calculated in exactly the same way as for Drell-Yan lepton pair production The predicted distributions for $W^{\pm}$, Z production in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV. Here we have defined $y > 0$ to be the direction of the incoming proton. Neglecting sea quark contributions to the cross section, a W^+ is obtained from the annihilation of a (valence) u quark from the proton and a (valence) $\bar{d}$ quark from the antiproton. Since u quarks in a proton are in general moving faster than d quarks, the W^+ bosons are produced preferentially in the proton direction. Likewise, W^- bosons are produced preferentially in the antiproton direction.



In fact the W *rapidity asymmetry*,

$$A_W(y) = \frac{d\sigma(W^+)/dy - d\sigma(W^-)/dy}{d\sigma(W^+)/dy + d\sigma(W^-)/dy} \ ,$$

provides a very sensitive measure of the relative shape of the u and d quark distributions In particular, A_W is strongly correlated with the *slope* of $d(x)/u(x)$. We can see this from a simple model calculation in which only valence quarks contribute to the production cross section. Then we have (adopting the notation $u(x) = f_u(x)$, using the fact that $u_p(x) = \bar{u}_{\bar{p}}(x)$ etc., and suppressing scale dependence)

$$A_W(y) \approx \frac{u(x_1)d(x_2) - d(x_1)u(x_2)}{u(x_1)d(x_2) + d(x_1)u(x_2)} = \frac{R(x_2) - R(x_1)}{R(x_2) + R(x_1)} \ ,$$

where $R(x) = d(x)/u(x)$ and

$$x_1 = x_0 \exp(+y) \ , \qquad x_2 = x_0 \exp(-y) \ , \qquad x_0 = M_W/\sqrt{s} \ .$$

For small y we can write $x_{1,2} = x_0 \ \pm \ yx_0 \ + \ \ldots$ and $R(x_{1,2}) = R(x_0) \ \pm \ yx_0 R'(x_0) \ + \ \ldots$, so that

$$A_W(y) \approx -yx_0 \ \frac{R'(x_0)}{R(x_0)} \ .$$

The asymmetry is therefore predicted to be linear in y for small y, with a coefficient proportional to the slope of $d(x)/u(x)$ evaluated at $x = M_W/\sqrt{s}$ (and $\mu^2 \sim M_W^2$). Unfortunately the W rapidity is difficult to measure accurately because of the undetected neutrino in the final state. What *can* be measured with high precision is the closely related charged lepton asymmetry $A_l(y)$. However the inclusion of the leptonic decay in the measurement provides an additional complication: because of the $V - A$ structure of the weak charged current, the charged lepton is not produced isotropically in the W rest frame. In particular, for the process

$$d(p_d) + \bar{u}(p_u) \to e^-(p_e) + \bar{\nu}(p_\nu) \ ,$$

where the momentum labels are indicated in brackets.

$$\begin{aligned} \overline{\sum} |\mathcal{M}(d\bar{u} \to e^- \bar{\nu})^2 \quad &= \quad 16 \left(\sqrt{2} G_F M_W^2\right)^2 |V_{ud}|^2 \\ &\quad \times \frac{(p_u \cdot p_e)^2}{((p_u + p_d)^2 - M_W^2)^2 + M_W^2 \Gamma_W^2} \ . \end{aligned}$$

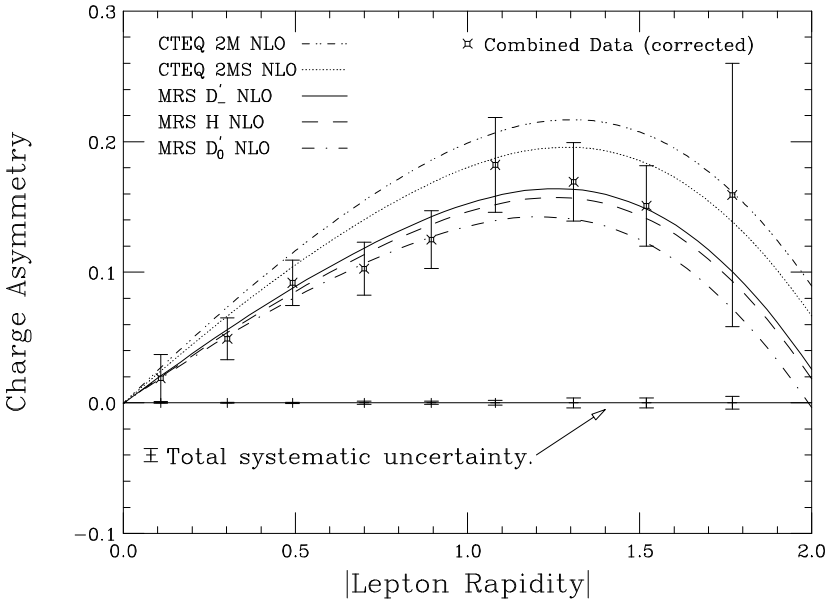
Likewise, for the charge-conjugate process $u\bar{d} \to e^+\nu$ we have exactly the same expression, where now p_u is the momentum of the incoming u quark etc.

If we define θ^* to be the $e^+(e^-)$ polar angle of emission in the $W^+(W^-)$ rest frame, measured with respect to the direction of the incident $\bar{p}(p)$, and if we assume that all incoming quarks (antiquarks) are constituents of the proton (antiproton), then for the above matrix elements we have

$$(p_u \cdot p_e)^2 \;=\; \frac{M_W^4}{16}\,(1+\cos\theta^*)^2 \;.$$

The cross section is therefore maximal when the outgoing electron (positron) moves in the direction of the incoming proton (antiproton). In fact there is a simple angular momentum argument for this. In the Standard Model, the W couples to negative helicity fermions and positive helicity antifermions. Angular momentum conservation therefore requires the outgoing fermion (electron) to preferentially follow the direction of the incoming fermion (quark), which is usually the direction of the incoming proton. This is in the *opposite* sense to the $W^\pm$ production asymmetry and so the charged lepton asymmetry is smaller than that of the W boson itself.

