Perturbative QCD: from the Tevatron to the LHC

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Fermilab

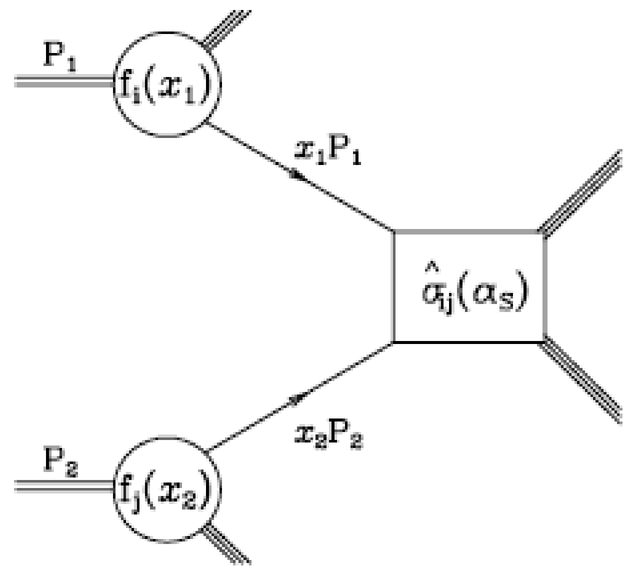
Menu

- Theoretical setup - Factorization (again?)
- Status of α_s and parton distributions.
- Recent results for the total top production.
- MCFM and comparison with Tevatron data.
- Theoretical advances in the calculation of one loop diagrams.

QCD improved parton model

Hard QCD cross section is represented as the convolution of a short distance cross-section and non-perturbative parton distribution functions.

Physical cross section is formally independent of $\mu_{F \text{ and }} \mu_R$



Physical cross section

Parton distribution function

Renormalization scale μ_R

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 \; f_i(x_1, \mu_F) f_j(x_2, \mu_F) \; \hat{\sigma}_{ij}(p_1, p_2, \alpha_S(\mu_R), Q^2, \mu_R, \mu_F).$$

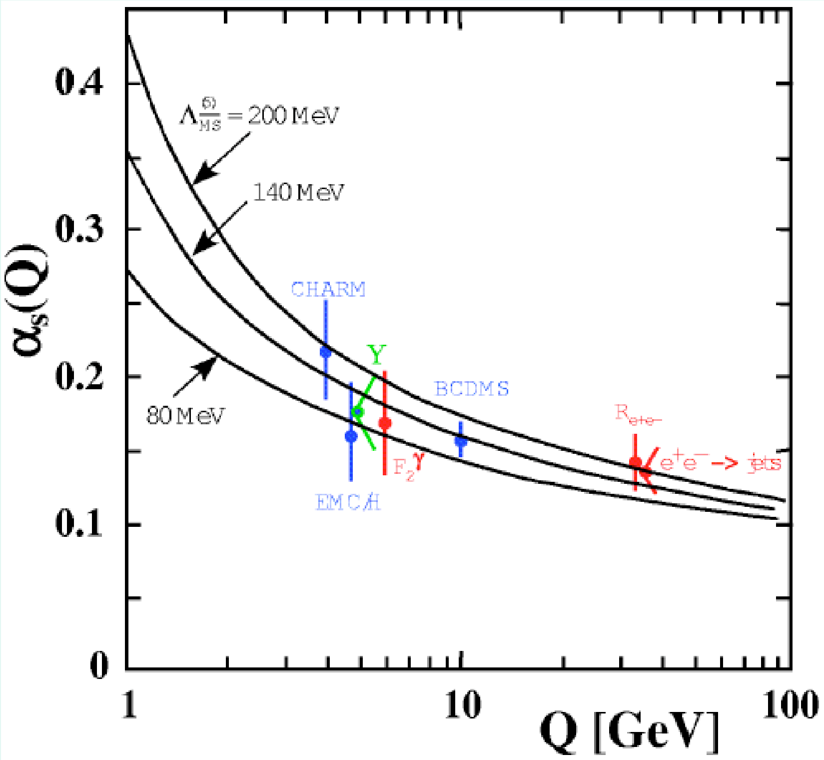
Factorization scale μ_F

Short distance cross section, calculated as a perturbation series in α_S

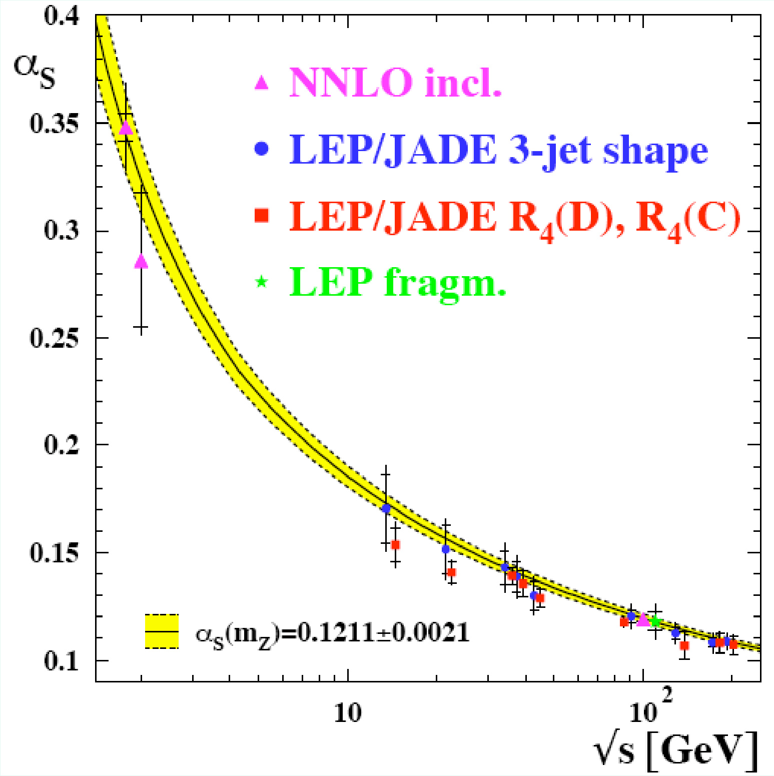
α_S

α_S is small(ish) at high energies because of the property of asymptotic freedom.

The role of LEP in determining the size of α_S has been crucial

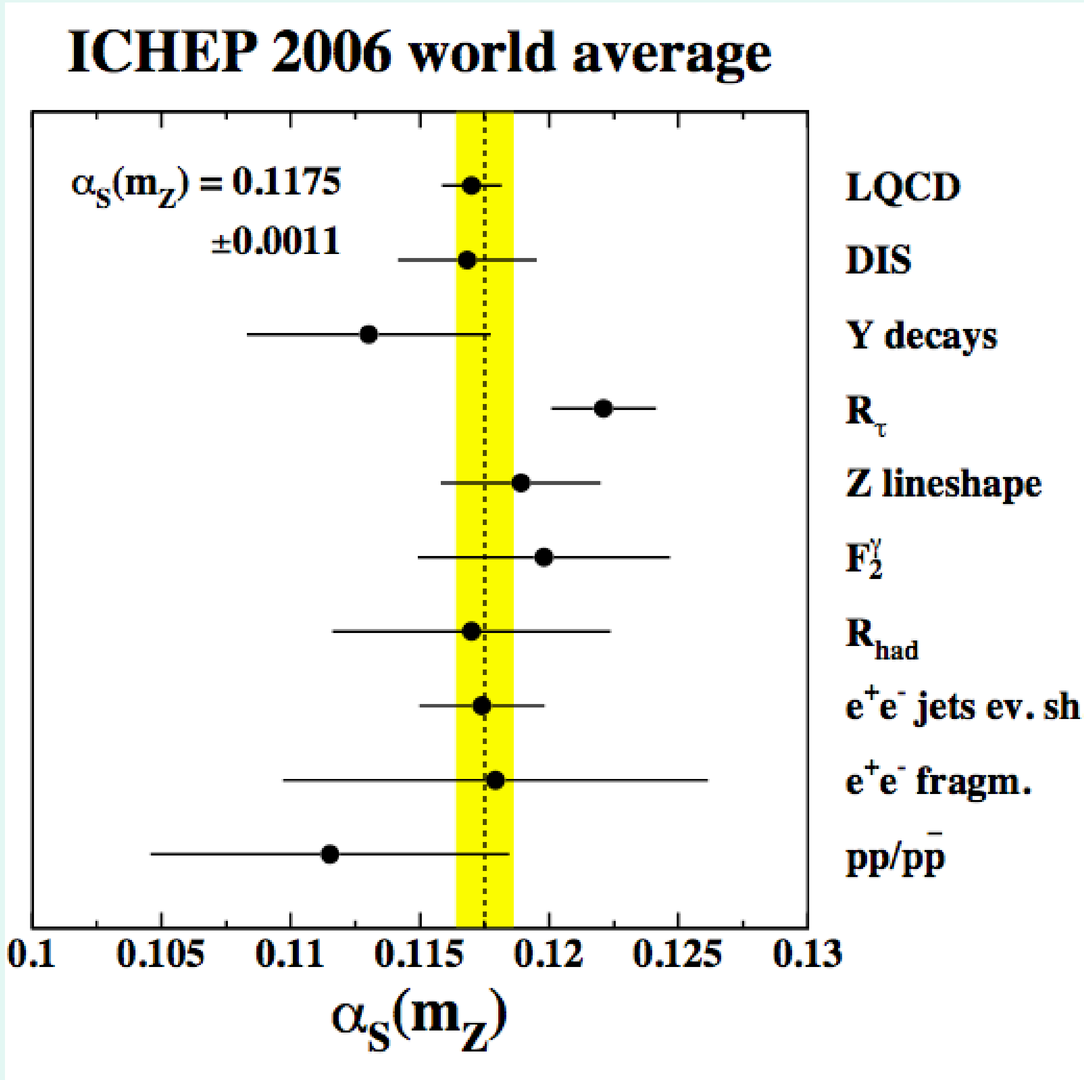


G. Altarelli 1989



S. Kluth EPS, 2007

S Kluth, hep-ex/0609020



α_s known to ~1%!   2006 World average $\alpha_s(M_z) = 0.1175 \pm 0.0011$

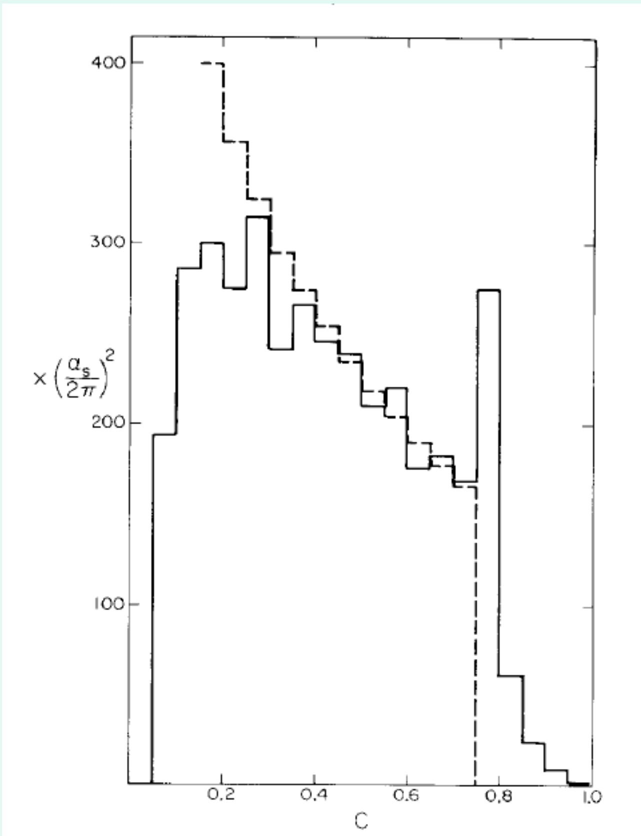
α_s from event shapes: Prehistory

- In 1980 RKE, D.A. Ross and Terrano considered jet shapes in e^+e^- annihilation.
- We introduced the subtraction method for the cancellation of real and virtual singularities.
- We calculated the NLO corrections to the C-parameter defined in terms of the eigenvalues of the 3x3 matrix θ. C is the coefficient of the linear term in the characteristic equation.

