QCD and Collider Physics
Lecture IV:Electroweak theory and the Higgs boson

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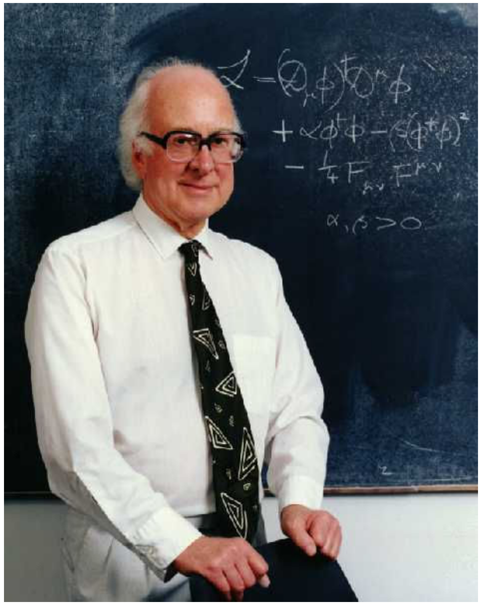
What is the Higgs boson?

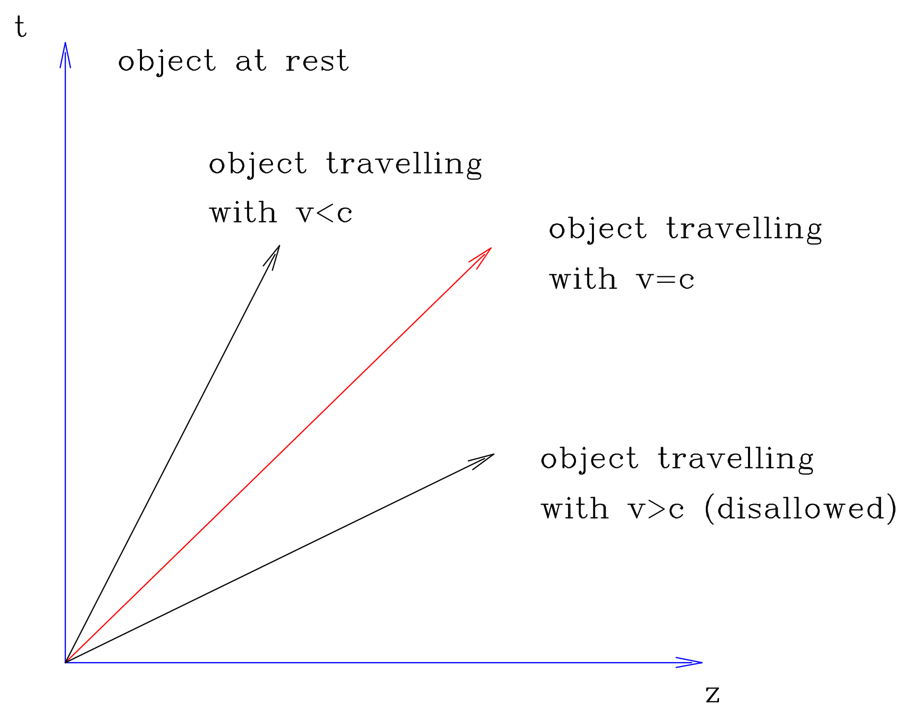
- In the standard model of elementary particles, the Higgs boson is an as yet undiscovered scalar particle.
- By spontaneous symmetry breakdown the quarks and weak vector bosons acquire mass.
- the Higgs boson is the remaining physical manifestation of the Higgs field.



‘It is worth noting that an essential feature of the type of theory which has been described in this note is the prediction of incomplete multiplets of scalar bosons. It is to be expected that this feature will also appear in theories in which the symmetry-breaking scalar fields are not elementary dynamic variables but bilinear combinations of Fermi fields’, Peter Higgs, PRL 13 508-509, (1964)

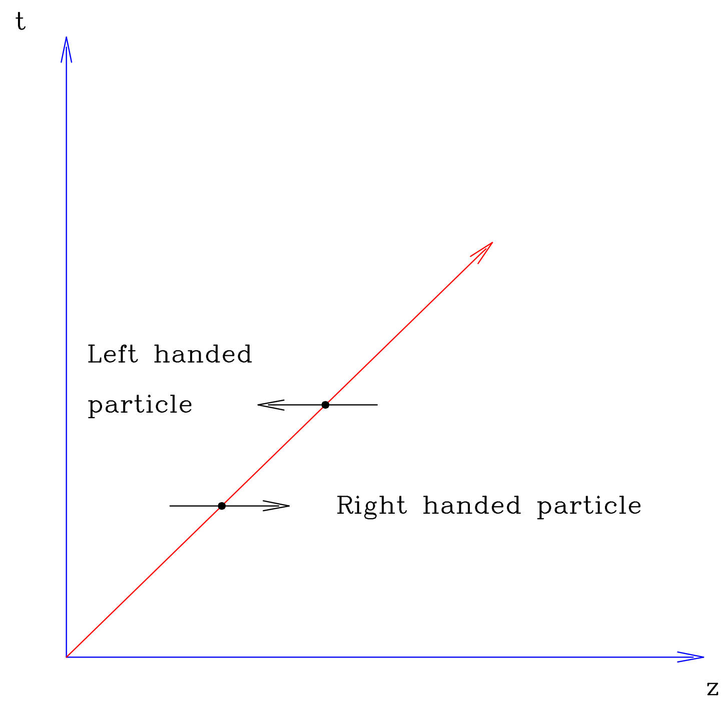
Massive particles and the light cone

- Work in units in which $v = c = 1$
- A massless particle remains on the light cone.
- A massive particle travels with a $v < c$

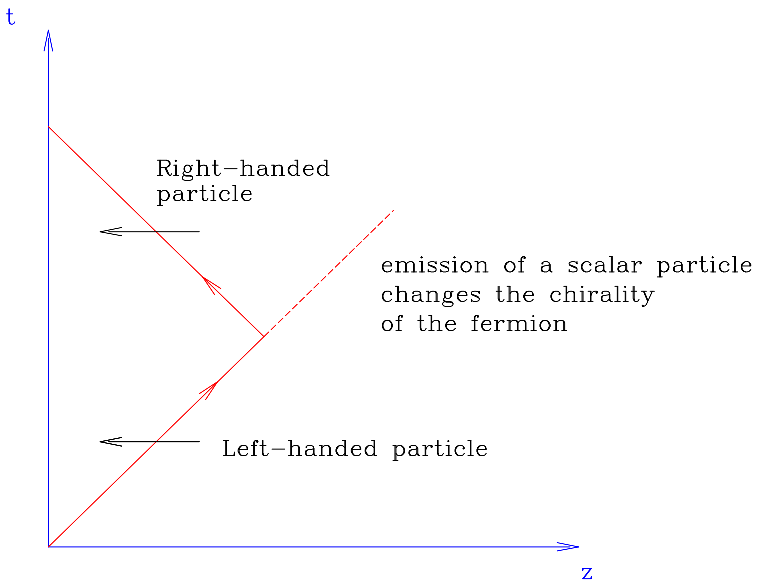


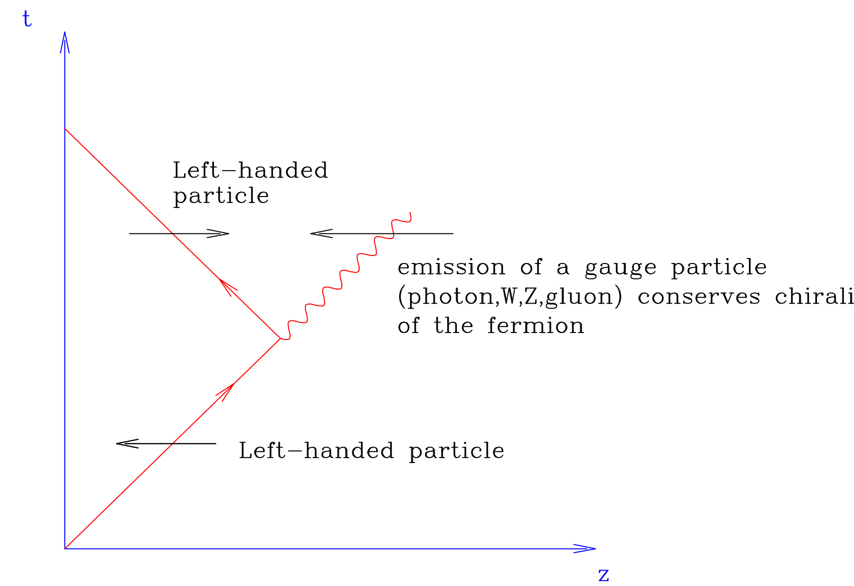
Massless fermion

- Massless fermions have definite chirality either left-handed or right-handed.
- Chirality is conserved under Lorentz transformations



Scalars, vectors and chirality

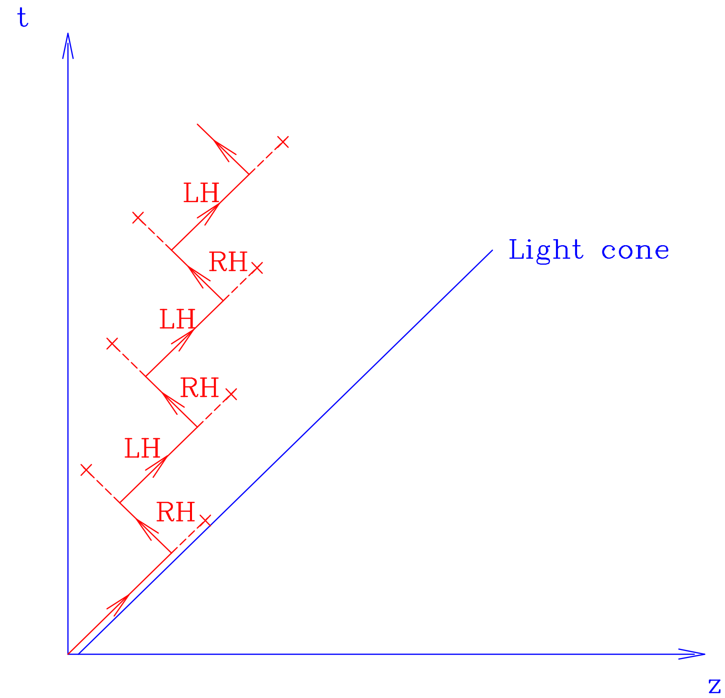




- Scalar interactions flip chirality
- Vector interactions conserve chirality

Generation of Fermion mass

- Propagation of a fermion through a Bose-Einstein condensate of scalars generates a Dirac mass.
- Dirac mass $-m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$.



- A freely propagating massive particle can always be written as a linear combination of massless left- and right-handed spinors.
- In the standard model, left-handed fields are in SU(2) doublets, right-handed field are in singlets.
- Condensate must carry $SU(2)_L$ charge.

Electroweak Lagrangian

In QED the Lagrangian for the photon field A^μ coupled to an electron field ψ is fixed by local gauge invariance.

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}(i\not{D} - m_e)\psi, \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \; .$$

D is the covariant derivative

$$D_\alpha = \partial_\alpha + ieA^\alpha \; .$$

The Lagrangian is invariant under independent redefinitions of the phase of the field ψ at each space-time point,

$$\psi'(x) = \exp i\theta(x) \; \psi(x),$$

provided that the gauge field A^μ transforms as

$$A'_\mu(x) = A_\mu(x) - \frac{1}{e} \partial_\mu \theta(x) \; .$$

There is a single gauge coupling e corresponding to this gauge group.

The Standard Electroweak Model

The Standard Electroweak Model is based on the more complicated group SU(2)⊗U(1). Initially the Lagrangian of this model contains three massless bosons, $W^i (i = 1, 2, 3)$, associated with the gauge group SU(2) and one massless boson, B, associated with the U(1) gauge group. The Lagrangian of the gauge bosons is written as

$$\mathcal{L} = -\frac{1}{4} W^{i\ \mu\nu} W^i_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \ ,$$

where the field strength tensors of the U(1) gauge field B and the SU(2) gauge fields W^i are

$$\begin{aligned} W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_W \epsilon^{ijk} W^j_\mu W^k_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \ , \end{aligned}$$

g_W being the SU(2) gauge coupling. Note that the vector bosons W^i already have interactions because of the non-Abelian nature of their symmetry group, SU(2). This is entirely analogous to the fact that gluons carry colour charge in QCD.
The coupling of the gauge fields to fermionic matter fields uses the covariant derivative,

$$D^\mu = \delta_{ij} \partial^\mu + i g_W (T \cdot W^\mu)_{ij} + i Y \delta_{ij} g'_W B^\mu$$

where g'_W is the U(1) gauge coupling.