As an example of how well the lepton asymmetry can discriminate between parton distributions, shows data from the CDF collaboration compared to the predictions using various sets of parton distributions. In principle, all of these describe deep inelastic structure function data equally well, and so the lepton asymmetry measurement provides an important additional constraint on the distributions in global fits



W mass measurement

- In the rest frame of the decaying W, the energy of the charged lepton is $M_W/2$. It can be used to measure the W mass.
- *transverse* momentum of the electron, also carries information on M_W.
- in the W rest frame the angular distribution of the electron is

$$\frac{1}{\sigma}\,\frac{d\sigma}{d\cos\theta^*} \;=\; \frac{3}{8}\,(1+\cos^2\theta^*)\;,$$

- If the W has zero transverse momentum

$$\cos\theta^* \;=\; \left(1-\frac{4p_{Te}^2}{M_W^2}\right)^{\frac{1}{2}}\;,$$

so that

$$\frac{1}{\sigma}\,\frac{d\sigma}{dp_{Te}^2} \;=\; \frac{3}{M_W^2}\,\left(1-\frac{4p_{Te}^2}{M_W^2}\right)^{-\frac{1}{2}}\,\left(1-\frac{2p_{Te}^2}{M_W^2}\right)\;.$$

- The distribution is strongly peaked at $p_{Te} = M_W/2$ (the *Jacobian peak*)
- to include the missing (neutrino) transverse momentum define the *transverse mass*

$$M_T^2 \;=\; 2|p_{Te}||p_{T\nu}|(1-\cos\Delta\phi_{e\nu})\;.$$

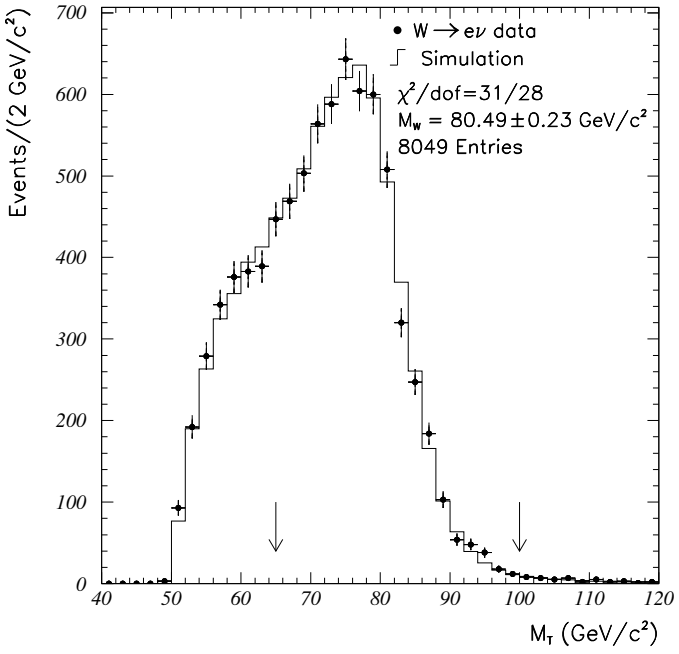
At leading order ($q\bar{q}' \to W \to e\nu$) and in the absence of any quark transverse momentum, we have $|p_{Te}| = |p_{T\nu}| = p^*$, $\Delta\phi_{e\nu} = \pi$, and so $M_T = 2|p_{Te}|$. The transverse mass distribution therefore also has a Jacobian peak, at $M_T = M_W$.

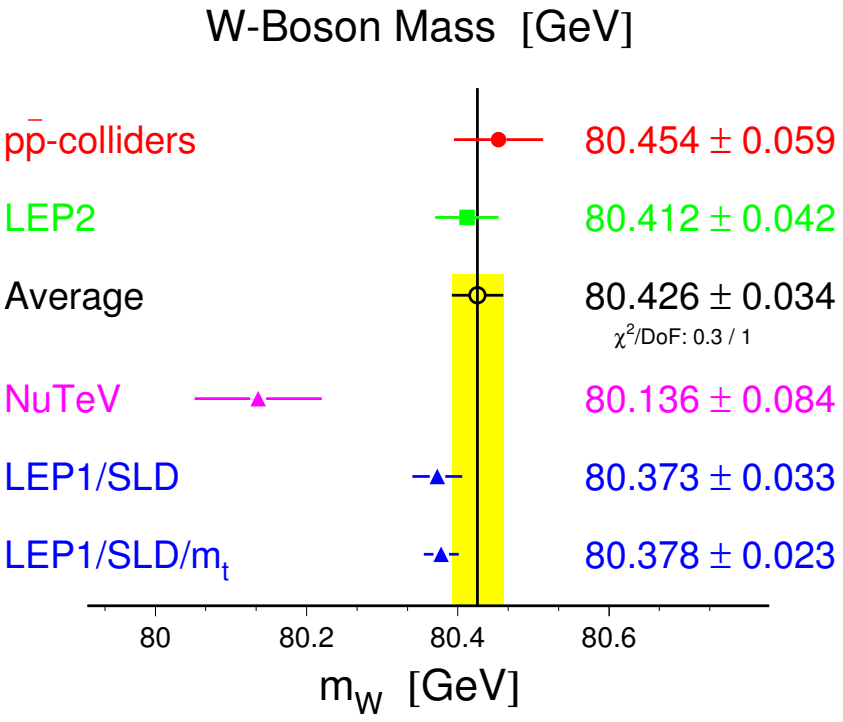
- the transverse mass distribution is that it is less sensitive to the transverse momentum (p_T^W) of the W boson. If p_T^W is small, the transverse momenta of the leptons in the laboratory and W centre-of-mass frames are related by a simple Galilean transformation:

$$\begin{aligned} p_{Te} &= p^* + \frac{1}{2}p_T^W \\ p_{T\nu} &= -p^* + \frac{1}{2}p_T^W\;. \end{aligned}$$

It is straightforward to show that, to leading order in p_T^W, the transverse mass is unchanged by such a transformation.

Current results



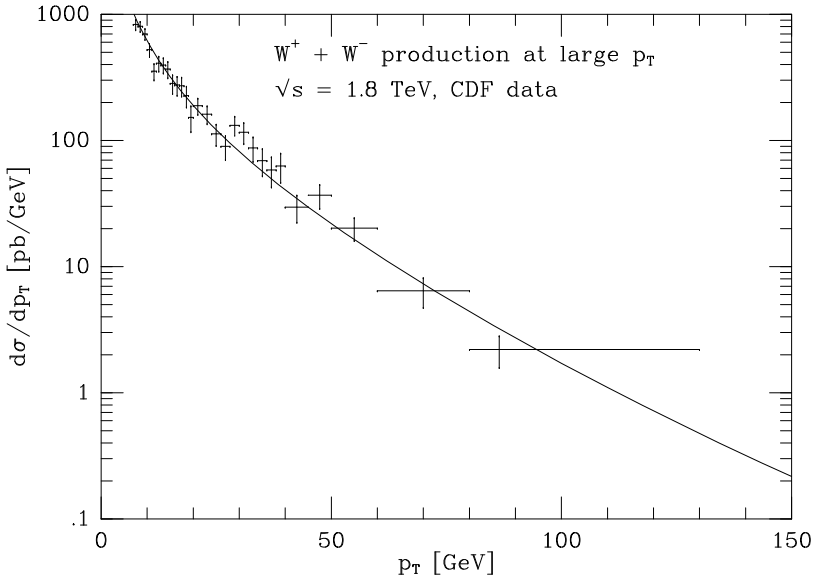


W and Z transverse momentum distributions

- most W and Z bosons (collectively denoted by V) are produced with relatively little transverse momentum, $p_T \ll M_V$. However, part of the total cross section corresponds to the production of large transverse momentum bosons. The relevant mechanisms are the $2 \to 2$ processes $q\bar{q} \to Vg$ and $qg \to Vq$.

$$\begin{aligned}
\overline{\sum}|\mathcal{M}^{q\bar{q}' \to Wg}|^2 &= \pi\alpha_s\sqrt{2}G_F M_W^2 |V_{qq'}|^2 \\
&\times \frac{8}{9}\,\frac{t^2+u^2+2M_W^2 s}{tu}\,, \\
\overline{\sum}|\mathcal{M}^{gq \to Wq'}|^2 &= \pi\alpha_s\sqrt{2}G_F M_W^2 |V_{qq'}|^2 \\
&\times \frac{1}{3}\,\frac{s^2+u^2+2tM_W^2}{-su}\,,
\end{aligned}$$

with similar results for the Z boson. The transverse momentum distributions $d\sigma/dp_T^2$ are obtained by convoluting these matrix elements with parton distributions in the usual way.



