Conventions for massless spinor products

$$\langle pq \rangle = \langle p - | q + \rangle, \quad [pq] = \langle p + | q - \rangle$$

$$\langle p \pm | \gamma_\mu | p \pm \rangle = 2p_\mu$$

$$\langle p + | q + \rangle = \langle p - | q - \rangle = \langle pp \rangle = [pp] = 0$$

$$\langle pq \rangle = -\langle qp \rangle, \quad [pq] = -[qp]$$

$$2|p \pm \rangle \langle q \pm | = \tfrac{1}{2}(1 \pm \gamma_5)\gamma^\mu \langle q \pm | \gamma_\mu | p \pm \rangle$$

$$\langle pq \rangle^* = -\mathrm{sign}(p \cdot q)[pq] = \mathrm{sign}(p \cdot q)[qp]$$

$$|\langle pq \rangle|^2 = \langle pq \rangle \langle pq \rangle^* = 2|p \cdot q| \equiv |s_{pq}|$$

$$\langle pq \rangle [qp] = 2p \cdot q \equiv s_{pq}$$

$$\langle p \pm | \gamma_{\mu_1} \ldots \gamma_{\mu_{2n+1}} | q \pm \rangle = \langle q \mp | \gamma_{\mu_{2n+1}} \ldots \gamma_{\mu_1} | p \mp \rangle$$

$$\langle p \pm | \gamma_{\mu_1} \ldots \gamma_{\mu_{2n}} | q \mp \rangle = -\langle q \pm | \gamma_{\mu_{2n}} \ldots \gamma_{\mu_1} | p \mp \rangle$$

$$\langle AB \rangle \langle CD \rangle = \langle AD \rangle \langle CB \rangle + \langle AC \rangle \langle BD \rangle$$

$$\langle A + | \gamma_\mu | B + \rangle \langle C - | \gamma^\mu | D - \rangle = 2[AD]\langle CB \rangle$$

$$\langle A \pm | \gamma^\mu | B \pm \rangle \; \gamma_\mu = 2 \left[\, |A \mp \rangle \langle B \mp | + |B \pm \rangle \langle A \pm | \, \right]$$

For polarization with momentum k and gauge vector b

$$\varepsilon^\mu_\pm(k, b) = \pm \frac{\langle k \pm | \gamma^\mu | b \pm \rangle}{\sqrt{2} \langle b \mp | k \pm \rangle}$$

Hence we have that

$$\varepsilon^\mu_+(k, b) = \frac{\langle k + | \gamma^\mu | b + \rangle}{\sqrt{2} \langle bk \rangle}, \quad \varepsilon^\mu_-(k, b) = \frac{\langle k - | \gamma^\mu | b - \rangle}{\sqrt{2} [kb]}$$

and

$$\gamma_\mu \varepsilon_+^\mu(k,b) = \frac{\sqrt{2}[|k-\rangle\langle b-| + |b+\rangle\langle k+|]}{\langle bk\rangle}$$

$$\gamma_\mu \varepsilon_-^\mu(k,b) = \frac{\sqrt{2}[|k+\rangle\langle b+| + |b-\rangle\langle k-|]}{[kb]}$$

The Weyl representation is most suitable at high energy

$$\gamma^0 = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}, \gamma^i = \begin{pmatrix} \mathbf{0} & -\sigma^i \\ \sigma^i & \mathbf{0} \end{pmatrix}, \gamma^5 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix},$$

The massless spinors solns of Dirac eqn are

$$u_+(k) = \begin{bmatrix} \sqrt{k^+} \\ \sqrt{k^-}e^{i\varphi_k} \\ 0 \\ 0 \end{bmatrix}, \; u_-(k) = \begin{bmatrix} 0 \\ 0 \\ \sqrt{k^-}e^{-i\varphi_k} \\ -\sqrt{k^+} \end{bmatrix},$$

where

$$e^{\pm i\varphi_k} \equiv \frac{k^1 \pm ik^2}{\sqrt{(k^1)^2 + (k^2)^2}} = \frac{k^1 \pm ik^2}{\sqrt{k^+k^-}}, \qquad k^\pm = k^0 \pm k^3.$$

In this representation the Dirac conjugate spinors are

$$\overline{u_+}(k) \equiv u_+^\dagger(k)\gamma^0 = [0, 0, \sqrt{k^+}, \sqrt{k^-}e^{-i\varphi_k}]$$

$$\overline{u_-}(k) = \left[\sqrt{k^-}e^{i\varphi_k}, -\sqrt{k^+}, 0, 0\right]$$

Normalization

$$u_\pm^\dagger u_\pm = 2k^0$$

Thus

$$\langle kb \rangle = \langle k - | b + \rangle = \sqrt{\frac{k^+}{b^+}}(b^1 - ib^2) - \sqrt{\frac{b^+}{k^+}}(k^1 - ik^2)$$

$$[kb] = \langle k + | b - \rangle = \sqrt{\frac{b^+}{k^+}}(k^1 + ik^2) - \sqrt{\frac{k^+}{b^+}}(b^1 + ib^2)$$

We may examine the particular representation of the polarization vectors by setting $k^\mu = (1,0,0,1), b^\mu = (1,1,0,0)$. With this definition we get

$$\varepsilon_+^\mu(k,b) = \frac{1}{\sqrt(2)}(0,1,-i,0) + \frac{1}{\sqrt{2}}k^\mu$$

$$\varepsilon_-^\mu(k,b) = \frac{1}{\sqrt(2)}(0,1,+i,0) + \frac{1}{\sqrt{2}}k^\mu$$