$SU(2) \otimes U(1)$

The matrices T are a representation of the SU(2) *weak isospin* algebra and the U(1) charge Y is called the *weak hypercharge*. In order to specify the coupling to matter we therefore have to choose the SU(2) representation, T, and the U(1) gauge charge, Y, for the matter fields.
The tensor ϵ^{ijk} appears because its components are the structure constants of SU(2):

$$[T^i, T^j] = i\epsilon^{ijk}T^k, \ \epsilon^{123} = 1 \ .$$

Defining $W_\mu^\pm = (W_\mu^1 \mp iW_\mu^2)/\sqrt{2}$ and $T^\pm = T^1 \pm iT^2$ we have

$$W_\mu \cdot T = W_\mu^3 T_3 + \frac{1}{\sqrt{2}} W_\mu^+ T^+ + \frac{1}{\sqrt{2}} W_\mu^- T^-$$

where the matrices $T^\pm$and T^3 satisfy the relations

$$\left[T^+, T^-\right] = 2T^3, \quad \left[T^3, T^\pm\right] = \pm T^\pm \ .$$

T^+ and T^- are the weak isospin raising and lowering operators. For example, in the doublet representation of SU(2) we have

$$T^3 = \left(\begin{array}{cc} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{array} \right), \ T^+ = \left(\begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array} \right), \ T^- = \left(\begin{array}{cc} 0 & 0 \\ 1 & 0 \end{array} \right) .$$

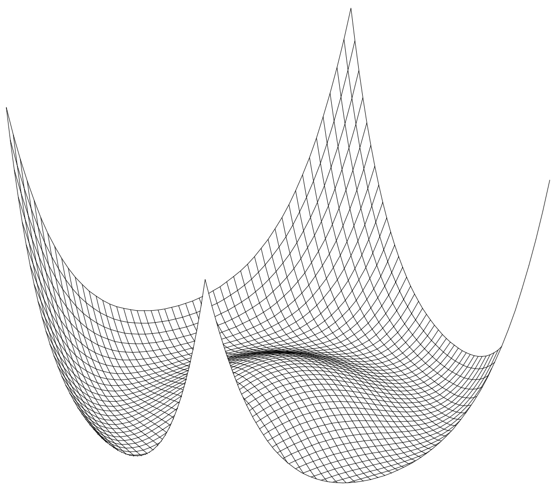
Standard Model Higgs boson

- Higgs field is a single SU(2) doublet of complex fields, with weak hypercharge $Y = \frac{1}{2}$.

$$\mathcal{L} = (D_\mu \phi)(D^\mu \phi^\dagger) - \mathcal{V}(\phi^\dagger \phi) \quad \text{where} \quad \phi = \left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right) .$$

The Higgs potential $\mathcal{V}$ is chosen to be of the form

$$\mathcal{V}(\phi^\dagger \phi) = \lambda \, (\phi^\dagger \phi)^2 - \mu^2 \phi^\dagger \phi \, .$$



Two dimensional representation of a function with 4 degrees of freedom

- The Higgs field ϕ develops a vacuum expectation value,

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v \end{array} \right) .$$

Mass for W and Z

- We may rewrite the Higgs field as

$$\phi = U^{-1}(\xi)\begin{pmatrix} 0 \\ (H+v)/\sqrt{2} \end{pmatrix}, \quad U(\xi) = \exp(-iT\cdot\xi/v)\ .$$

- Note that we still have four real degrees of freedom, (three ξs and one H), equivalent to the two complex fields.
- We now make a gauge transformation of the form

$$\begin{aligned} \phi &\quad\to\quad U(\xi)\phi \\ T\cdot W^\mu &\quad\to\quad UT\cdot W^\mu U^{-1} + \frac{i}{g_W}(\partial^\mu U)U^{-1}\ . \end{aligned}$$

- The ξ degrees of freedom no longer appear in the Higgs Lagrangian. They will reappear as the longitudinal modes of the massive gauge bosons.
- This gauge is called the *unitary gauge*.
- The Higgs boson H is the only remaining dynamical field.

- The Higgs Lagrangian now becomes

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu H \partial^\mu H - \mathcal{V}\left(\frac{(v+H)^2}{2}\right) \\ &+ \frac{(v+H)^2}{8}\chi^\dagger(2g_W T\cdot W_\mu + g'_W B_\mu)(2g_W T\cdot W^\mu + g'_W B^\mu)\chi \ ,\end{aligned}$$

where χ is a unit vector along the direction of the vacuum expectation value,

$$\chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} .$$

- We therefore have three generators of the SU(2)$\otimes$U(1) symmetry which are spontaneously broken.
- Goldstone's theorem would lead us to expect three massless bosons.
- The massless Goldstone modes provide the extra longitudinal degrees of freedom necessary to change the quanta of the vector fields from massless to massive bosons.

- examine the terms quadratic in the vector boson fields:

$$\mathcal{L}_{\mathsf{M}} = \frac{v^2}{8}\Big[(g_W W^3_\mu - g'_W B_\mu)(g_W W^{3\,\mu} - g'_W B^\mu) + 2g_W^2 W^-_\mu W^{+\,\mu}\Big]\,.$$

- The propagator for the B and W^3 fields is not diagonal.
- redefine the electrically neutral fields by introducing new fields A_μ and Z_μ which propagate independently,

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}\,,$$

- the angle θ_W (the Weinberg or electroweak mixing angle) is fixed by the relative strengths of the coupling constants:

$$\sin^2\theta_W = \frac{g'^{\,2}_W}{g_W^2 + g'^{\,2}_W} \simeq 0.23\,.$$

- the quadratic terms in the vector boson fields become

$$\mathcal{L}_{\mathrm{M}} = \frac{g_W^2 v^2}{4} W_\mu^+ W^{-\,\mu} + \frac{(g_W^2 + g_W'^{\,2}) v^2}{8} Z_\mu Z^\mu \; .$$

  We therefore find that the W and Z bosons have acquired masses, given by

$$M_W = \frac{1}{2} v g_W, \quad M_Z = \frac{1}{2} v \sqrt{g_W^2 + g_W'^{\,2}} \equiv \frac{M_W}{\cos\theta_W} \; .$$

- The Higgs mechanism is responsible for the breaking of SU(2)$_L\times$U(1) gauge symmetry down to QED and giving mass to the W and Z.
- The photon remains massless.
- Three of the four degrees of freedom of the Higgs field give rise to longitudinal modes of the W^+, W^- and Z^0. The remaining degree of freedom is the Higgs boson.
- The mass of the Higgs boson, $M_h = \sqrt{2\lambda} v$, is undetermined.

Gauge boson coupling to fermions

The coupling of the gauge bosons to fermions which preserves gauge symmetry is specified by the covariant derivative. Before symmetry breaking, the coupling of the fermions to the vector bosons is given by

$$\mathcal{L} = \bar{\psi}_R i(\partial\!\!\!/ + ig'_W Y_R B\!\!\!/)\psi_R + \bar{\psi}_L i(\partial\!\!\!/ + ig_W T \cdot W\!\!\!\!/ + ig'_W Y_L B\!\!\!/)\psi_L \ .$$

The U(1) charges of the left- and right-handed fermions, Y_L and Y_R, are chosen to satisfy the relation $Q = T^3 + Y$,

Fermion			T^3_L	Y_L	T^3_R	Y_R	Q_f
u	c	t	$+\frac{1}{2}$	$+\frac{1}{6}$	0	$+\frac{2}{3}$	$+\frac{2}{3}$
d	s	b	$-\frac{1}{2}$	$+\frac{1}{6}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
ν_e	ν_μ	ν_τ	$+\frac{1}{2}$	$-\frac{1}{2}$	-	-	0
e^-	μ^-	τ^-	$-\frac{1}{2}$	$-\frac{1}{2}$	0	-1	-1

For the case of leptons the left-handed fields are put into doublets

$$\psi_L = \gamma_L \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \gamma_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \gamma_L \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

where as before

$$\gamma_R = \frac{1}{2}(1+\gamma_5), \ \ \gamma_L = \frac{1}{2}(1-\gamma_5).$$

The right-handed fields are all SU(2) singlets:

$$\psi_R = \gamma_R e^-, \ \gamma_R \mu^-, \ \gamma_R \tau^- \ .$$

Similarly the quarks form three left-handed doublets

$$\psi_L = \gamma_L \begin{pmatrix} u \\ d' \end{pmatrix}, \gamma_L \begin{pmatrix} c \\ s' \end{pmatrix}, \gamma_L \begin{pmatrix} t \\ b' \end{pmatrix},$$

and the right-handed quarks are again singlets.

The interaction Lagrangian can be expressed in terms of physical fields by substituting for B and W^3

$$\begin{aligned}
\mathcal{L} &= \sum_f \bar{\psi}_f \left(i\partial\!\!\!/ - m_f - g_W \frac{m_f H}{2M_W} \right) \psi_f \\
&- \frac{g_W}{2\sqrt{2}} \sum_f \bar{\psi}_f (\gamma^\mu (1-\gamma_5) T^+ W_\mu^+ + \gamma^\mu (1-\gamma_5) T^- W_\mu^-) \psi_f \\
&- e \sum_f Q_f \bar{\psi}_f A\!\!\!/ \psi_f - \frac{g_W}{2\cos\theta_W} \sum_f \bar{\psi}_f \gamma^\mu (V_f - A_f \gamma_5) \psi_f Z_\mu \ .
\end{aligned}$$

The couplings of the fermions to the Z boson are

$$\begin{aligned}
V_f &= T_f^3 - 2Q_f \sin^2\theta_W \ , \\
A_f &= T_f^3 \ ,
\end{aligned}$$

where Q_f is the charge of the fermion in units of the positron electric charge e. The values of e and the weak SU(2) charge g_W are related by

$$e = g_W \sin\theta_W \ .$$

Determination of g_W

- To relate the electroweak parameters to those of low-energy weak interactions, consider the amplitude for the decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu$,

$$\frac{-ig_W^2}{2}\bar{u}(\nu_\mu)\gamma^\alpha\gamma_L u(\mu)\;\bar{u}(e)\gamma^\beta\gamma_L v(\overline{\nu_e})\;\left(-g_{\alpha\beta}+\frac{k_\alpha k_\beta}{M_W^2}\right)\frac{1}{k^2-M_W^2}\,.$$

- At low energies this reproduces the Fermi model,

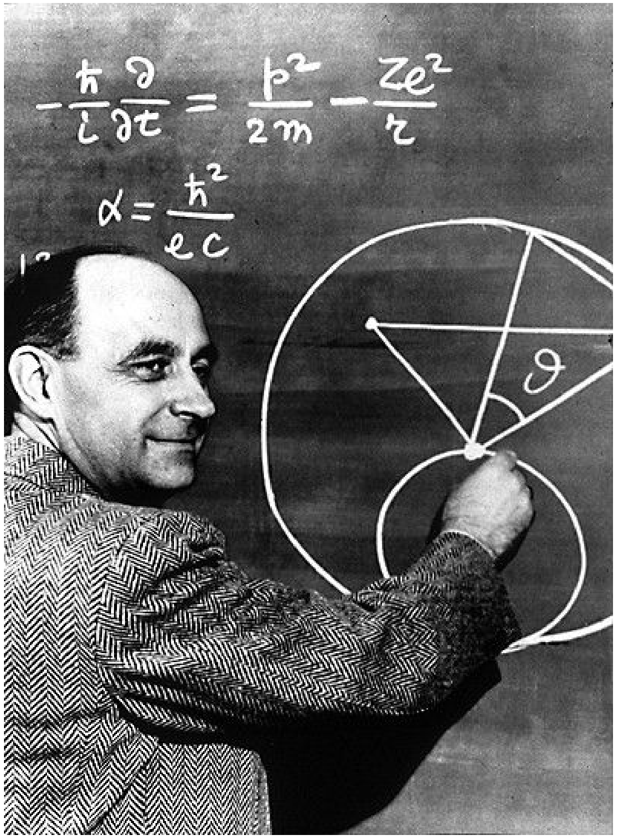
$$\frac{iG_F}{\sqrt{2}}\bar{u}(\nu_\mu)\gamma^\alpha(1-\gamma_5)u(\mu)\;\bar{u}(e)\gamma_\alpha(1-\gamma_5)v(\overline{\nu_e})\;,$$

provided that the weak charge g_W is related to the Fermi constant G_F as follows:

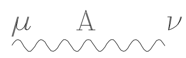
$$\frac{G_F}{\sqrt{2}}=\frac{g_W^2}{8M_W^2}\,.$$

The Fermi scale

- The Fermi constant introduces a new scale of physics.
- $v = 1/\sqrt{(G_F\sqrt{2})} = 246$ GeV.



Feynman rules unitary gauge

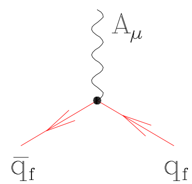


$$[-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}]\,\frac{i}{q^2}$$

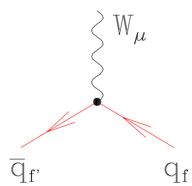
$$[-g^{\mu\nu}+\frac{q^\mu q^\nu}{M^2}]\,\frac{i}{(q^2-M^2)}$$

$$\frac{i}{\not q-m}$$

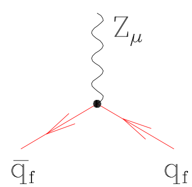
$$\frac{i}{(q^2-M_H^2)}$$



$$-ieQ_f\gamma_\mu$$



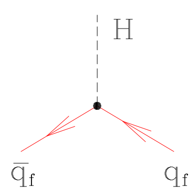
$$\frac{-ig_w}{2\sqrt{2}}\,\gamma^\mu(1-\gamma_5)(T^+)_{f'f} \qquad g_w=\frac{e}{\sin\theta_w}$$



$$\frac{-ig_w}{2\cos\theta_W}\,\gamma^\mu(V_f-A_f\gamma_5) = \frac{-ig_w}{\cos\theta_W}\,\gamma^\mu(r_f\gamma_R+l_f\gamma_L)$$

$$(V_f=T_f^3-2Q_f\sin^2\theta_W,\ A_f=T_f^3)$$

$$(r_f=-Q_f\sin^2\theta_W,\ l_f=T_f^3-Q_f\sin^2\theta_W)$$

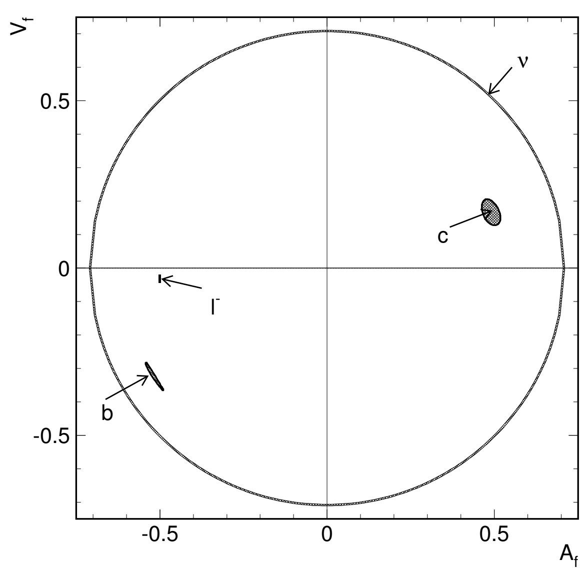


$$\frac{-ig_w m_f}{2M_W}$$

Couplings of the Z's

Fermions			Q_f	V_f	A_f
u	c	t	$+\frac{2}{3}$	$(+\frac{1}{2} - \frac{4}{3}\sin^2\theta_W) \sim +0.191$	$+\frac{1}{2}$
d	s	b	$-\frac{1}{3}$	$(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W) \sim -0.345$	$-\frac{1}{2}$
ν_e	ν_μ	ν_τ	0	$\frac{1}{2}$	$+\frac{1}{2}$
e	μ	τ	-1	$(-\frac{1}{2} + 2\sin^2\theta_W) \sim -0.036$	$-\frac{1}{2}$

- The numerical values of V_f are for $\sin^2\theta_W = 0.232$.
- Dramatic confirmation of the standard model. Vector couplings small for charged leptons.