- Data on the p_T distribution of the W boson compared with the next-to-leading-order QCD prediction.

The poles at $t = 0$ and $u = 0$ in the matrix elements cause the leading-order theoretical cross section to diverge as $p_T \to 0$. The leading behaviour at small p_T comes from the emission of a soft ($k^\mu \to 0$) gluon in the process $q\bar{q} \to Vg$. Schematically (with $M = M_W$ or M_Z)

$$\frac{d\sigma^R}{dp_T^2} = \alpha_s \left(A \, \frac{\ln(M^2/p_T^2)}{p_T^2} + B \, \frac{1}{p_T^2} + C(p_T^2) \right) ,$$

where A and B are calculable coefficients and C is an integrable function.

Virtualcorrections to $q\bar{q} \to V$ only contribute at $p_T = 0$, $d\sigma^V/dp_T^2 \propto \delta(p_T^2)$, and their contribution to the differential distribution essentially introduces a 'plus prescription' on the singular parts of the gluon emission cross section:

$$\frac{d\sigma^{R+V}}{dp_T^2} = \alpha_s \left(A \left[\frac{\ln(M^2/p_T^2)}{p_T^2} \right]_+ + B \left[\frac{1}{p_T^2} \right]_+ + \overline{C}(p_T^2) \right) ,$$

such that the integrated contribution is finite, [a]

$$\int dp_T^2 \, \frac{d\sigma^{R+V}}{dp_T^2} = \alpha_s \int dp_T^2 \, \overline{C}(p_T^2) \, .$$

[a] Recall that $\int_0^1 dx [f(x)]_+ = 0$.

An important ingredient missing from the above derivation is the *non-perturbative* contribution to the p_T distribution. At very small p_T, the intrinsic transverse motion of the quarks and gluons inside the colliding hadrons cannot be neglected. This non-perturbative contribution has to be combined with the perturbative large p_T tail. A simple convolution in transverse-momentum space of the perturbative distribution with non-perturbative intrinsic k_T distributions is one method of doing this.
For $p_T \gg \langle k_T \rangle$ a purely perturbative approach should be adequate. However, if at the same time $p_T \ll M$ then higher-order terms in the perturbation series cannot be neglected. In particular, the emission of multiple soft gluons becomes important and the leading contributions at each order have the form

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} \simeq \frac{1}{p_T^2}\left[A_1\alpha_s \ln\frac{M^2}{p_T^2} + A_2\alpha_s^2 \ln^3\frac{M^2}{p_T^2} + \ldots + A_n\alpha_s^n \ln^{2n-1}\frac{M^2}{p_T^2} + \ldots\right],$$

where the A_i are calculable coefficients of order unity. The higher-order terms in the series are evidently important when

$$\alpha_s \ln^2\frac{M^2}{p_T^2} > 1 .$$

Taking into account the relative magnitude of the A_n coefficients, this corresponds to p_T

Indeed the resummed double leading logarithms give

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} \simeq \frac{d}{dp_T^2} \exp\left(-\frac{\alpha_s s}{2\pi} C_F \ln^2 \frac{M^2}{p_T^2}\right) ,$$

which vanishes at $p_T = 0$.

- the production of a W or Z boson with $p_T \approx 0$ does *not* require that all emitted gluons are soft, merely that their *vector* transverse momentum sum is small.
- the double-leading-logarithm result omits contributions from the multiple emission of soft gluons with $k_{Ti} \sim p_T$ and $\sum_i \vec{k}_{Ti} = \vec{p}_T$. Such additional contributions 'fill in' the dip at $p_T \approx 0$.

A more complete analysis of the small p_T distribution requires proper treatment of transverse momentum conservation in multiple gluon emission. This is achieved by introducing the two-dimensional impact parameter vector $\vec{b}$, which is the Fourier conjugate of $\vec{p}_T$, and writing

$$\delta^{(2)}\left(\sum_{i=1}^{n}\vec{k}_{Ti}-\vec{p}_T\right)=\frac{1}{(2\pi)^2}\int d^2b\; e^{-i\vec{b}\cdot\vec{p}_T}\;\prod_{i=1}^{n}e^{i\vec{b}\cdot\vec{k}_{Ti}}\;,$$

for the emission of n soft gluons. The Sudakov form factor then appears in b-space after resumming large $\ln(b^2M^2)$ logarithms.

- a complete formalism has been developed for taking account of all large $\ln(M^2/p_T^2)$ logarithms in the perturbation series at small p_T.

In particular, the result for the small $p_T(Z)$ distribution is

$$\frac{d\sigma}{dp_T^2} \simeq \sum_q \sigma_0^{q\bar{q}} \frac{1}{2} \int_0^\infty db\, b\, J_0(bp_T)\, \exp(-S(b, M_Z))$$

$$\int_0^1 dx_1\, dx_2\, \delta(x_1 x_2 - \frac{M_Z^2}{s}) \left[q(x_1, (b_0/b)^2)\, \bar{q}(x_2, (b_0/b)^2) + (q \leftrightarrow \bar{q})\right] ,$$

where

$$\sigma_0^{q\bar{q}} = \pi\sqrt{2} G_F M_Z^2 (V_q^2 + A_q^2)/(3s)$$

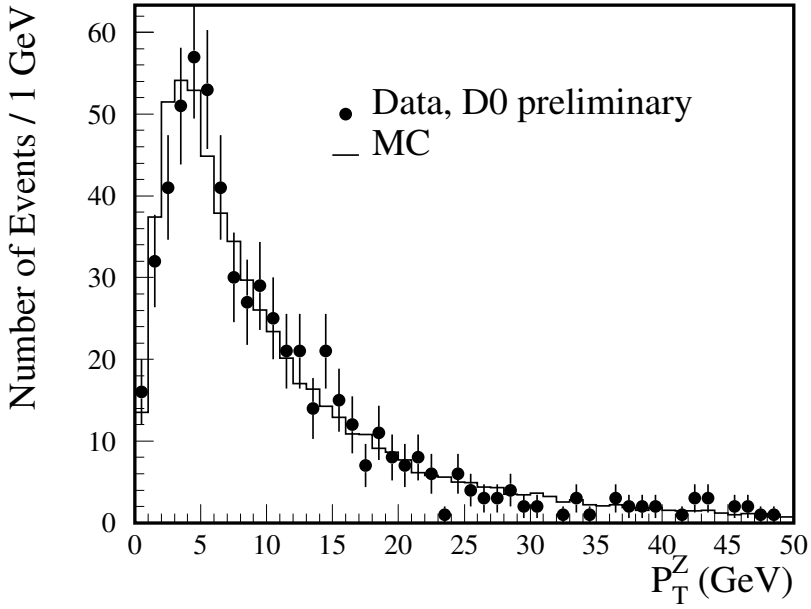
and $b_0 = 2\exp(-\gamma_E)$ ($\gamma_E = 0.5772\ldots$ is the Euler constant). The Sudakov form factor in b-space is $\exp(-S)$ where