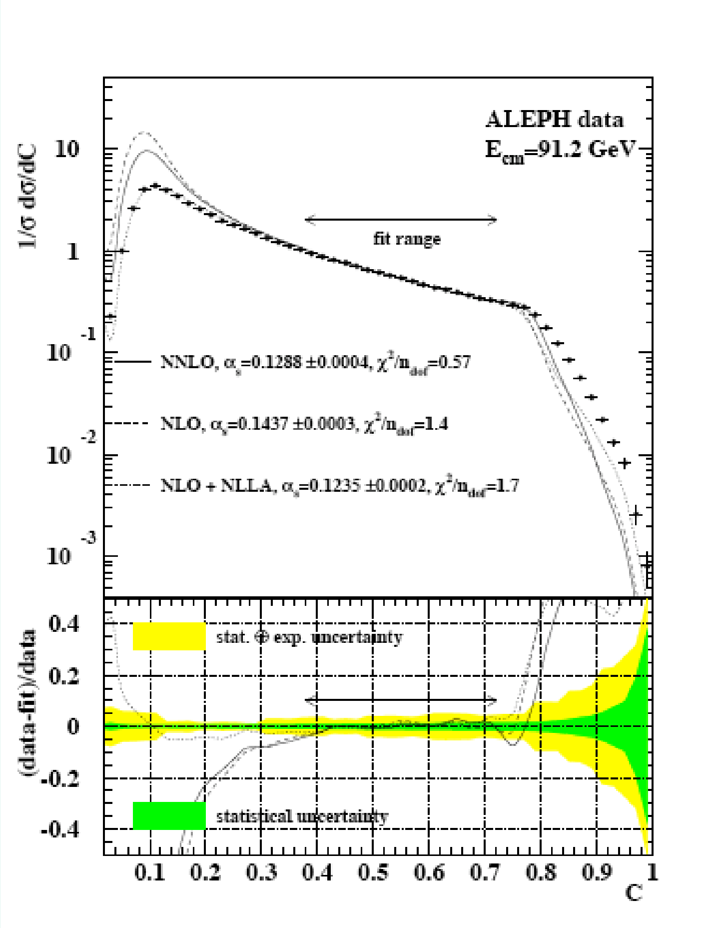
$$\theta^{ij} = \sum_a \frac{P_a^i P_a^j}{|P_a|} \Big/ \sum_a |P_a|$$

$$\lambda^3 - \lambda^2 + \tfrac{1}{3} C\lambda - \tfrac{1}{27} D = 0$$

C parameter



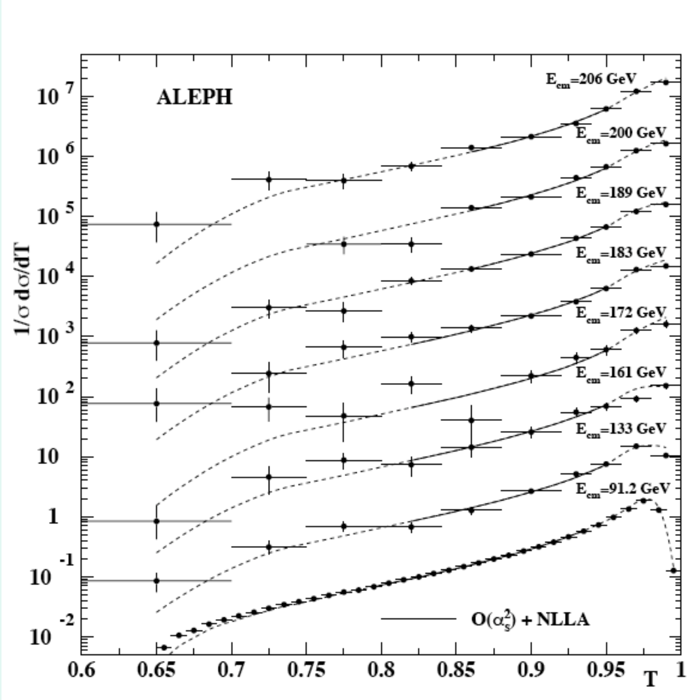
In 1980 the NLO corrections were found to be large. The dashed curve shows LO result multiplied by $(1+ 4.45\ \alpha_s)$.



Dissertori et al, arXiv:0712.0327v2

NNLO results for thrust

The thrust is defined as the maximum of directed momentum
$$T = \max_{\mathbf{n}} \frac{\sum_i \mathbf{p}_i \cdot \mathbf{n}}{\sum_i |\mathbf{p}_i|}$$



Defining τ=1-T, perturbative expansion of thrust is

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \delta(\tau) + \frac{\alpha_S}{2\pi}A(\tau) + \left(\frac{\alpha_S}{2\pi}\right)^2 B(\tau) + \left(\frac{\alpha_S}{2\pi}\right)^3 C(\tau)$$

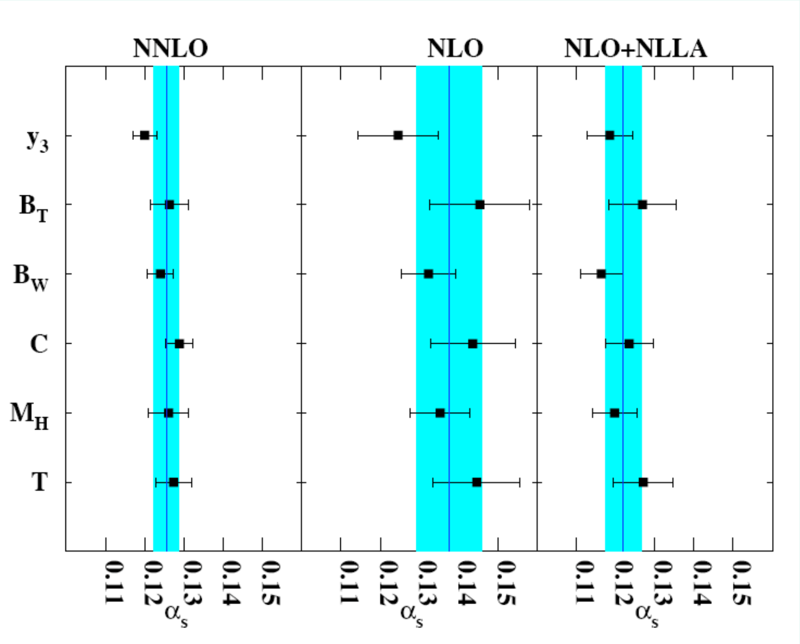
A=LO, Farhi, 1977
B=NLO, Kunszt 1980
C=NNLO, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 2007

After many years of work the calculation of the $O(\alpha_s^3)$ term was completed by Gehrmann et al.
The first time a subtraction scheme has been implemented at NNLO.

Dissertori et al, arXiv 0712.0327

Determination of α_s from NNLO

- Spread between different observables is reduced because of different event shape variables have different NNLO corrections.
- Scale variation uncertainty reduced by a factor 2 with respect to NLO.
- Scale variation error still the largest.



$$\alpha_s(M_Z^2) = 0.1240 \ \pm\ 0.0008\,(\mathrm{stat})\ \pm\ 0.0010\,(\mathrm{exp})\ \pm\ 0.0011\,(\mathrm{had})\ \pm\ 0.0029\,(\mathrm{theo})$$

Resummation results for thrust

Expression for thrust contains large logarithms for small τ

$$R(\tau) = \int_0^\tau d\tau' \frac{1}{\sigma_0} \frac{d\sigma}{d\tau'} \quad \lim_{\tau \to 0} \quad 1 + \frac{2\alpha_S}{3\pi} \left[-2\ln^2\tau - 3\ln\tau \right] + \ldots$$

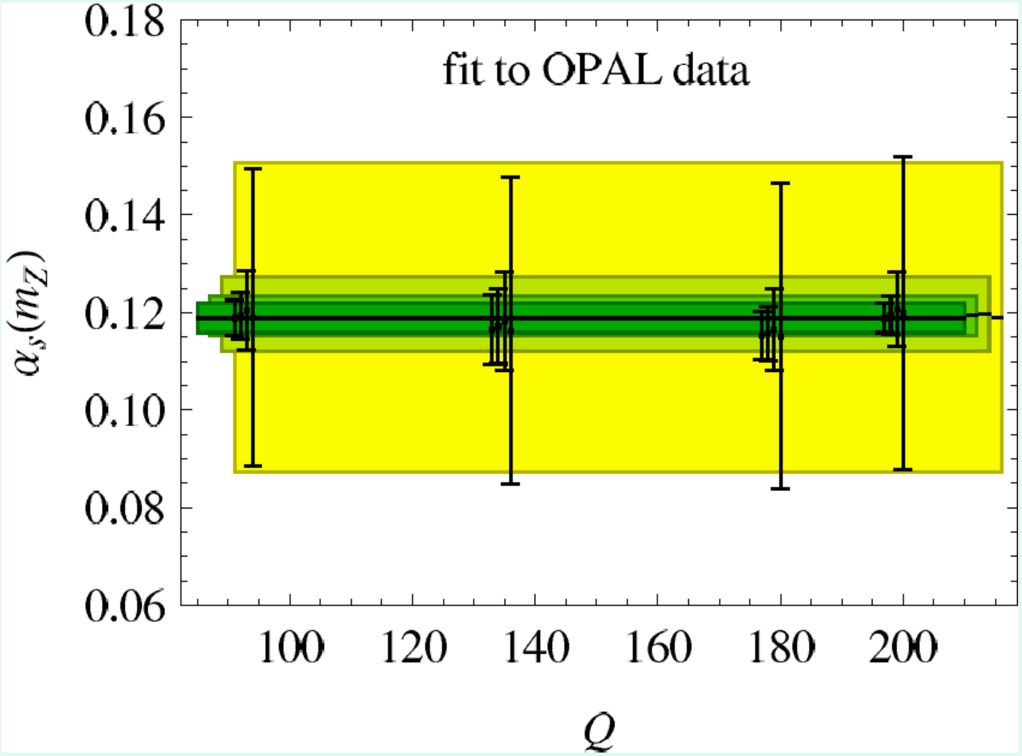
$$\begin{array}{rcl} \alpha_S^n \ln^{2n} \tau & : & \mathrm{LL} \\ \alpha_S^n \ln^{2n-1} \tau & : & \mathrm{NLL} \\ \alpha_S^n \ln^{2n-2} \tau & : & \mathrm{N^2LL} \\ \alpha_S^n \ln^{2n-3} \tau & : & \mathrm{N^3LL} \end{array}$$