The masses of the fermions

Fermions also acquire mass through their interactions with the condensate.

$$L_Y = h_U \overline{q_L} u_R (\tilde{\phi}) + h_D \overline{q_L} d_R \phi + h_E \overline{l_L} e_R \phi + h.c.$$

- The Yukawa interactions break the global flavour symmetry and give masses to the fermions.
- The Higgs boson coupling to fermions is proportional to the mass $g_w m_f / (2 M_w)$.
- The top mass is exactly at the Fermi scale, (significance?)

h_u	2×10^{-5}	h_c	9×10^{-3}	h_t	1
h_d	4×10^{-5}	h_s	8×10^{-4}	h_b	3×10^{-2}
h_e	3×10^{-6}	h_μ	6×10^{-4}	h_τ	1×10^{-2}

Tree graphs

- Tree level prediction

$$
\begin{aligned}
M_W^2 &= \frac{\pi\alpha}{\sqrt{2}G_\mu}\frac{1}{s_W^2} \\
M_Z^2 &= \frac{\pi\alpha}{\sqrt{2}G_\mu}\frac{1}{s_W^2 c_W^2}\,.
\end{aligned}
$$

- Using $G_\mu = 1.16637(1)10^{-5}$GeV^2, $\alpha^{-1} = 137.035\,999\,6(50)$ and $s_W^2 = 0.2253 \pm 0.0021$ from neutrino DIS

$$
\begin{aligned}
M_W &= 78.54 \pm 0.4 \text{ GeV} \\
M_Z &= 89.23 \pm 0.3 \text{ GeV}
\end{aligned}
$$

The experimentally measured values are

$$
\begin{aligned}
M_W &= 80.451 \pm 0.061\text{GeV} \\
M_Z &= 91.1876 \pm 0.0021\text{GeV}
\end{aligned}
$$

Precision of data requires inclusion of loop effects.

Input parameters

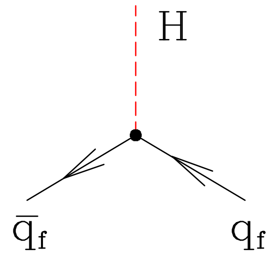
- Yukawa couplings fix quark and charged lepton masses.
- Primary parameters are the two couplings, g_W and g'_W and v, the vacuum expectation value of the Higgs field and M_H the mass of the Higgs boson.
- Mass of the Higgs enters only peripherally into low energy phenomenology, so three main inputs g_W, g'_W and v.
- trade these three inputs for the three most precisely measured parameters.

$$\begin{aligned} \alpha^{-1} &= 137.035\,989\,6(50) \\ G_F &= 1.166\,39(1) \times 10^{-5}\ \text{GeV}^{-2} \\ M_Z &= 91.188\,5(22)\ \text{GeV} \end{aligned}$$

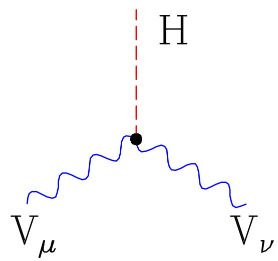
- tree level relationships to the more fundamental parameters of the SM are $(\sin^2\theta_W = g'^{\,2}_W/(g'^{\,2}_W + g^2_W)$

$$\alpha = \frac{g_W^2 \sin^2\theta_W}{4\pi}\,, G_F = \frac{1}{\sqrt{2}v^2}\,, M_Z = \frac{\frac{1}{2} g_W v}{\cos\theta_W}$$

Couplings of the SM Higgs boson



$$\frac{-ig_w m_f}{2M_W} \qquad = \quad -i\ \ m_f/v$$



$$+ig_{VH}M_W g_{\mu\nu}$$

$$(g_{WH}=g_W,\ \ g_{ZH}=g_W/\cos^2\theta_W)$$

- Couplings to Fermions are proportional to the Fermion mass.
- SM Higgs will prefer to decay into $b\bar{b}$ or $\tau^-\tau^+$ rather than light quarks or leptons.
- Couplings to W and Z have a normal weak strength

$H \to W^+W^-$

We find that the tree-level invariant matrix element squared for the process

$$H \to W^+W^- \ ,$$

summed over final-state polarizations, is given by

$$\sum |\mathcal{M}|^2 = \frac{g_W^2 M_H^4}{4M_W^2}\left(1 - 4\frac{M_W^2}{M_H^2} + 12\frac{M_W^4}{M_H^4}\right) .$$

The Lorentz-invariant phase space for decay into two particles is

$$d\Phi_2(P, p_1, p_2) = \frac{1}{8\pi}\frac{2|\mathbf{p_1}|}{M_P}\frac{d\Omega}{4\pi} \ ,$$

where $\mathbf{p_1}$ is the three-momentum of the decay products in the rest frame of the decaying particle, which has mass M_P, and $d\Omega$ is the element of solid angle. Including the initial-state normalization factor of $1/(2M_H)$, we obtain the final result for the Higgs partial width into W bosons:

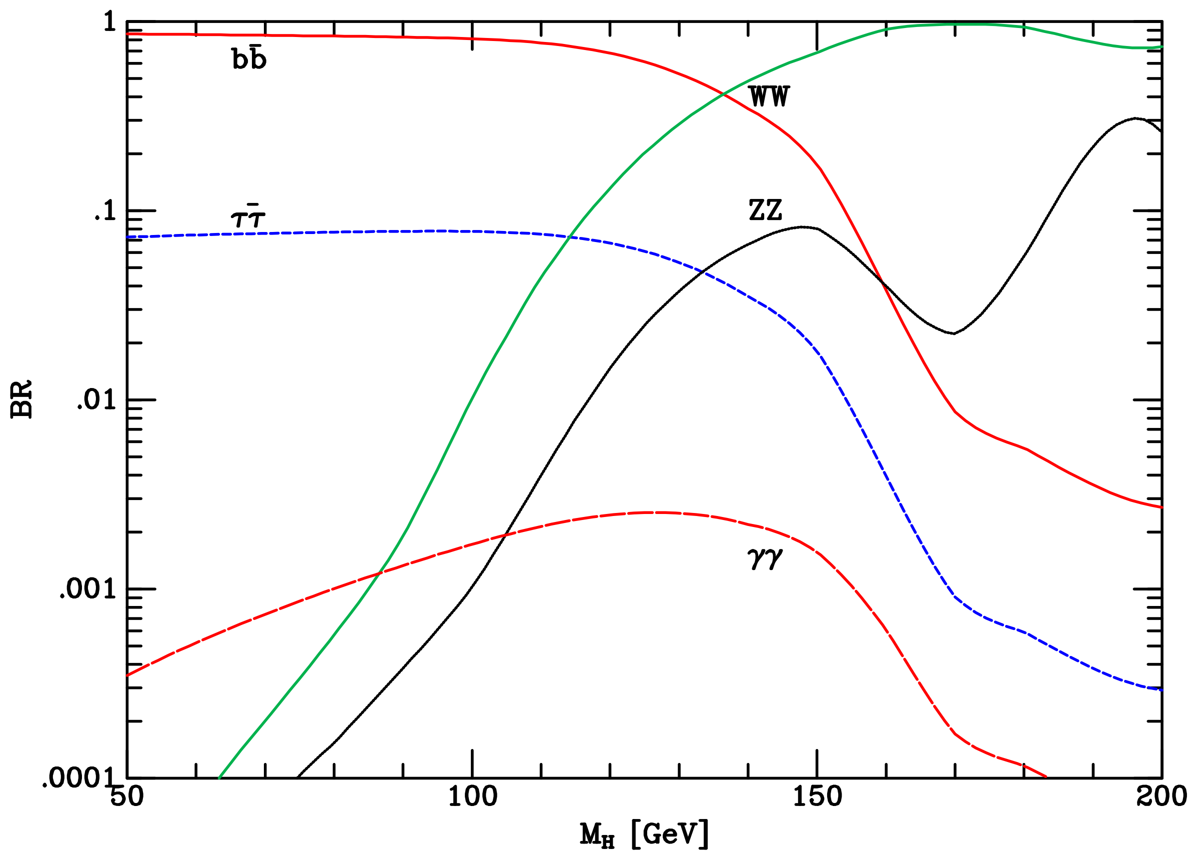
$$\Gamma(H \to W^+W^-) \ = \ \frac{g_W^2 M_H^3}{64\pi M_W^2}\left(1 - \frac{4M_W^2}{M_H^2}\right)^{\frac{1}{2}}\left(1 - \frac{4M_W^2}{M_H^2} + \frac{12M_W^4}{M_H^4}\right).$$

Note that for $M_H \gg M_W$ the tree-level Higgs width grows and eventually becomes larger than its mass,

$$\Gamma(H \to W^+W^-) = \left(\frac{M_H^3}{M_U^2}\right), \quad M_U = 1.74 \text{ TeV} .$$

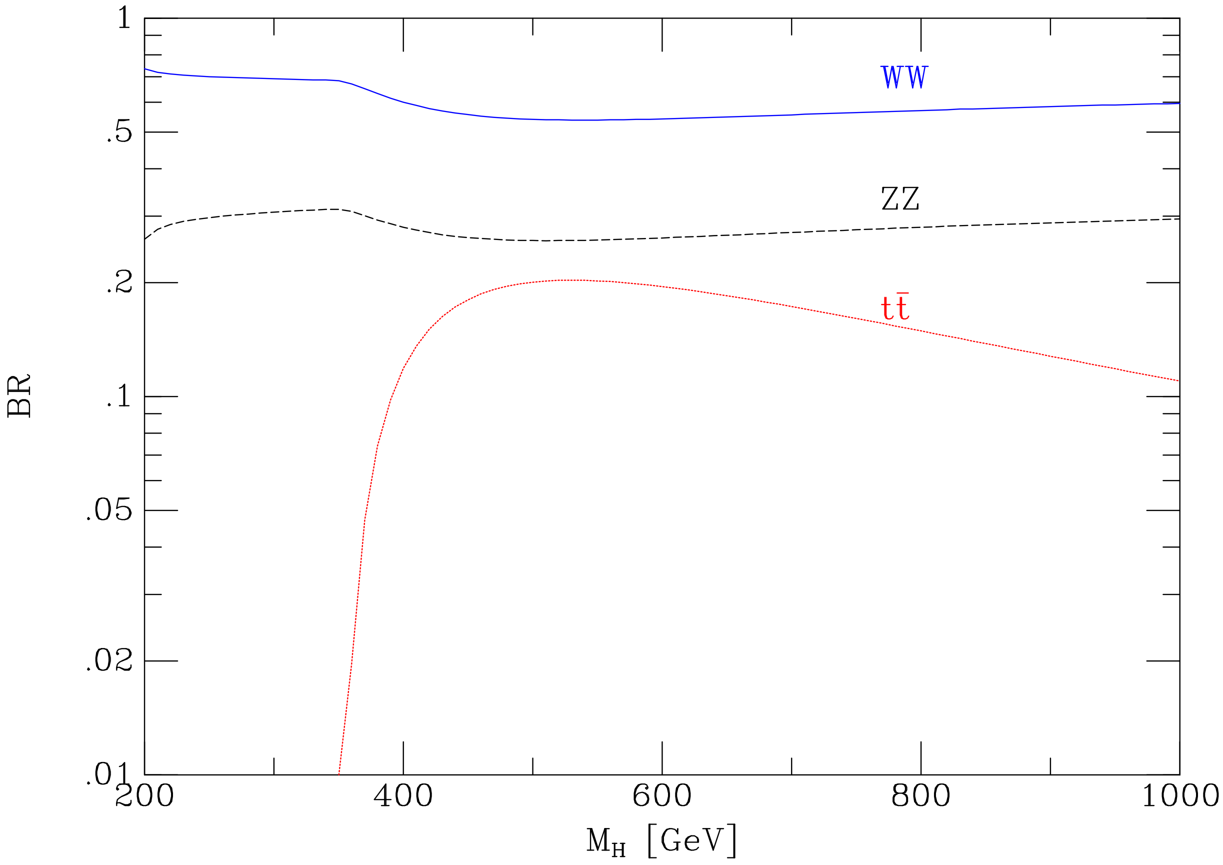
The scale at which $\Gamma_H > M_H$ is set by M_U.

