Progress in QCD

Frontiers in Contemporary Physics, Nashville

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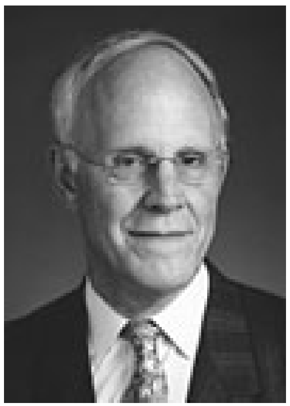
Fermilab

QCD after the prize (2004)

“for the discovery of asymptotic freedom in the theory of the strong interaction”



’a large body of significant advances . . . and are the work of not just three people but a great many scientists, . . . This is really a prize for that whole community’, – David Politzer, Nobel Lecture.

Running coupling

- Consider dimensionless physical observable R which depends on a single large energy scale, $Q \gg m$ where m is any mass. Then we can set $m \to 0$ (assuming this limit exists), and dimensional analysis suggests that R should be independent of Q.
- This is not true in quantum field theory. Calculation of R as a perturbation series in the coupling requires renormalization to remove ultraviolet divergences. This introduces a second mass scale μ — point at which subtractions which remove divergences are performed. R depends on the ratio Q/μ and is not constant. The renormalized coupling α_S also depends on μ.
- But μ is arbitrary! Therefore, if we hold bare coupling fixed, R cannot depend on μ. Since R is dimensionless, it can only depend on Q^2/μ^2 and the renormalized coupling α_S.

Running coupling II

$$\mu^2 \frac{d}{d\mu^2} R\left(\frac{Q^2}{\mu^2}, \alpha_S\right) \equiv \left[\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_S}{\partial \mu^2} \frac{\partial}{\partial \alpha_S}\right] R = 0 \; .$$

- Introducing

$$\tau = \ln\left(\frac{Q^2}{\mu^2}\right), \quad \beta(\alpha_S) = \mu^2 \frac{\partial \alpha_S}{\partial \mu^2} \; ,$$

we have

$$\left[-\frac{\partial}{\partial \tau} + \beta(\alpha_S) \frac{\partial}{\partial \alpha_S}\right] R = 0.$$

Running coupling III

- This renormalization group equation is solved by defining running coupling $\alpha_S(Q)$:

$$\tau = \int_{\alpha_S}^{\alpha_S(Q)} \frac{dx}{\beta(x)} \;, \quad \alpha_S(\mu) \equiv \alpha_S \;.$$

$$\frac{\partial \alpha_S(Q)}{\partial \tau} = \beta(\alpha_S(Q)) \;, \quad \frac{\partial \alpha_S(Q)}{\partial \alpha_S} = \frac{\beta(\alpha_S(Q))}{\beta(\alpha_S)} \;.$$

  and hence $R(Q^2/\mu^2, \alpha_S) = R(1, \alpha_S(Q))$. Thus all scale dependence in R comes from running of $\alpha_S(Q)$.

- We shall see QCD is asymptotically free: $\alpha_S(Q) \to 0$ as $Q \to \infty$. Thus for large Q we can safely use perturbation theory. Then knowledge of $R(1, \alpha_S)$ to fixed order allows us to predict variation of R with Q.

β *function in perturbation theory*

- Running of the QCD coupling α_S is determined by the β function,
- The β-function of QCD is found to be negative.

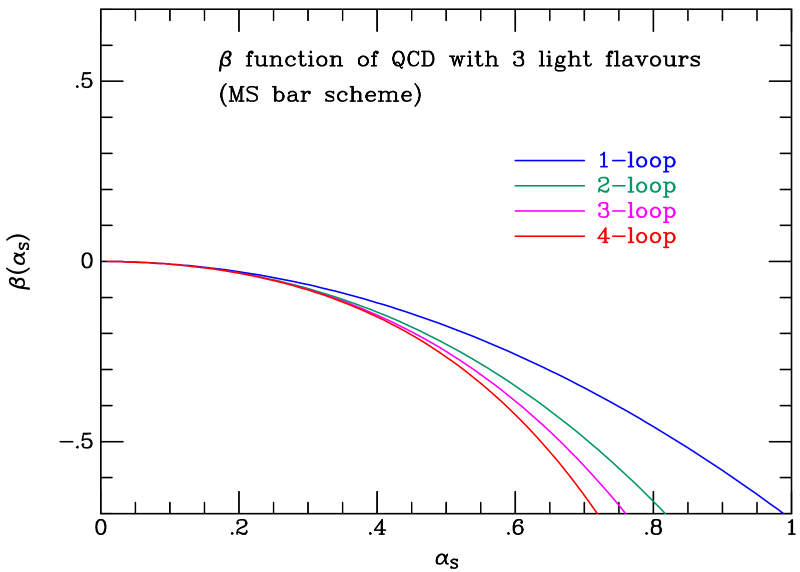
$$\beta(\alpha_S) = -b\alpha_S^2(1 + b'\alpha_S) + \mathcal{O}(\alpha_S^4)$$

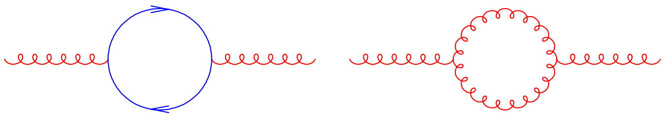
$$b = \frac{(11C_A - 2n_{lf})}{12\pi}, \quad b' = \frac{(17C_A^2 - 5C_A n_{lf} - 3C_F n_{lf})}{2\pi(11C_A - 2n_{lf})},$$

where n_{lf} is number of “active” light flavors.

Results of explicit calculation

Terms up to $\mathcal{O}(\alpha_S^5)$ are known.





Roughly speaking, quark loop diagram (a) contributes negative n_{lf} term in b, while gluon loop (b) gives positive C_A contribution, which makes β function negative overall.

Running of the coupling

- From previous slide,

$$\frac{\partial \alpha_S(Q)}{\partial \tau} = -b\alpha_S^2(Q)\Big[1 + b'\alpha_S(Q)\Big] + \mathcal{O}(\alpha_S^4).$$

  Neglecting b' and higher coefficients gives

$$\alpha_S(Q) = \frac{\alpha_S(\mu)}{1 + \alpha_S(\mu)b\tau}\ , \quad \tau = \ln\left(\frac{Q^2}{\mu^2}\right).$$

- As Q becomes large, $\alpha_S(Q)$ decreases to zero: this is asymptotic freedom. Notice that sign of b is crucial.
- the decrease of α_S is quite slow – as the inverse power of a logarithm.

Renormalization Group resummation

What type of terms does the solution of the renormalization group equation take into account in the physical quantity R?
Assume that R has perturbative expansion

$$R = \alpha_S + \mathcal{O}(\alpha_S^2)$$

Solution $R(1, \alpha_S(Q))$ can be re-expressed in terms of $\alpha_S(\mu)$:

$$\begin{aligned} R(1, \alpha_S(Q)) &= \alpha_S(\mu) \sum_{j=0}^{\infty} (-1)^j (\alpha_S(\mu) b\tau)^j \\ &= \alpha_S(\mu) \Big[1 - \alpha_S(\mu) b\tau + \alpha_S^2(\mu)(b\tau)^2 + \ldots \Big] \end{aligned}$$

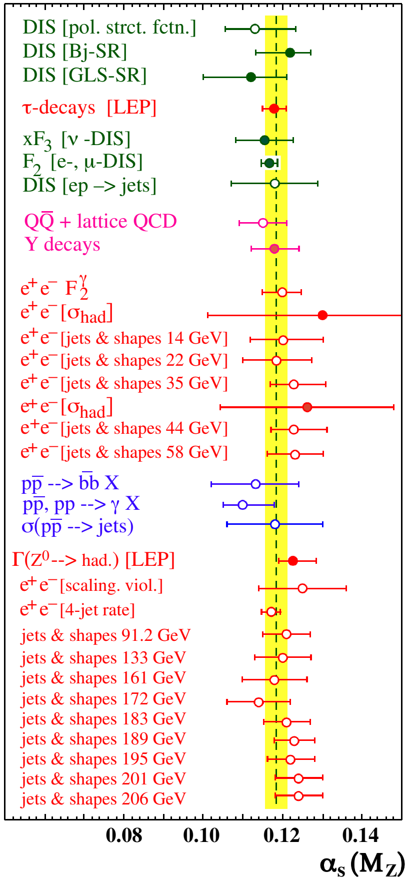
Logarithms of Q^2/μ^2 which are automatically resummed by using the running coupling. Neglected terms have fewer logs per power of α_S.

