Dilogarithms

$$\mathrm{Li}_2(x) = -\int_0^x \frac{dx}{x}\ln(1-x) = \frac{x}{1^2}+\frac{x^2}{2^2}+\frac{x^3}{3^2}+\ldots \text{when} |x| \leq 1 \tag{1}$$

$$\mathrm{Li}_2(1) = \frac{\pi^2}{6} \tag{2}$$

$$\mathrm{Li}_2(-1) = -\frac{\pi^2}{12} \tag{3}$$

$$\mathrm{Li}_2(\frac{1}{2}) = \frac{\pi^2}{12} - \frac{1}{2}\ln^2 2 \tag{4}$$

$$\mathrm{Li}_2(-x) + \mathrm{Li}_2(-1/x) = -\frac{\pi^2}{6} - \frac{1}{2}\ln^2 x, \quad x > 0 \tag{5}$$

$$\mathrm{Li}_2(x) + \mathrm{Li}_2(1/x) = \frac{\pi^2}{3} - \frac{1}{2}\ln^2 x - i\pi \ln x, \quad x > 1 \tag{6}$$

$$\mathrm{Li}_2(x) + \mathrm{Li}_2(1-x) = \frac{\pi^2}{6} - \ln x \ln(1-x) \tag{7}$$

$$\mathrm{Li}_2(x) + \mathrm{Li}_2(-\frac{x}{1-x}) = -\frac{1}{2}\ln^2(1-x), \quad x < 1 \tag{8}$$

$$\mathrm{Li}_2(x)+\mathrm{Li}_2(\frac{x}{x-1}) = \frac{\pi^2}{2}-\frac{1}{2}\ln^2(x-1)+i\pi\ln(\frac{x-1}{x^2}), \quad x > 1 \tag{9}$$

$$\mathrm{Li}_2(1-x)-\mathrm{Li}_2(1/x) = \frac{1}{2}\ln x \ln(\frac{x}{(x-1)^2})-\frac{\pi^2}{6}, \quad x > 1 \tag{10}$$

$$\mathrm{Li}_2(\frac{1}{1+x})-\mathrm{Li}_2(-x) = \frac{\pi^2}{6}-\frac{1}{2}\ln(1+x)\ln(\frac{1+x}{x^2}), \quad x > 0 \tag{11}$$