Higgs decay Branching ratios



- for $m_h \leq 135$ the dominant decay is $H \to b\bar{b}$
- for $m_h \geq 135$ the dominant decay is $H \to WW^*$

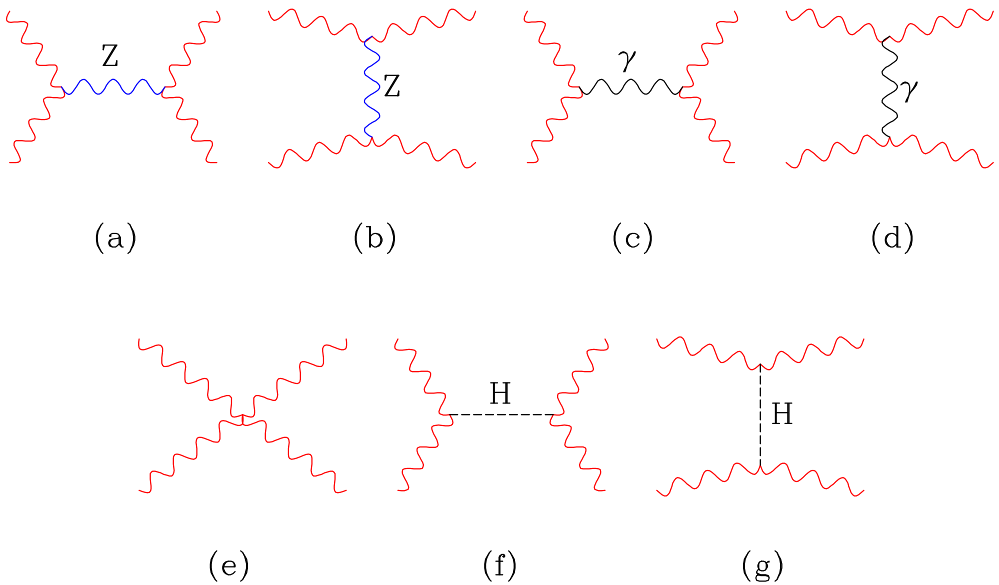
High mass Higgs branching ratios



- High mass Higgs predominantly decays to vector boson pairs

The Higgs boson and vector boson scattering

- Strong cancellations in the high-energy behaviour of amplitudes in the SM.
- $W^+(p_+) + W^-(p_-) \to W^+(q_+) + W^-(q_-)$



- the scattering of longitudinal bosons, is responsible for the leading behaviour at high energy.
- In the centre-of-mass frame the longitudinal polarization vectors for the W bosons are

$$
\begin{aligned}
\varepsilon_L(p_\pm) &= \left(\frac{p}{M_W}, 0, 0, \pm\frac{E}{M_W}\right) , \\
\varepsilon_L(q_\pm) &= \left(\frac{p}{M_W}, 0, \pm\frac{E}{M_W}\sin\theta, \pm\frac{E}{M_W}\cos\theta\right) .
\end{aligned}
$$

- Because of the longitudinal polarizations, the high-energy behaviour of individual graphs in grows with the centre-of-mass momentum like

$$T(s,t) = A\left(\frac{p}{M_W}\right)^4 + B\left(\frac{p}{M_W}\right)^2 + C\ . \tag{1}$$

- On summing over graphs, the term A cancels without including the Higgs graphs, (f) and (g).
- The full cancellation of the B term involves the Higgs boson in an essential way.
- The result is

$$T(s,t) \quad = \quad -g_W^2 \frac{M_H^2}{4M_W^2}\left[\frac{s}{s-M_H^2} + \frac{t}{t-M_H^2}\right]$$

- The scattering amplitude can still be too large to be consistent limitation due to unitarity.
- It is convenient to perform a partial wave expansion,

$$T(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta)\ ,$$

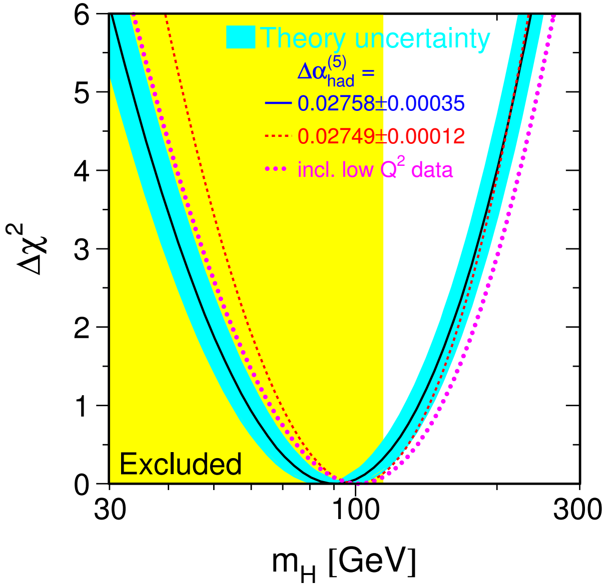
The Higgs mass and perturbative unitarity

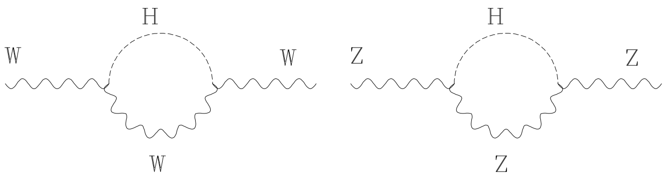
- The unitarity constraint (conservation of probability) is $|a_J| < 1$. The result for the a_0 partial wave amplitude is

$$a_0(s) \quad \rightarrow \quad -\frac{G_F M_H^2}{4\pi\sqrt{2}} \; .$$

- This implies that $M_H^2 < 4\sqrt{2}\pi/G_F \approx 1$ TeV.
- The Higgs mass can exceed the value given by the bound, but in that case its properties cannot be analysed using perturbation theory.
- Measurements of the scattering of longitudinal vector bosons at a high-energy hadron collider, will either discover the Higgs boson or see strong interactions of the electroweak bosons, or both.

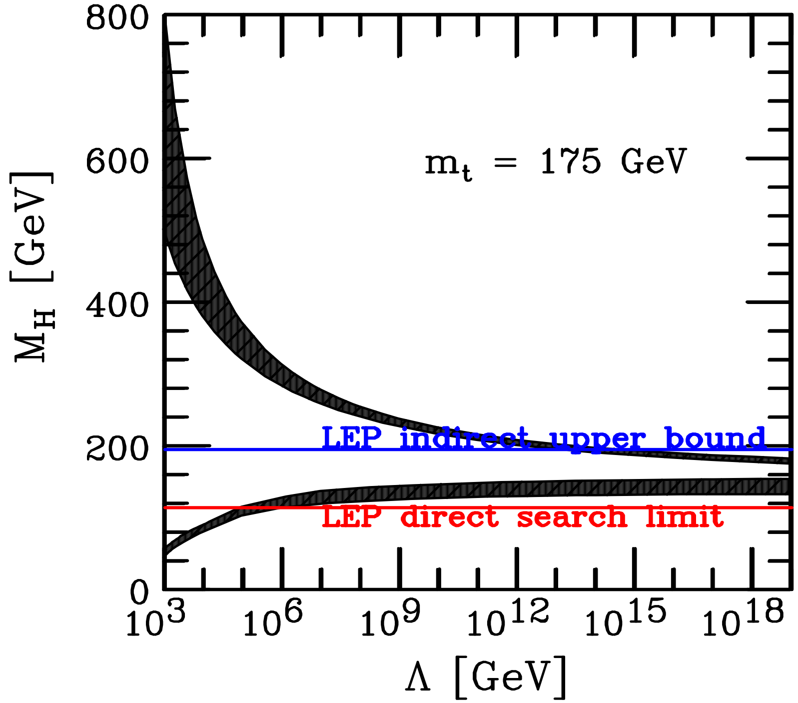
Indirect bounds from precision electroweak





- Sensitivity to the Higgs mass enters through loop corrections.
- The dependence on the Higgs mass is logarithmic.
- $m_h < 211$ GeV at 95% C.L (Winter 2003)
- Strong preference for a low mass Higgs.
- Note that more than half the χ^2-region is excluded by the direct search limit.

Theoretical Limits on the m_h

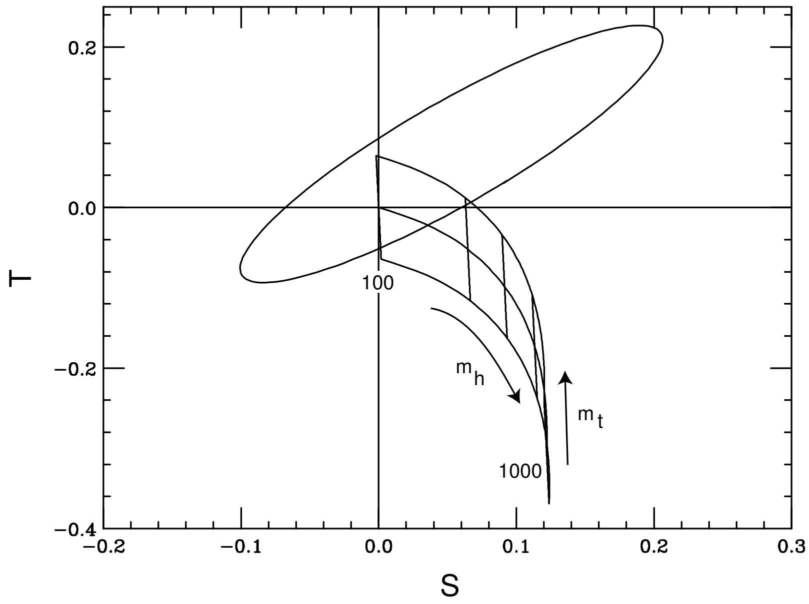


- Upper limit on M_h from Landau Pole.
- Lower limit on M_h from vacuum stability, ie when $\mathcal{V}(v) > \mathcal{V}(0)$.
- Lower limit is weakened for lower m_t and/or only requiring only vacuum metastability.

$$\frac{d\lambda}{d\ln\mu} \sim \frac{3}{2\pi^2}(\lambda^2 + \frac{1}{2}\lambda h_t^2 - \frac{1}{4}h_t^4)$$

Evading light Higgs boson bounds

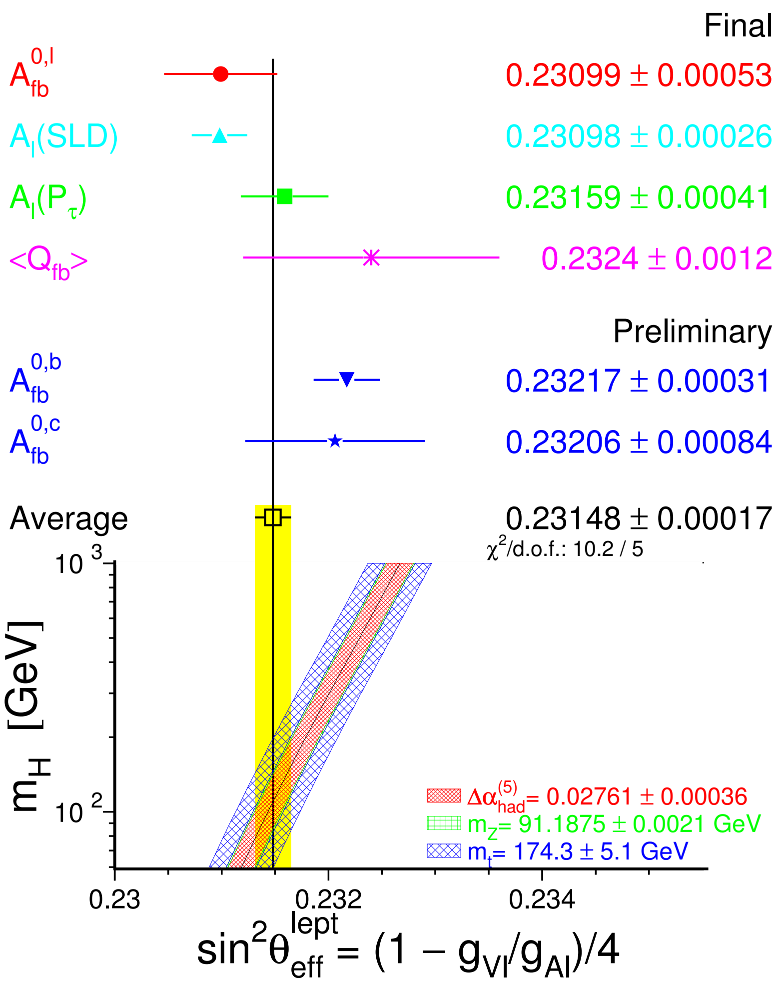
Peskin and Wells



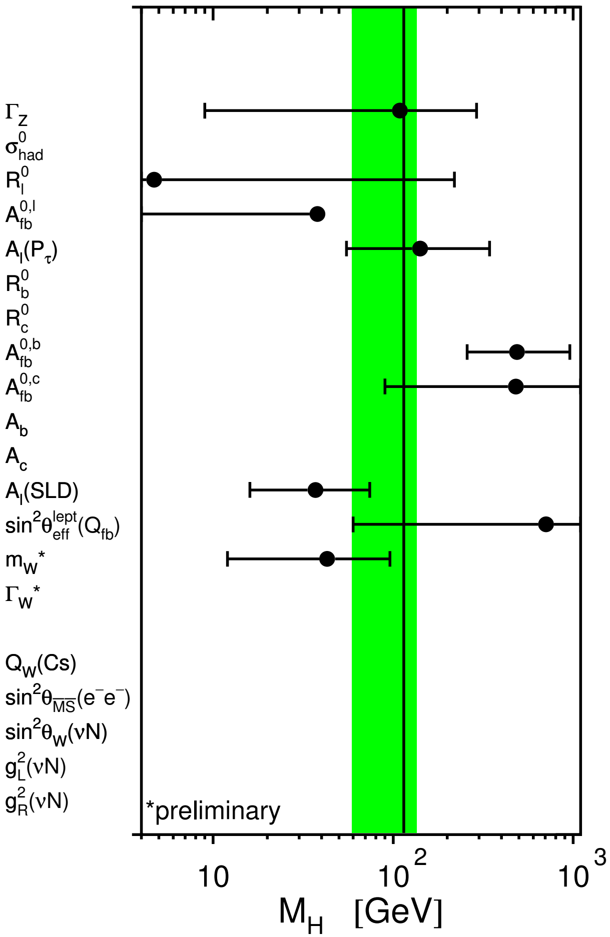
- Models evading light Higgs bounds are constrained in terms of ΔS, (ΔT), the weak isospin conserving (violating) contributions to W and Z loop diagrams.
- Although one cannot foreclose on the Heavy Higgs option, consistency with the low energy constraints frequently requires additional low energy signatures.

Are the precision electroweak bounds credible?

- Effective weak mixing angles determined from lepton data alone, and from lepton and quark data differ by 3.3 standard deviations, (Drees, Lepton-Photon 2001)



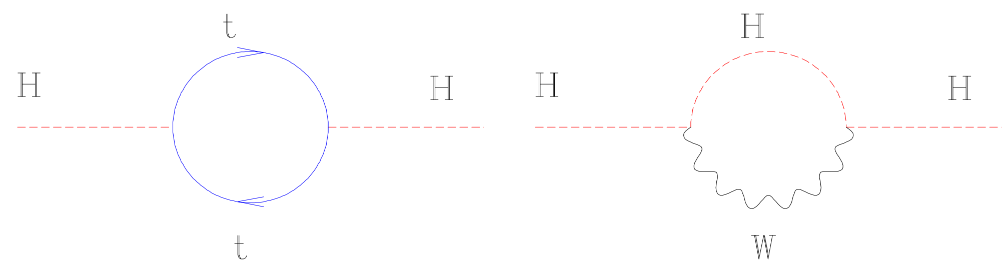
Higgs mass from separate measurements



- Considerable scatter in the values predicted for the Higgs mass. Support for $m_H > 114$ GeV only for A^b_{FB}.
- Recent decrease in the W mass, $m_W = 80.426(34)$ has improved the consistency of the standard model
- Take the precision bounds for the Higgs mass with a grain of salt?