$$\begin{aligned} S(b, Q) &= \int_{(b_0/b)^2}^{Q^2} \frac{dq^2}{q^2} \left[\ln \frac{Q^2}{q^2} A(\alpha_s(q^2)) + B(\alpha_s(q^2))\right] , \\ A(\alpha_s) &= \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n A^{(n)} , \\ B(\alpha_s) &= \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n B^{(n)} . \end{aligned}$$

The $A^{(n)}$ and $B^{(n)}$ coefficients for $n = 1, 2$ have been calculated.

The small-p_T distribution can, in principle, be used as a test of the resummed QCD perturbation series. In practice, however, there are some difficulties — for example, some non-perturbative cut-off or smearing must be included to make the b integral converge at large b and to avoid infra-red problems from evaluating α_s and the parton distributions at low scales. This introduces a significant theoretical uncertainty.

- The theoretical prediction incorporates both the resummed higher-order contributions at small p_T, and the exact next-to-leading-order correction at large p_T.



Kinematics of Jets

- Choose the final state variables which transform simply under longitudinal boosts, (rapidity y, the transverse momentum p_T,azimuthal angle ϕ, transverse mass $m_T = \sqrt{p_T^2 + m^2}$.

$$\begin{aligned} p^\mu &= (E, p_x, p_y, p_z) \\ &= \left(m_T \cosh y, p_T \sin\phi, p_T \cos\phi, m_T \sinh y\right) \ , \end{aligned}$$

$$y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right),$$

- Rapidity is additive under longitudinal boosts. Rapidity differences are boost invariant.
- Pseudorapidity variable η, coincides with the rapidity in the $m \to 0$ limit.

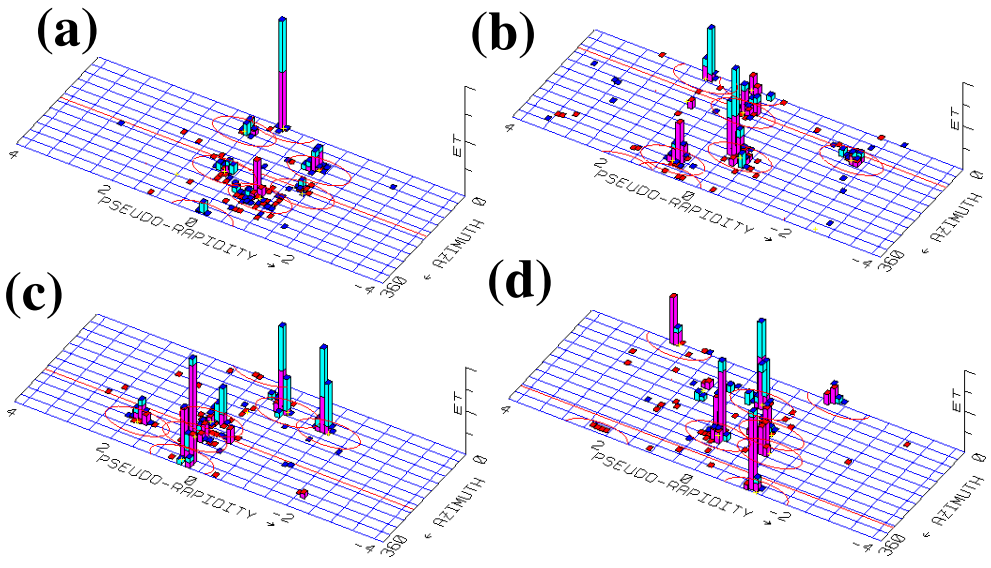
$$\eta = -\ln \tan(\theta/2) \ ,$$

Since E is directly measured in the calorimeter we also use

$$E_T = E \sin\theta,$$

- Examples of (Lego) plots of E_T vs. pseudorapidity and azimuth.
- These events are four jet events from CDF.

Lego plots

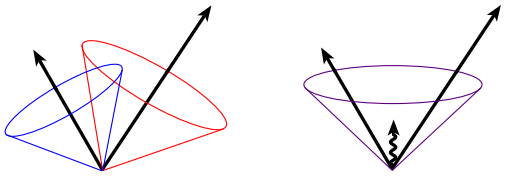


Jet algorithms

- From previous plots we see that jets exist as localized clusters of energy.
- There is no 'best' jet definition, although there are better and worse.
- Good jet definitions are
  - ⋆ Fully specified
  - ⋆ Theoretically well-behaved
  - ⋆ Detector independent
  - ⋆ Order independent
- For hadron-hadron collisions, the most commonly used definition is of the cone type: a jet is defined to be in the directions which maximizes the transverse energy E_T in a 'cone' of radius R, where

$$R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \ .$$

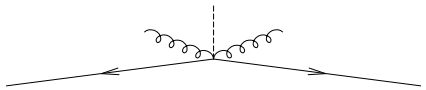
- In the two-dimensional η, ϕ plane, curves of constant R are circles around the axis of the jet.
- Cone jet has many problems, e.g. jet classification is not stable under emission of soft radiation.



- In addition, since a global maximization is too costly in computer time,

k_T-jet algorithm

- The basic idea is of the k_T jet algorithm to use the transverse momentum rather than the invariant mass as clustering parameter.
- Below is an event classified as a three jet event by the mass (JADE) clustering algorithm