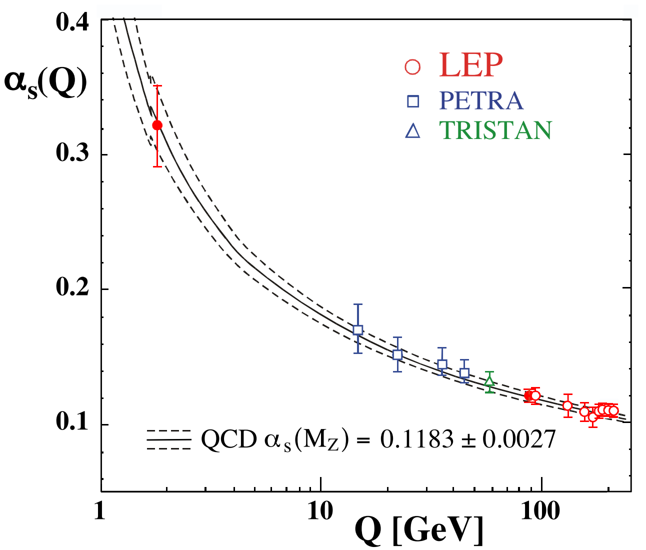
Current experimental results on α_S

Bethke,hep-ph/0407021

α_S is large at current scales. Higher order corrections are important.

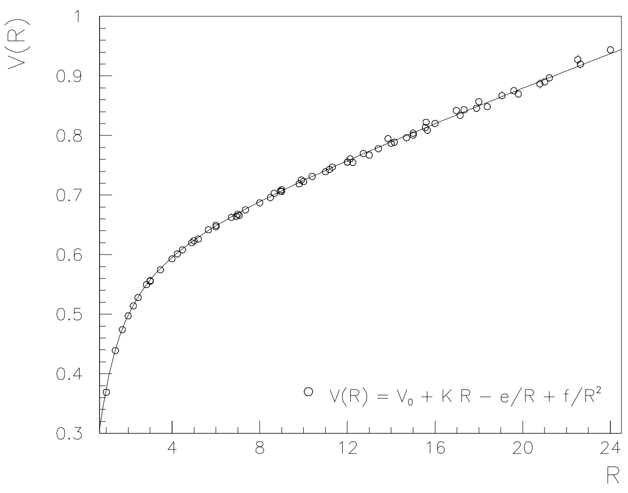
$\alpha_S(M_Z) = 0.1182 \pm 0.0027$, $\overline{\text{MS}}$, NNLO





Non-perturbative QCD

- Corresponding to asymptotic freedom at high momentum scales, we have infra-red slavery: $\alpha_S(Q)$ becomes large a low momenta, (long distances). Perturbation theory is not reliable for large α_S, so non-perturbative methods, (e.g. lattice) must be used.



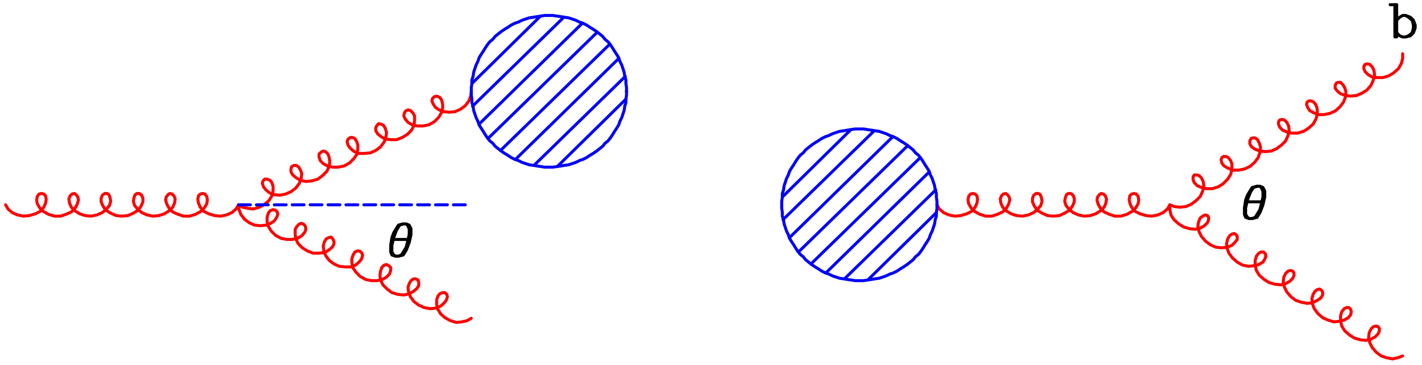
- Results on QCD potential from lattice QCD show a modified Coulomb potential at short distances and a linear potential at large distances.

Non-perturbative effects

- Confinement: partons (quarks and gluons) found only in colour singlet bound states, hadrons, size $\sim$ 1 fm. If we try to separate them it becomes energetically favourable to create extra partons from the vacuum.
- Hadronization: partons produced in short distance interactions re-organize themselves to make the observed hadrons.

Infrared divergences

- Even in high-energy, short-distance regime, long-distance aspects of QCD cannot be ignored. Soft or collinear gluon emission gives infrared divergences in PT. Light quarks ($m_q \ll \Lambda$) also lead to divergences in the limit $m_q \to 0$ (mass singularities).



(a) (b)

Spacelike branching

- Spacelike branching: gluon splitting on incoming line (a)

$$p_b^2 = -2E_aE_c(1-\cos\theta) \leq 0 \ .$$

Propagator factor $1/p_b^2$ diverges as $E_c \to 0$ (soft singularity) or $\theta \to 0$ (collinear or mass singularity). If a and b are quarks, inverse propagator factor is

$$p_b^2 - m_q^2 = -2E_aE_c(1-v_a\cos\theta) \leq 0 \ ,$$

Hence $E_c \to 0$ soft divergence remains; collinear enhancement becomes a divergence as $v_a \to 1$, i.e. when quark mass is negligible. If emitted parton c is a quark, vertex factor cancels $E_c \to 0$ divergence.

Timelike branching

- Timelike branching: gluon splitting on outgoing line (b)

$$p_a^2 = 2E_b E_c (1 - \cos\theta) \geq 0 \ .$$

Diverges when either emitted gluon is soft (E_b or $E_c \to 0$) or when opening angle $\theta \to 0$. If b and/or c are quarks, collinear/mass singularity in $m_q \to 0$ limit. Again, potential soft quark divergences cancelled by vertex factor.

