Quantum Chromodynamics at the LHC Lecture II: Hard scattering cross sections

Frascati Spring School: Bruno Touschek May, 2006

Keith Ellis

`ellis@fnal.gov`

Fermilab/CERN

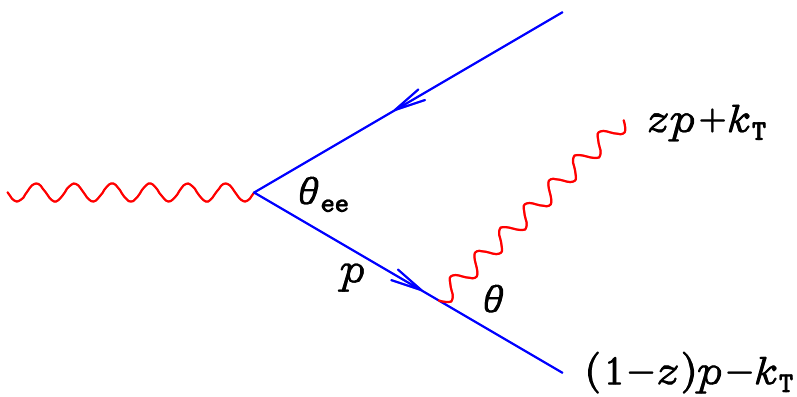
Slides available from http://theory.fnal.gov/people/ellis/Talks/

Hard scattering cross sections

- Angular ordering (left over from lecture 1)
- W production
  - DY cross section
  - Subtraction method
- W + jet production
- $t\bar{t}$ and single top production
- Combining NLO corrections and parton showers

Chudakov effect

- Angular ordering is coherence effect common to all gauge theories. In QED it causes Chudakov effect – suppression of soft bremsstrahlung from e^+e^- pairs, which has simple explanation in old-fashioned (time-ordered) perturbation theory.



- Consider emission of soft photon at angle θ from electron in pair with opening angle $\theta_{ee} < \theta$. For simplicity assume $\theta_{ee}, \theta \ll 1$.
- Transverse momentum of photon is $k_T \sim zp\theta$ and energy imbalance at $e \to e\gamma$ vertex is

$$\Delta E \sim k_T^2/zp \sim zp\theta^2 \ .$$

- Time available for emission is $\Delta t \sim 1/\Delta E$. In this time transverse separation of pair will be $\Delta b \sim \theta_{ee}\Delta t$.

Chudakov effect

- For non-negligible probability of emission, photon must resolve this transverse separation of pair, so

$$\Delta b > \lambda/\theta \sim (zp\theta)^{-1}$$

  where λ is photon wavelength.

- This implies that

$$\theta_{ee}(zp\theta^2)^{-1} > (zp\theta)^{-1} \ ,$$

  and hence $\theta_{ee} > \theta$. Thus soft photon emission is suppressed at angles larger than opening angle of pair, which is angular ordering.

- Photons at larger angles cannot resolve electron and positron charges separately – they see only total charge of pair, which is zero, implying no emission.
- More generally, if i and j come from branching of parton k, with (colour) charge $\boldsymbol{Q}_k = \boldsymbol{Q}_i + \boldsymbol{Q}_k$, then radiation outside angular-ordered cones is emitted coherently by i and j and can be treated as coming directly from (colour) charge of k.

Coherent branching

- Angular ordering provides basis for coherent parton branching formalism, which includes leading soft gluon enhancements to all orders.
- In place of virtual mass-squared variable t in earlier treatment, use angular variable

$$\zeta = \frac{p_b \cdot p_c}{E_b\, E_c} \simeq 1 - \cos\theta$$

as evolution variable for branching $a \to bc$, and impose angular ordering $\zeta' < \zeta$ for successive branchings. Iterative formula for n-parton emission becomes

$$d\sigma_{n+1} = d\sigma_n \frac{d\zeta}{\zeta} dz \frac{\alpha_S}{2\pi} \hat{P}_{ba}(z)\ .$$

Coherent branching

- In place of virtual mass-squared cutoff t_0, must use angular cutoff ζ_0 for coherent branching. This is to some extent arbitrary, depending on how we classify emission as unresolvable. Simplest choice is

$$\zeta_0 = t_0/E^2$$

  for parton of energy E.
- For radiation from particle i with finite mass-squared t_0, radiation function becomes

$$\omega^2 \left(\frac{p_i \cdot p_j}{p_i \cdot q \, p_j \cdot q} - \frac{p_i^2}{(p_i \cdot q)^2} \right) \simeq \frac{1}{\zeta} \left(1 - \frac{t_0}{E^2 \zeta} \right) \; ,$$

  so angular distribution of radiation is cut off at $\zeta = t_0/E^2$. Thus t_0 can still be interpreted as minimum virtual mass-squared.
- With this cutoff, most convenient definition of evolution variable is not ζ itself but rather

$$\tilde{t} = E^2 \zeta \geq t_0 \; .$$

Coherent branching

Angular ordering condition $\zeta_b, \zeta_c < \zeta_a$ for timelike branching $a \to bc$ (a outgoing) becomes

$$\tilde{t}_b < z^2 \tilde{t}\,, \qquad \tilde{t}_c < (1-z)^2 \tilde{t}$$

where $\tilde{t} = \tilde{t}_a$ and $z = E_b/E_a$. Thus cutoff on z becomes

$$\sqrt{t_0/\tilde{t}} < z < 1 - \sqrt{t_0/\tilde{t}}\,.$$

- Neglecting masses of b and c, virtual mass-squared of a and transverse momentum of branching are

$$t = z(1-z)\tilde{t}\,, \qquad p_t^2 = z^2(1-z)^2\tilde{t}\,.$$

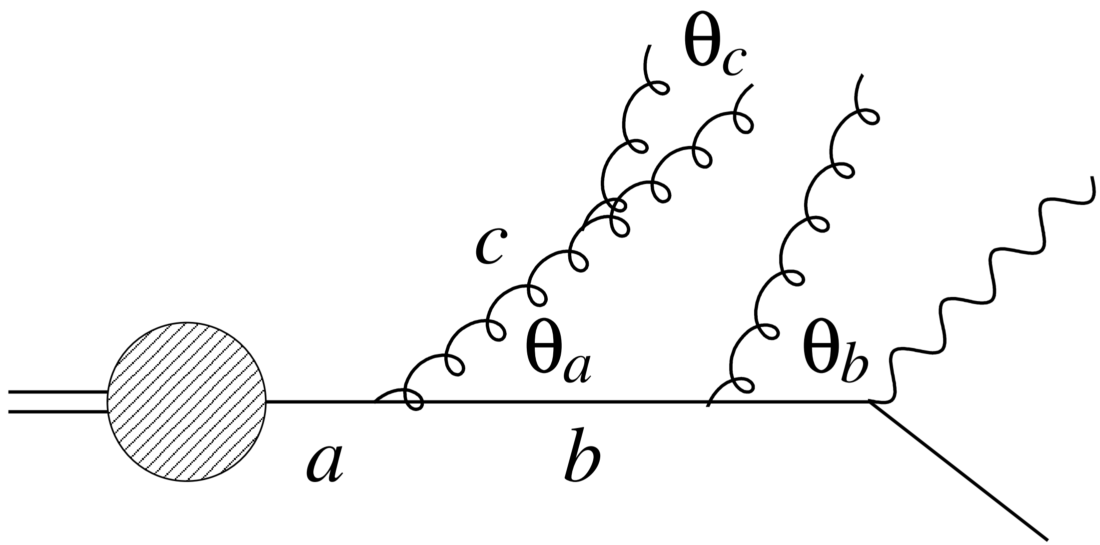
Thus for coherent branching Sudakov form factor of quark becomes

$$\tilde{\Delta}_q(\tilde{t}) = \exp\left[-\int_{4t_0}^{\tilde{t}} \frac{dt'}{t'} \int_{\sqrt{t_0/t'}}^{1-\sqrt{t_0/t'}} \frac{dz}{2\pi} \alpha_S(z^2(1-z)^2 t') \hat{P}_{qq}(z)\right]$$

At large $\tilde{t}$ this falls more slowly than form factor without coherence, due to the suppression of soft gluon emission by angular ordering.

Coherent branching

- Note that for spacelike branching $a \to bc$ (a incoming, b spacelike), angular ordering condition is



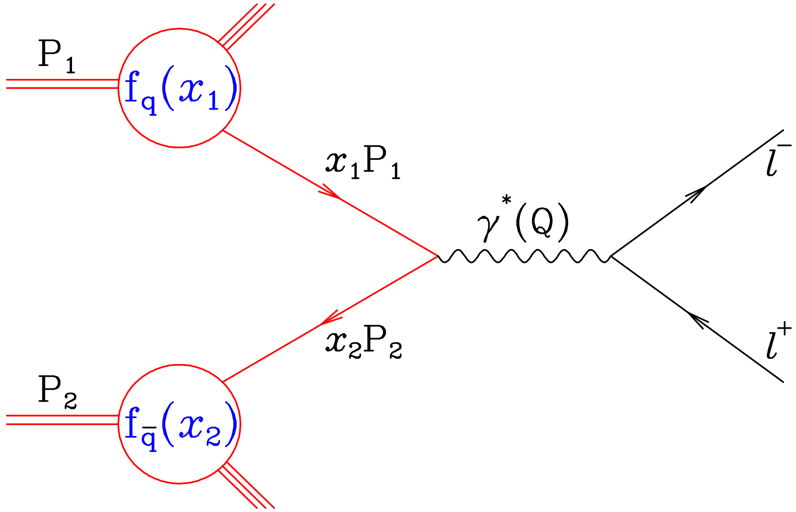
$$\theta_b > \theta_a > \theta_c \ ,$$

and so for $z = E_b/E_a$ we now have

$$\tilde{t}_b > z^2 \tilde{t}_a \ , \quad \tilde{t}_c < (1-z)^2 \tilde{t}_a \ .$$

- Thus we can have either $\tilde{t}_b > \tilde{t}_a$ or $\tilde{t}_b < \tilde{t}_a$, especially at small z — spacelike branching becomes disordered at small x.