Shortcomings of the Standard Higgs model

- When we attempt to interpret the Higgs model as more than a low energy effective theory we run into problems



- Because of quadratically divergent contributions to the Higgs self mass, we need to introduce a cutoff. In general there is no reason for the Higgs mass to have the value it does; it is renormalized to the value of the cutoff.
- Thus if we want the SM model to be valid up to the Planck mass, we would expect the Higgs mass to be naturally of that order.
- The SM Higgs is a low energy effective theory.
- There could exist supersymmetric partners which cancel the divergences.
- In the little Higgs scenario the divergences are postponed to one order higher, by introducing new cancelling Higgs degrees of freedom.

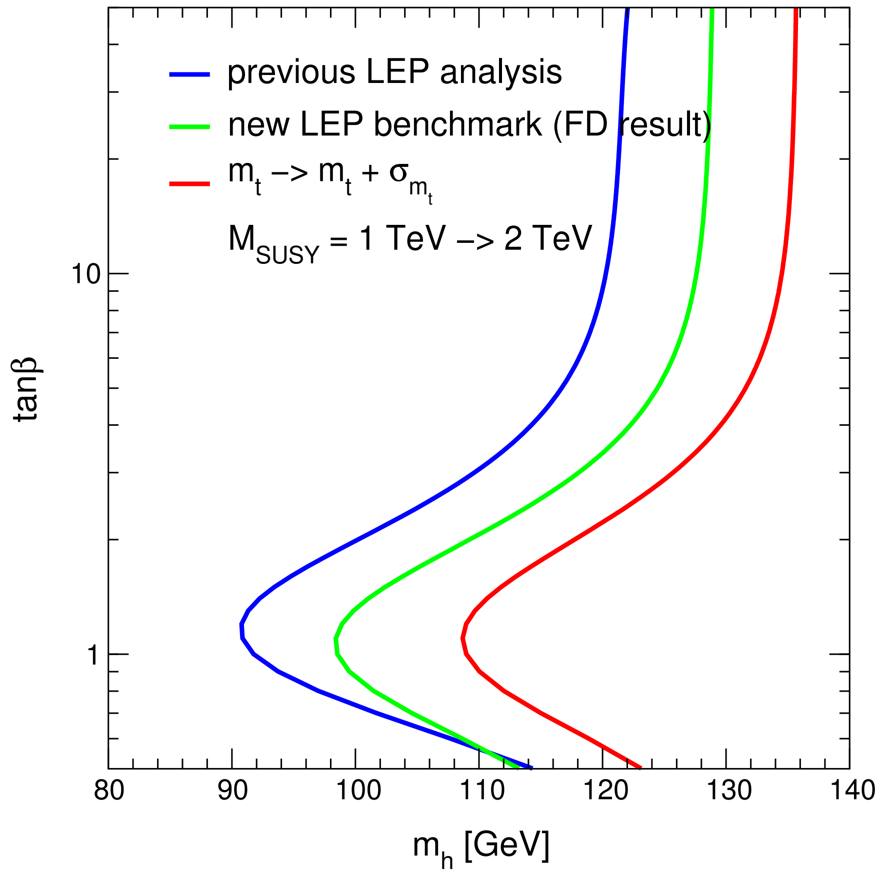
Higgs particles in the MSSM

- Mass scales in supersymmetric theories which are set by the supersymmetry breaking scale are often elusive.
- An exception is the Higgs particle in the MSSM, whose mass is quite constrained.
- MSSM contains two doublets of Higgs fields
  Physical states: $h^0, H^0, A^0, H^\pm$, Goldstones: $G^0, G^\pm$
  Input parameters $\tan\beta, M_A^2$
  $m_{H,h}^{2,\text{tree}} = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2+M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}\right]$
- At tree level $m_h < M_Z$
- Dominant corrections are from $t - \tilde{t}$-sector, for $\tan\beta > 4$

$$m_h^2 \sim m_Z^2 + \frac{3G_F m_t^4}{\sqrt{2}\pi^2} \ln\left(m_{\tilde{t}}^2/v^2\right)$$

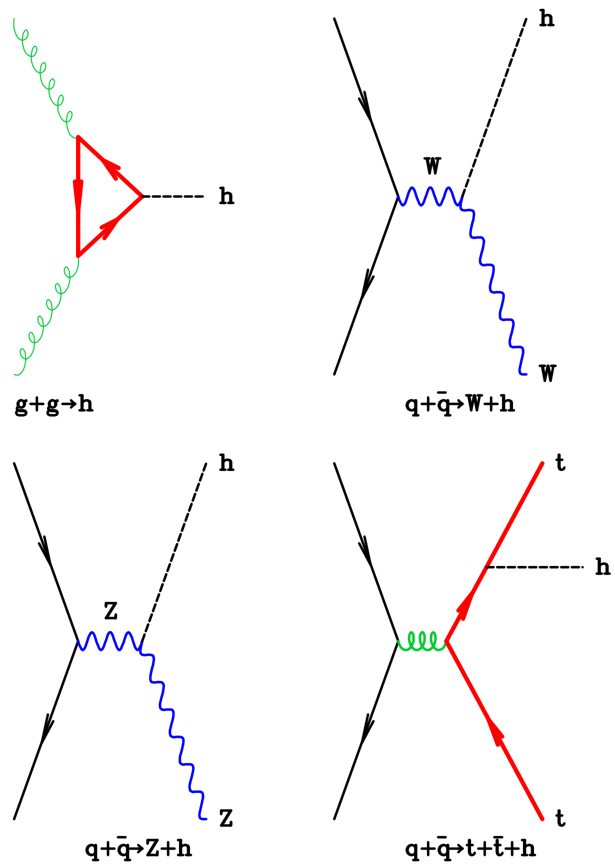
Upper limit on m_h in MSSM

Heinemayer, Hollik, Weiglein



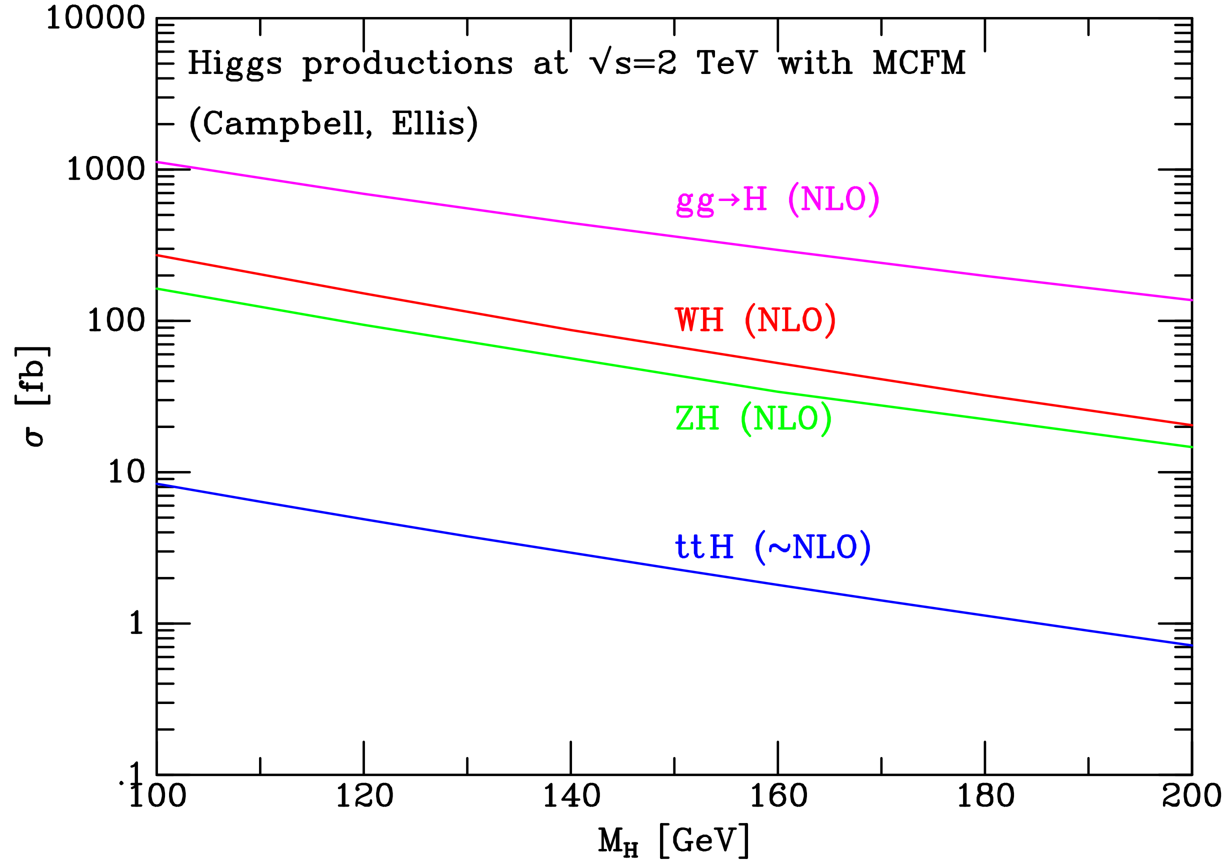
- Two loop calculation $m_h \leq 135$ GeV
- Upper limit for MSSM two doublet model sensitive to value of m_t
- Models with additional representations of particles extend the limit to 205 GeV. Quiros,Espinosa

Higgs production mechanisms Tevatron



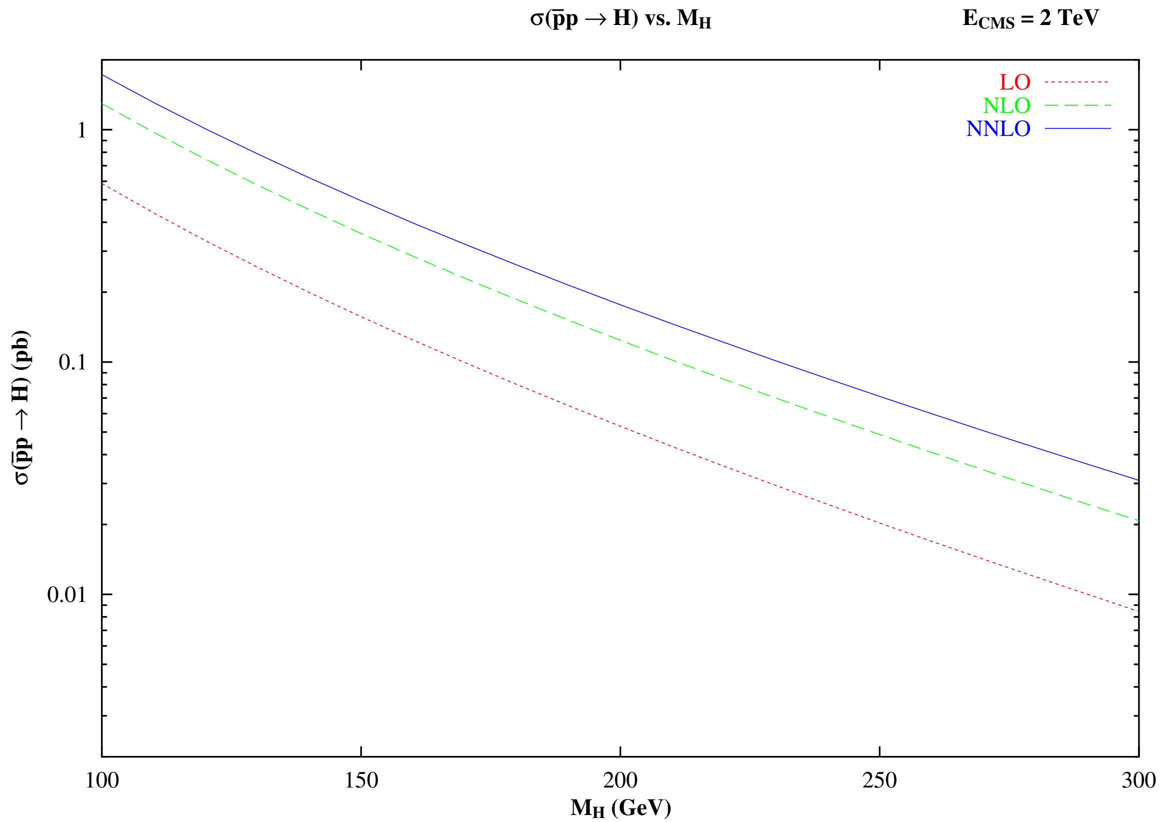
- $gg \longrightarrow H$
- $q\bar{q} \longrightarrow WH$
- $q\bar{q} \longrightarrow ZH$
- $q\bar{q} \longrightarrow t\bar{t}H$

Higgs boson production at 2 TeV



- Higgs cross sections are not so small at the Tevatron.
- Hence the issue of background is crucial.
- Associated production cross sections are more than twenty times smaller than gluon fusion cross section.