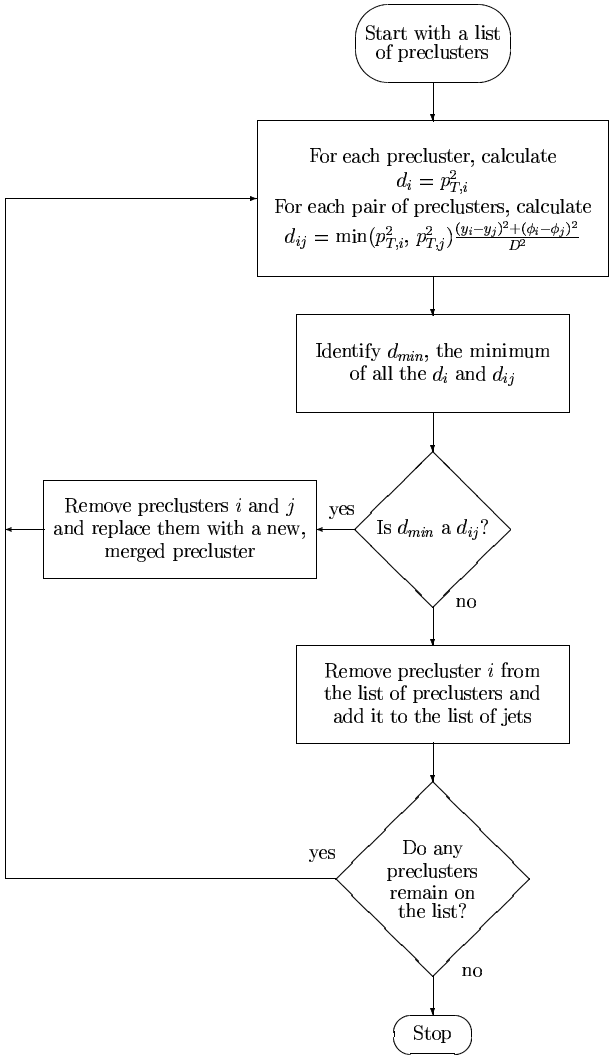


- For a more natural jet classification we use, instead of using $y_{ij} = M^2/s$

$$\begin{aligned} y_{ij} &= 2\min\{E_i^2, E_j^2\}(1-\cos\theta_{ij})/s \\ &\sim k_{T\ ij}^2/s \text{ for } \theta_{ij} \to 0 \end{aligned}$$

- Theoretical benefits stem from the fact that soft hadrons produced coherently by the fragmentation of hard partons should be assigned to the jet nearest in angle. Jets have a natural radius depending on the hardness of the jet itself.

Tevatron Run II Jet algorithm



Results on W+jets

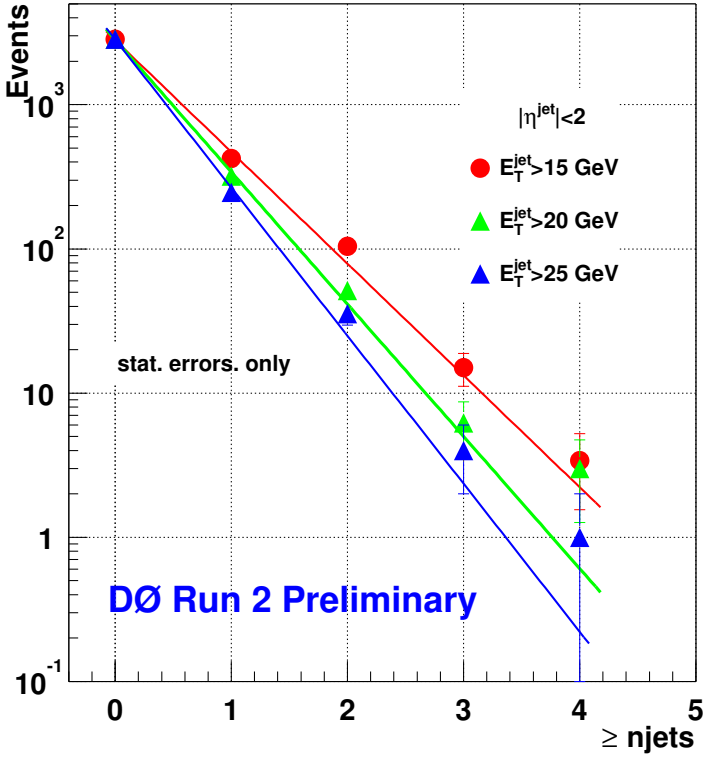
- One of the most important Standard Model processes in high-energy hadron-hadron collisions is the production of a W or Z with accompanying hadronic (quark or gluon) jets.
- Most 'new physics' processes, for example the production of heavy quarks, Higgs bosons and supersymmetric particles, can be mimicked by the production of vector bosons in association with jets.
- decompose the total W cross section into its multijet components:

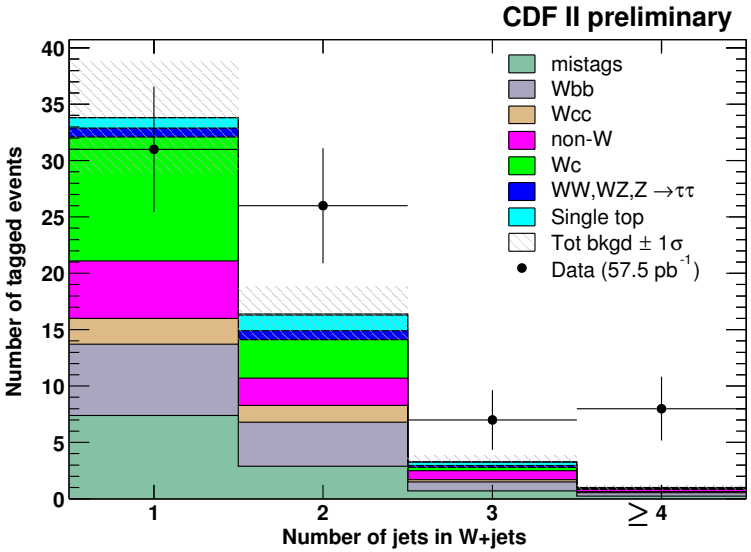
$$\sigma_W = \sigma_{W+0j} + \sigma_{W+1j} + \sigma_{W+2j} + \sigma_{W+3j} + \ldots$$

where, schematically,

$$\begin{aligned}
\sigma_{W+0j} &= a_0 + \alpha_s a_1 + \alpha_s^2 a_2 + \ldots \\
\sigma_{W+1j} &= \alpha_s b_1 + \alpha_s^2 b_2 + \ldots \\
\sigma_{W+2j} &= \alpha_s^2 c_2 + \ldots \\
\ldots & \quad .
\end{aligned}$$