Infrared divergences

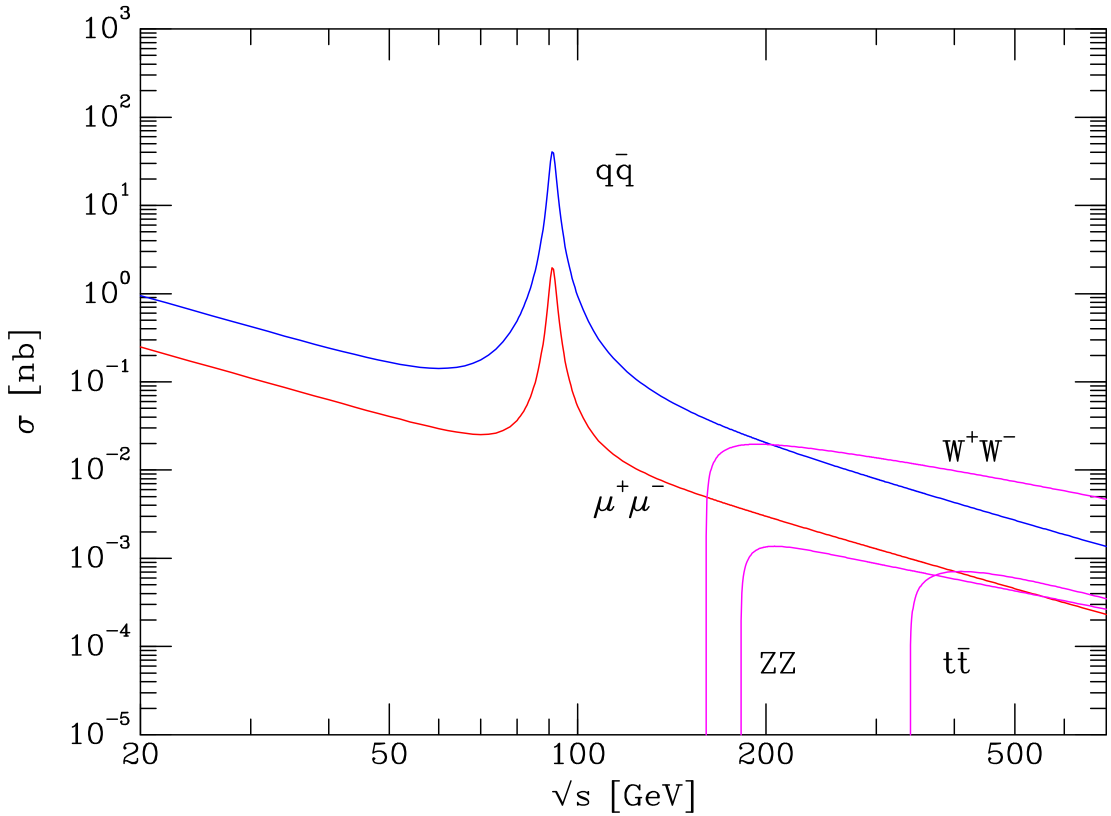
- There are similar infrared divergences in loop diagrams, associated with soft and/or collinear configurations of virtual partons within region of integration of loop momenta.
- Infrared divergences indicate dependence on long-distance aspects of QCD not correctly described by PT. Divergent (or enhanced) propagators imply propagation of partons over long distances. When distance becomes comparable with hadron size ~ 1 fm, quasi-free partons of perturbative calculation are confined/hadronized non-perturbatively, and apparent divergences disappear.

Infrared divergences

- Can still use PT to perform calculations, provided we limit ourselves to two classes of observables:
  - ⋆ Infrared safe quantities, i.e. those insensitive to soft or collinear branching. Infrared divergences in PT calculation either cancel between real and virtual contributions or are removed by kinematic factors. Such quantities are determined primarily by hard, short-distance physics; long-distance effects give power corrections, suppressed by inverse powers of a large momentum scale.
  - ⋆ Factorizable quantities, i.e. those in which infrared sensitivity can be absorbed into an overall non-perturbative factor, to be determined experimentally.
- In either case, infrared divergences must be *regularized* during PT calculation, even though they cancel or factorize in the end. Dimensional regularization analogous to that used for ultraviolet divergences is used. Divergences are give rise to powers of $1/\epsilon$.

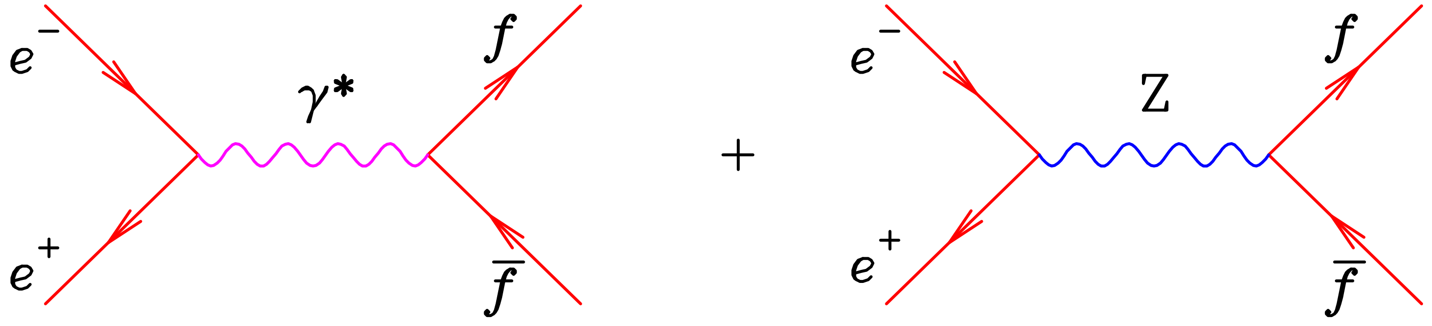
e^+e^- *annihilation cross section*

- $e^+e^- \to \mu^+\mu^-$ is a fundamental electroweak processes. Same type of process, $e^+e^- \to q\bar{q}$, will produce hadrons. Cross sections are roughly proportional.



Total cross section

- Since formation of hadrons is non-perturbative, how can PT give hadronic cross section? This can be understood by visualizing event in space-time:
- e^+ and e^- collide to form γ or Z^0 with virtual mass $Q = \sqrt{s}$. This fluctuates into $q\bar{q}$, $q\bar{q}g$,..., occupy space-time volume $\sim 1/Q$. At large Q, rate for this short-distance process given by PT.

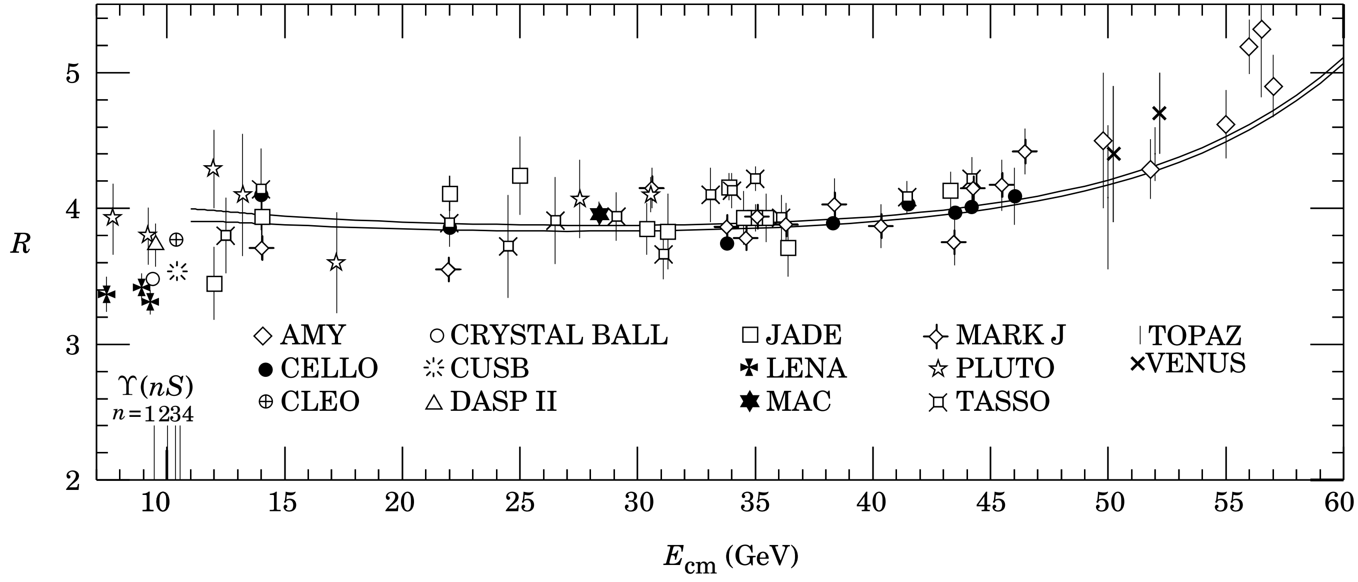


- Subsequently, at much later time $\sim 1/\Lambda$, produced quarks and gluons form hadrons. This modifies outgoing state, but occurs too late to change original probability for event to happen.