Catani et al 1993
Becher, Schwartz, 2008
Becher, Schwartz, 2008

Becher and Schwartz, arXiv:0803.0342

Matched result for α_s in N^3LLA+NNLO

- N^3LLA resummation performed using soft collinear effective theory.
- Expansion of resummed result near kinematic endpoint provides a check of fixed order NNLO calculation.
- For the first time the scale variation error is not the largest error.



$$\alpha_s(m_Z) = 0.1172 \pm 0.0010(\mathrm{stat}) \pm 0.0008(\mathrm{sys}) \pm 0.0012(\mathrm{had}) \pm 0.0012(\mathrm{pert})$$

Parton distribution functions

Measurement of the non-perturbative parton distributions at lower energies allow extrapolations to higher values of μ and lower values of x using the DGLAP equation

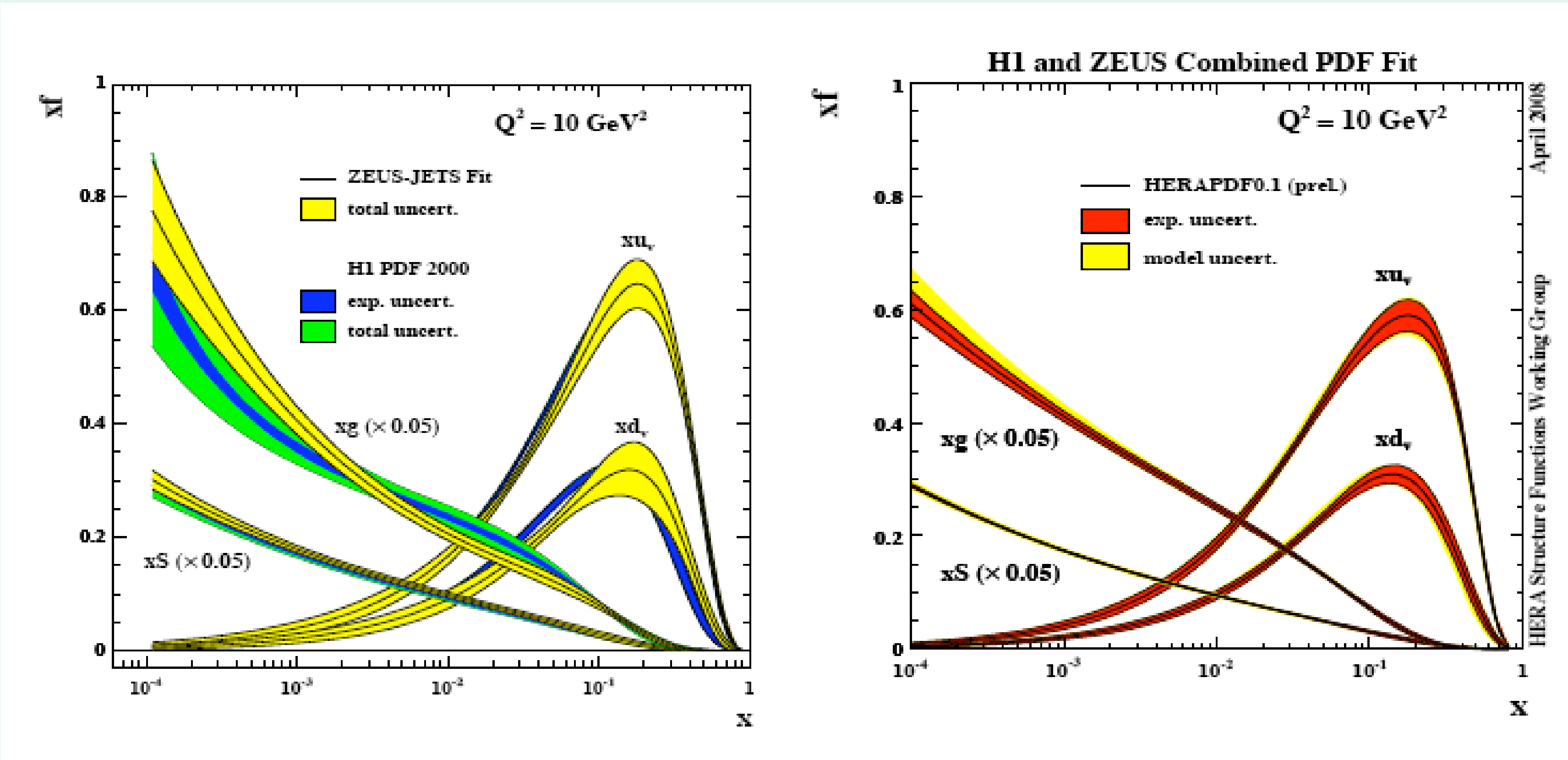
$$\frac{\partial}{\partial \ln \mu^2} f_i(x,t) = \frac{\alpha_S(\mu)}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{ij}\left(\frac{x}{\xi}, \alpha_S(\mu)\right) f_j(\xi, \mu)$$

The evolution kernel is calculable as a perturbation series in α_s

$$P_{ij}(z,\alpha_S) = \underbrace{P_{ij}^{(0)}(z)}_{\text{LO}} + \underbrace{\frac{\alpha_S}{2\pi} P_{ij}^{(1)}(z)}_{\text{NLO}} + \underbrace{\left(\frac{\alpha_S}{2\pi}\right)^2 P_{ij}^{(2)}(z)}_{\text{NNLO}} \ldots$$

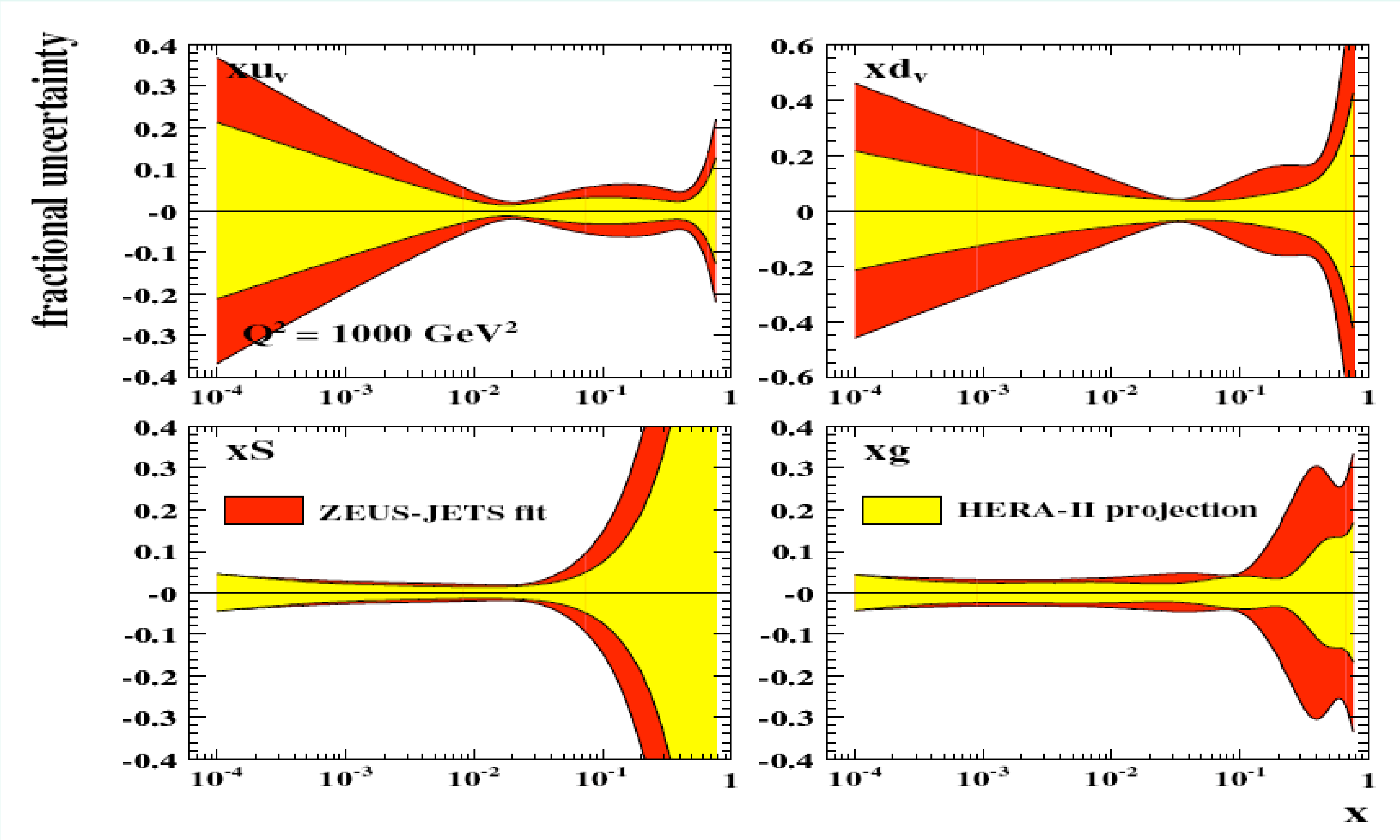
NNLO is known completely. (Moch et al, hep-ph/0403192) arXiv:0903.3861