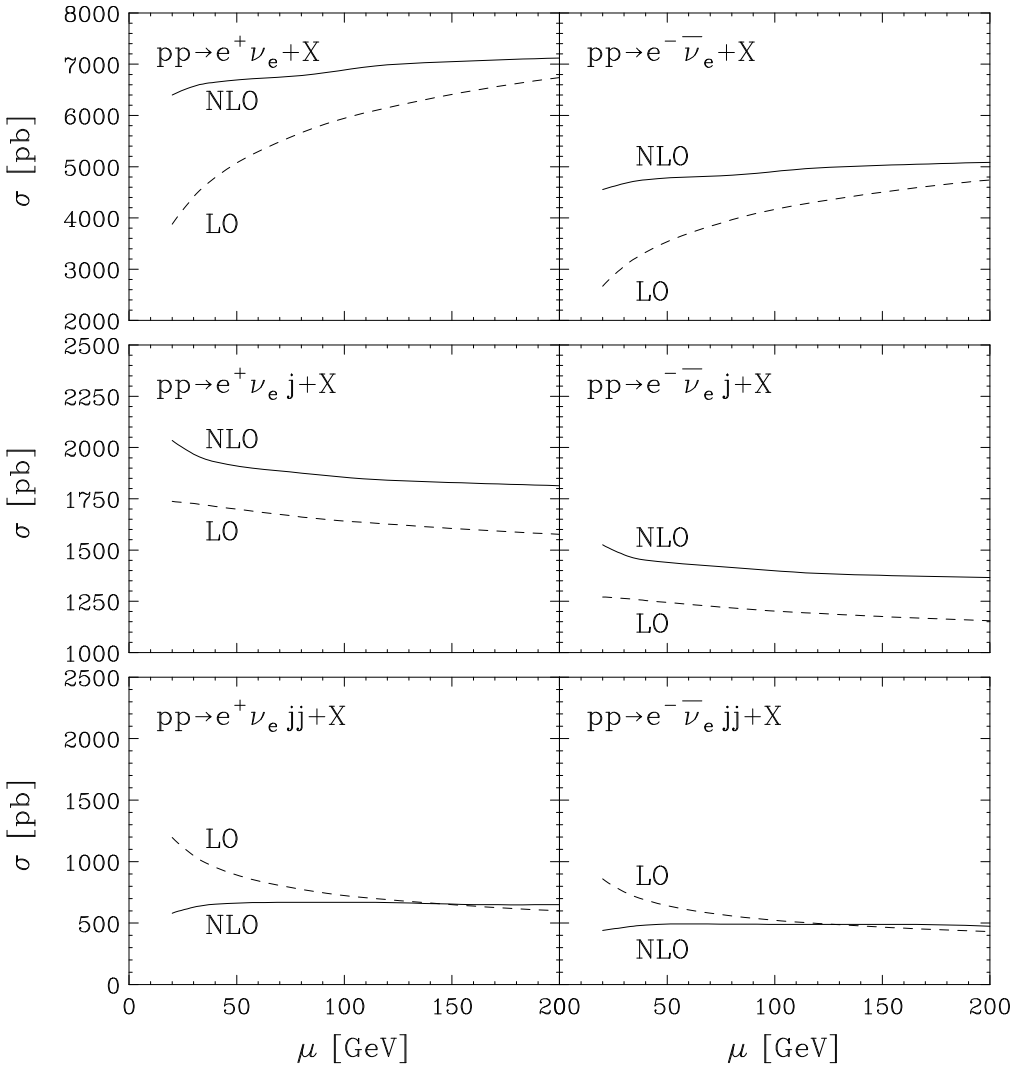
- The $a_i, b_i, c_i, \ldots$ coefficients in these expansions are in general functions of the jet-definition parameters, in particular the cone size used to cluster the partons into jets, and the transverse momentum, rapidity and separation cuts imposed on the jets/clusters.
- The leading contributions to the cross sections, $a_0, b_1, c_2, \ldots$, can be calculated from the matrix elements for the 'tree-level' parton processes:


Events
10^3
10^2
10
1
10^{-1}
$|\eta^{jet}|<2$
$E_T^{jet}>15$ GeV
$E_T^{jet}>20$ GeV
$E_T^{jet}>25$ GeV
stat. errors. only
DØ Run 2 Preliminary
0
1
2
3
4
5
≥ njets

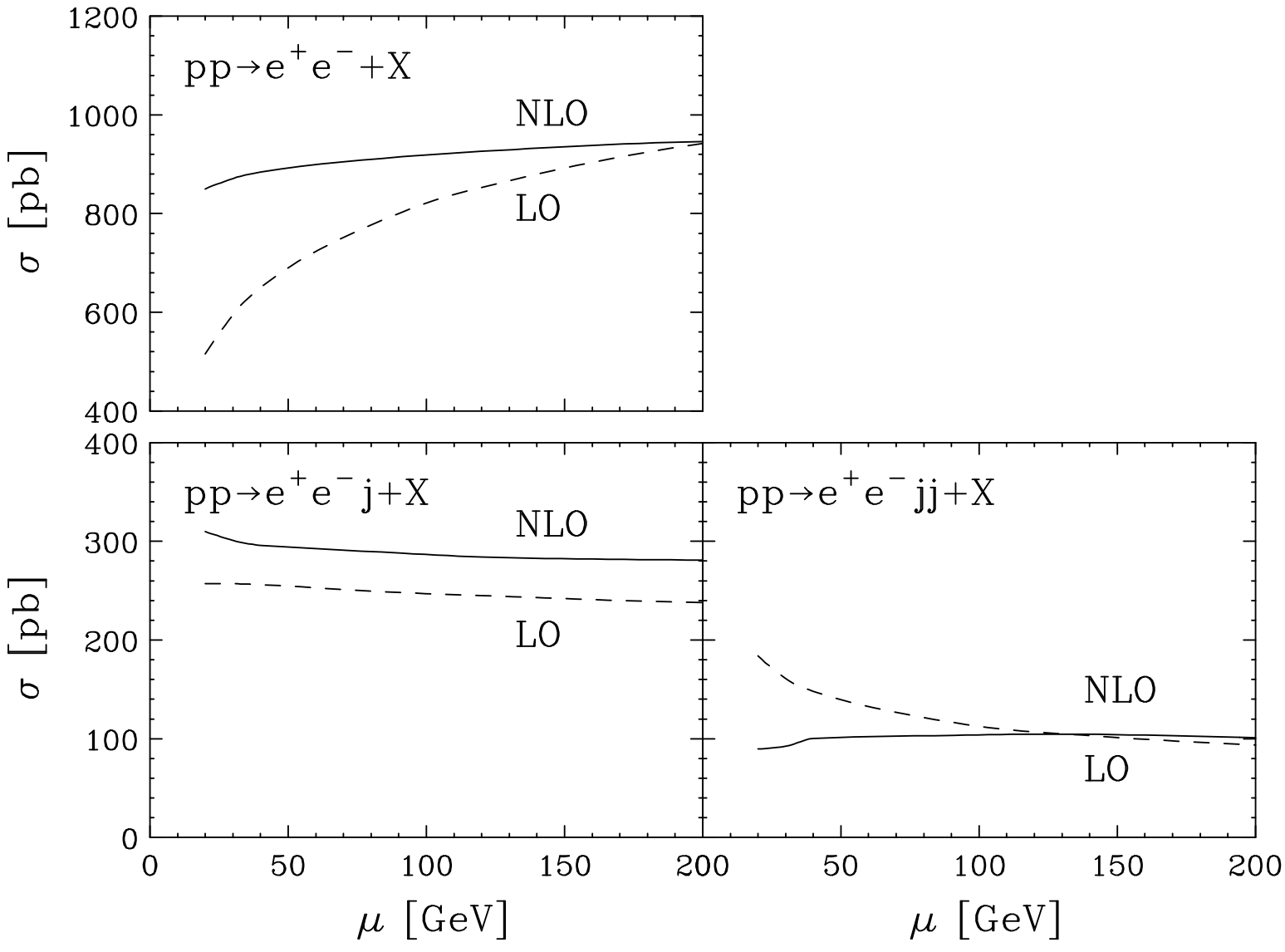

CDF II preliminary
Number of tagged events
40
35
30
25
20
15
10
5
0
mistags
Wbb
Wcc
non-W
Wc
WW,WZ,Z →ττ
Single top
Tot bkgd ± 1σ
Data (57.5 pb^{-1})
1
2
3
≥ 4
Number of jets in W+jets