Total cross section II

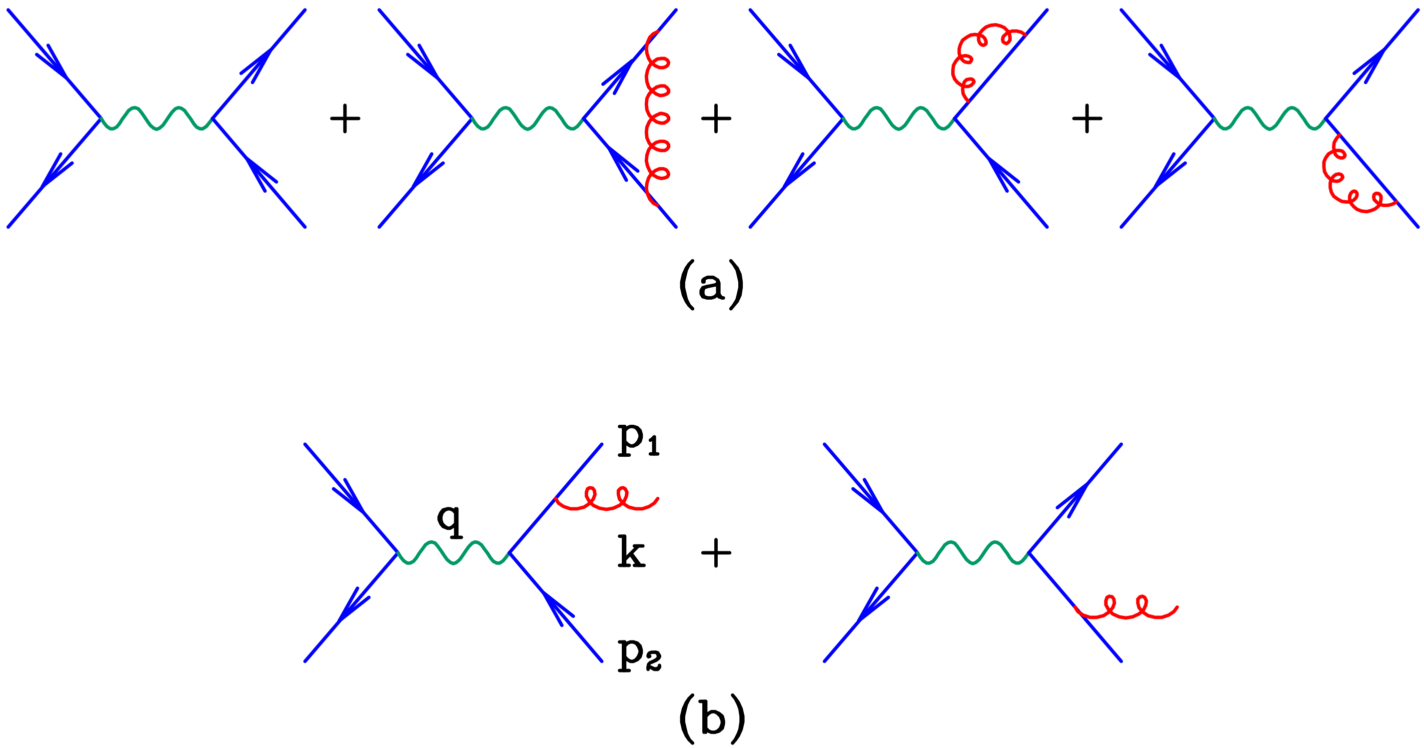
- Well below Z^0, process $e^+e^- \to f\bar{f}$ is purely electromagnetic, with lowest-order (Born) cross section (neglecting quark masses) $\sigma_0 = \frac{4\pi\alpha^2}{3s}\, Q_f^2$
  Thus ($3 = N =$ number of possible $q\bar{q}$ colours)

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\sum_q \sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q Q_q^2\ .$$



α_S corrections

- Measured cross section is about 5% higher than σ_0, due to QCD corrections. For massless quarks, corrections to R and R_Z are equal. To $\mathcal{O}(\alpha_S)$ we have:



Real emission diagrams

- Write 3-body phase-space integration as

$$d\Phi_3 = [...] d\alpha\, d\beta\, d\gamma\, dx_1\, dx_2 \ ,$$

α, β, γ are Euler angles of 3-parton plane,

$$\begin{aligned} x_1 &= 2p_1 \cdot q/q^2 = 2E_q/\sqrt{s}, \\ x_2 &= 2p_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{s}. \end{aligned}$$

- Applying Feynman rules and integrating over Euler angles:

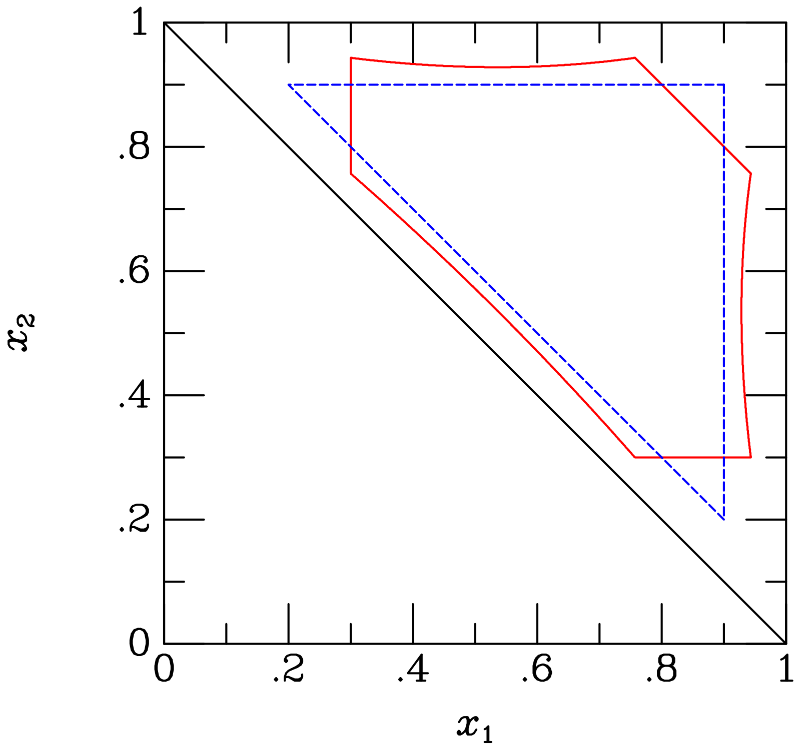
$$\sigma^{q\bar{q}g} = 3\sigma_0 C_F \frac{\alpha_S}{2\pi} \int dx_1\, dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \ .$$

- Integral divergent at $x_{1,2} = 1$:

Integration region

$0 \le x_1, x_2, x_3 \le 1$
where $x_3 = 2k \cdot q/q^2 = 2E_g/\sqrt{s} = 2 - x_1 - x_2$.

$$
\begin{aligned}
1 - x_1 &= \frac{1}{2} x_2 x_3 (1 - \cos\theta_{qg}) \\
1 - x_2 &= \frac{1}{2} x_1 x_3 (1 - \cos\theta_{\bar{q}g})
\end{aligned}
$$



- Divergences: collinear when $\theta_{qg} \to 0$ or $\theta_{\bar{q}g} \to 0$;
- soft when $E_g \to 0$, i.e. $x_3 \to 0$. Singularities are not physical – simply indicate breakdown of PT when energies and/or invariant masses approach QCD scale Λ.