Comparison of H1 and Zeus



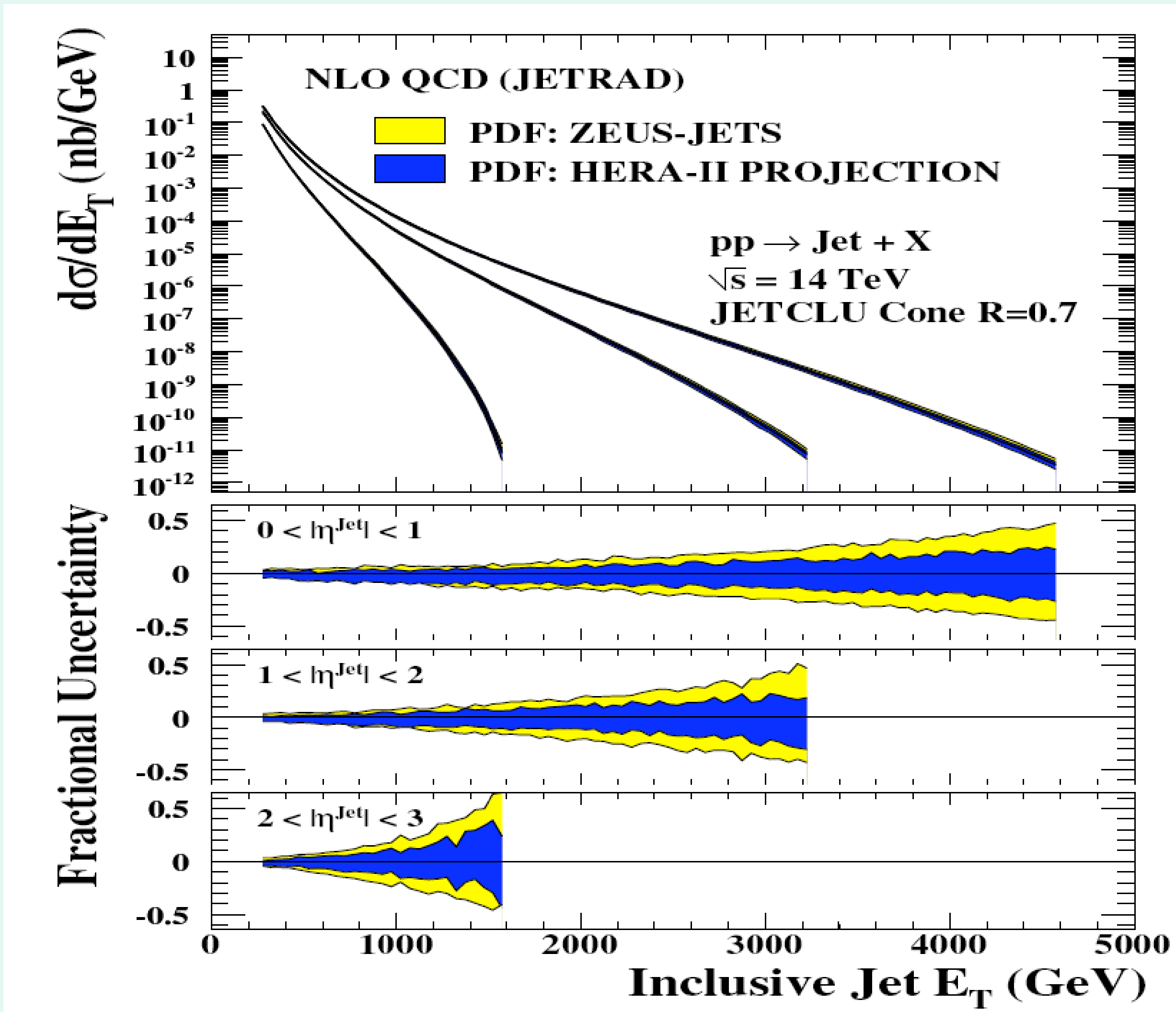
Some of the differences are understood (inclusion of BCDMS at large x (ZEUS) ; inclusion of jet data for mid x gluon (H1))

Projected parton model uncertainties after HERAII

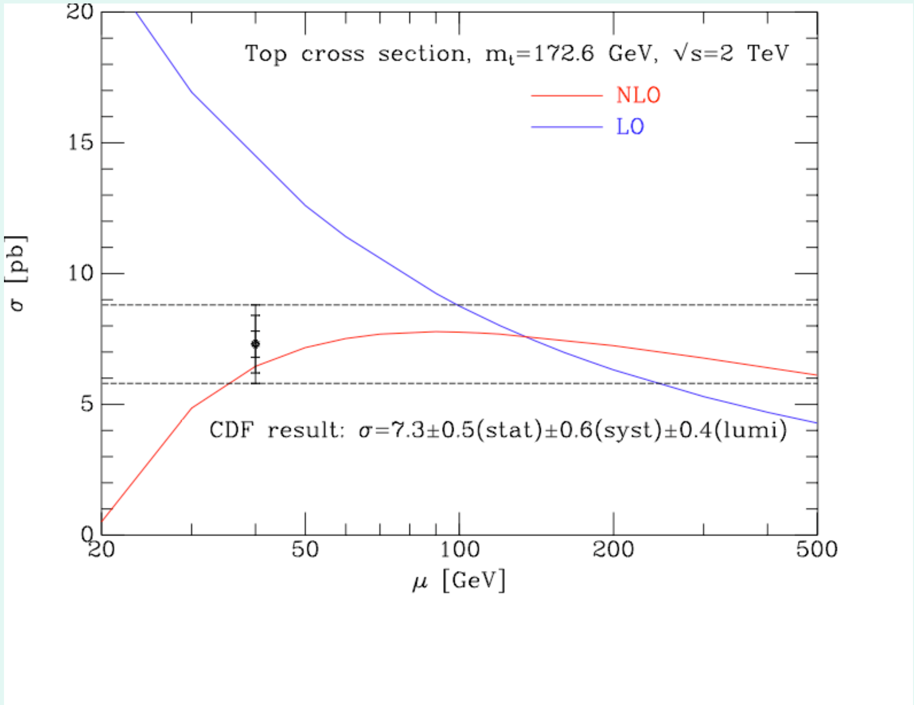


HERA for LHC workshop,hep-ph/0601012

…and consequent improvement on uncertainty of jet cross section



Why NLO?



In order to get ~10% accuracy we need to include NLO.

- Less sensitivity to unphysical input scales, (eg. renormalization and factorization scales)
- NLO first approximation in QCD which gives an idea of suitable choice for μ.
- NLO has more physics, parton merging to give structure in jets, initial state radiation, more species of incoming partons enter at NLO.
- A necessary prerequisite for more sophisticated techniques which match NLO with parton showering.

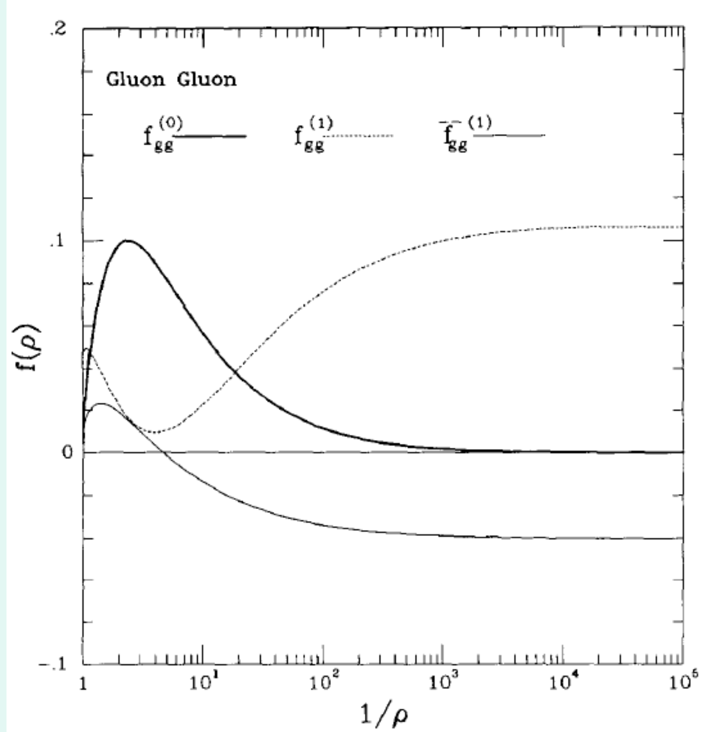
Nason,Dawson,Ellis

Top total cross section at NLO

Short distance cross section given by

$$\hat{\sigma}_{ij}(s, m^2, \mu^2) = \frac{\alpha_S^2(\mu^2)}{m^2} f_{ij}\left(\rho, \frac{\mu^2}{m^2}\right)$$

$$\rho = \frac{4m^2}{s}, \quad \beta = \sqrt{1-\rho}$$



$$f_{ij}(\rho, \mu^2/m^2) = f_{ij}^{(0)}(\rho) + g^2(\mu^2)\left[f_{ij}^{(1)}(\rho) + \bar{f}_{ij}^{(1)}(\rho)\ln(\mu^2/m^2)\right]$$

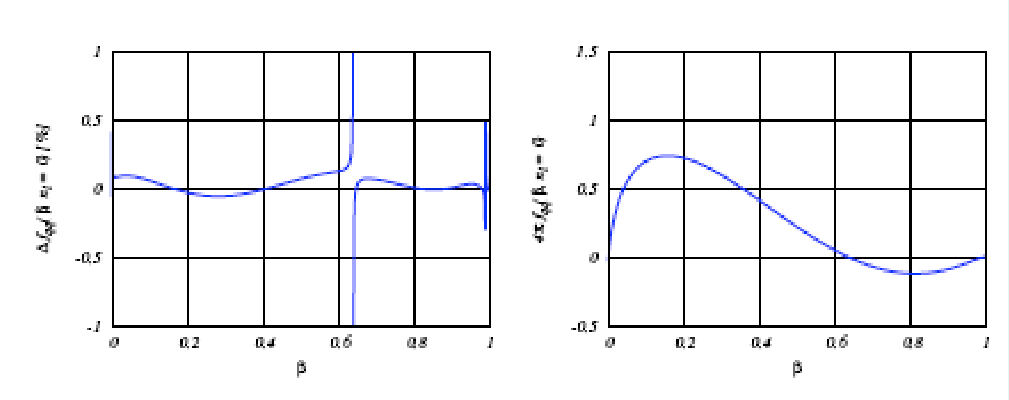
NDE 1988: $f_{ij}^{(0)}$ and $\bar{f}_{ij}^{(1)}$ calculated analytically,
$f_{ij}^{(1)}$ provided as a numerical fit with accuracy better than 1%

Czakon-Mitov arXiv:0811.4119

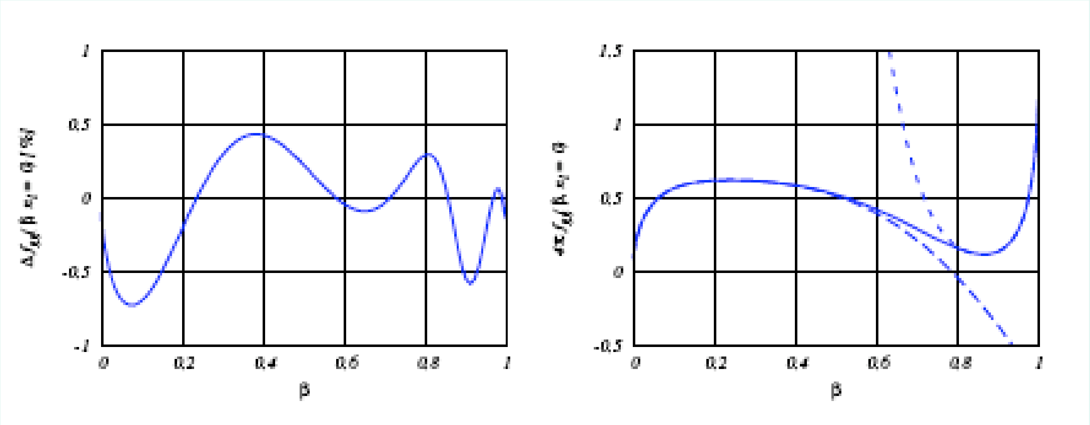
Total cross section for top: Analytic results (2008)

- Czakon and Mitov have analytic results for the total cross section.
- Results agree with NDE fit within stated tolerance (1%).

