Troisième Cycle de Physique en Suisse Romande
Exercises for Lecture 2
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1) Using the solutions to the massless Dirac equation given in the first lecture show that

$$\bar{u}_{+\,i}(p)u_{+\,j}(p) = \frac{1}{2}\Big[(1+\gamma_5)\not{p}\Big]_{ij} \tag{1}$$

$$\bar{u}_{-\,i}(p)u_{-\,j}(p) = \frac{1}{2}\Big[(1-\gamma_5)\not{p}\Big]_{ij} \tag{2}$$

$$\tag{3}$$

where $\not{p} = \gamma^\mu p_\mu$ and hence that, if we sum over spins (helicities)

$$\sum_s \bar{u}_{s\,i}(p)u_{s\,j}(p) = \Big[\not{p}\Big]_{ij} \tag{4}$$

This relation is used to reduce squares of matrix elements involving quarks to traces.

2) Calculate the width of the W boson. Use the sum over the three polarizations

$$\sum_{polarizations} \varepsilon(q)\varepsilon^*(q) = \Big[-g^{\mu\nu} + \frac{q^\mu q^\nu}{M_W^2}\Big] \tag{5}$$

to perform the average over the polarizations of the W boson. The lowest order matrix element is given by ($q = p_1 + p_2$)

$$M = -i\frac{g_W}{2\sqrt{2}}\bar{u}(p_1)\gamma^\mu(1-\gamma_5)u(p_2)\varepsilon_\mu(q) \tag{6}$$

where $\frac{g_W}{8M_W^2} = \frac{G_F}{2}$

Show that the square of the matrix element summed (averaged) over the initial (final) polarizations is

$$\overline{\sum}|M|^2 = \frac{1}{3}g_W^2 M_W^2 \tag{7}$$

and hence that the result for the partial widths

$$\Gamma(W^+ \to f\bar{f}) = C\frac{G_F M_W^3}{6\sqrt{2}\,\pi} \tag{8}$$

C is a color normalization factor which is one for leptons and 3 for quarks. The total width is obtained by summimg over three flavours of leptons and two generations of colored quarks. What is the branching fraction of a W boson into leptons? (see ESW, page 290).