We shall examine here the calculation of the vertex function n-dimensions.

$$\Gamma^\mu(p,p') = (-ie)(-ig)^2 \int \frac{d^n k}{(2\pi)^n} \bar{u}(p')\gamma^\alpha t^A \frac{i(\hat{k}+\hat{p'})}{(p'+k)^2+i\varepsilon}\gamma^\mu \frac{i(\hat{k}+\hat{p})}{(p+k)^2+i\varepsilon}\gamma_\alpha t^A u(p)\frac{-i}{k^2} \quad (1)$$

The first step is to collect the overall factors and to introduce the Feynman parameters for the three denominators.

$$\frac{1}{ABC} = 2\int_0^1 d\alpha d\beta d\gamma \frac{1}{[A\alpha + B\beta + C\gamma]^3} \quad (2)$$

Using the relation $t^A t^A = C_F I$ we may write

$$\Gamma^\mu(p,p') = X \int \frac{d^n k}{(2\pi)^n} \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \frac{\bar{u}(p')\gamma^\alpha(\hat{k}+\hat{p'})\gamma^\mu(\hat{k}+\hat{p})\gamma_\alpha u(p)}{[k^2+2\alpha p\cdot k + 2\beta p'\cdot k + i\varepsilon]^3} \quad (3)$$

where $X = -2eg^2 C_F$. We now simplify this equation by shifting the momentum in the normal way $k_s = k + \alpha p + \beta p'$ leading to the expression

$$\begin{aligned}\Gamma^\mu(p,p') &= X \int \frac{d^n k}{(2\pi)^n} \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \\ &\times \frac{\bar{u}(p')\gamma^\alpha(\hat{k_s} - \alpha\hat{p} + (1-\beta)\hat{p'})\gamma^\mu(\hat{k_s} + (1-\alpha)\hat{p} - \beta\hat{p'})\gamma_\alpha u(p)}{[k_s^2 + \alpha\beta q^2 + i\varepsilon]^3} \quad (4)\end{aligned}$$

where $q^2 = -2p\cdot p'$. We shall proceed to the simplificatiom of the numerator terms, where we can immediately drop terms which are even in k_s which vanish under symmetric integration.

$$\begin{aligned}&\bar{u}(p')\gamma^\alpha(\hat{k_s} + (1-\beta)\hat{p'} - \alpha\hat{p})\gamma^\mu(\hat{k_s} + (1-\alpha)\hat{p} - \beta\hat{p'})\gamma_\alpha u(p) \\ =\ &\bar{u}(p')\Big[\gamma^\alpha\hat{k_s}\gamma^\mu\hat{k_s}\gamma_\alpha + \gamma^\alpha((1-\beta)\hat{p'} - \alpha\hat{p})\gamma^\mu((1-\alpha)\hat{p} - \beta\hat{p'})\gamma_\alpha\Big]u(p) \quad (5)\end{aligned}$$

The five remaining terms in the above expression can be simplified as follows.

$$\begin{aligned}&+\ \bar{u}(p')\gamma^\alpha\hat{k_s}\gamma^\mu\hat{k_s}\gamma_\alpha u(p) \\ &-\ \beta(1-\beta)\bar{u}(p')\gamma^\alpha\hat{p'}\gamma^\mu\hat{p'}\gamma_\alpha u(p) \\ &-\ \alpha(1-\alpha)\bar{u}(p')\gamma^\alpha\hat{p}\gamma^\mu\hat{p}\gamma_\alpha u(p) \\ &+\ (1-\alpha)(1-\beta)\bar{u}(p')\gamma^\alpha\hat{p'}\gamma^\mu\hat{p}\gamma_\alpha u(p) \\ &+\ \alpha\beta\bar{u}(p')\gamma^\alpha\hat{p}\gamma^\mu\hat{p'}\gamma_\alpha u(p) \quad (6)\end{aligned}$$

which give

$$\begin{aligned}&+\ \frac{4k_s^2}{n}(1-\epsilon)^2\ \bar{u}(p')\gamma^\mu u(p) \\ &+\ 0 \\ &+\ 0 \\ &+\ 4(1-\alpha)(1-\beta)\ p\cdot p'\ \bar{u}(p')\gamma^\mu u(p) \\ &-\ 4\epsilon\ \alpha\beta\ p\cdot p'\ \bar{u}(p')\gamma^\mu u(p) \quad (7)\end{aligned}$$

where we have made extensive use of the fact that $p \cdot p = p' \cdot p' = 0$ and the equations of motion of the massless spinors $\hat{p}u(p) = \bar{u}(p')\hat{p'} = 0$. In addition we have used the relations valid in n-dimensions

$$\gamma^\alpha \gamma^\mu \gamma_\alpha = -2(1-\epsilon)\gamma^\mu \tag{8}$$

Thus our final result for the numerator factor is

$$\begin{aligned} - \quad & \frac{-4k_s^2}{n}(1-\epsilon)^2\ \bar{u}(p')\gamma^\mu u(p) \\ - \quad & 2q^2\ (1-\alpha-\beta+\alpha\beta(1-\epsilon))\ \bar{u}(p')\gamma^\mu u(p) \end{aligned} \tag{9}$$

THe formula which allows us to do the integration is as follows, $n = 4 - 2\epsilon$.

$$\begin{aligned} & \int \frac{d^n k}{(2\pi)^n} \frac{(-k^2)^r}{\Big[-k^2 + C - i\varepsilon\Big]^m} = \\ & \frac{i(4\pi)^\epsilon}{16\pi^2}\left[C - i\varepsilon\right]^{2+r-m-\epsilon} \frac{\Gamma(r+n/2)}{\Gamma(n/2)} \frac{\Gamma(m-r-2+\epsilon)}{\Gamma(m)}. \end{aligned} \tag{10}$$

Using this formula the result for the first UV divergent piece is

$$\frac{iX}{16\pi^2}\Big(\frac{4\pi}{-q^2 - i\varepsilon}\Big)^\epsilon \frac{4}{n}(1-\epsilon)^2 \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \frac{1}{(\alpha\beta)^\epsilon} \frac{\Gamma(1+\frac{n}{2})}{\Gamma(\frac{n}{2})} \frac{\Gamma(\epsilon)}{\Gamma(3)} \tag{11}$$

Performing the integration we get

$$\frac{iX}{16\pi^2}\bar{u}(p')\gamma^\mu u(p)\Big(\frac{4\pi}{-q^2 - i\varepsilon}\Big)^\epsilon \frac{(1-\epsilon)}{\epsilon(1-2\epsilon)} \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \tag{12}$$