Comparison with NDE fit for q-qbar



Comparison with NDE fit for g-g



Czakon-Mitov. arXiv 0811.4119

Resummation of Threshold Logarithms

NLO calculation can be used to determine coefficient in resummed formula. Despite accuracy of fit , constant coefficient C_3 was not well determined in fit.

$$f_{gg}^{(1)}(\rho) = \frac{1}{4\pi^2} f_{gg}^{(0)}(\rho) \left[\frac{11\pi^2}{84\beta} + 6\ln^2(8\beta^2) - \frac{183}{7}\ln(8\beta^2) + C_3 + \ln(\frac{\mu^2}{m^2})(\frac{35}{3} - 6\ln(4\beta^2)) \right]$$

$$C_3 = \frac{1111}{21} + \frac{15}{7}\ln 2 - 6\ln^2 2 - \frac{283\pi^2}{168} \approx 34.88, \quad \text{c.f. } C_3^{\mathrm{NDE}} = 37.25$$

Phenomenological impact on the cross section, not yet published, but it will probably lead to a modest decrease ~1% (Nason, private communication).

Resummation of Threshold Logs

Resummation performed in moment space, separately for each color channel

$$\sigma_N(m^2) = \int_0^1 d\rho\, \rho^{N-1} \sigma(\rho, m^2)$$

$$\sigma_{\mathbf{1},\mathbf{8}}(N) = \hat{\sigma}^{(LO)}_{\mathbf{1},\mathbf{8}}(N)\sigma^{H}_{\mathbf{1},\mathbf{8}}(N)\Delta_{\mathbf{1},\mathbf{8}}(N)$$

$$\begin{aligned}\Delta(N) &\sim \left[1 + \sum_{n=1}^{\infty} \alpha_S^n \sum_{m=1}^{2n} c_{n,m} \ln^m N\right] \\ &= \exp\left[\ln N g^{(1)}(\alpha_S \ln N) + g^{(2)}(\alpha_S \ln N) + \alpha_S g^{(3)}(\alpha_S \ln N) + \ldots\right]\end{aligned}$$

$$LL \qquad\qquad NLL \qquad\qquad NNLL$$

Cacciari et al , arXiv 0804.2800

Top at LHC: Uncertainty budget

Best prediction (without the update in coefficient C_3)

$$\sigma_{t\bar{t}}^{\mathrm{NLO+NLL}}(\mathrm{LHC}, m_t = 171\ \mathrm{GeV}, \mathrm{CTEQ6.5}) = 908\ {}^{+82(9.0\%)}_{-85(9.3\%)}\ (\mathrm{scales})\ {}^{+30(3.3\%)}_{-29(3.2\%)}\ (\mathrm{PDFs})\ \ \mathrm{pb}$$

LO and NLO

$$\sigma_{t\bar{t}}^{\mathrm{NLO}}(\mathrm{LHC}, m_t = 171\ \mathrm{GeV}, \mathrm{CTEQ6.5}) = 875\ {}^{+102(11.6\%)}_{-100(11.5\%)}\ (\mathrm{scales})\ {}^{+30(3.4\%)}_{-29(3.3\%)}\ (\mathrm{PDFs})\ \ \mathrm{pb}$$

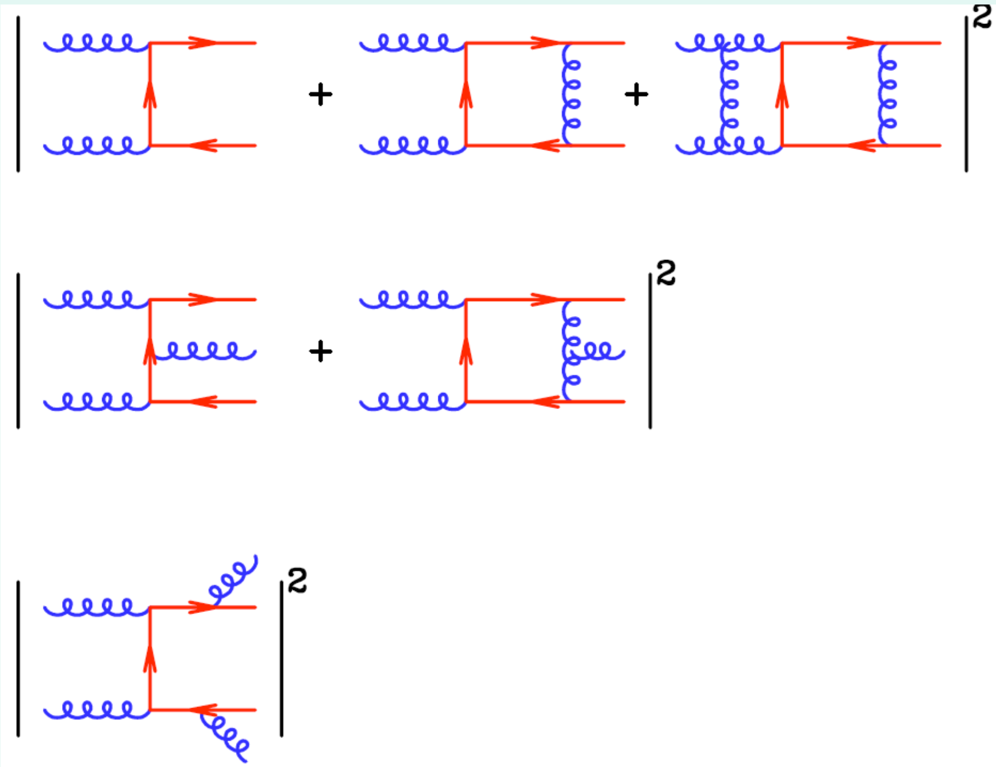
$$\sigma_{t\bar{t}}^{\mathrm{LO}}(\mathrm{LHC}, m_t = 171\ \mathrm{GeV}, \mathrm{CTEQ6.5}) = 583\ {}^{+165(28.2\%)}_{-120(20.7\%)}\ (\mathrm{scales})\ {}^{+20(3.4\%)}_{-19(3.3\%)}\ (\mathrm{PDFs})\ \ \mathrm{pb}$$

Top mass uncertainty --> 5 $\Delta m_t / m_t \sim 5\%$

- Scale uncertainty dominant at LHC.
- Limited benefit for scale uncertainty from NLL resummation.
- Provides motivation for NNLO calculation, reduction of scale uncertainty to 3%?, Moch-Uwer et al , arXiv 0804.1476

Progress on the NNLO Top quark cross section

- Motivation: Scale dependence is dominant error at LHC.
- Standard candle for gg flux.



Loop-by-loop, Anastasiou, 0809.1355
Korner et al, arXiv:, 0802.0106, 0809.3980
2-loop amplitudes, qqbar
Czakon, arXiv:0803.1400

Tt+jet, Dittmaier arXiv:0810.0452

MCFM
A NLO parton level generator

- pp ➔ W/Z
- pp ➔ W+Z, WW, ZZ
- pp ➔ W/Z + 1 jet
- pp ➔ W/Z +2 jets
- pp ➔ t W
- pp ➔ tX (s&t channel)
- pp ➔ tt
- pp ➔ W/Z+H
- pp (gg) ➔ H
- pp➔(gg) ➔ H + 1 jet
- pp➔(gg) ➔ H + 2 jets
- pp(VV) ➔ H +2 jets
- pp ➔ W/Z +b , W+c
- pp ➔ W/Z +bb

Processes calculated at NLO, but no automatic procedure for including new processes.

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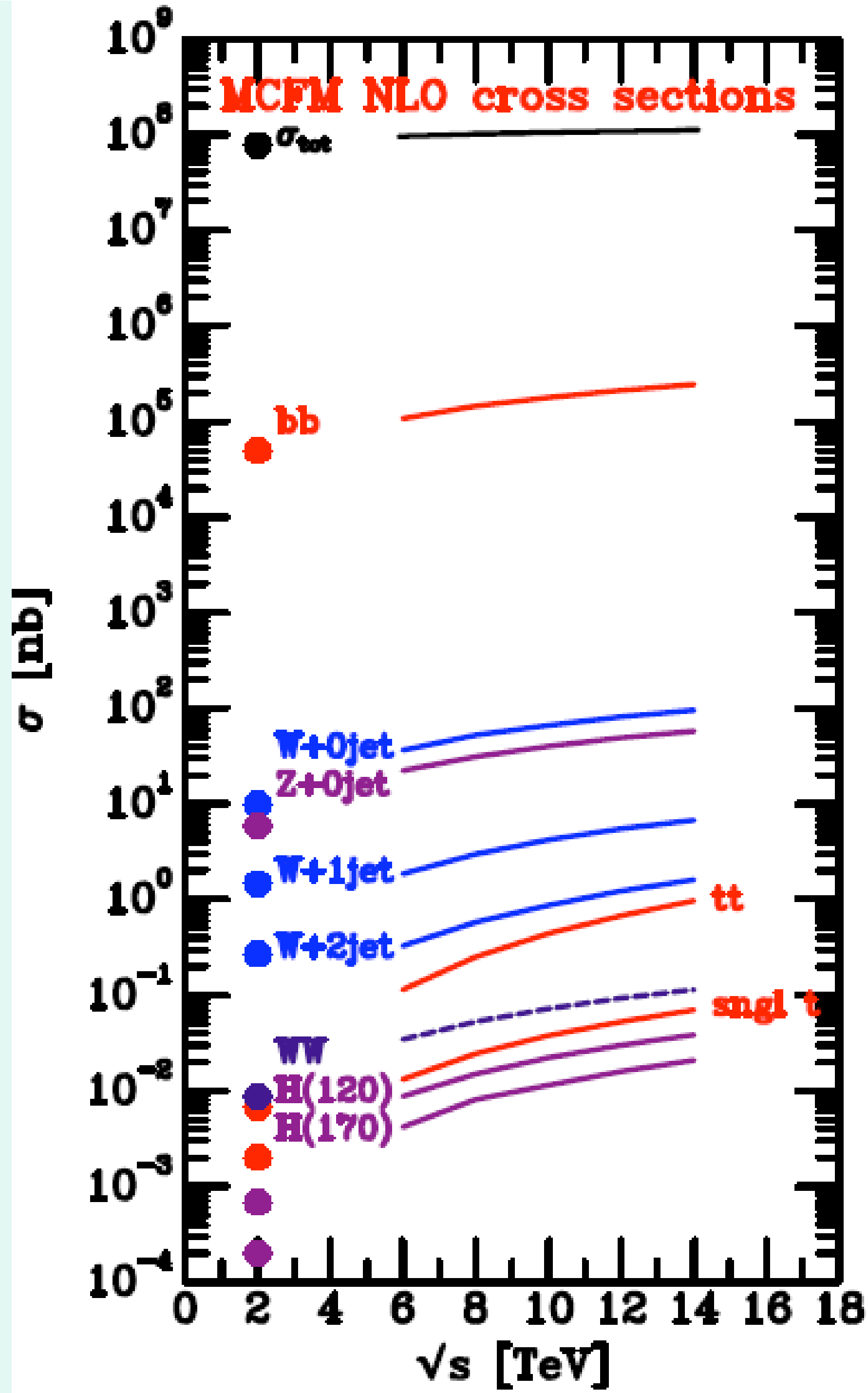
Code available at http://mcfm.fnal.gov