Real contribution integrated

- Collinear and/or soft regions do not in fact make important contribution to R. To see this, make integrals finite using dimensional regularization, $D = 4 - 2\epsilon$. Then

$$\sigma^{q\bar{q}g} = \frac{2\sigma_0 \alpha_S H(\epsilon)}{\pi} \int \frac{dx_1 dx_2}{P(x_1, x_2)} \Big[\frac{(1-\epsilon)(x_1^2 + x_2^2) + 2\epsilon(1-x_3)}{[(1-x_1)(1-x_2)]} - 2\epsilon \Big]$$

where $H(\epsilon) = \frac{3(1-\epsilon)(4\pi)^{2\epsilon}}{(3-2\epsilon)\Gamma(2-2\epsilon)} =$
$1 + \mathcal{O}(\epsilon)$ and $P(x_1, x_2) = \Big[(1-x_1)(1-x_2)(1-x_3)\Big]^{\epsilon}$

$$\sigma^{q\bar{q}g} \;=\; 2\sigma_0 \frac{\alpha_S}{\pi} \, H(\epsilon) \, \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + \mathcal{O}(\epsilon) \right] \, .$$

- Soft and collinear singularities are regulated, appearing instead as poles at $D = 4$.

Virtual gluon contributions

- Virtual gluon contributions (a): using dimensional regularization again

$$\sigma^{q\bar{q}} \;=\; 3\sigma_0 \left\{1 + \frac{2\alpha_S}{3\pi}\, H(\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon)\right]\right\} .$$

- Adding real and virtual contributions, poles cancel and result is finite as $\epsilon \to 0$. R is an infrared safe quantity.

$$R \;=\; 3\sum_q Q_q^2 \left\{1 + \frac{\alpha_S}{\pi} + \mathcal{O}(\alpha_S^2)\right\} .$$

- Coupling α_S evaluated at renormalization scale μ. UV divergences in R cancel to $\mathcal{O}(\alpha_S)$, so coefficient of α_S independent of μ.

Scale dependence

- At $\mathcal{O}(\alpha_S^2)$ and higher, UV divergences make coefficients renormalization scheme dependent:

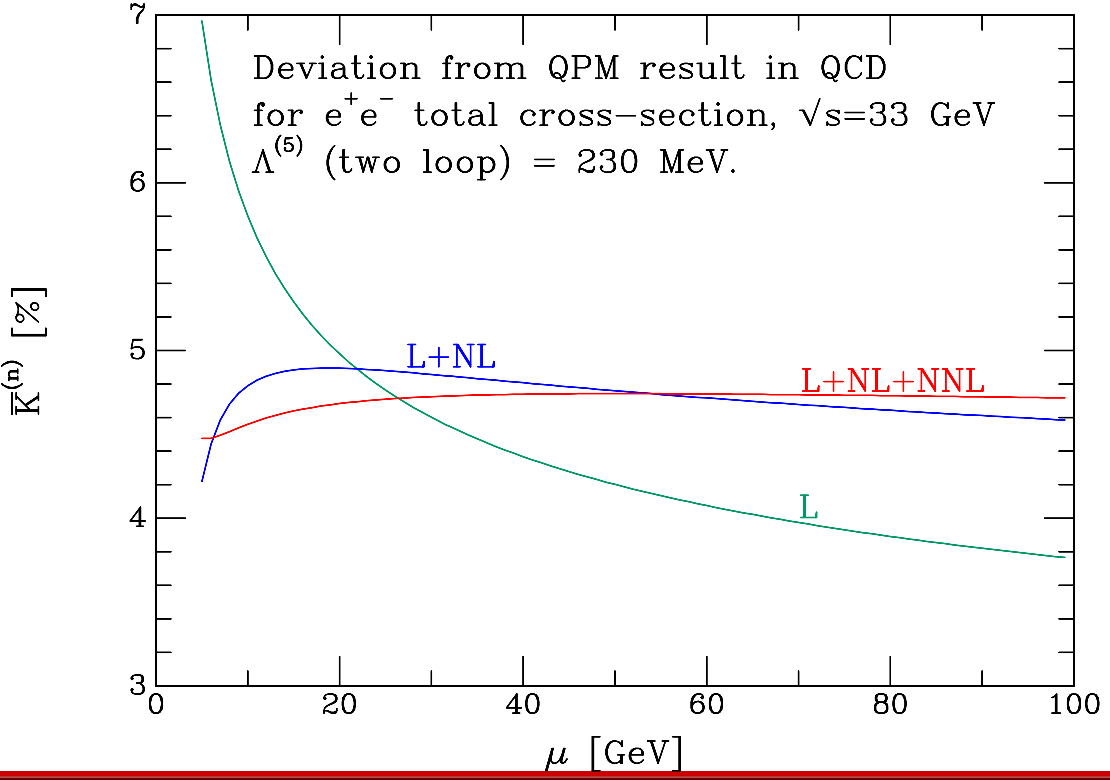
$$
\begin{aligned}
R &= 3\,K_{QCD}\sum_q Q_q^2\,, \\
K_{QCD} &= 1+\frac{\alpha_S(\mu^2)}{\pi}+\sum_{n\geq 2} C_n\left(\frac{s}{\mu^2}\right)\left(\frac{\alpha_S(\mu^2)}{\pi}\right)^n
\end{aligned}
$$

- In the $\overline{\text{MS}}$ scheme with scale $\mu=\sqrt{s}$,

$$
\begin{aligned}
C_2(1) &= \frac{365}{24}-11\zeta(3)-[11-8\zeta(3)]\frac{n_{lf}}{12} \\
&\simeq 1.986-0.115 n_{lf}
\end{aligned}
$$

Scale dependence

- Scale and scheme dependence tends to cancel as more terms are included. Scale change at $\mathcal{O}(\alpha_S^n)$ induces changes at $\mathcal{O}(\alpha_S^{n+1})$. The more terms are added, the more stable is prediction with respect to changes in μ.



Scale dependence II

- Residual scale dependence is an important source of uncertainty in QCD predictions. One can vary scale over some ‘physically reasonable’ range, e.g. $\sqrt{s}/2 < \mu < 2\sqrt{s}$, to try to quantify this uncertainty, but there is no real substitute for a full higher-order calculation.

DIS: a factorizable quantity

- If we calculate the Deep Inelastic Scattering (DIS) off a quark in perturbation series we find a logarithm

$$F_2 \sim 1 + \frac{\alpha_S}{2\pi} P \ln(Q^2/p^2)$$

where p^2 is an infrared cutoff.

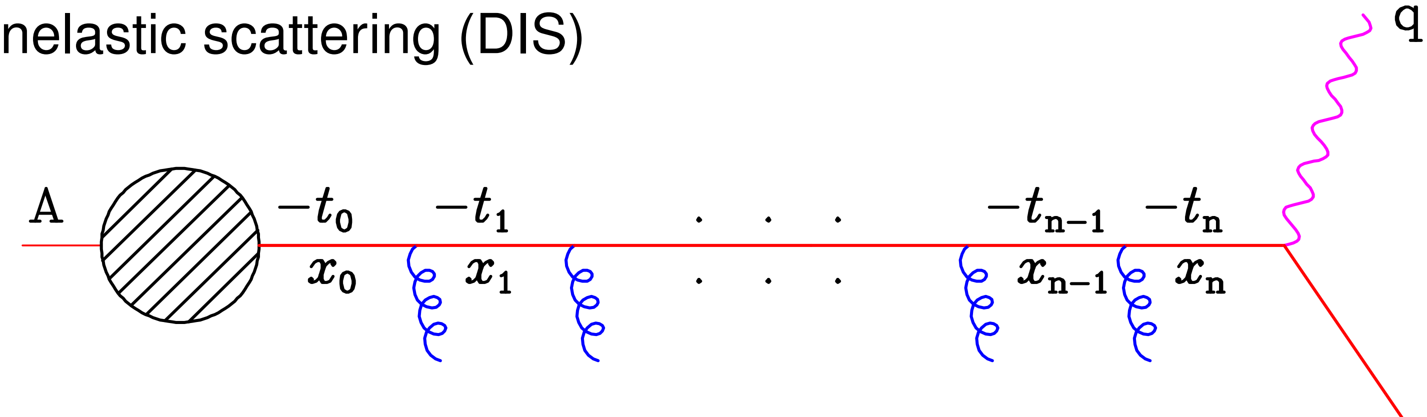
- The strategy is to factorize the answer at a scale μ^2

$$F_2 \sim (1 + \frac{\alpha_S}{2\pi} P \ln(Q^2/\mu^2)) \otimes (1 + \frac{\alpha_S}{2\pi} P \ln(\mu^2/p^2)) + O(\alpha_S^2)$$