Lepton-pair production



- Mechanism for Lepton pair production, W-production, Z-production, Vector-boson pairs, ...
- Collectively known as the Drell-Yan process.
- Color average $1/N$.

$$\frac{d\hat{\sigma}}{dQ^2} = \frac{\sigma_0}{N} \, Q_q^2 \, \delta(\hat{s} - Q^2), \qquad \sigma_0 = \frac{4\pi\alpha^2}{3Q^2}, \quad \text{cf } e^+e^- \text{ annihilation.}$$

In the CM frame of the two hadrons, the momenta of the incoming partons are

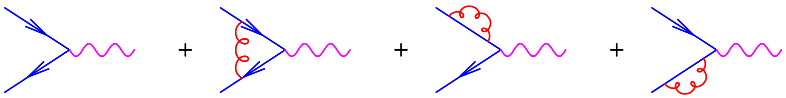
$$p_1 = \frac{\sqrt{s}}{2}(x_1, 0, 0, x_1), \quad p_2 = \frac{\sqrt{s}}{2}(x_2, 0, 0, -x_2) \, .$$

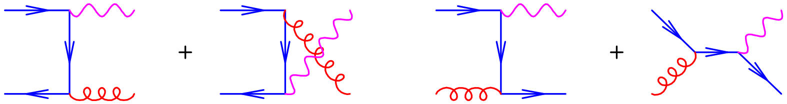
The square of the $q\bar{q}$ collision energy $\hat{s}$ is related to the overall hadron-hadron collision energy by $\hat{s} = (p_1 + p_2)^2 = x_1 x_2 s$. The parton-model cross section for this process is:

$$\begin{aligned}
\frac{d\sigma}{dM^2} &= \int_0^1 dx_1 dx_2 \sum_q \{f_q(x_1) f_{\bar{q}}(x_2) + (q \leftrightarrow \bar{q})\} \frac{d\hat{\sigma}}{dM^2}(q\bar{q} \to l^+ l^-) \\
&= \frac{\sigma_0}{Ns} \int_0^1 \frac{dx_1}{x_1} \frac{dx_2}{x_2} \, \delta(1-z) \left[\sum_q Q_q^2 \{f_q(x_1) f_{\bar{q}}(x_2) + (q \leftrightarrow \bar{q})\} \right] .
\end{aligned}$$

- For later convenience we have introduced the variable $z = \frac{Q^2}{\hat{s}} = \frac{Q^2}{x_1 x_2 s}$.
- The sum here is over quarks only and the $\bar{q}q$ contributions are indicated explicitly.

Next-to-leading order


(a)


(b) (c)

- The contribution of the real diagrams (in four dimensions) is

$$|M|^2 \sim g^2 C_F \left[\frac{u}{t} + \frac{t}{u} + \frac{2Q^2 s}{ut} \right] = g^2 C_F \left[\left(\frac{1+z^2}{1-z} \right) \left(\frac{-s}{t} + \frac{-s}{u} \right) - 2 \right]$$

  where $z = Q^2/s$, $s + t + u = Q^2$.
- Note that the real diagrams contain collinear singularities, $u \to 0, t \to 0$ and soft singularities, $z \to 1$.
- The coefficient of the divergence is the unregulated branching probability $\hat{P}_{qq}(z)$.
- Ignore for simplicity the diagrams with incoming gluons.

- Control the divergences by continuing the dimensionality of space-time, $d = 4 - 2\epsilon$, (technically this is dimensional reduction). Performing the phase space integration, the total contribution of the real diagrams is

$$\begin{aligned}\sigma_R &= \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} c_\Gamma \Bigg[\Big(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{\pi^2}{3}\Big)\delta(1-z) - \frac{2}{\epsilon} P_{qq}(z) \\ &- 2(1-z) + 4(1+z^2)\Big[\frac{\ln(1-z)}{1-z}\Big]_+ - 2\frac{1+z^2}{(1-z)}\ln z\Bigg]\end{aligned}$$

with $c_\Gamma = (4\pi)^\epsilon / \Gamma(1-\epsilon)$.

- The contribution of the virtual diagrams is

$$\sigma_V = \delta(1-z)\left[1 + \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} c'_\Gamma \Big(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 6 + \pi^2\Big)\right]$$

$c'_\Gamma = c_\Gamma + O(\epsilon^3)$

- Adding it up we get in dim-reduction

$$\sigma_{R+V} = \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} c_\Gamma \left[\left(\frac{2\pi^2}{3} - 6\right)\delta(1-z) - \frac{2}{\epsilon} P_{qq}(z) - 2(1-z) + 4(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{(1-z)}\ln z\right]$$

- The divergences, proportional to the branching probability , are universal.
- We will factorize them into the parton distributions. We perform the mass factorization by subtracting the counterterm

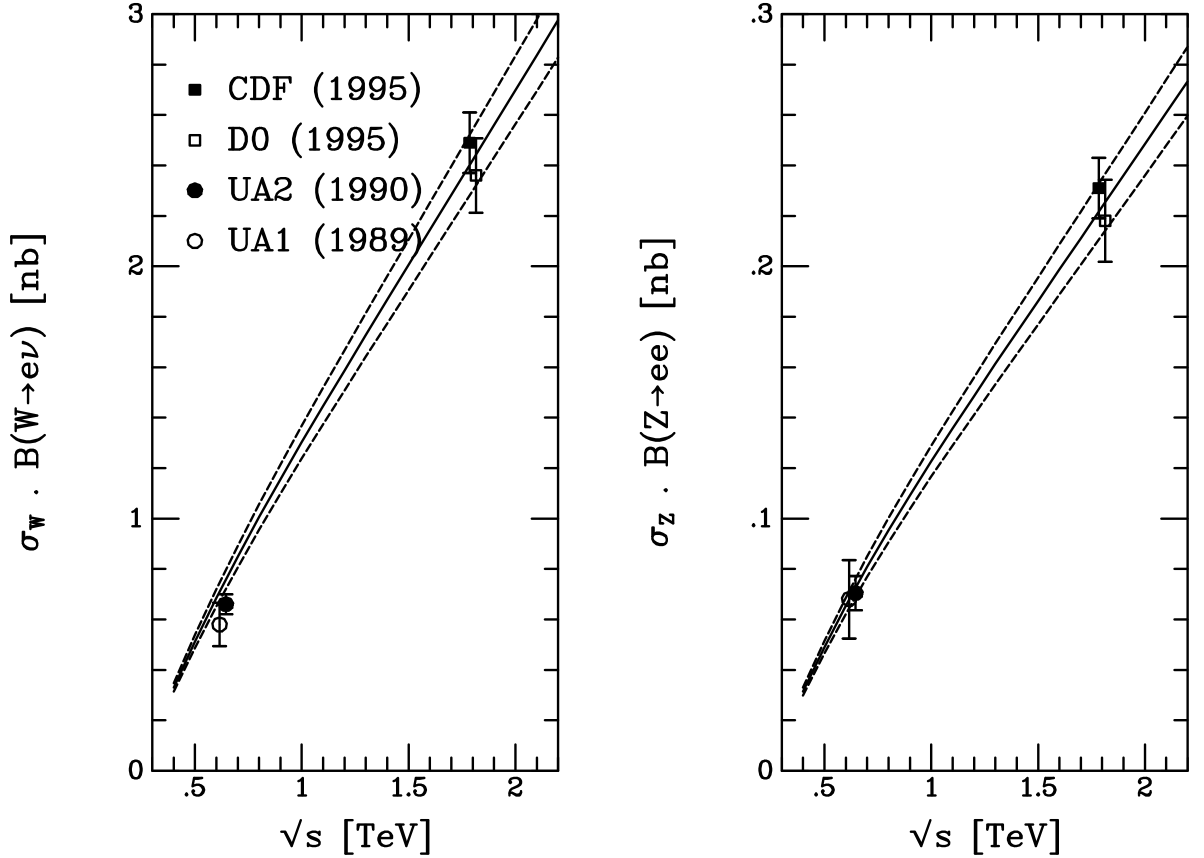
$$2\frac{\alpha_S}{2\pi} C_F \left[\frac{-c_\Gamma}{\epsilon} P_{qq}(z) - (1-z) + \delta(1-z)\right]$$

(The finite terms are necessary to get us to the $\overline{MS}$-scheme).

$$\hat{\sigma} = \frac{\alpha_S}{2\pi} C_F \left[\left(\frac{2\pi^2}{3} - 8\right)\delta(1-z) + 4(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{(1-z)}\ln z + 2P_{qq}(z)\ln\frac{Q^2}{\mu^2}\right]$$

- Similar correction for incoming gluons.

Application to W, Z production



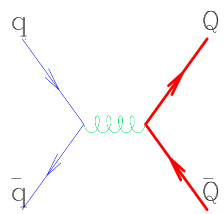
- Agreement with NLO theory is good (three curves estimate theoretical error).
- LO curves (not shown) lie about 25% too low.

Heavy quark production, leading order

The leading-order processes for the production of a heavy quark Q of mass m in hadron-hadron collisions

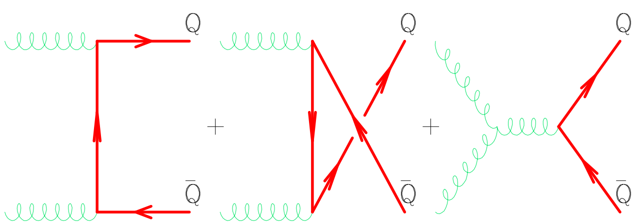
$$
\begin{array}{ll}
(a) & q(p_1)+\overline{q}(p_2)\to Q(p_3)+\overline{Q}(p_4) \\
(b) & g(p_1)+g(p_2)\to Q(p_3)+\overline{Q}(p_4)
\end{array}
$$

where the four-momenta of the partons are given in brackets.