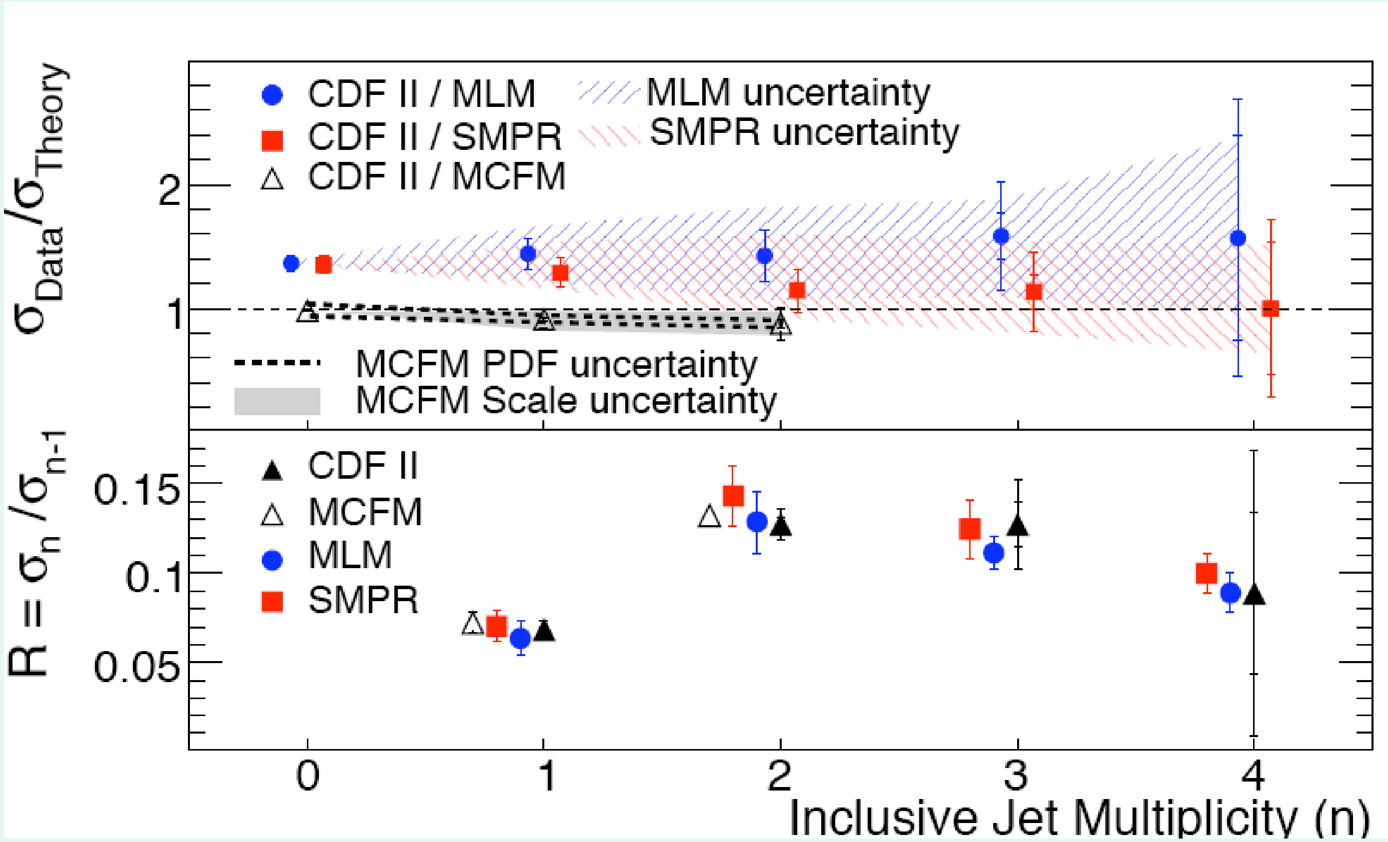
Current version 5.4 (March 11, 2009)

The big picture

- MCFM contains the best predictions for many processes of relevance for Tevatron and LHC.
- LHC will be a great machine because of the increase of both energy and luminosity wrt to the Tevatron.
- Dramatic growth with energy of gluon-induced processes (eg tt).



W+n jet rates from CDF

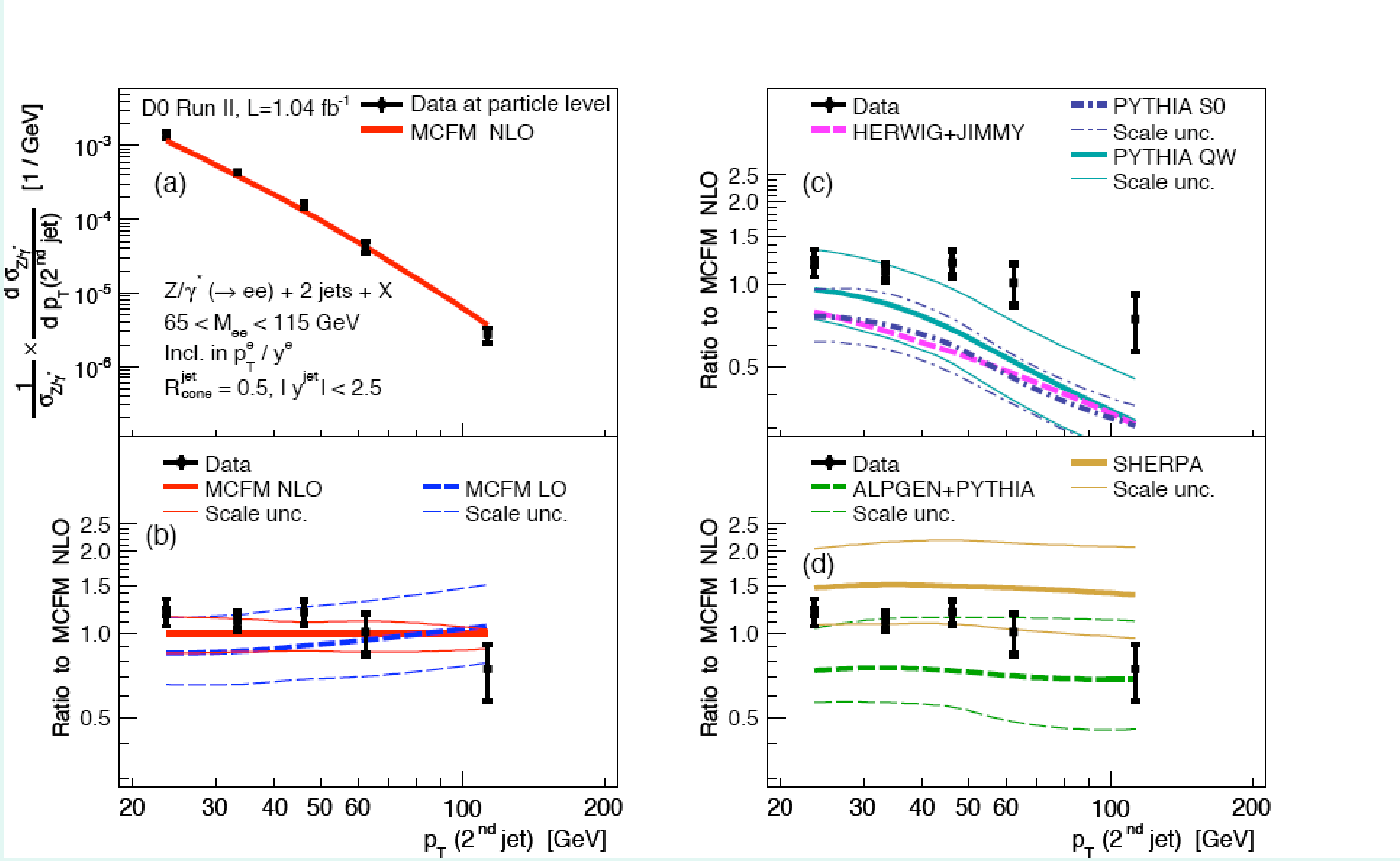


Both uncertainty on rates and deviation of Data/Theory from 1 are smaller than other calculations. The ratio R agrees well for all theory calculations, but only available from MCFM with small error for n≤2.

MCFM RKE, Campbell

D0 arXiv:0903.1748

New Z + jets results from D0



- MCFM, LO and NLO agrees with data;
- shower-based generators show significant differences with data;
- matrix element + parton shower models agree in shape, but with larger normalization uncertainties.

Extension to multi-leg processes

- At the LHC we are interested in processes with many jets; these have standard model backgrounds involving many legs.
- The NLO calculation of multi-leg processes is pressing because the dependence on the unphysical scales is so strong.
- We need both efficient methods to calculate tree diagrams and efficient methods to calculate loops.

The calculation of one loop amplitudes

- The classical paradigm for the calculation of one-loop diagrams was established in 1979.
- Complete calculation of one-loop scalar integrals
- Reduction of tensors one-loop integrals to scalars.

Nuclear Physics B153 (1979) 365–401
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SCALAR ONE-LOOP INTEGRALS

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Received 16 January 1979

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ONE-LOOP CORRECTIONS FOR e^+e^- ANNIHILATION INTO $\mu^+\mu^-$ IN THE WEINBERG MODEL

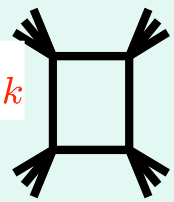
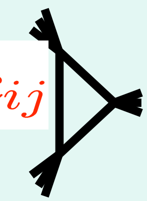
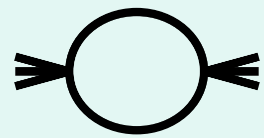
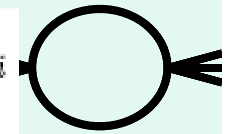
G. PASSARINO* and M. VELTMAN
Institute for Theoretical Physics, University of Utrecht, Utrecht, The Netherlands

Received 22 March 1979

Neither will be adequate for present-day purposes.

Basis set of scalar integrals

Any one-loop amplitude can be written as a linear sum of scalar box-, triangle-, bubble- and tadpole-integrals.

$\mathcal{A}_N(\{p_i\}) = \sum d_{ijk}$  $+ \sum c_{ij}$  $+ \sum b_i$  $+ \sum_i a_i$ 

In the context of NLO calculations, scalar higher point functions, can always be expressed as sums of box integrals. Passarino, Veltman - Melrose ('65)

•Scalar hexagon can be written as a sum of six pentagons.

•For the purposes of NLO calculations, the scalar pentagon can be written as a sum of five boxes.