- The idea that we can separate the high frequency and low frequency parts of a process is quite common in Physics; the proof in non-abelian gauge theory is quite challenging.
- After factorization the parton distributions are scale dependent

DGLAP equation

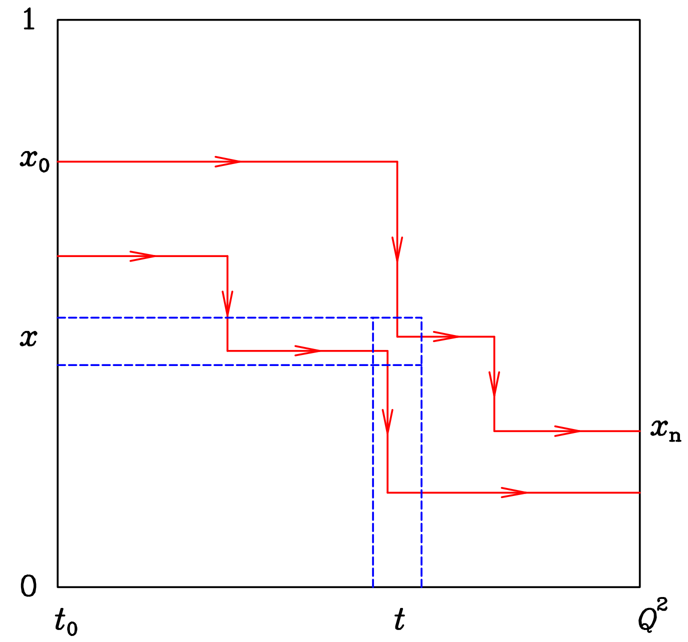
- Consider enhancement of higher-order contributions due to multiple small-angle parton emission, for example in deep inelastic scattering (DIS)



- Incoming quark from target hadron, initially with low virtual mass-squared $-t_0$ and carrying a fraction x_0 of hadron's momentum, moves to more virtual masses and lower momentum fractions by successive small-angle emissions, and is finally struck by photon of virtual mass-squared $q^2 = -Q^2$.
- Cross section depends on Q^2 and on momentum fraction distribution of partons seen by virtual photon at this scale, $D(x, Q^2)$.

DGLAP equation pictorially

- Represent sequence of branchings by path in (t, x)-space. Each branching is a step downwards in x, at a value of t equal to (minus) the virtual mass-squared after the branching.
- At $t = t_0$, paths have distribution of starting points $D(x_0, t_0)$ characteristic of target hadron at that scale. Then distribution $D(x, t)$ of partons at scale t is just the x-distribution of paths at that scale.



Quarks and gluons

- For several different types of partons, must take into account different processes by which parton of type i can enter or leave the element $(\delta t, \delta x)$. This leads to coupled DGLAP evolution equations of form

$$t\frac{\partial}{\partial t}D_i(x,t) = \sum_j \int_x^1 \frac{dz}{z}\frac{\alpha_S}{2\pi}P_{ij}(z)D_j(x/z,t)\ .$$

$$\begin{aligned}
P^{(0)}_{qq}(z) &= \hat{P}_{qq}(z)_+ = C_F\left(\frac{1+z^2}{1-z}\right)_+ \\
P^{(0)}_{qg}(z) &= \hat{P}_{qg}(z) = T_R\left[z^2+(1-z)^2\right] \\
P_{gg}(z) &= 2C_A\left[\left(\frac{z}{1-z}\right)_+ + \frac{1-z}{z} + z(1-z)\right] + b_0\delta(1-z)\ , \\
P_{gq}(z) &= P_{g\bar{q}}(z) = \hat{P}_{gq}(1-z) = C_F\frac{1+(1-z)^2}{z}\ .
\end{aligned}$$

Solution by moments

- Given $D_i(x,t)$ at some scale $t = t_0$, factorized structure of DGLAP equation means we can compute its form at any other scale.
- One strategy is to take moments (Mellin transforms) with respect to x:

$$\tilde{D}_i(N,t) = \int_0^1 dx\, x^{N-1}\, D_i(x,t)\ .$$

- After Mellin transformation, convolution in DGLAP equation becomes simply a product:

$$t\frac{\partial}{\partial t}\tilde{D}_i(x,t) = \sum_j \gamma_{ij}(N,\alpha_S)\tilde{D}_j(N,t)$$

where moments of splitting functions are anomalous dimensions,γ_{ij}

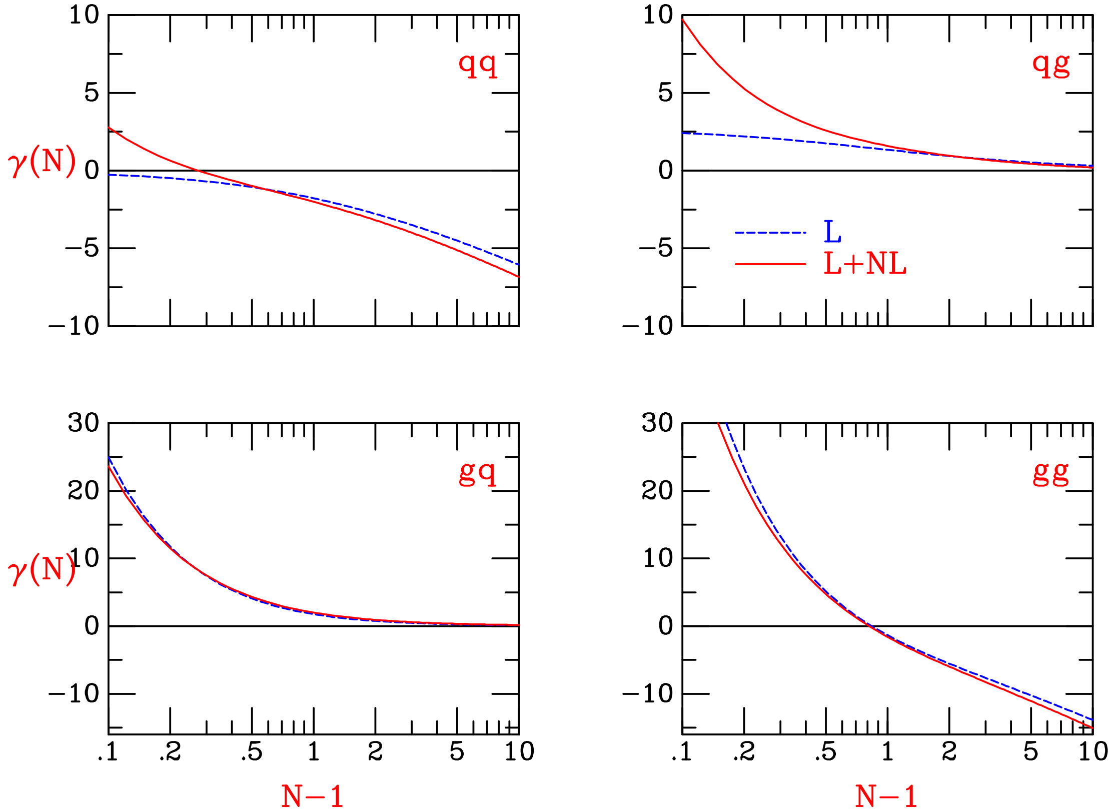
$$\gamma_{ij}(N,\alpha_S) = \sum_{n=0}^{\infty} \gamma_{ij}^{(n)}(N)\left(\frac{\alpha_S}{2\pi}\right)^{n+1}\ ,\ \gamma_{ij}^{(0)}(N) = \tilde{P}_{ij}(N) = \int_0^1 dz\, z^{N-1}\, P_{ij}(z)$$