(a)

Process	$\overline{\sum}\|\mathcal{M}\|^2/g^4$
$q\,\overline{q}\to Q\,\overline{Q}$	$\frac{4}{9}\left(\tau_1^2+\tau_2^2+\frac{\rho}{2}\right)$
$g\,g\to Q\,\overline{Q}$	$\left(\frac{1}{6\tau_1\tau_2}-\frac{3}{8}\right)\left(\tau_1^2+\tau_2^2+\rho-\frac{\rho^2}{4\tau_1\tau_2}\right)$



(b)

$\overline{\sum}$ indicates averaged (summed) over initial (final) colours and spins

We have introduced the following notation for the ratios of scalar products:

$$\tau_1 = \frac{2p_1.p_3}{\hat{s}}, \quad \tau_2 = \frac{2p_2.p_3}{\hat{s}}, \quad \rho = \frac{4m^2}{\hat{s}}, \quad \hat{s} = (p_1 + p_2)^2.$$

- The short-distance cross section is obtained from the invariant matrix element in the usual way:

$$d\hat{\sigma}_{ij} = \frac{1}{2\hat{s}} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \overline{\sum} |\mathcal{M}_{ij}|^2.$$

  The first factor is the flux factor for massless incoming particles. The other terms come from the phase space for $2 \to 2$ scattering.

- In terms of the rapidity $y = \frac{1}{2} \ln((E + p_z)/(E - p_z))$ and transverse momentum, p_T, the relativistically invariant phase space volume element of the final-state heavy quarks is

$$\frac{d^3 p}{E} = dy \; d^2 p_T \; .$$

The result for the invariant cross section may be written as

$$\frac{d\sigma}{dy_3 dy_4 d^2 p_T} = \frac{1}{16\pi^2 \hat{s}^2} \sum_{ij} x_1 f_i(x_1, \mu^2)\; x_2 f_j(x_2, \mu^2) \overline{\sum} |\mathcal{M}_{ij}|^2 .$$

x_1 and x_2 are fixed if we know the transverse momenta and rapidity of the outgoing heavy quarks. In the centre-of-mass system of the incoming hadrons we may write

$$\begin{aligned} p_1 &= \frac{1}{2}\sqrt{s}(x_1, 0, 0, x_1) \\ p_2 &= \frac{1}{2}\sqrt{s}(x_2, 0, 0, -x_2) \\ p_3 &= (m_T \cosh y_3, p_T, 0, m_T \sinh y_3) \\ p_4 &= (m_T \cosh y_4, -p_T, 0, m_T \sinh y_4). \end{aligned}$$

Applying energy and momentum conservation, we obtain

$$x_1 = \frac{m_T}{\sqrt{s}}\left(e^{y_3} + e^{y_4}\right), x_2 = \frac{m_T}{\sqrt{s}}\left(e^{-y_3} + e^{-y_4}\right), \hat{s} = 2m_T^2(1 + \cosh \Delta y).$$

The quantity $m_T = \surd(m^2 + p_T^2)$ is the transverse mass of the heavy quarks and $\Delta y = y_3 - y_4$ is the rapidity difference between them.

In these variables the leading order cross section is

$$\frac{d\sigma}{dy_3 dy_4 d^2 p_T} = \frac{1}{64\pi^2 m_T^4 (1+\cosh(\Delta y))^2} \sum_{ij} x_1 f_i(x_1,\mu^2)\, x_2 f_j(x_2,\mu^2) \overline{\sum} |\mathcal{M}_{ij}|^2 .$$

Expressed in terms of m, m_T and Δy, the matrix elements for the two processes are

$$\overline{\sum} |\mathcal{M}_{q\overline{q}}|^2 = \frac{4g^4}{9} \Big(\frac{1}{1+\cosh(\Delta y)} \Big) \Big(\cosh(\Delta y) + \frac{m^2}{m_T^2} \Big) ,$$

$$\overline{\sum} |\mathcal{M}_{gg}|^2 = \frac{g^4}{24} \Big(\frac{8\cosh(\Delta y) - 1}{1+\cosh(\Delta y)} \Big) \Big(\cosh(\Delta y) + 2\frac{m^2}{m_T^2} - 2\frac{m^4}{m_T^4} \Big).$$

- As the rapidity separation Δy between the two heavy quarks becomes large

$$\overline{\sum} |\mathcal{M}_{q\overline{q}}|^2 \sim \text{ constant}, \quad \overline{\sum} |\mathcal{M}_{gg}|^2 \sim \exp \Delta y \ .$$

- The cross section is damped at large Δy and heavy quarks produced by $q\bar{q}$ annihilation are more closely correlated in rapidity those produced by gg fusion.

Applicability of perturbation theory?

- Consider the propagators in the diagrams.

$$\begin{aligned}
(p_1+p_2)^2 = &\quad 2p_1.p_2 &= 2m_T^2\big(1+\cosh\Delta y\big)\ , \\
(p_1-p_3)^2 - m^2 = &\quad -2p_1.p_3 &= -m_T^2\big(1+e^{-\Delta y}\big)\ , \\
(p_2-p_3)^2 - m^2 = &\quad -2p_2.p_3 &= -m_T^2\big(1+e^{\Delta y}\big)\ .
\end{aligned}$$

  Note that the propagators are all off-shell by a quantity of least of order m^2.

- Thus for a sufficiently heavy quark we expect the methods of perturbation theory to be applicable. It is the mass m (which by supposition is very much larger than the scale of the strong interactions Λ) which provides the large scale in heavy quark production. We expect corrections of order Λ/m
- This does not address the issue of whether the charm or bottom mass is large enough to be adequately described by perturbation theory.

Heavy quark production in $O(\alpha_S^3)$

In NLO heavy quark production m is the heavy quark mass.

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 \; \hat{\sigma}_{ij}(x_1 x_2 S, m^2, \mu^2) F_i(x_1, \mu^2) F_j(x_2, \mu^2)$$

$$\hat{\sigma}_{i,j}(\hat{s}, m^2, \mu^2) = \sigma_0 c_{ij}(\hat{\rho}, \mu^2)$$

where $\hat{\rho} = 4m^2/\hat{s}, \bar{\mu}^2 = \mu^2/m^2, \sigma_0 = \alpha_S^2(\mu^2)/m^2$ and $\hat{s}$ in the parton total c-of-m energy squared. The coupling satisfies

$$\frac{d\alpha_S}{d\ln\mu^2} = -b_0 \frac{\alpha_S^2}{2\pi} + O(\alpha_S^3), \; b_0 = \frac{11N - 2n_f}{6}$$

$$c_{ij}\left(\rho, \frac{\mu^2}{m^2}\right) = c_{ij}^{(0)}(\rho) + 4\pi\alpha_S(\mu^2)\left[c_{ij}^{(1)}(\rho) + \overline{c}_{ij}^{(1)}(\rho)\ln(\frac{\mu^2}{m^2})\right] + O(\alpha_S^2)$$

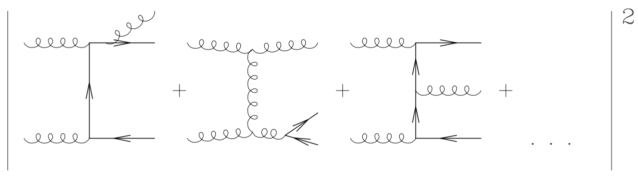
The lowest-order functions $c_{ij}^{(0)}$ are obtained by integrating the lowest order matrix elements

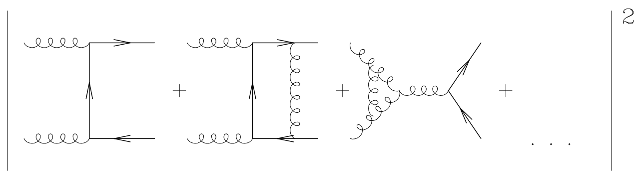
$$
\begin{aligned}
c_{q\overline{q}}^{(0)}(\rho) &= \frac{\pi\beta\rho}{27}\left[(2+\rho)\right] , \\
c_{gg}^{(0)}(\rho) &= \frac{\pi\beta\rho}{192}\left[\frac{1}{\beta}\left[\rho^2+16\rho+16\right]\ln\left(\frac{1+\beta}{1-\beta}\right)-28-31\rho\right] , \\
c_{gq}^{(0)}(\rho) &= c_{g\overline{q}}^{(0)}(\rho) = 0 ,
\end{aligned}
$$

and $\beta = \sqrt{1-\rho}$.

- The functions $c_{ij}^{(0)}$ vanish both at threshold ($\beta \to 0$) and at high energy ($\rho \to 0$).
- Note that the quark-gluon process is zero in lowest order, but is present in higher orders.