Vector boson + n jets

- Next-to-leading-order corrections are at present known only for the $W + 0, 1$ and 2 jet production cross sections
- results for LHC

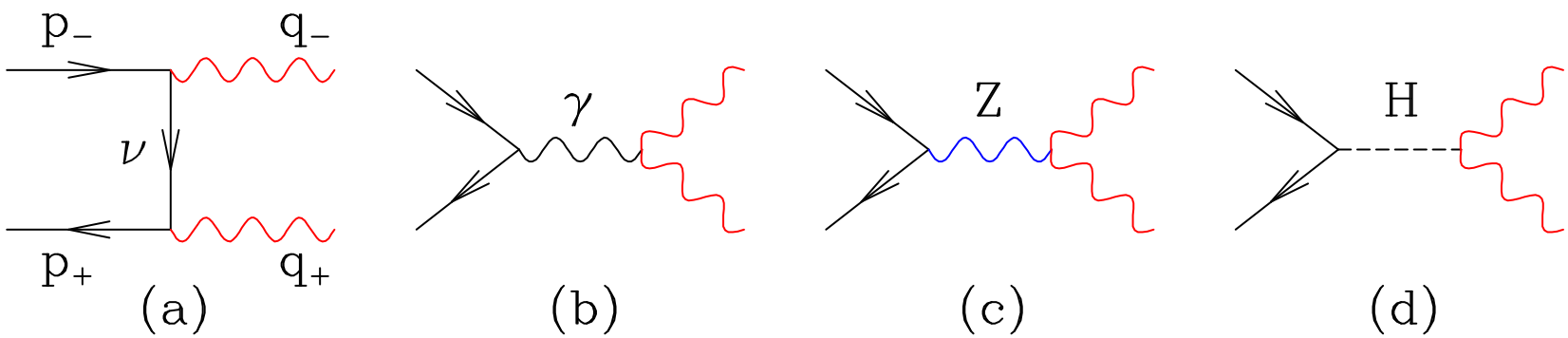


Results on Z + jet production



Vector boson pair production in e^+e^-

- gauge cancellations amongst the couplings have an enormous effect on the high-energy behaviour of WW scattering.



- consider the process $e^-e^+ \to W^-W^+$
- The outgoing vector bosons can have longitudinal or transverse polarizations.
- the longitudinal polarization is responsible diagram-by-diagram for the leading behaviour at high energy.
- We choose a frame in which the outgoing momenta are directed along the z axis. Then the polarization vectors are

$$\begin{aligned} \varepsilon_L(q_\pm) &= \left(\frac{q}{M_W}, 0, 0, \pm\frac{E}{M_W}\right) \\ &\sim \frac{1}{M_W}\left(q_\pm - \frac{M_W^2}{2E^2} q_\mp\right) . \end{aligned}$$

- the individual diagrams will lead to amplitudes which grow like E^2.

If we are only interested in the leading terms we may contract each vector boson with the leading term. The contribution of diagram (a) is

$$M = \frac{(-ig_W)^2}{8}\bar{v}(-p_+)\not{\varepsilon}(q_+)(1-\gamma_5)\frac{i}{\not{p}_- - \not{q}_-}\not{\varepsilon}(q_-)(1-\gamma_5)u(p_-) \ .$$

Inserting the leading term for the polarizations and dropping non-leading terms, we obtain

$$M = -i\frac{(-ig_W)^2}{8M_W^2}\bar{v}(-p_+)(\not{q}_+ - \not{q}_-)(1-\gamma_5)u(p_-) \ .$$

The contribution of the photon exchange diagram (b),

$$\begin{aligned} M &= (-ig_W)(ig_W)Q_e \sin^2\theta_W \bar{v}(-p_+)\gamma^\rho u(p_-)\frac{-ig_{\rho\alpha}}{(q_+ + q_-)^2} \\ &\times V^{\alpha\beta\delta}(q_+ + q_-, -q_-, -q_+)\varepsilon_\beta(q_-)\varepsilon_\delta(q_+) \ . \end{aligned}$$

The contribution of the Z exchange diagram (c) is

$$\begin{aligned} M &= \frac{(-ig_W)(ig_W)}{2}\bar{v}(-p_+)\gamma^\rho(V_e - A_e\gamma_5)u(p_-) \\ &\times \frac{-ig_{\rho\alpha}}{(q_+ + q_-)^2 - M_Z^2} \\ &\times V^{\alpha\beta\delta}(q_+ + q_-, -q_-, -q_+)\varepsilon_\beta(q_-)\varepsilon_\delta(q_+) \end{aligned}$$

where

$$V^{\alpha\beta\delta}(p,q,r) = g^{\alpha\beta}(p^\delta - q^\delta) + g^{\beta\delta}(q^\alpha - r^\alpha) + g^{\delta\alpha}(r^\beta - p^\beta) \ .$$

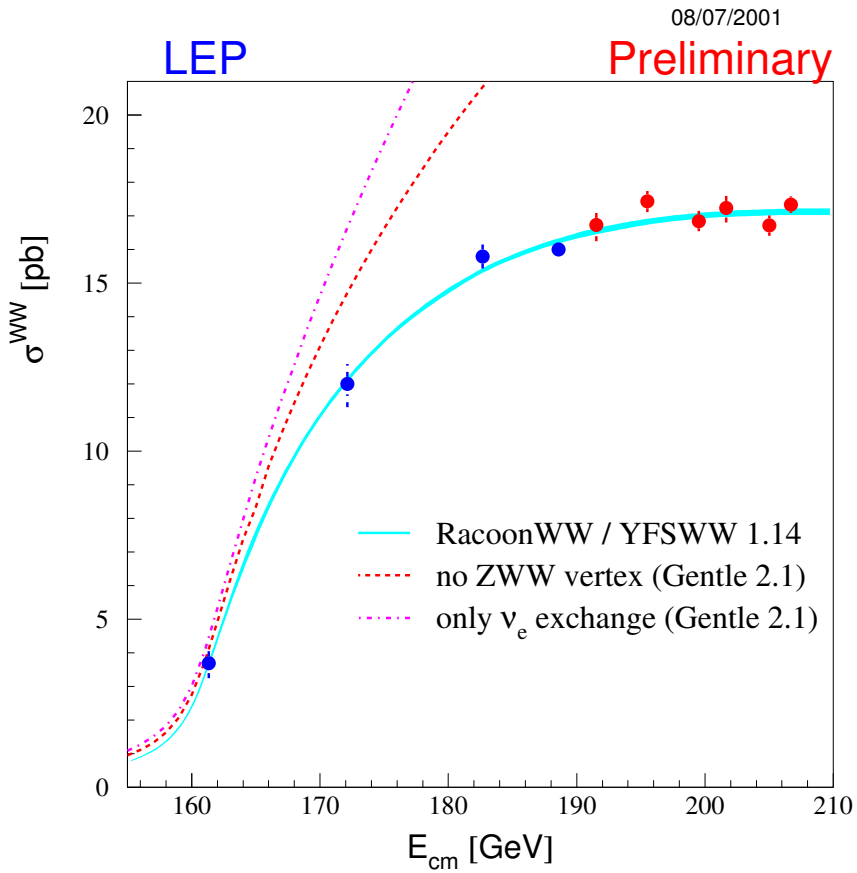
- using the approximate longitudinal polarization vector,