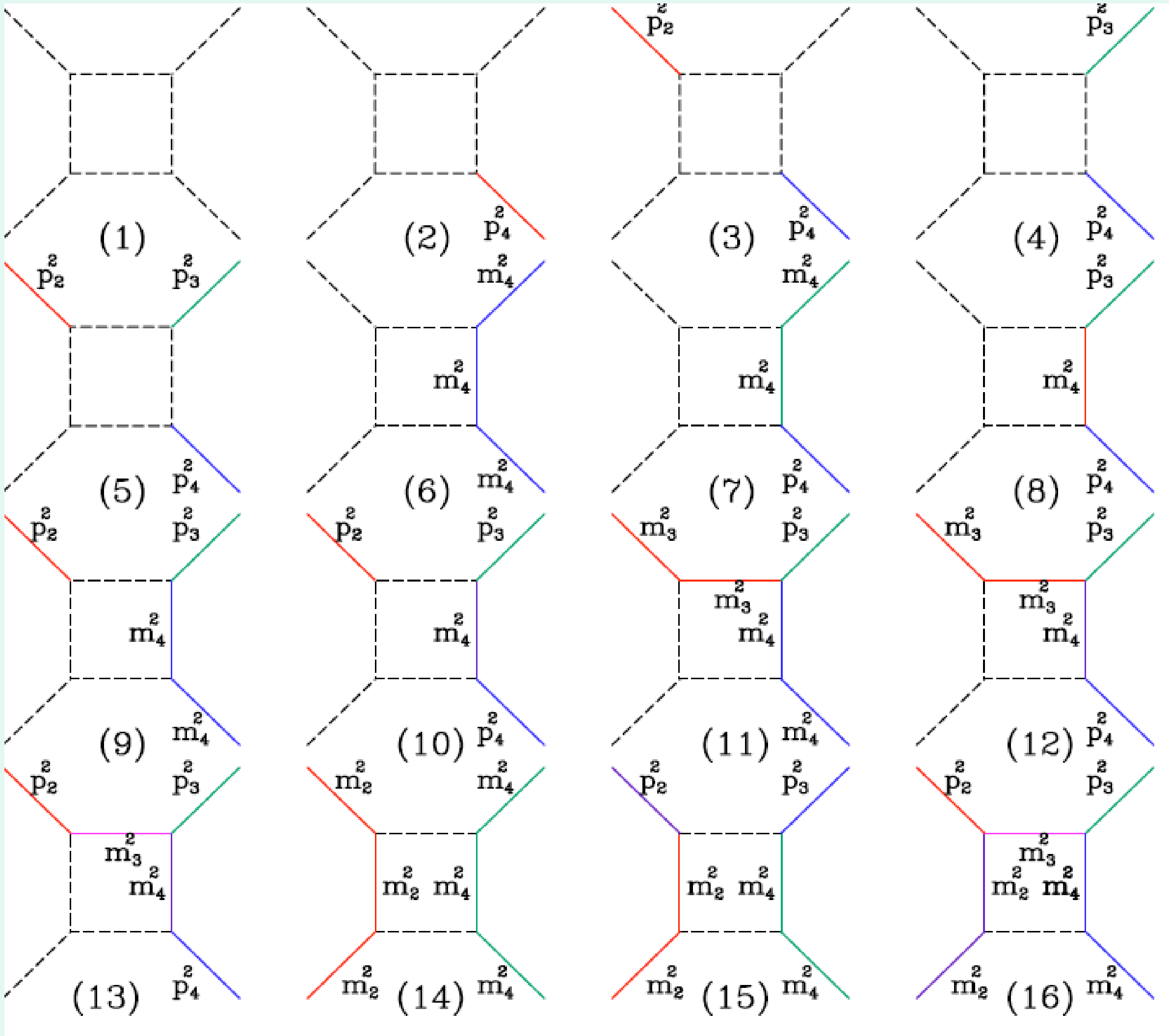
•In addition to the finite integrals we need integrals containing infrared and collinear divergences.

Scalar one-loop integrals

- ‘t Hooft and Veltman’s integrals contain internal masses; however in QCD many lines are (approximately) massless. The consequent soft and collinear divergences are regulated by dimensional regularization.
- Analytic results are given for boxes, triangles, bubbles and tadpoles, including the cases with one or more vanishing internal masses at http://qcdloop.fnal.gov
- Fortran 77 code is provided which calculates an arbitrary scalar box, triangle, bubble or tadpole integral.
- Problem of one-loop scalar integrals for NLO calculations is completely solved numerically and analytically!

Basis set of sixteen divergent box integrals

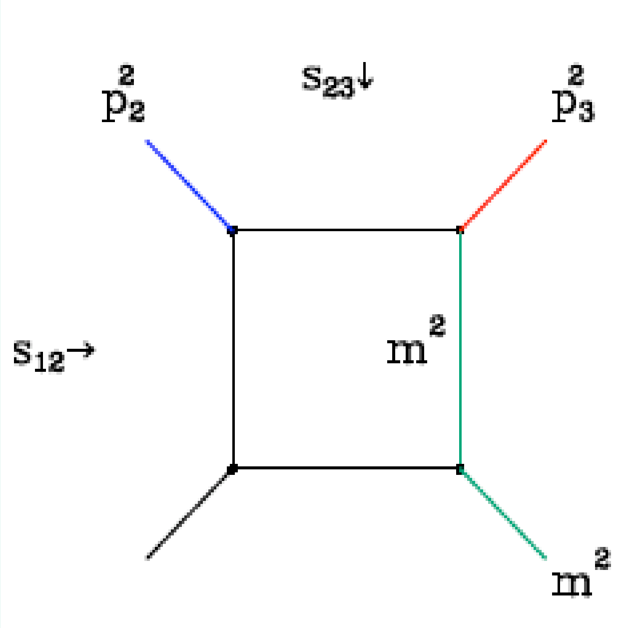
RKE, Zanderighi



Dashed lines massless, lines of same colour have same virtuality/mass

Example of box integral from qcdloop.fnal.gov

Basis set of 16 basis integrals allows the calculation of any divergent box diagram.
Result given in the spacelike region.
Analytic continuation as usual by
$s_{ij} \Rightarrow s_{ij} + i\,\varepsilon$



$$I_4^{\{D=4-2\epsilon\}}(0,p_2^2,p_3^2,m^2;s_{12},s_{23};0,0,0,m^2) = \frac{1}{s_{12}(s_{23}-m^2)}\left[\frac{1}{2\epsilon^2} - \frac{1}{\epsilon}\ln\left(\frac{s_{12}}{p_2^2}\frac{(m^2-s_{23})}{\mu m}\right)\right.$$
$$\left. + \;\; \mathrm{Li}_2\left(1+\frac{(m^2-p_3^2)(m^2-s_{23})}{m^2 p_2^2}\right) + 2\,\mathrm{Li}_2\left(1-\frac{s_{12}}{p_2^2}\right) + \frac{\pi^2}{12} + \ln^2\left(\frac{s_{12}}{p_2^2}\frac{(m^2-s_{23})}{\mu m}\right)\right] + \mathcal{O}(\epsilon)\,.$$

Limit $p_3^2 = 0$ can be obtained from this result, (limit $p_2^2 = 0$ cannot)

Determination of coefficients of scalar integrals

Feynman diagrams may not be the answer as the number of legs increases. There are too many diagrams with cancellations between them.

Process	Amplitude	# of diagrams at 1 loop
$t\bar{t}$	$t\bar{t}gg$	30
$t\bar{t}$+1 jet	$t\bar{t}ggg$	341
$t\bar{t}$+2 jets	$t\bar{t}gggg$	4341
$t\bar{t}$+3 jets	$t\bar{t}ggggg$	63800

Semi-numerical methods based on unitarity offer great promise. "Semi-numerical" because the integral containing the divergences is determined analytically, but its coefficient is determined numerically.

Unitarity for one-loop diagrams

- Important steps include:-
- First modern use of the idea Bern, Dixon,Kosower
- Cuts w.r.t. to loop momenta give (box) coefficients directly Cachazo, Britto, Feng
- OPP tensor reduction scheme, Ossola, Pittau, Papadopoulos
- Integrating the OPP procedure with unitarity Ellis, Giele, Kunszt
- D-dimensional unitarity Giele, Kunszt, Melnikov

Giele, Kunszt,Melnikov, arXiv 0801.2237

Unitarity in D-dimensions

- The theory contains divergences which we regulate dimensionally. Divergences give poles as ε =(4-D)/2 ->0
- Calculate the unitarity cuts numerically in integer dimensions D>4. Internal degrees of freedom are taken to be D_s dimensional.

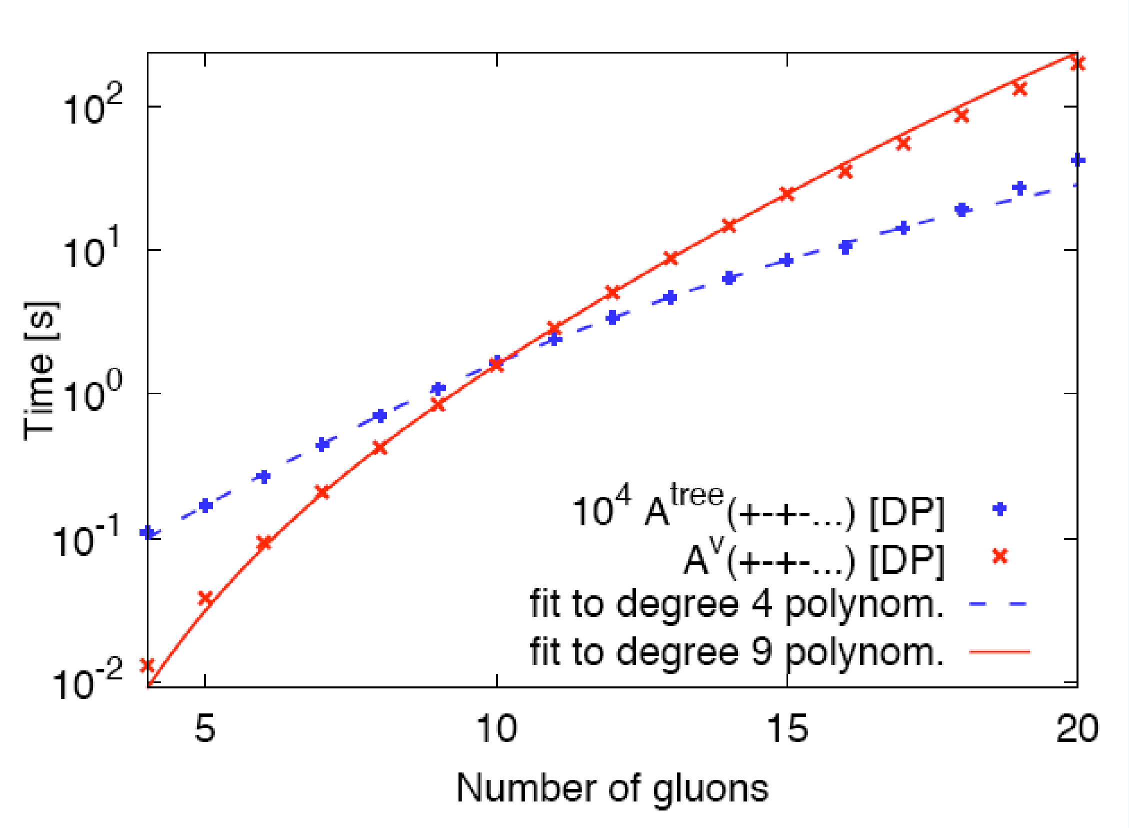
$$\mathcal{N}^{D_s}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l)$$

- Dependence on D_s is linear so we calculating in a two different integer dimensions and extrapolate to ε=0
- Only the length of the loop momentum in the extra dimension is relevant so we can treat the loop momentum as five-dimensional.