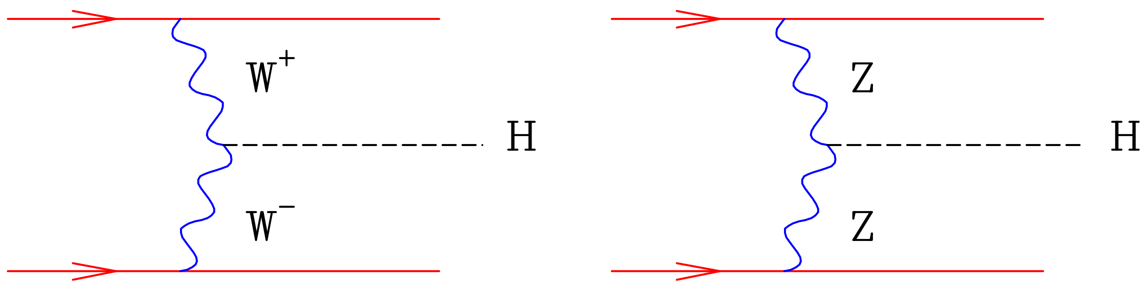
Next-to-next-to-leading order



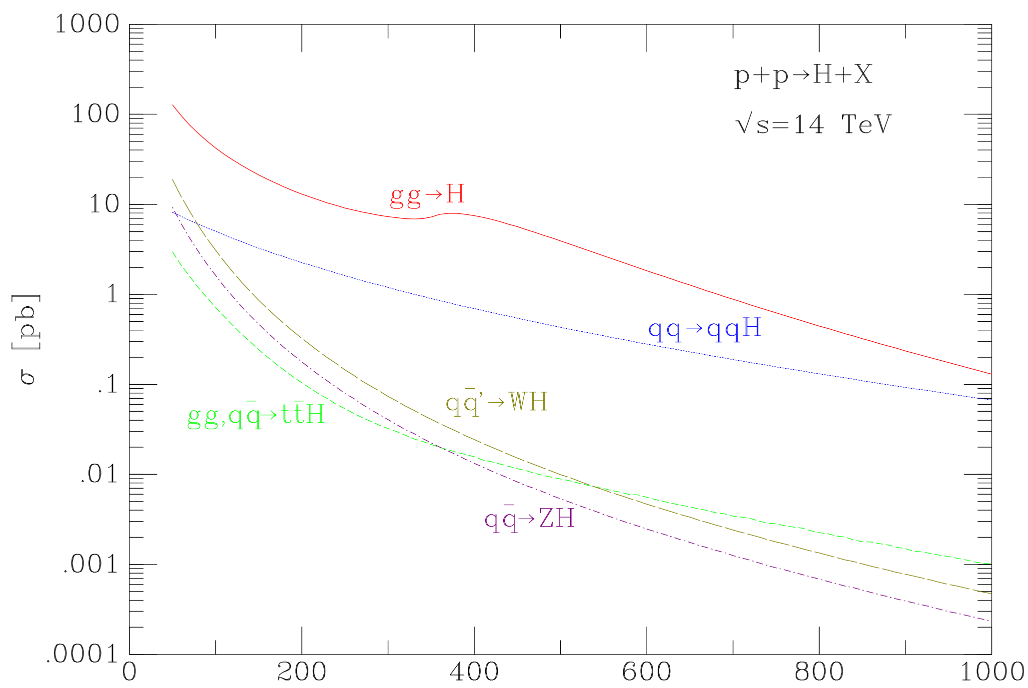
- Complete NNLO result, Harlander,Kilgore(hep-ph/0201206), Anastasiou and Melnikovhep-ph/0207004
- Substantial increase in cross section
- Uses MRST NNLO (partial) structure functions, from hep-ph/0201127.

Higgs production at LHC

- In addition at the LHC another mechanism is also important.

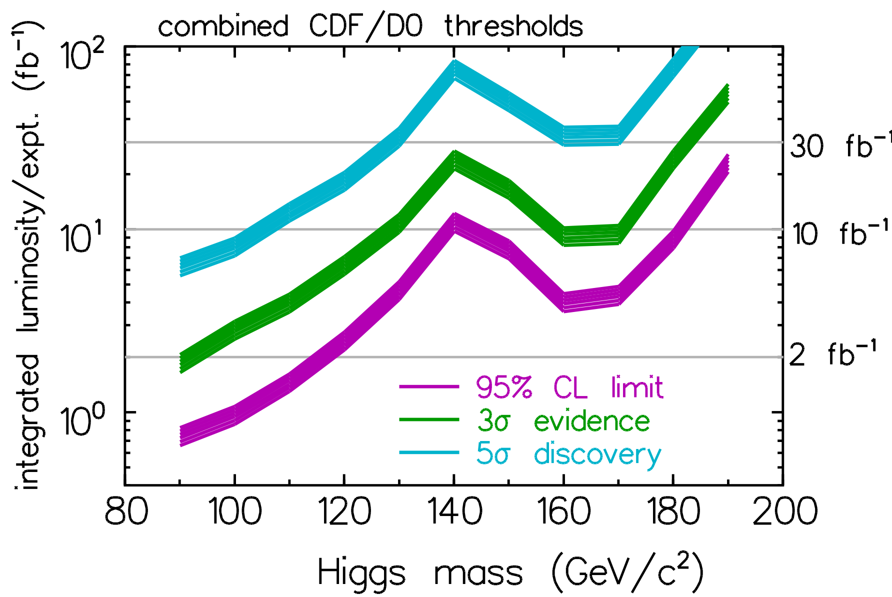


- Vector boson fusion $qq \longrightarrow Hqq$
- Characteristic signature - forward jets.



Tevatron overview

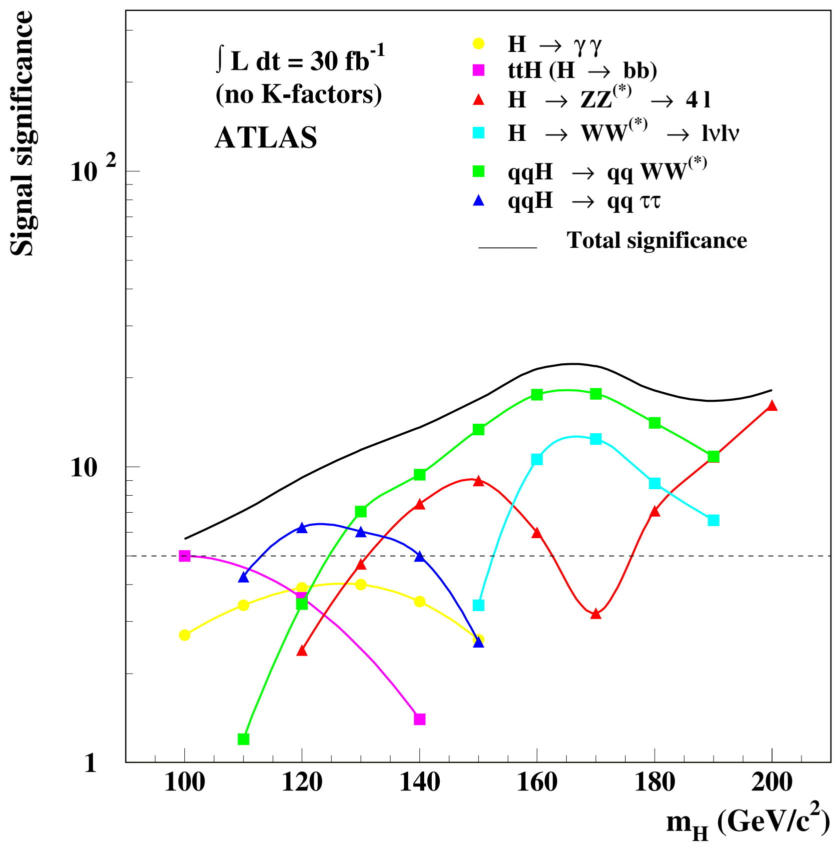
production	decay	M_H range[GeV]	mass peak
$gg \to H$	W^*W	150-200+	$\times$
$q\bar{q} \to WH$	W^*W	150-170	$\times$
$q\bar{q} \to WH$	$b\bar{b}$	110-170	$\surd$
$q\bar{q} \to ZH$	$b\bar{b}$	110-170	$\surd$
$p\bar{p} \to t\bar{t}H$	$b\bar{b}$	100-120	$\surd$



LHC overview

production	decay	M_H range[GeV]	mass peak
$gg \to H$	$\gamma\gamma$	110-140	$\surd$
	W^+W^-	150-200+	?
	$ZZ \to 4l$	> 130	$\surd$
$pp \to Hjj$	$\tau^+\tau^-$	110-140	$\surd$
	W^+W^-	120-200+	($\surd$)
	ZZ	120-200+	$\surd$
	$\gamma\gamma$	110-140	$\surd$
	invisible	?	$\times$
$q\bar{q} \to ZH$	invisible	110-170	$\times$
$p\bar{p} \to t\bar{t}H$	$b\bar{b}$	100-120	$\surd$
$p\bar{p} \to t\bar{t}H$	W^+W^-	130-200+	$\times$

LHC overview



Higgs search at the Tevatron

- Studies using LO Monte Carlos and other event generators show that for a Higgs in the mass range of 100-130 GeV, the most promising channels for discovery at Run II are associated Higgs production.

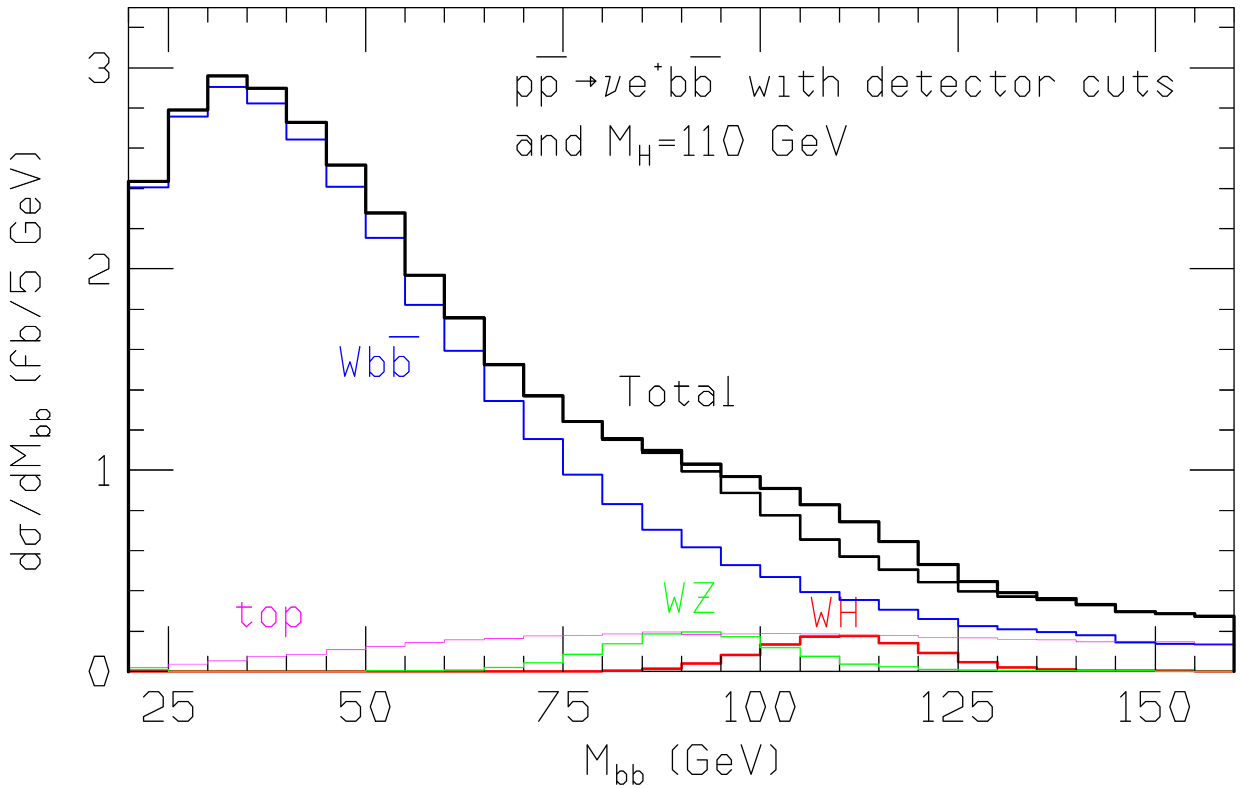
$$p\bar{p} \longrightarrow W(\to e\nu)H(\to b\bar{b})$$
$$p\bar{p} \longrightarrow Z(\to \nu\bar{\nu}, \ell\bar{\ell})H(\to b\bar{b})$$

- Mass range interesting for the MSSM.

- Backgrounds for the WH signal: $\begin{aligned} p\bar{p} &\longrightarrow W\, g^\star(\to b\bar{b}) \\ p\bar{p} &\longrightarrow W\, Z/\gamma^\star(\to b\bar{b}) \end{aligned}$

$$\begin{aligned} p\bar{p} &\longrightarrow t(\to bW^+)\bar{t}(\to \bar{b}W^-) \\ p\bar{p} &\longrightarrow W^{\pm *}(t(\to bW^+)\bar{b}) \\ qg &\longrightarrow q' t(\to bW^+)\bar{b} \end{aligned}$$

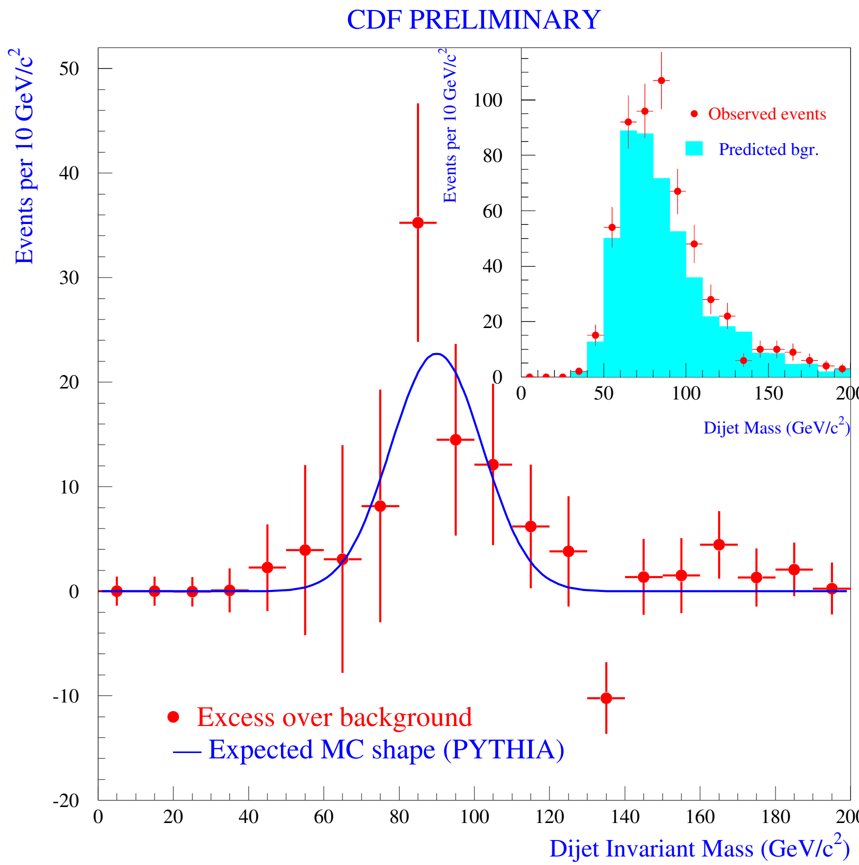
Signal and Backgrounds for $m_h = 110$ *GeV*