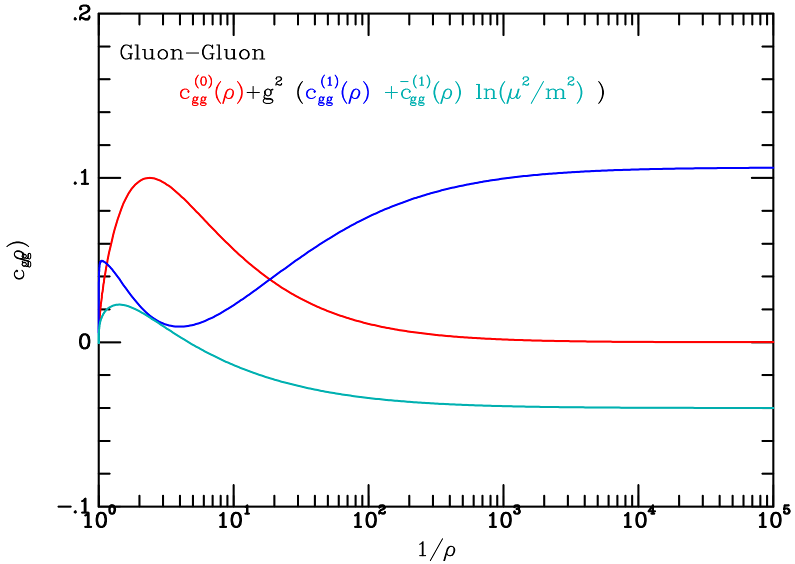
- The functions $c_{ij}^{(1)}$ are also known

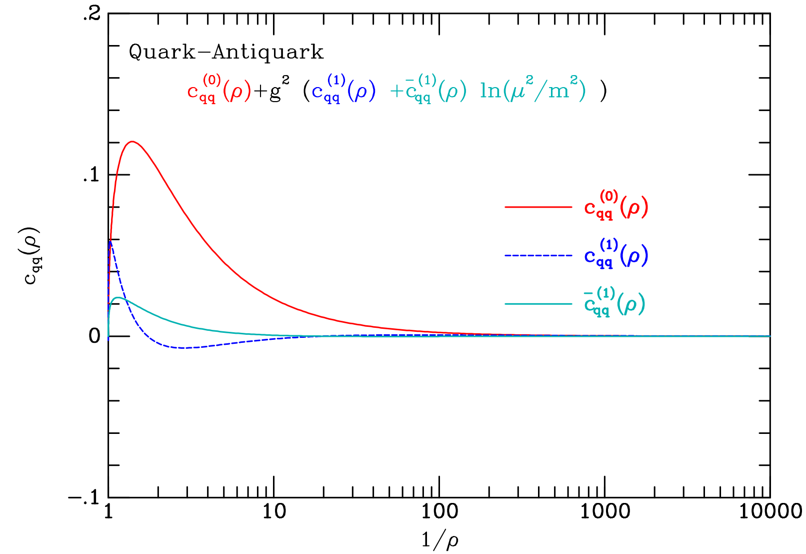


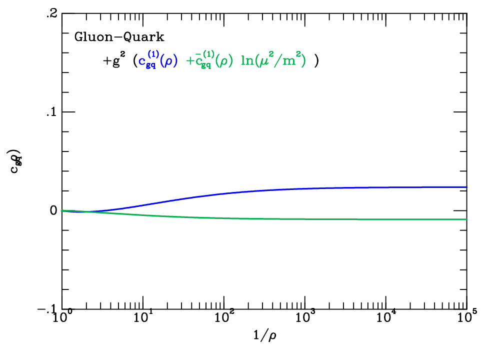


- Examples of higher-order corrections to heavy quark production.
- In order to calculate the c_{ij} in perturbation theory we must perform both renormalization and factorization of mass singularities. The subtractions required for renormalization and factorization are done at mass scale μ.

Higher order results, $c_{ij}^{(1)}$







μ dependence

μ is an unphysical parameter. The physical predictions should be invariant under changes of μ at the appropriate order in perturbation theory. If we have performed a calculation to $O(\alpha_S^3)$, variations of the scale μ will lead to corrections of $O(\alpha_S^4)$,

$$\mu^2 \frac{d}{d\mu^2} \sigma = O(\alpha_S^4).$$

- The term $\overline{c}^{(1)}$, which controls the μ dependence of the higher-order perturbative contributions, is fixed in terms of the lower-order result $c^{(0)}$:

$$\begin{aligned}
\overline{c}_{ij}^{(1)}(\rho) \quad = \quad & \frac{1}{8\pi^2}\Bigg[4\pi b c_{ij}^{(0)}(\rho) - \int_\rho^1 dz_1 \sum_k c_{kj}^{(0)}(\frac{\rho}{z_1}) P_{ki}^{(0)}(z_1) \\
& - \int_\rho^1 dz_2 \sum_k c_{ik}^{(0)}(\frac{\rho}{z_2}) P_{kj}^{(0)}(z_2)\Bigg].
\end{aligned}$$

In obtaining this result we have used the renormalization group equation for the running coupling

$$\mu^2 \frac{d}{d\mu^2} \alpha_S(\mu^2) = -b\alpha_S^2 + \ldots$$

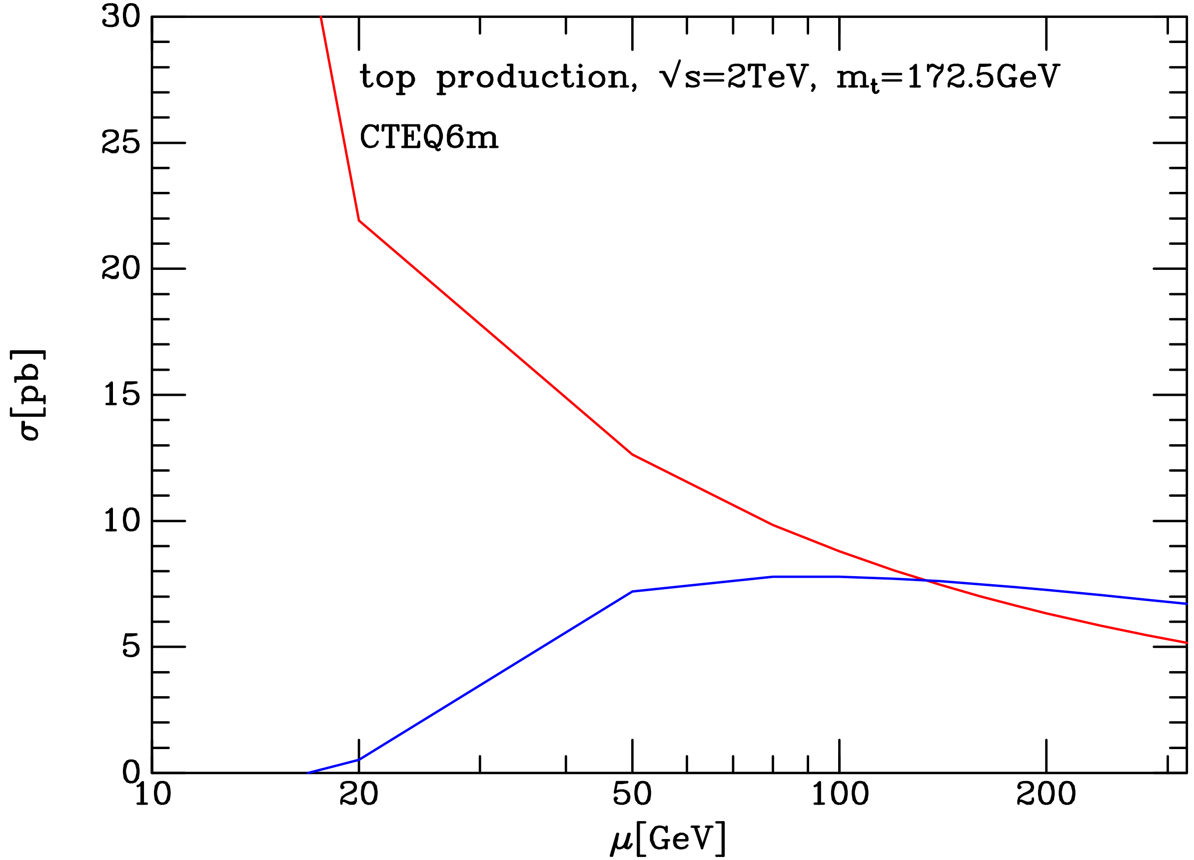
and the lowest-order form of the GLAP equation

$$\mu^2 \frac{d}{d\mu^2} f_i(x,\mu^2) = \frac{\alpha_S(\mu^2)}{2\pi} \sum_k \int_x^1 \frac{dz}{z} P_{ik}^{(0)}(z) f_k(\frac{x}{z},\mu^2) + \ldots .$$

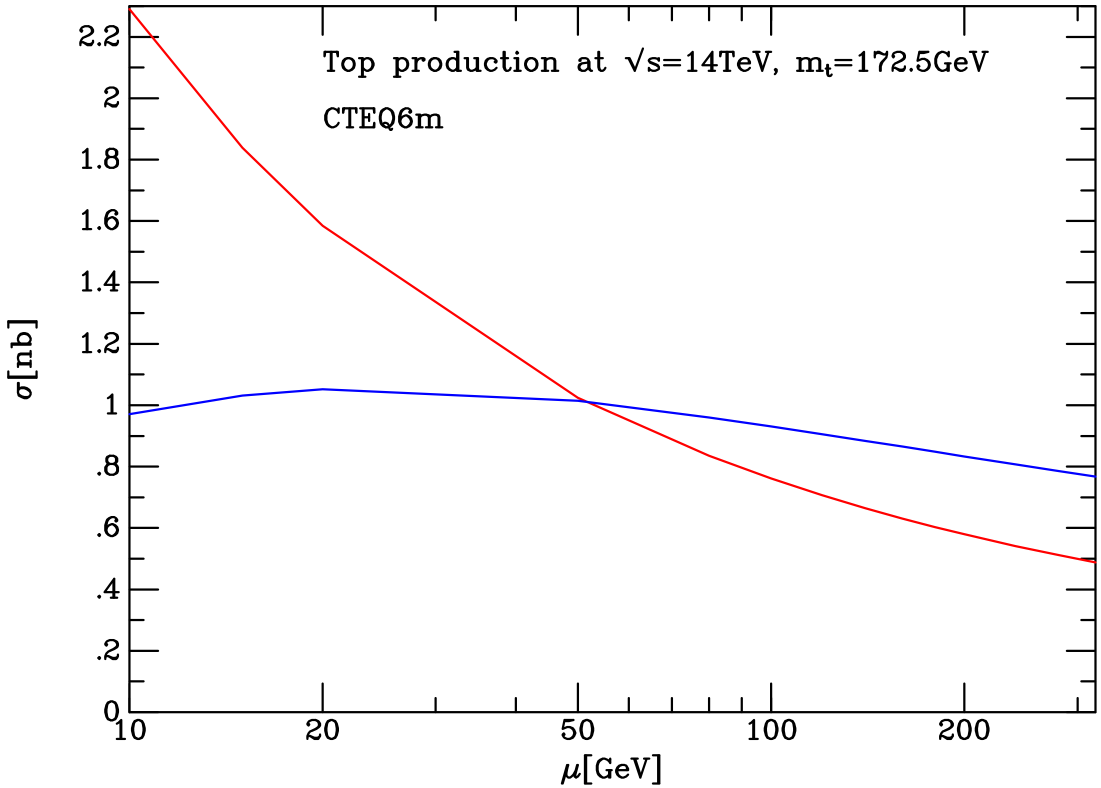
This illustrates an important point which is a general feature of renormalization group improved perturbation series in QCD. The coefficient of the perturbative correction depends on the choice made for the scale μ, but the scale dependence changes the result in such a way that the physical result is independent of that choice. Thus the scale dependence is formally small because it is of higher order in α_S. This does not assure us that the scale dependence is actually *numerically* small for all series. A pronounced dependence on the scale μ is a signal of an untrustworthy perturbation series.