Giele, Zanderighi arXiv:0805.2152

One loop calculation of pure gluon amplitudes

Time to calculate one-loop amplitude scales as N^9 as expected. For small numbers of legs N=4,5,6 the times are of the order of 10's of milliseconds



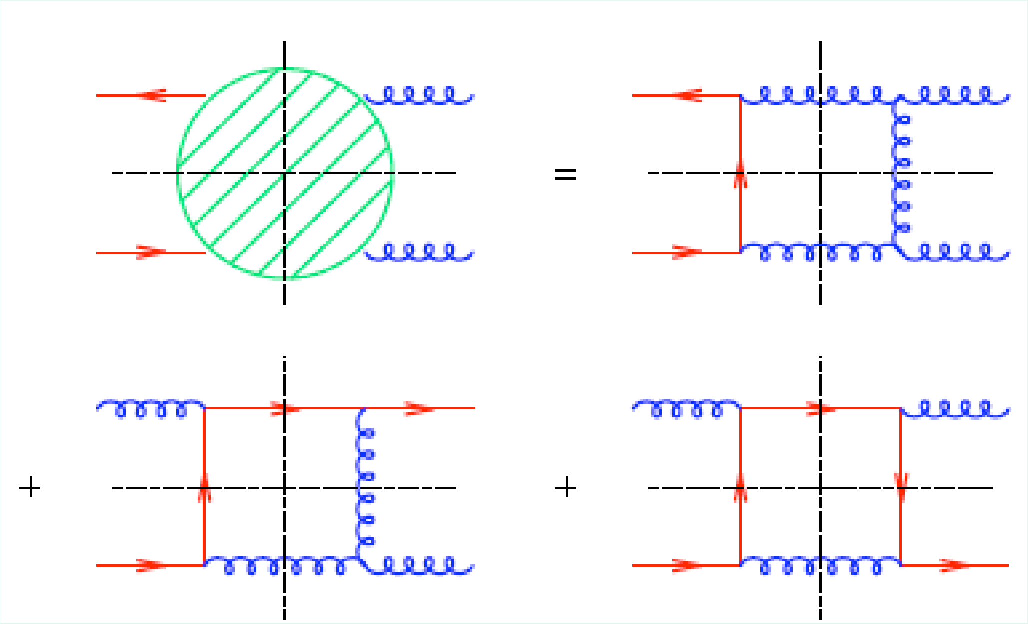
4g:Ellis-Sexton(1985)
5g:Bern-Dixon-Kosower(1993)
6g:Ellis-Giele-Zanderighi(2006)

Ellis, Giele, Kunszt Melnikov:arXiv:0806.3467

Generalized unitarity and massive fermions

We have calculated the one-loop amplitudes for ttgg and ttggg as a proof of principle that the method can be extended to massive particles. Calculation times are longer than pure gluon amplitudes

Thus ttgg ~ 10 ms (cf 1ms for gggg) and ttggg ~ 40 ms.



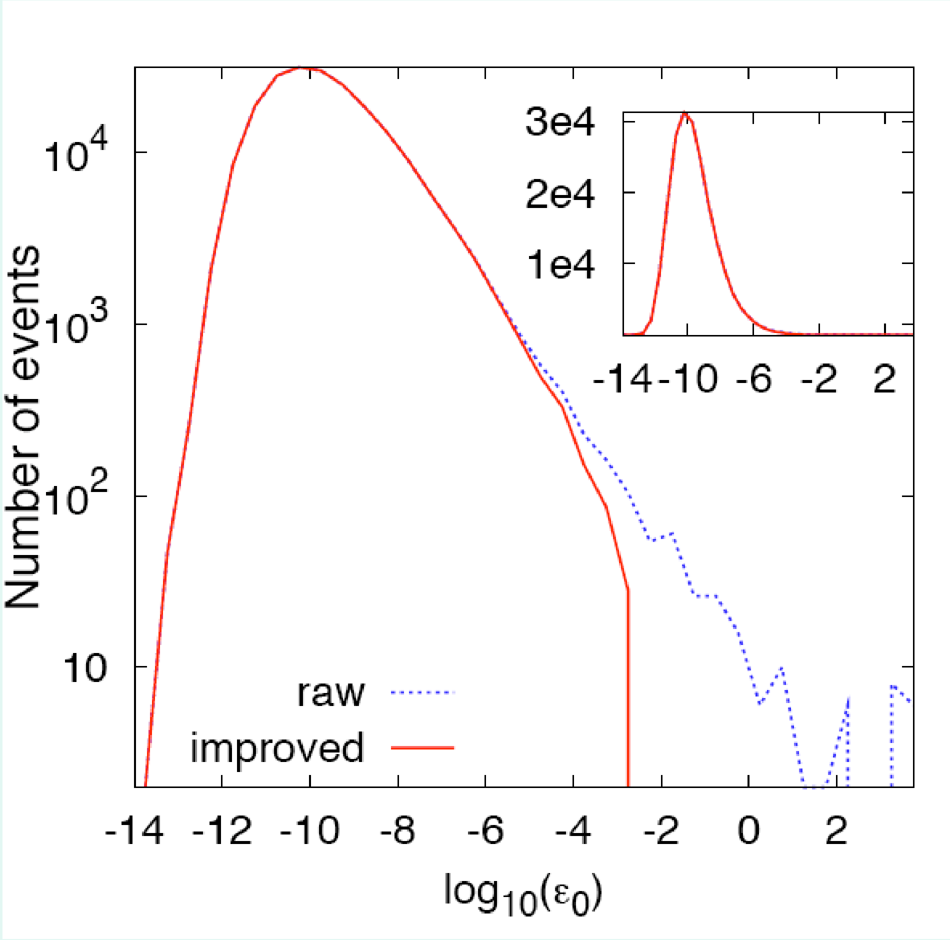
EGKMZ, arXiv:0810.2762

W+qqggg amplitudes

Numerical stability assured by computation, (where necessary) in quadruple precision.

$\varepsilon_0 = |(DP-QP)/QP|$

Evaluation times are 45-50 msec per leading color primitive on 2.33 GHz pentium Xeon machine.



One-loop amplitude summary

- There are a number of groups which use unitarity and OPP ideas to perform one-loop calculations (Berger et al, OPP, Lazopoulos, Giele & Winter).
- The F90 Rocket program (Ellis, Melnikov, Zanderighi) can compute results at one loop for:-
- N gluon scattering amplitudes
- two quarks (massless and massive) + N gluons,
- W-boson + two quarks + N gluons,
- W-boson + four quarks + 1 gluon
- tt+N gluons, ttqq+ N gluons (EGKM +Schulze)
- Note that extension to arbitrary number of gluons (using Berends-Giele recursion), and the proven ability to deal with massive fermions.

RKE, Melnikov,Zanderighi arXiv:0901.4101

W + 3 jets

- Here I report on recent calculations of W+3-jet rate at hadron colliders
- The calculation represents a proof of principle for the unitarity-based methods and is challenging with traditional methods, (1480 1-loop diagrams)
- W+3 jets is phenomenologically relevant because of Tevatron measurements, single top, SUSY searches, …
- More generally the rates for vector boson + jets production at the LHC are important as backgrounds to BSM processes.

RKE, Melnikov, Zanderighi, arXiv:0901.4101

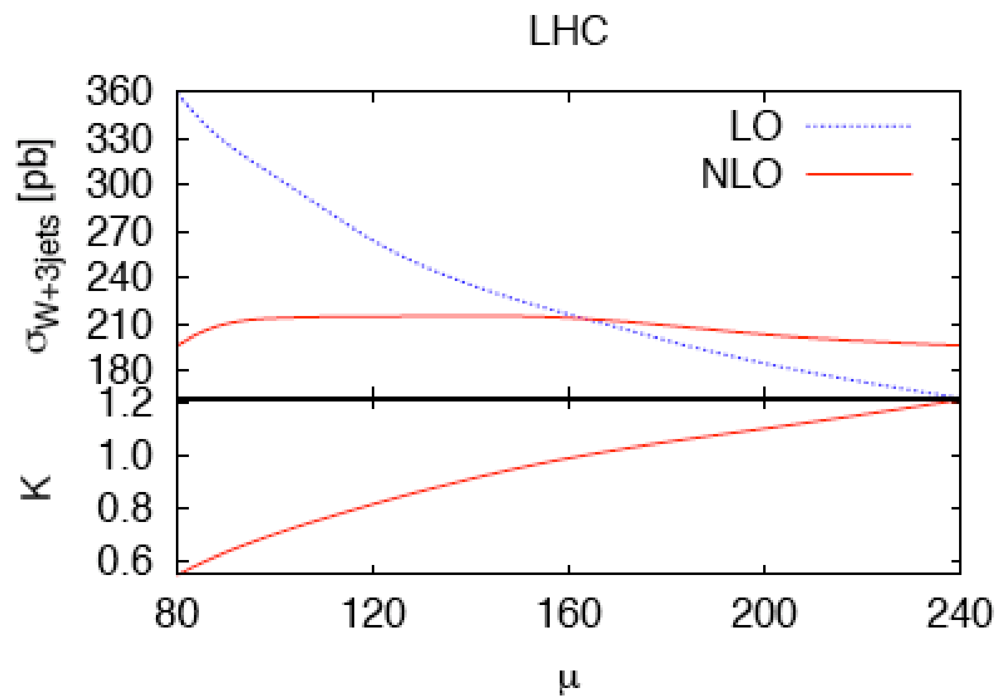
W+3 jets: First NLO QCD results

- We simplify the problem by working at large N_c (N_c is the number of colors) and by keeping only the two quark channels qqW+ggg
- These are 10-30 percent approximations, so the phenomenology is rather preliminary.
- Virtual corrections are computed using a grid determined from the leading order computation
- Dipole subtraction is used for the real emission corrections.

RKE, Melnikov,Zanderighi arXiv:0901.4101

W+3 jets: First NLO QCD results for LHC

Inclusive W+3jets + K factor
p_T>50GeV, $|\eta|$ <3, R=0.7,
$\sqrt{S}$=14 TeV



This calculation displays the standard improvement of scale dependence.

Detailed phenomenology at the 10% level will have to await the inclusion of all processes.

Summary

- $\alpha_S(M_Z)$ is known to < 1% and parton distributions are known well enough to predict most cross sections to 20%, $(0.005<x<0.3)$.
- Theoretical error in jet shapes in e^+e^- annihilation is now for the first time smaller than the experimental error.
- New analytic results on total top production at NLO. Drive to complete NNLO calculation.
- At high p_T, parton level integrators, such as MCFM, can do an adequate job of describing data with smaller theoretical errors than other methods.

Summary (continued)

- Open theoretical problem in calculating multi-leg processes at NLO has been the calculation of one-loop amplitudes
- All one-loop integrals for QCD are known.
- Unitarity based methods have achieved important results for one-loop amplitudes, these methods are now being tested in real physical calculations.
- The hope is to have several semi-automatic methods of calculating one-loop amplitudes.

Collaborators (1974-2009)

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