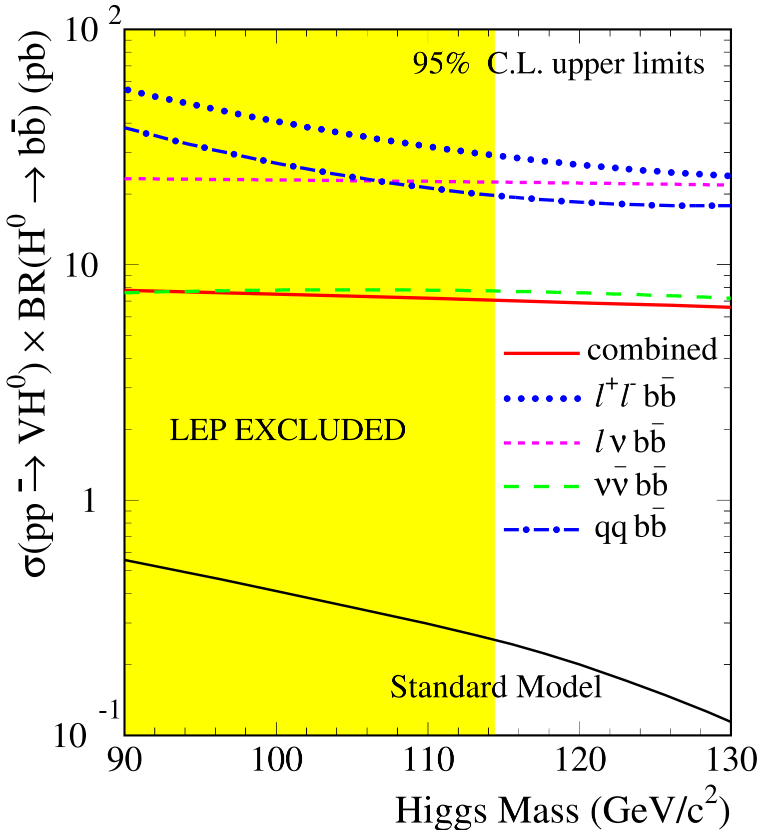
- Double b-tagging efficiency of $\epsilon_{b\bar{b}} = 0.45$
- We have to understand a cocktail of backgrounds, both in normalization and shape. ($WZ \equiv WH$).

$b\bar{b}$ resolution

- The $b\bar{b}$ resolution and b-jet energy scale are crucial.
- In Run II both of these will be *measured* using the decay $Z \to b\bar{b}$.
- A proof of principle is provided by the Run I observation of $Z \to b\bar{b}$ shown below. Curve has an RMS width, $\Delta = 12.3$ GeV



Current limits from CDF and D0



CDF, hep-ex/0503039

- Upper limits on the production cross sections in the various channels for Run I.
- Sensitivity is still an order of magnitude away from that required in the SM.

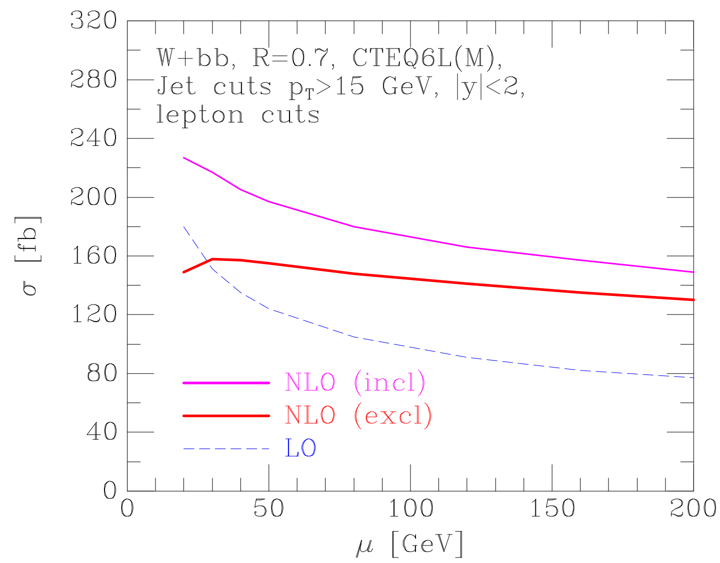
Measuring the $Wb\bar{b}$ background

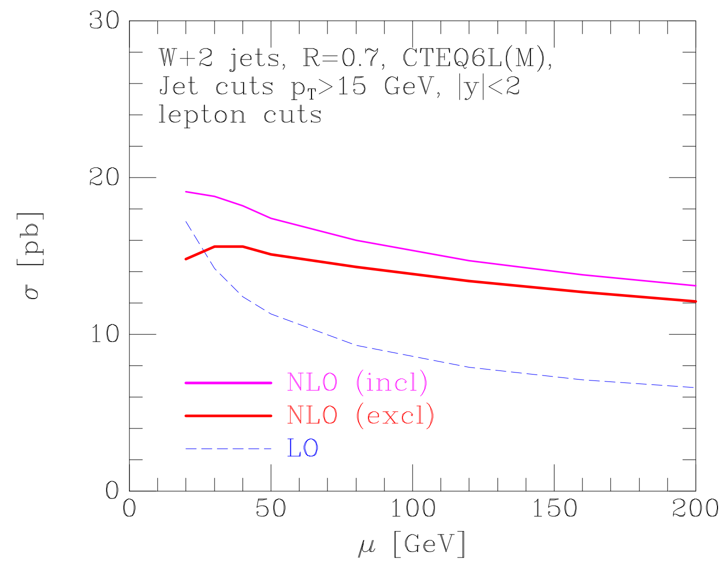
- How can theory best assist in determining the $Wb\bar{b}$ background.
- $Wb\bar{b}$ background can be measured using CDF's 'Method 2':

$$\sigma(Wb\bar{b}) = \left[\frac{\sigma(Wb\bar{b})}{\sigma(W+2\ \mathrm{jet})}\right]_{MC} \times [\sigma(W+2\ \mathrm{jet})]_{\mathrm{data}}$$

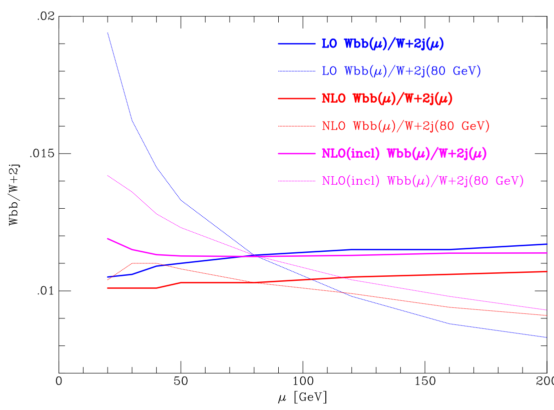
- We can calculate the ratio at NLO in MCFM, a next to leading order Monte Carlo (mcfm.fnal.gov).
- One sees a much reduced scale dependence in each of the cross-sections at NLO, but . . .
  - ⋆ If we choose the same scales in the numerator and denominator, is the ratio also stable?
  - ⋆ If the same scale is not appropriate, is this ratio useful? $Wb\bar{b}$ is simply gluon-splitting at LO, suggesting a different renormalization scale may be appropriate.

Scale dependence - $W b\bar{b}$ vs. $W + 2$ jets



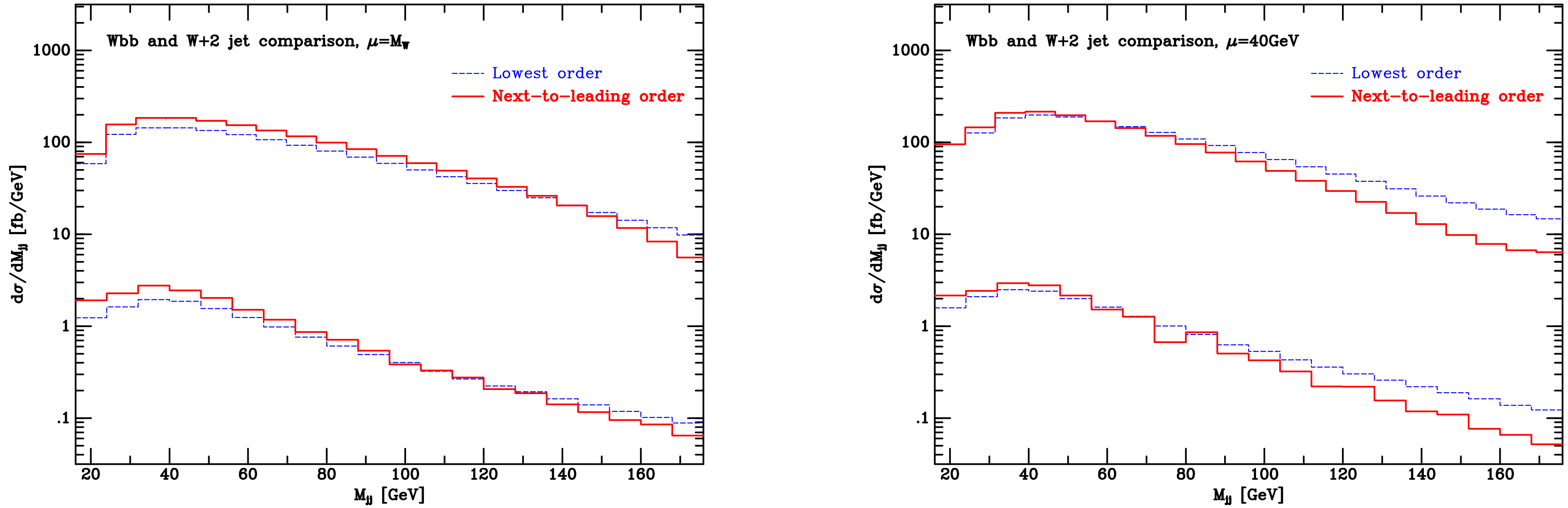


Ratio is much more stable at NLO, whether or not the same scale is used for $W b\bar{b}$ as for $W + 2$ jets.



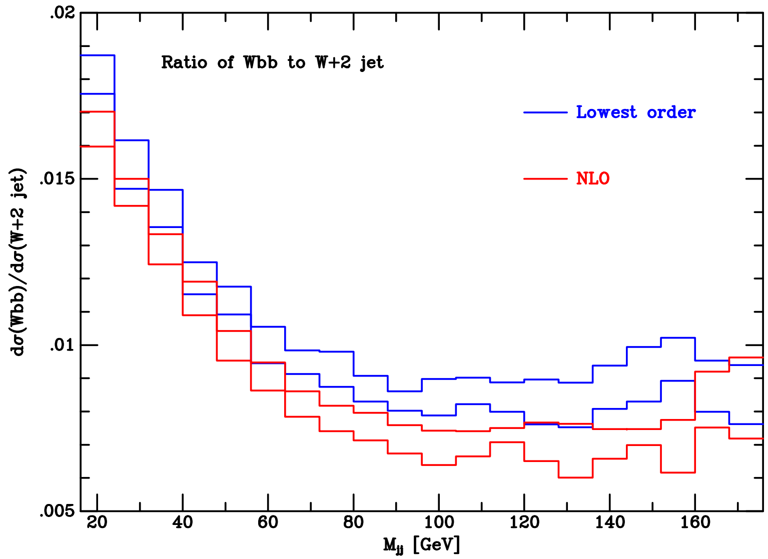
m_{JJ} distributions

- $Wb\bar{b}$ and $W + 2$ jet distributions appear very similar in shape at both LO and NLO. The shapes change when moving to a lower scale, with a depletion in the cross-section at high M_{jj}.



Heavy flavour fraction vs. m_{JJ}

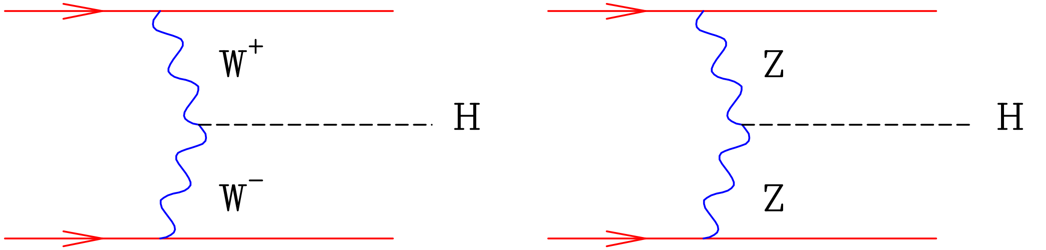
- Look at the variation of the ratio as the scale is changed (in both numerator and denominator) from ~ 30 GeV up to ~ 160 GeV.



- The ratio of b-tagged to untagged jets changes little at NLO and appears to be predicted very well by perturbation theory.
- The fraction peaks at low M_{jj}, but in the reliable domain $M_{jj} > 60$ GeV, the value is fairly constant $\sim 0.8\%$.

Theoretical issues at the LHC

- At the LHC the vector boson fusion process represents 20% of the cross section.



- It may have significant advantages over the dominant gluon-gluon fusion in terms of background.
- At $m_H \sim 120$ GeV

$$
\begin{array}{l}
pp \to H \quad \to \quad \tau^+\tau^- \\
\qquad\qquad\qquad\quad \hookrightarrow \mu^- + Y \\
\qquad\qquad\quad \hookrightarrow e^+ + X
\end{array}
$$

- At $m_H > 140$ GeV

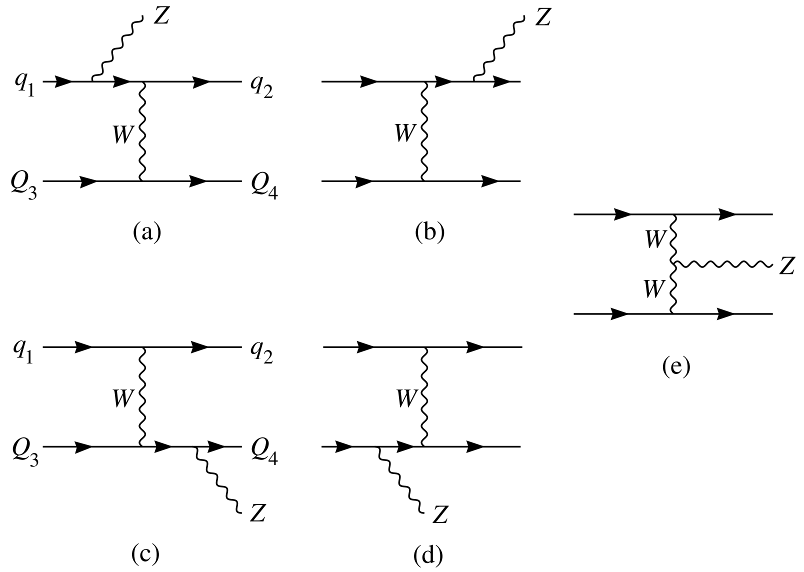
$$
\begin{array}{l}
pp \to H \quad \to \quad W^+W^{-*} \\
\qquad\qquad\qquad\quad \hookrightarrow \mu^- \bar{\nu} \\
\qquad\qquad\quad \hookrightarrow \nu e^+
\end{array}
$$

Vector boson fusion process

- The vector boson fusion process produces jets in the forward direction.
- Because of the color singlet nature of the exchanged bosons, jet activity in the central region is reduced.
- This can be investigated in perturbation theory, using say the Z+2 jet process. Z+ 2jet events can be produced either by Vector boson fusion, (little central jet activity) or by colored parton exchange (predominantly central activity).