Scale dependence in top production

- Inclusion of the higher order terms leads to a stabilization of the top cross section.



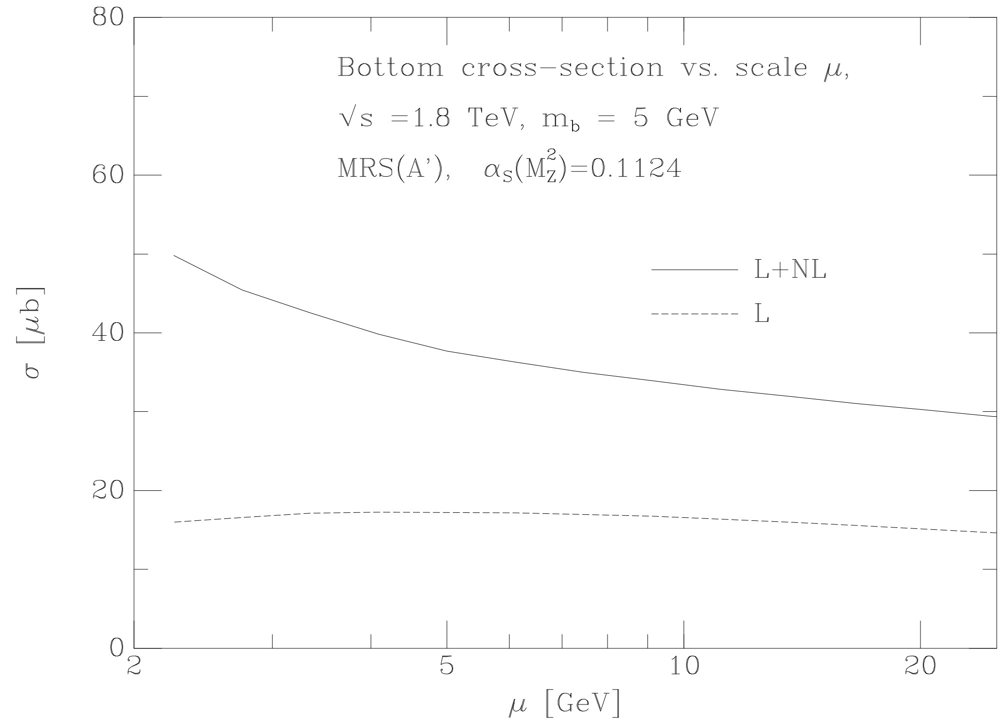
Top production at LHC



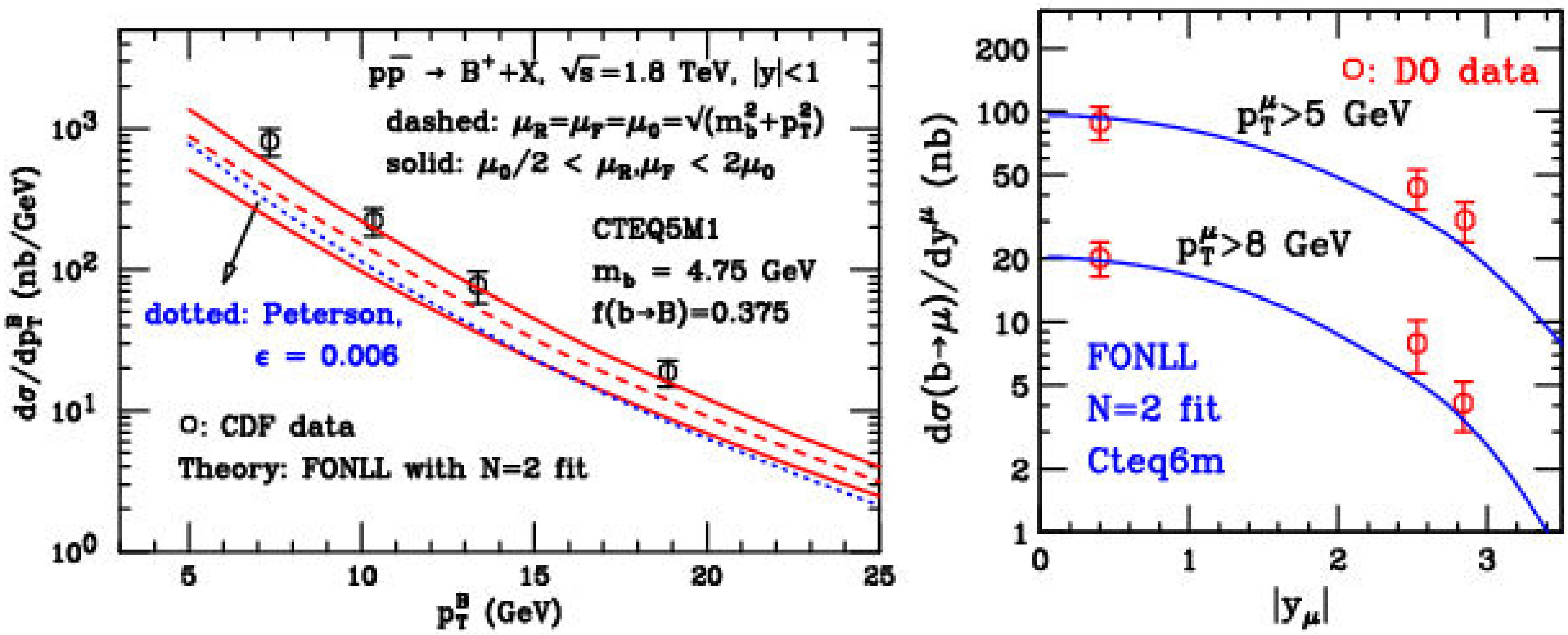
- At LHC top cross section is more than 100 times bigger than at Tevatron.

Scale dependence in bottom production

- The perturbation series for bottom quark production is not well behaved.
- The lowest order cross section is almost μ independent because of an accidental cancellation between the fall-off of α_S and the increase of the gluon distributions with increasing μ.



Beauty production at CDF

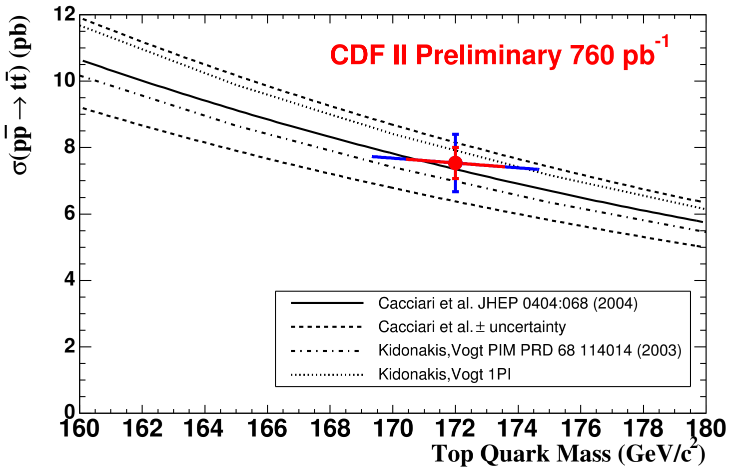


- To compare the b-quark production prediction with the experimental data on B-meson production, we need to include a fragmentation function.
- The data agrees with the upper range of the theoretical prediction. Given the status of the perturbation theory for b-quark production, this is a positive outcome.