Anomalous Dimensions

- From above expressions for $P_{ij}(z)$ we find

$$
\begin{aligned}
\gamma_{qq}^{(0)}(N) &= C_F\left[-\frac{1}{2}+\frac{1}{N(N+1)}-2\sum_{k=2}^{N}\frac{1}{k}\right]\\
\gamma_{qg}^{(0)}(N) &= T_R\left[\frac{(2+N+N^2)}{N(N+1)(N+2)}\right]\\
\gamma_{gq}^{(0)}(N) &= C_F\left[\frac{(2+N+N^2)}{N(N^2-1)}\right]\\
\gamma_{gg}^{(0)}(N) &= 2C_A\left[-\frac{1}{12}+\frac{1}{N(N-1)}+\frac{1}{(N+1)(N+2)}-\sum_{k=2}^{N}\frac{1}{k}\right]\\
&- \frac{2}{3}n_{lf}T_R\ .
\end{aligned}
$$

Form of anomalous dimension matrix



- Rapid growth at small N in gq and gg elements at lowest order
- $\ln N$ behaviour at large N in qq and gg elements

Scaling violation

- Consider combination of parton distributions which is flavour non-singlet, e.g. $D_V = D_{q_i} - D_{\bar{q}_i}$ or $D_{q_i} - D_{q_j}$. Then mixing with the flavour-singlet gluons drops out and solution for fixed α_S is

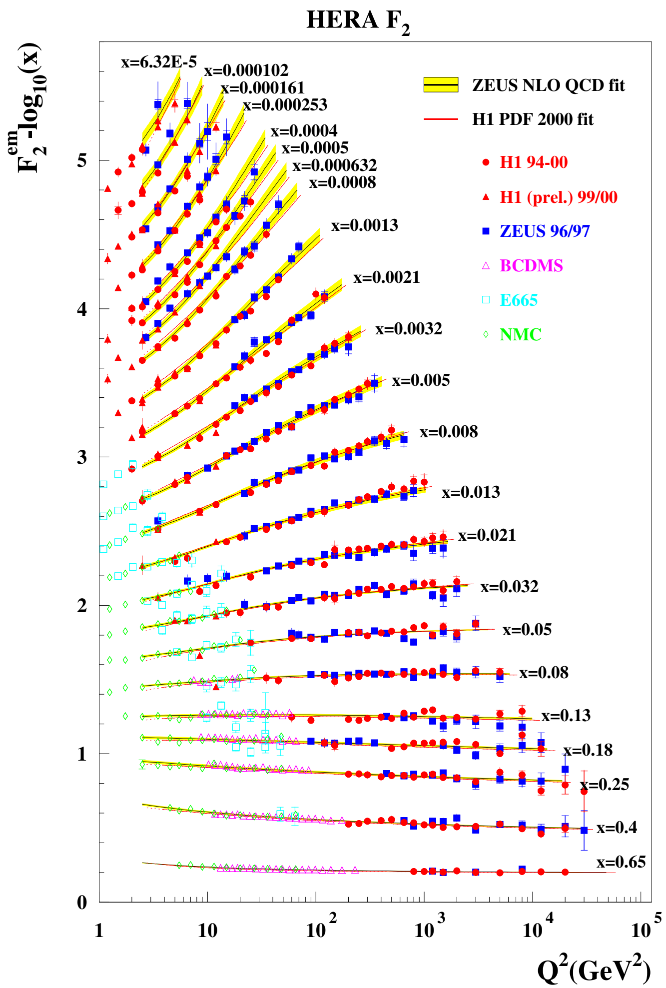
$$\tilde{D}_V(N,t) = \tilde{D}_V(N,t_0)\left(\frac{t}{t_0}\right)^{\gamma_{qq}(N,\alpha_S)} \quad ,$$

- We see that dimensionless function D_V, instead of being scale-independent function of x as expected from dimensional analysis, has scaling violation.
- For running coupling $\alpha_S(t)$, scaling violation is power-behaved in $\ln t$ rather than t. Using $\alpha_S(t) = 1/b\ln(t/\Lambda^2)$,

$$\tilde{D}_V(N,t) = \tilde{D}_V(N,t_0)\left(\frac{\alpha_S(t_0)}{\alpha_S(t)}\right)^{d_{qq}(N)}$$

, where $d_{qq}(N) = \gamma_{qq}^{(0)}(N)/2\pi b$.

Combined data on F2 proton



Now $d_{qq}(1) = 0$ and $d_{qq}(N) < 0$ for $N \geq 2$. Thus as t increases V *decreases* at large x and *increases* at small x. Physically, this is due to increase in the phase space for gluon emission by quarks as t increases, leading to loss of momentum. This is clearly visible in data:

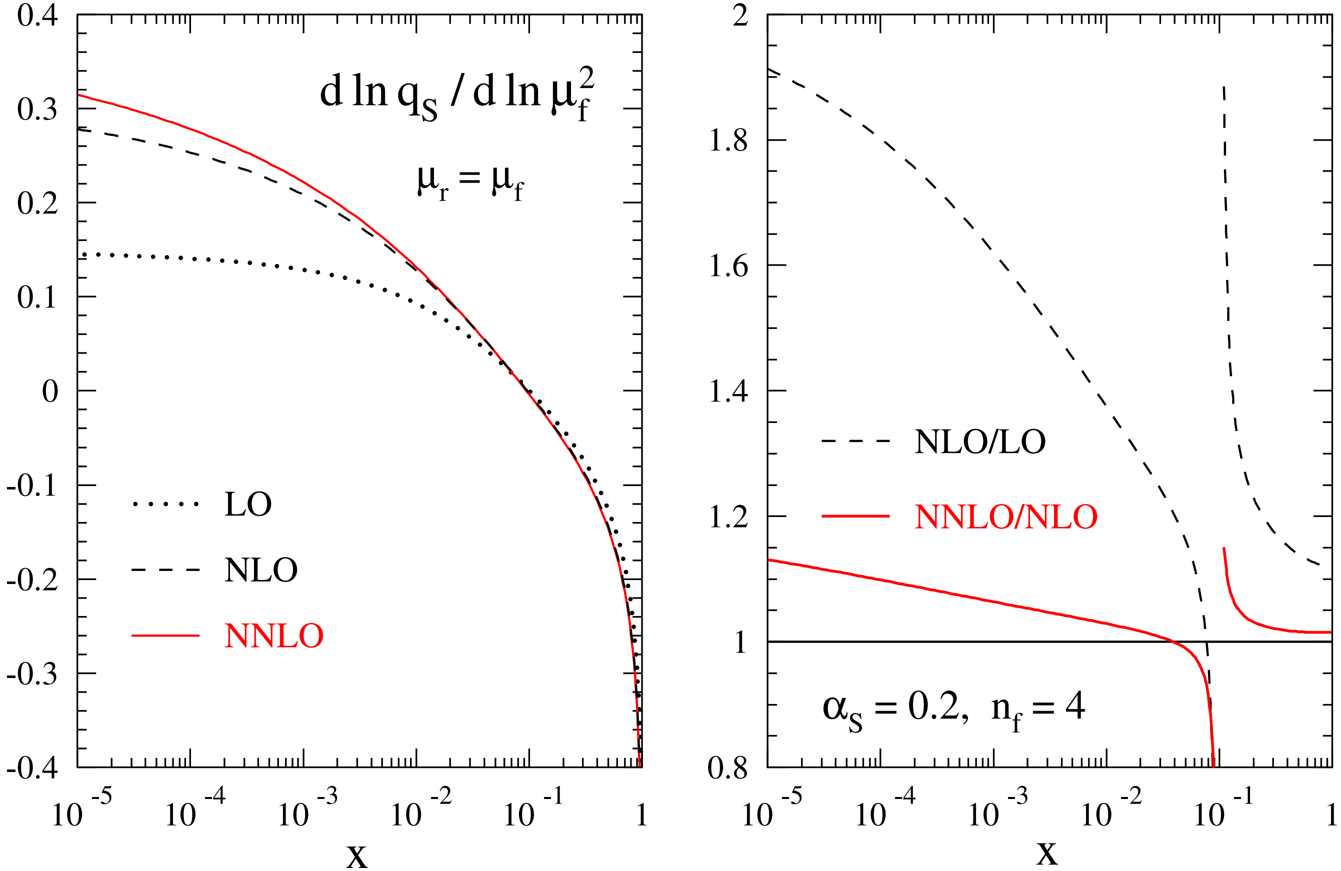
Deep Inelastic scattering at NNLO

Moch,Vogt,Vermaseren

- Current status is that splitting function is known to NNLO:
  $P(x, \alpha_S) = P^{(0)} + \alpha_S P^{(1)} + \alpha_S^2 P^{(2)} + \ldots$
- Coefficient function: $\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S \hat{\sigma}^{(1)} + \alpha_S^2 \hat{\sigma}^{(2)}$
- Need to know both the coefficient function and the splitting function to the same order for a valid prediction.
- We can now make consistent NNLO predictions for Tevatron and LHC quantities.
- New results on the coefficient function for the longitudinal structure function at appropriate order (2005)

Evolution of quarks

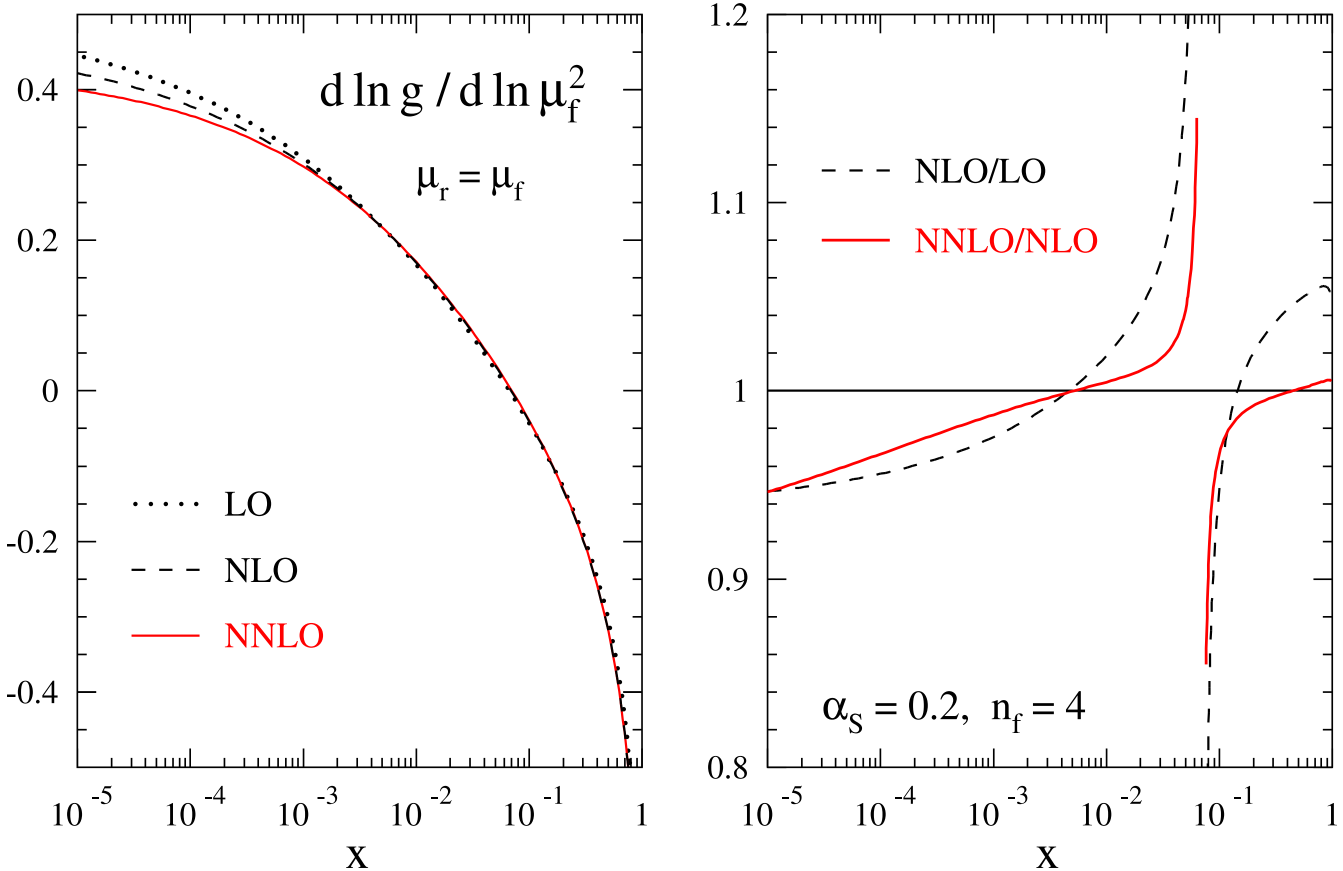
Moch, Vogt, Vermaseren



- Stability of perturbation series improved.

Evolution of gluons

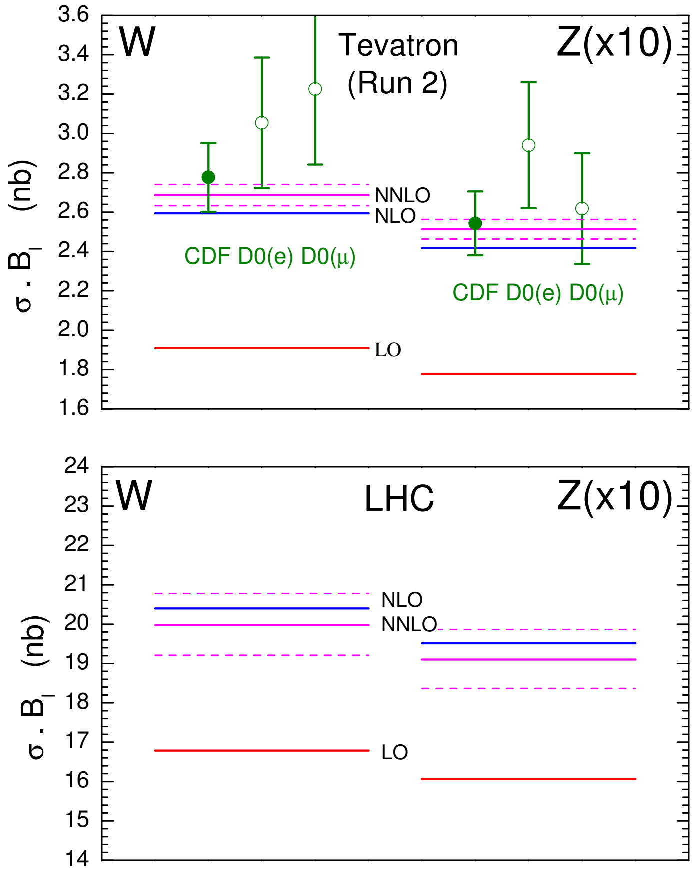
Moch,Vogt,Vermaseren



- Stability of perturbation series confirmed (small x) and improved (large x).

W and Z production at NNLO

Martin et al, MRST



- Large correction at NLO, indicates that we need NNLO to inspire confidence in stability of prediction.
- Good agreement with Tevatron data.
- 4% theoretical uncertainty at LHC.
- W and Z cross sections can be used as luminosity monitor at LHC.

Conclusion

- We have seen examples of the both types of calculable quantities in QCD, (infra-red safe and factorizable).
- In the next lecture I will outline the challenge of applying perturbative methods to collider experiments at the Tevatron and the LHC