Vector boson fusion at the LHC

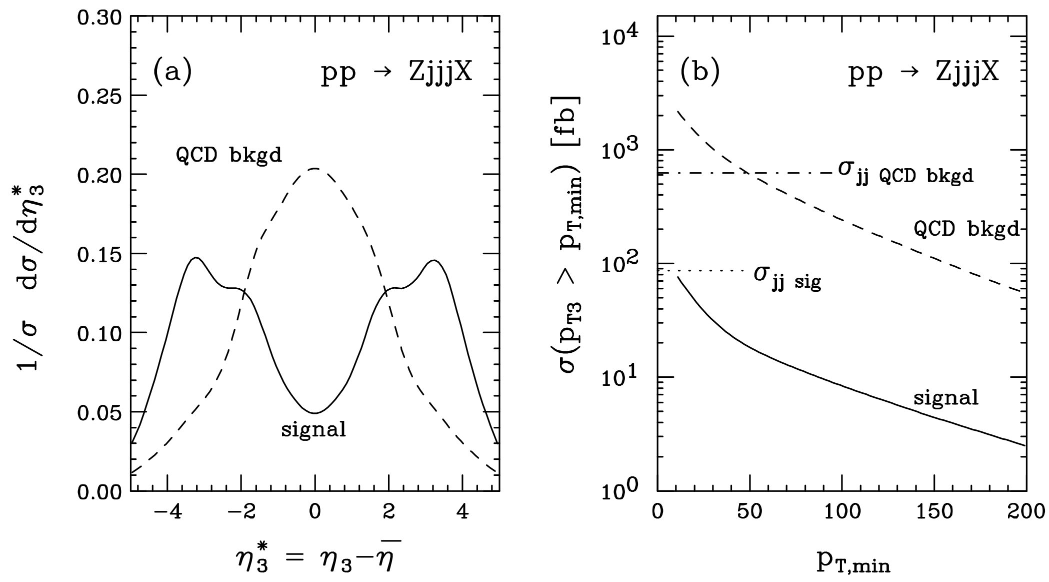
- Consider for example Z production with two jets at the LHC



- Vector boson fusion diagrams as well as normal QCD production.

Central jet activity veto

- Vector boson fusion diagrams lead to forward jets, with litte activity in the central region

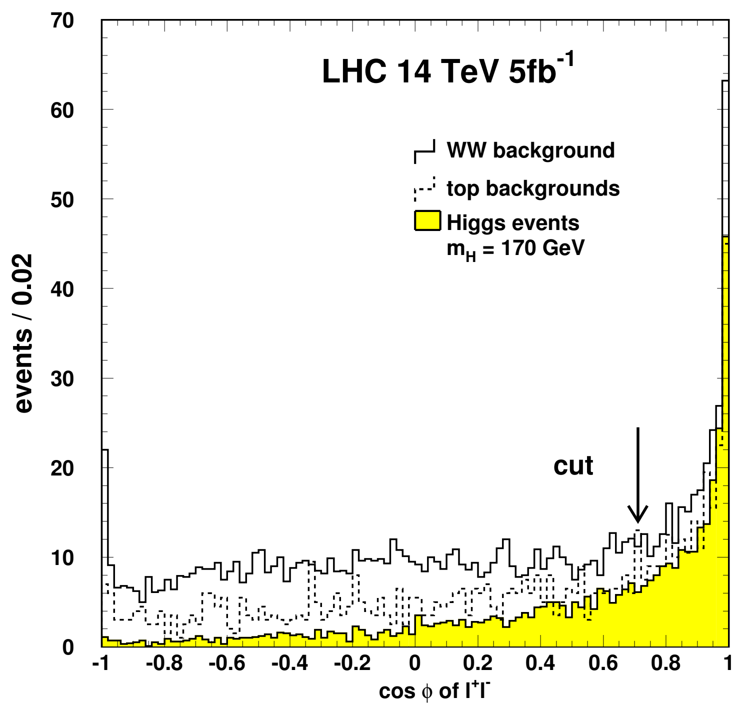


Rainwater, Zeppenfeld

- We are led to consider the corresponding diagram for Higgs production.

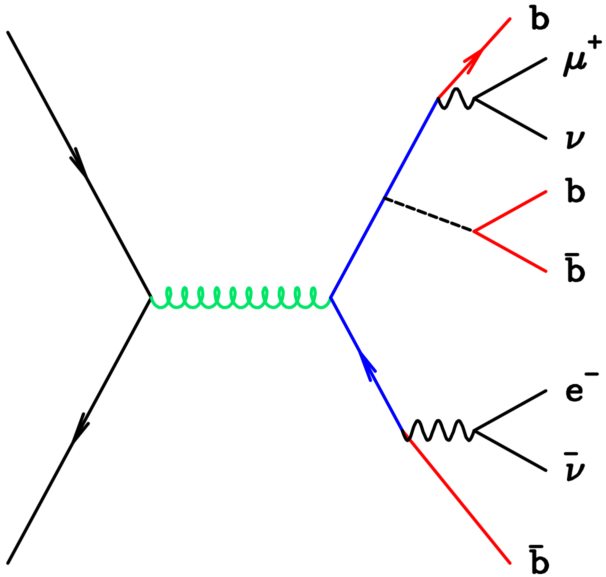
Cuts for the decay WW^*

- Because of the spin 0 nature of the Higgs there is a correlation between the directions of the vector bosons.
- Because of the V-A interactions, this correlation is preserved in the decay.
- Matrix element squared is proportional to $p_{e^-} \cdot p_\nu \; p_{\mu^+} \cdot p_{\bar{\nu}} \sim (1 + \cos(\theta_+ - \theta_-))^2$
- In the rest frame of the Higgs this product is maximum when the charged leptons are parallel. ϕ is the azimuthal angle between the charged leptons (Dittmar-Dreiner).



$t\bar{t}H$

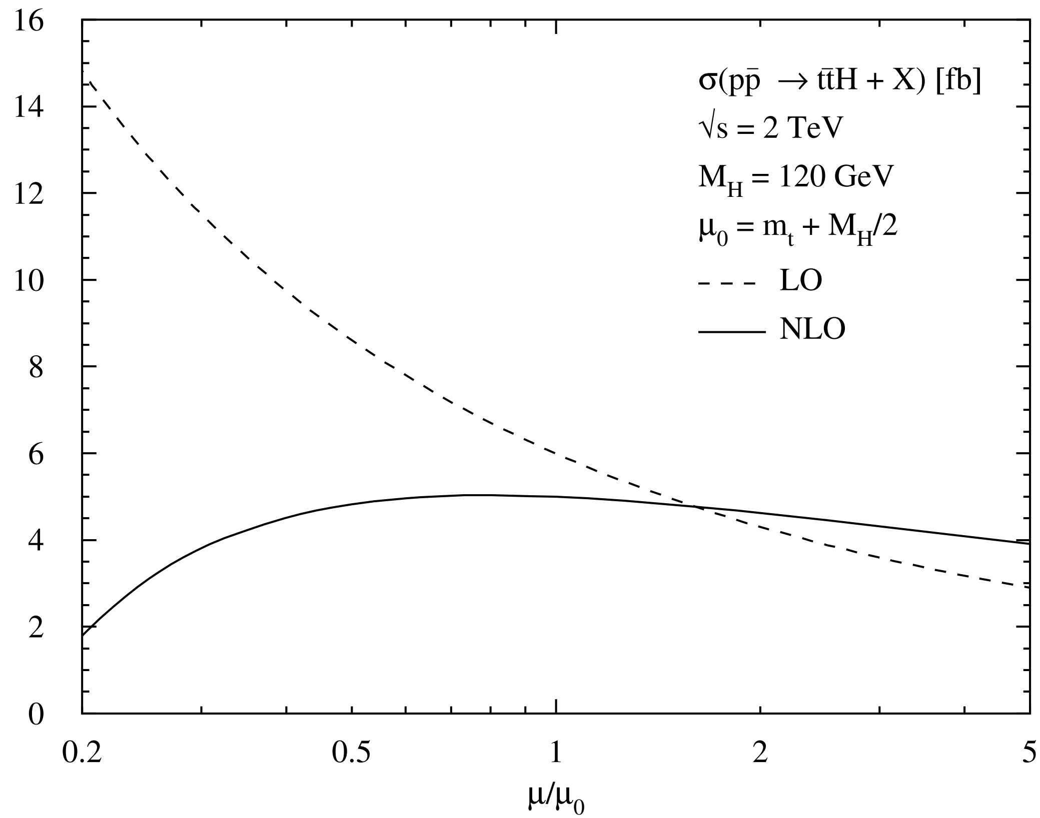
- The production of a Higgs boson in association with a $t\bar{t}$-pair gives a dramatic signature. Goldstein et al



- For $m_h < 135$ GeV the final state contains 4 b-quarks.
- Assume a 70% b-tagging efficiency
  92% probability to tag $\geq$ 2-jets
  65% probability to tag $\geq$ 3-jets
  24% probability to tag all 4 b-jets

$t\bar{t}H$-cross section

- The cross section has been calculated to NLO $\sigma \sim 10^{(1.85-M_h\,[\text{GeV}]/100)}$ [fb] Beenakker et al, Dawson et al



- The radiative corrections reduce the scale uncertainty of the cross section.

$t\bar{t}H$-backgrounds (Goldstein et al)

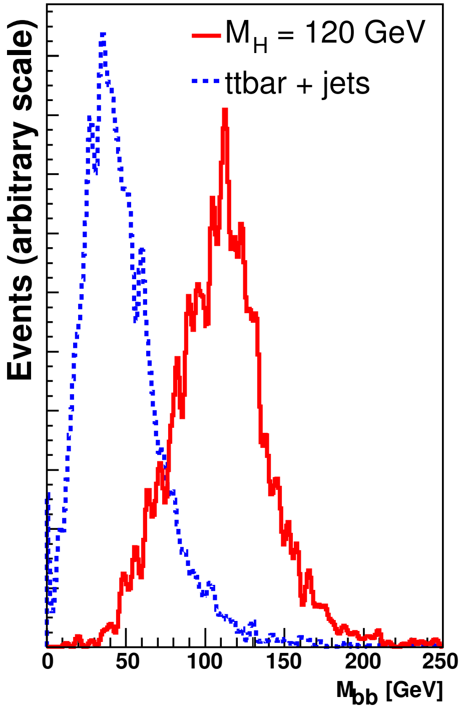
- Background estimates include $p_T > 20$ [GeV] and are uncertain by $\pm 50\%$.

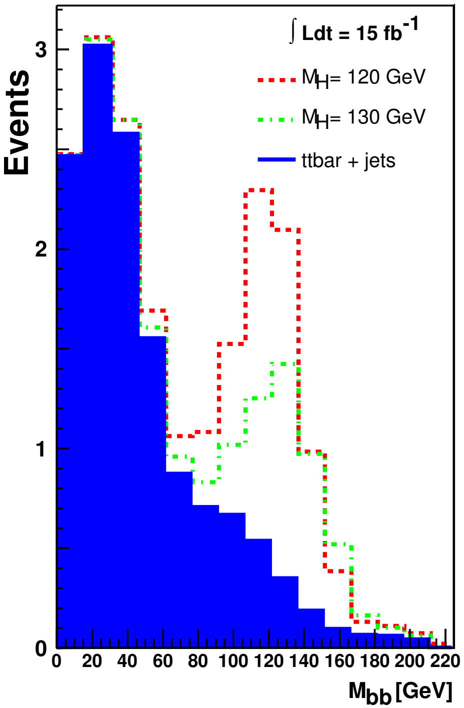
Signal and Backgrounds	σ [fb]
$t\bar{t}H, H \to b\bar{b}$	4
$t\bar{t} + jj$	1030
$t\bar{t} + b\bar{b}(c\bar{c})$	27
$t\bar{t} + Z, Z \to b\bar{b}$	1.5
$WZ + jj, Z \to b\bar{b}$	10

- In 15 fb^{-1}, 3-b tags gives 19 signal and 140 bkgd, 4-b tags gives 7 signal and 40 background.

$t\bar{t}H$-signal

- Form invariant masses of all combinations of jets
- Separation in 4th highest mass combination.
- Mass distribution after top mass reconstruction.
- Conclude that 15fb^{-1} gives a 2.8 σ effect at $M_h = 120$ GeV.





Conclusions

- The Luminosity targets for the Standard model Higgs at the Tevatron and LHC are well understood.
- Improved luminosity is needed at the Tevatron to get into the Higgs game in a significant way.
- For the Tevatron it is important to have an experimental determination of the $Wb\bar{b}$ background, perhaps by relating them to $Wb\bar{b}$ events at lower $m_{b\bar{b}}$ or W + 2jet events.
- For the LHC vector boson fusion is the most important channel at lower masses, (ie below Vector boson pair threshold)
- The SM Higgs model has a number of shortcomings. It will undoubtedly not be the last particle of the Standard model, but more likely the first of the model which supersedes it. The Higgs boson should not be viewed as an end, but as a beginning; in natural models it appears accompanied by other physics.
- We are assured that there will be a lot of fun ahead of us.