Top production

- All the information on the top quark is still rather limited and crude
- Within errors agreement between three generation theory and experiment

	Experiment	Theory
m_t	172.5 ± 2.3 GeV	$178.9 + 12.0 - 9.0$ GeV
$BR(t \to Wb)/BR(t \to Wq)$	$1.11^{+0.21}_{-0.26}$	≈ 1
$BR(t \to W_0 b)$	$0.74^{+0.22}_{-0.34}$	≈ 0.7
$BR(t \to W_+ b)$	< 0.27 (95%cl)	≈ 0



Single Top production

- CDF has measured

$$\frac{BR(t \to Wb)}{BR(t \to Wq)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} = 1.11^{+0.21}_{-0.26}$$

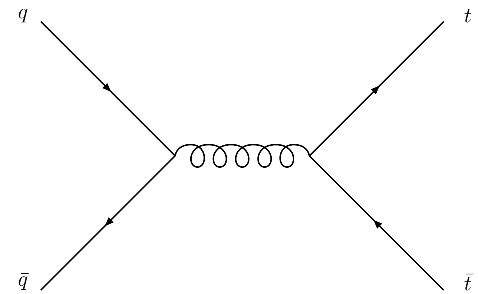
  If we assume just three generations of quarks, unitarity of the CKM matrix implies that the denominator is equal to one, so that we can extract

$$|V_{tb}| = 1.11^{+0.10}_{-0.13}$$

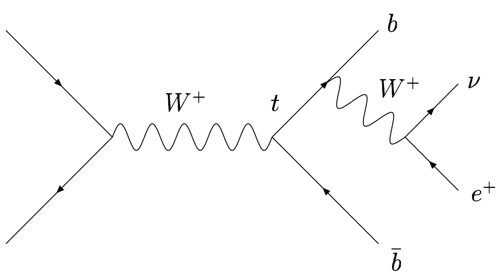
- But assuming unitarity we know already that $V_{tb} = 0.9990 - -0.9993$.
- The current CDF measurement shows that $|V_{tb}| \gg |V_{td}|, |V_{ts}|$.
- For a real measurement of V_{tb} we must look at the electroweak production of a top quark.

Producing the top quark

- The top quark was discovered in Run I of the Tevatron by producing it in pairs:

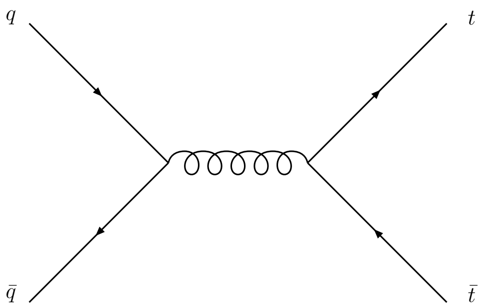


- However, it should also be possible to produce it singly in Run II, for example:

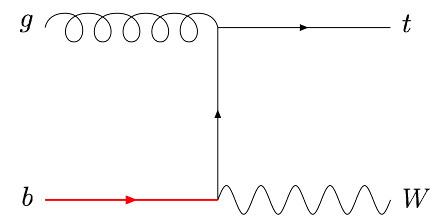


- This is especially interesting since it would yield information about the weak interaction of top quarks (V_{tb}).

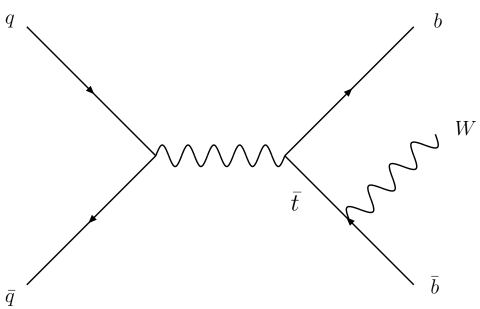
Top production rates



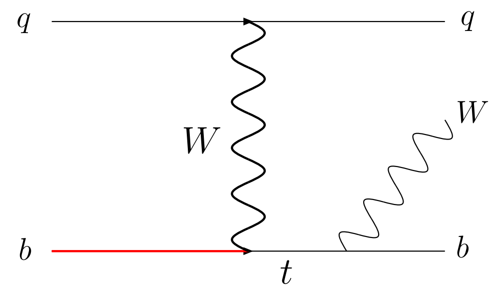
6 pb
720 pb



0.14 pb
66 pb



0.8 pb
10 pb

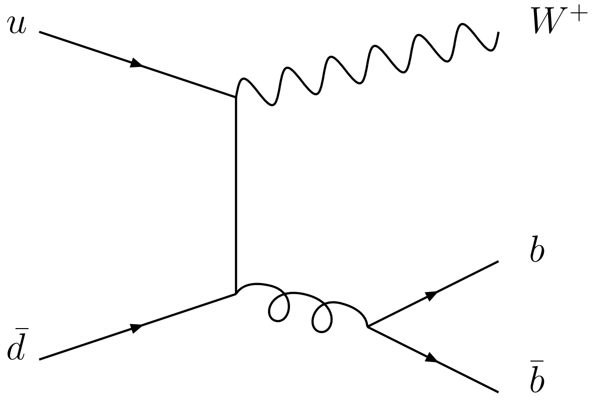


1.8 pb
240 pb

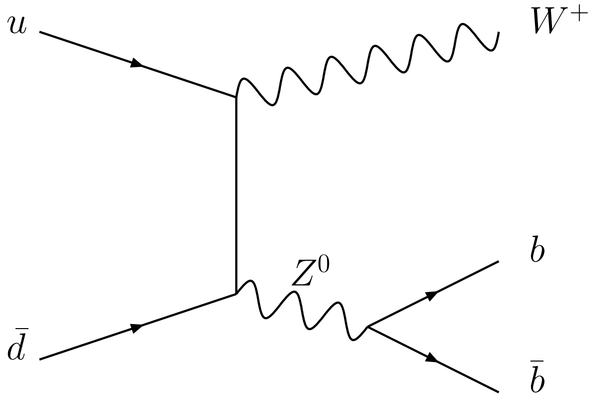
- All cross-sections are known to NLO (Tevatron / LHC)
- The total single top cross-section is smaller than the $t\bar{t}$ rate by about a factor of two, at both machines

Experimental signature

- The experimental “signature” is an event which contains a top quark – identified by the combined mass of its decay products – and which also has two jets containing b-quarks. These can be distinguished from other jets around 50% of the time.
- Observed events such as these can also be the result of other basic processes. These backgrounds include, for example:



$Wb\bar{b}$

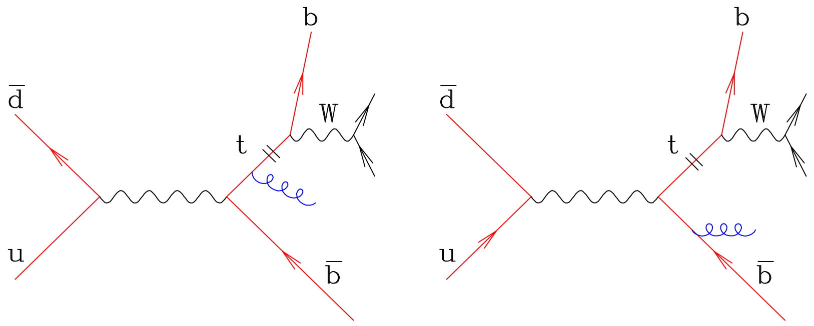


$WZ(\rightarrow b\bar{b})$

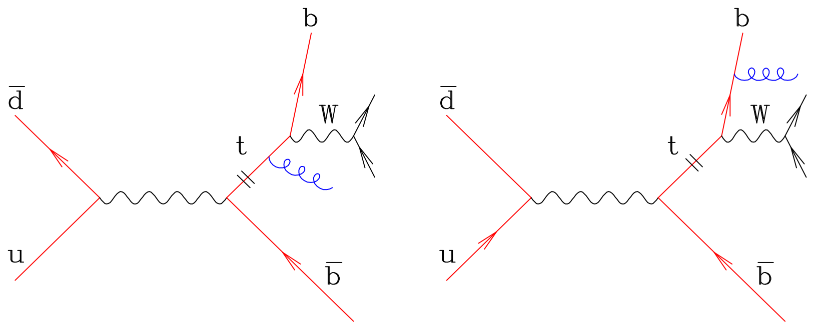
- MCFM can calculate the signal and backgrounds at NLO.

Inclusion of decay

- Results had previously been presented without including the decay of the top quark. Without it, predictions for some quantities used in Tevatron search strategies are impossible
- Final state radiation that enters at next-to-leading order is possible in either the production or decay phase:

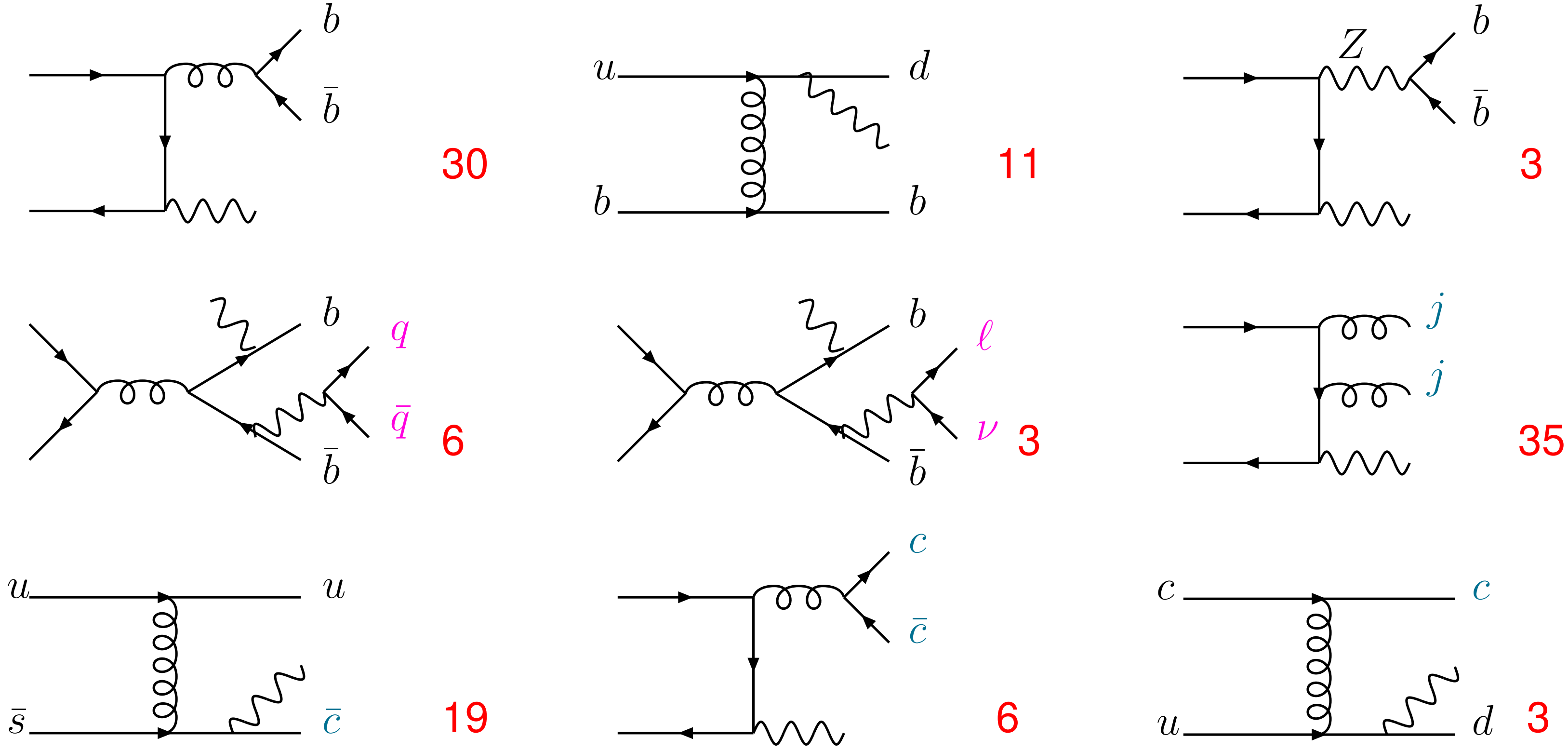


production



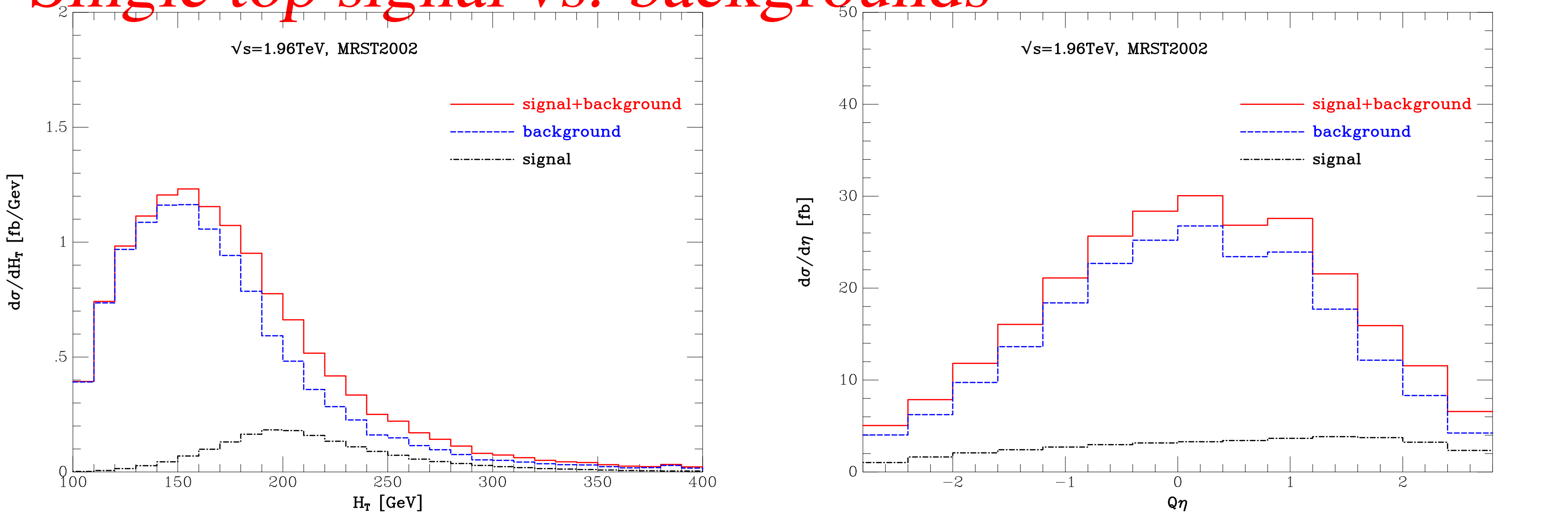
decay

Backgrounds



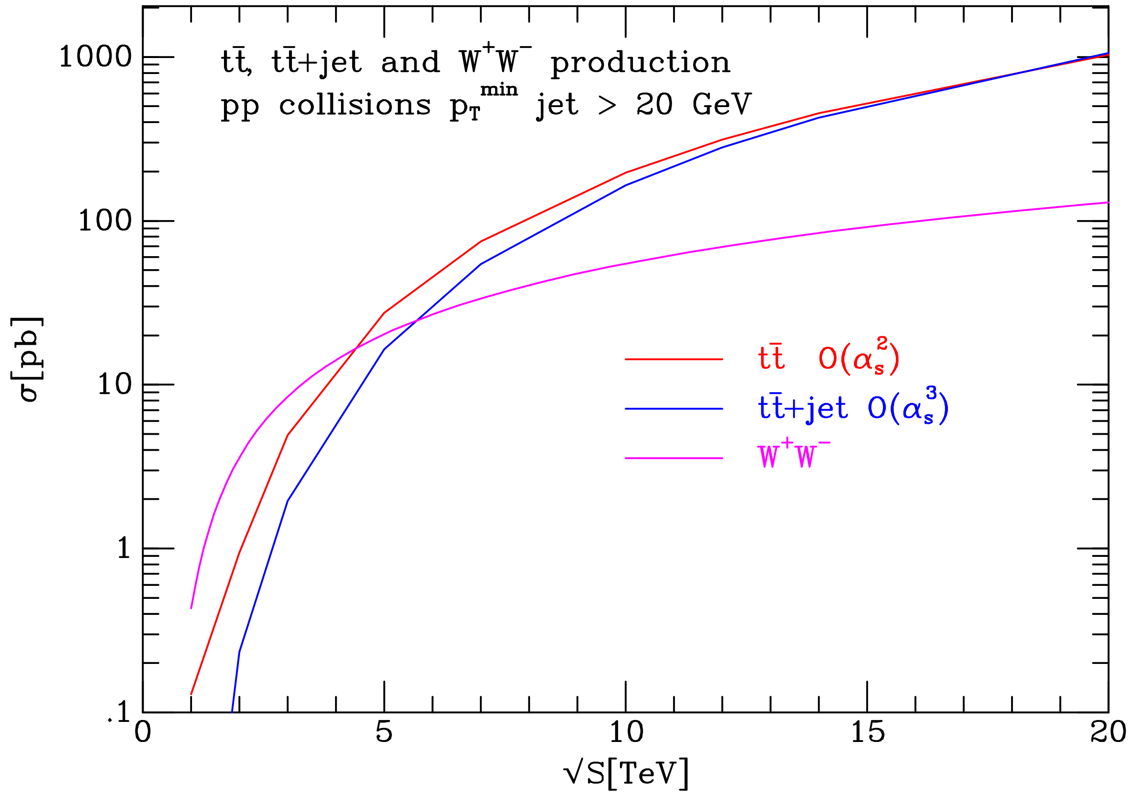
- Cross-sections in fb include nominal tagging efficiences and mis-tagging/fake rates. Calculated with MCFM, most at NLO
- Rates are 7 fb and 11 fb for s- and $t-$channel signal

Single top signal vs. backgrounds



- $H_T =$ scalar sum of jet, lepton and missing E_T
- $Q\eta$ is the product of the lepton charge and the rapidity of the untagged jet, useful for picking out the t-channel process
- Signal:Background (with our nominal efficiencies) is about $1 : 6$
  – a very challenging measurement indeed. Production in this mode has not yet been observed at Fermilab.
- it will take 1.5 fb^{-1} to have evidence (3σ) for single top from a single experiment at the Tevatron (Gresele, Moriond 2006).

Top +jet production at LHC



- $t\bar{t}$+jet cross section same as $t\bar{t}$ cross section; Radiation probability is one.
- Note that a $p_T = 20$ GeV jet can be adequately described using the soft approximation.
- The W^+W^- cross section is also shown, (subject to gauge cancellation)

General calculational method for NLO

- Direct integration is good for the total cross section, but for differential distributions, (to which we want to apply cuts), we need a Monte Carlo method.
- We use a general subtraction procedure at NLO.
- at NLO the cross section for two initial partons a and b and for m outgoing partons, is given by

$$\sigma_{ab} = \sigma_{ab}^{LO} + \sigma_{ab}^{NLO}$$

where

$$\begin{aligned} \sigma_{ab}^{LO} &= \int_m d\sigma_{ab}^{B} \\ \sigma_{ab}^{NLO} &= \int_{m+1} d\sigma_{ab}^{R} + \int_m d\sigma_{ab}^{V} \end{aligned}$$

the singular parts of the QCD matrix elements for real emission, corresponding to soft and collinear emission can be isolated in a process independent manner

Calculational method (cont)

- One can use this the construct a set of counterterms

$$d\sigma^{ct} = \sum_{ct} \int_m d\sigma^B \otimes \int_1 dV_{ct}$$

  where $d\sigma^B$ denotes the appropriate color and spin projection of the Born-level cross section, and the counter-terms are independent of the details of the process under consideration.

- these counterterm cancel all non-integrable singularities in $d\sigma^R$, so that one can write

$$\sigma_{ab}^{NLO} = \int_{m+1} [d\sigma_{ab}^R - d\sigma_{ab}^{ct}] + \int_{m+1} d\sigma_{ab}^{ct} + \int_m d\sigma_{ab}^V$$

  The phase space integration in the first term can be performed numerically in four dimensions.