$$\begin{aligned} &V^{\alpha\beta\delta}(q_+ + q_-, -q_-, -q_+)\varepsilon_\beta(q_-)\varepsilon_\delta(q_+) \\ &= -\frac{(q_+ + q_-)^2}{2M_W^2}\left[q_+^\alpha - q_-^\alpha\right] + O(1) \ . \end{aligned}$$

The result for the sum of the Z and γ exchange contributions, after inserting the values for V_e and A_e, is

$$M = i\frac{(-ig_W)^2}{8M_W^2}\bar{v}(-p_+)(\not{q}_+ - \not{q}_-)(1-\gamma_5)u(p_-) \ ,$$

Experimental observation of W-pairs at LEPII

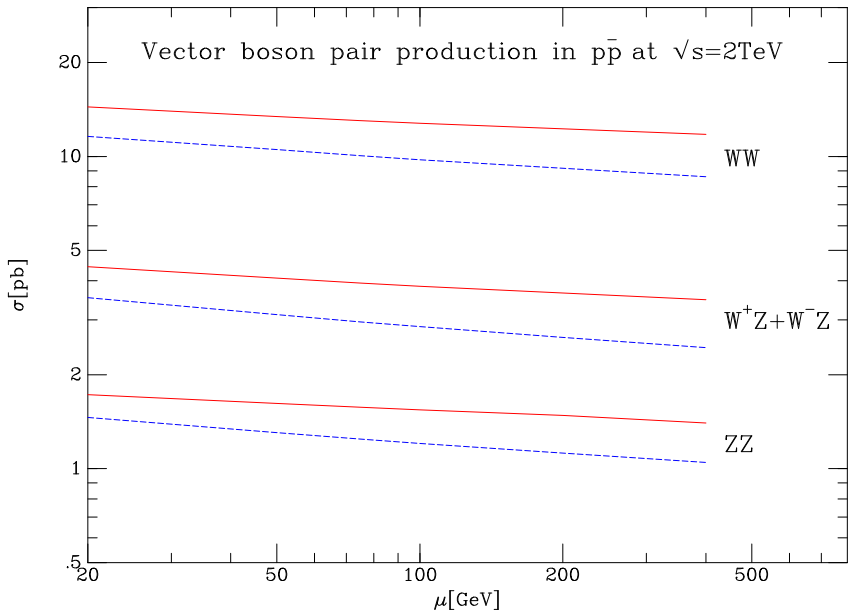
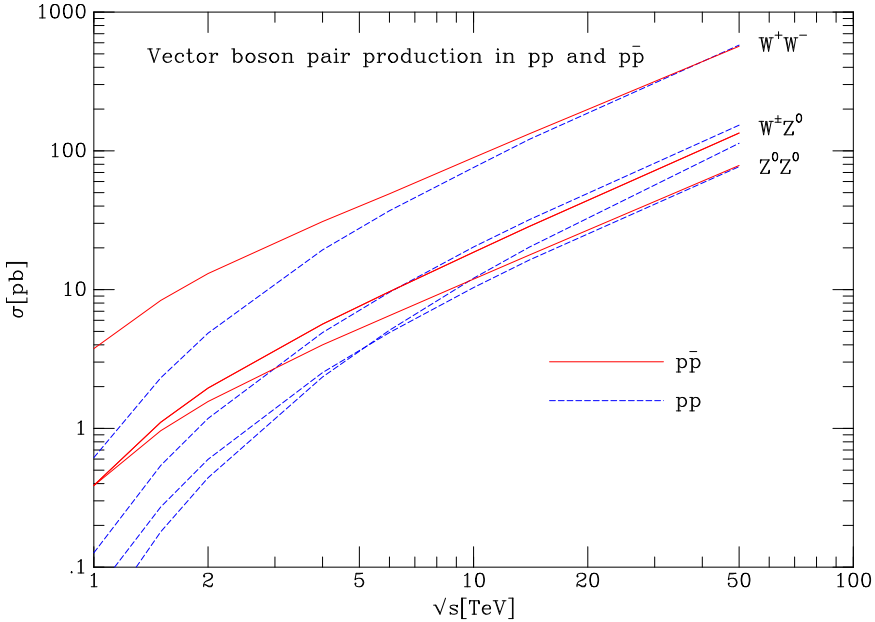


Vector boson production in hadronic collisions

- WW pair production is also observed at the Tevatron

	CDF Run II Summer 2003 Preliminary $\mathcal{L} = 126\ \mathrm{pb}^{-1}$			
Source	ee	$\mu\mu$	$e\mu$	$\ell\ell$
Drell-Yan e^+e^-	0.40 ± 0.19	0	0.051 ± 0.054	0.45 ± 0.1
Drell-Yan $\mu^+\mu^-$	0	0.49 ± 0.22	0.35 ± 0.17	0.84 ± 0.2
Drell-Yan $\tau^+\tau^-$	0.024 ± 0.010	0.022 ± 0.009	0.067 ± 0.025	0.11 ± 0.0
WZ	0.030 ± 0.004	0.057 ± 0.007	0.11 ± 0.01	0.19 ± 0.0
Fake	0.16 ± 0.04	0.16 ± 0.09	0.40 ± 0.11	0.72 ± 0.1
$t\bar{t}$	0.0039 ± 0.0030	0.0054 ± 0.0035	0.020 ± 0.009	0.029 ± 0.0
Total Background	0.61 ± 0.19	0.74 ± 0.24	1.00 ± 0.21	2.34 ± 0.3
$WW \to \mathrm{dileptons}$	1.61 ± 0.37	1.36 ± 0.31	3.92 ± 0.89	6.89 ± 1.5
Run 2 Data	2	1	2	5

$$\sigma_{meas}^{p\bar{p}\to WW} = 5.1^{+5.4}_{-3.6}\ \ (\mathrm{stat})\ \ \pm\ \ 1.3\ \ (\mathrm{syst})\ \ \pm\ \ 0.3\ \ (\mathrm{lumi})\ \ \mathrm{pb}\ \ .$$

$$\sigma_{theo:NLO}^{p\bar{p}\to WW} = 13.25 \pm 0.25\ \ \mathrm{pb}\ \ \ \ \mathrm{MCFM}\ \ \mathrm{hep-ph/9905386}$$

Theoretical expectations