Matrix element counter-event for W production

In the soft limit $p_5 \to 0$ we have

$$|M_1(p_1,p_2,p_3,p_4,p_5)|^2 = g^2 C_F \frac{p_1 \cdot p_2}{p_1 \cdot p_5 \; p_2 \cdot p_5} |M_0(p_1,p_2,p_3,p_4)|^2$$

- Eikonal factor can be associated with radiation from a given leg by partial fractioning

$$\frac{p_1 \cdot p_2}{p_1 \cdot p_5 \; p_2 \cdot p_5} = [\frac{p_1 \cdot p_2}{p_1 \cdot p_5 + p_2 \cdot p_5}][\frac{1}{p_1 \cdot p_5} + \frac{1}{p_2 \cdot p_5}]$$

- including the collinear contributions, singular as $p_1 \cdot p_5 \to 0$, the matrix element for the counter event has the structure

$$|M_1(p_1,p_2,p_3,p_4,p_5)|^2 = \frac{g^2}{x_a p_1 \cdot p_5} \hat{P}_{qq}(x_a) |M_0(x_a p_1, p_2, \tilde{p}_3, \tilde{p}_4)|^2$$

where $1 - x_a = (p_1 \cdot p_5 + p_2 \cdot p_5)/p_1 \cdot p_2$ and $\hat{P}_q q(x_a) = C_F(1+x^2)/(1-x)$

Subtraction method for NLO

- For event $q(p_1) + \bar{q}(p_2) \to W^+(\nu(p_3) + e^+(p_4)) + g(p_5)$ with $p_1 + p_2 = \sum_{i=3}^{5} p_i$ we generate a counter event $q(x_a p_1) + \bar{q}(p_2) \to W^+(\nu(\tilde{p}_3) + e^+(\tilde{p}_4))$ and $x_a p_1 + p_2 = \sum_{i=3}^{4} \tilde{p}_i$ with $1 - x_a = (p_1 \cdot p_5 + p_2 \cdot p_5)/p_1 \cdot p_2$.
- A Lorentz transformation is performed on all j final state momenta $\tilde{p}_j = \Lambda^\mu_\nu p^\nu_j, j = 3, 4$ such that $\tilde{p}^\mu_j \to p^\mu_j$ for p_5 collinear or soft.
- The longitudinal momentum of p_5 is absorbed by rescaling with x.
- The other components of the momentum, p_5 are absorbed by the Lorentz transformation.
- In terms of these variables the phase space has a convolution structure,

$$d\phi^{(3)}(p_1, p_2; p_3, p_4, p_5) = \int_0^1 \; dx \; d\phi^{(2)}(p_2, x p_1; \tilde{p_3}, \tilde{p_4})[dp_5(p_1, p_2, x)]$$

where

$$[dp_5(p_1, p_2, x_a)] = \frac{d^d p_5}{(2\pi)^3} \delta^+(p_5^2) \Theta(x) \Theta(1 - x) \delta(x - x_a)$$

MCFM overview

John Campbell and R.K. Ellis

- Parton level cross-sections predicted to NLO in α_S

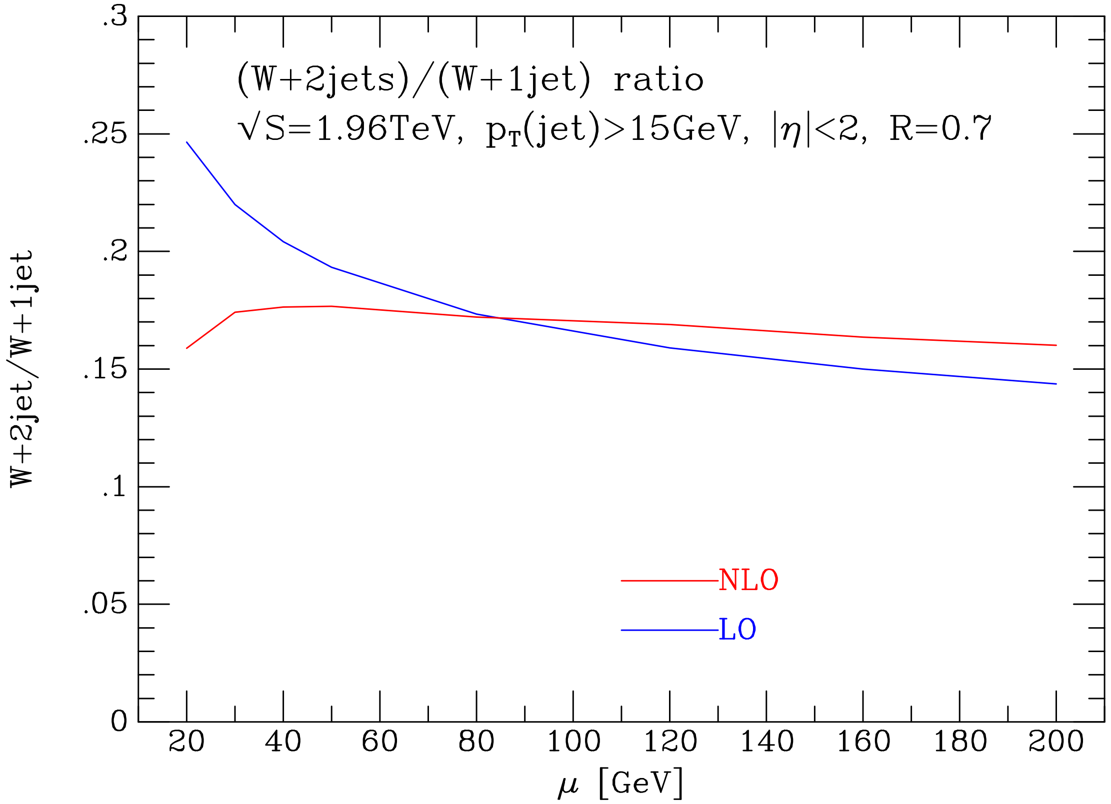
$p\bar{p} \to W^\pm/Z$	$p\bar{p} \to W^+ + W^-$
$p\bar{p} \to W^\pm + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^\pm + \gamma$	$p\bar{p} \to W^\pm/Z + H$
$p\bar{p} \to W^\pm + g^\star\ (\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W^\pm/Z +$ 1 jet	$p\bar{p} \to W^\pm/Z +$ 2 jets
$p\bar{p}(gg) \to H$	$p\bar{p}(gg) \to H +$ 1 jet
$p\bar{p}(VV) \to H +$ 2 jets	$p\bar{p} \to t + X$
$pp \to t + W$	

$\oplus$ less sensitivity to μ_R, μ_F, rates are better normalized, fully differential distributions.

$\ominus$ low particle multiplicity (no showering), no hadronization, hard to model detector effects

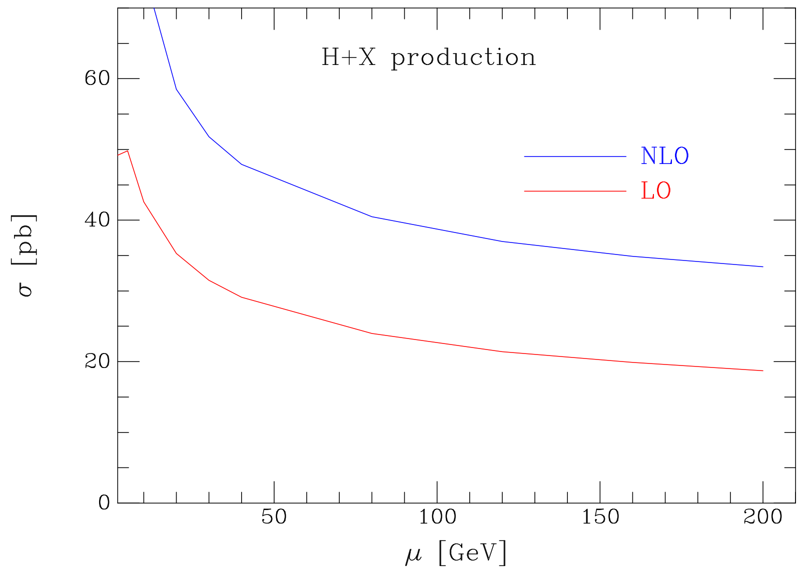
MCFM:examples

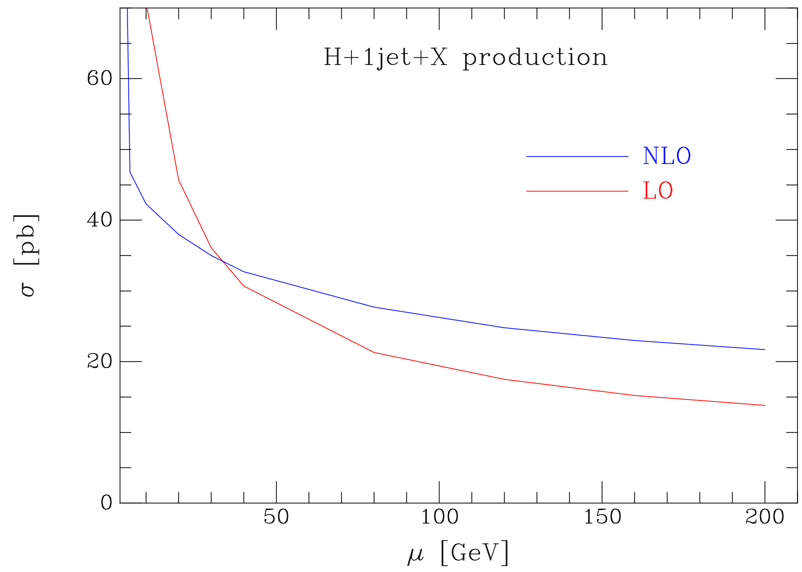
- (W+2 jet)/(W+1 jet)



MCFM examples

- Production of a $m_H = 120$ GeV Higgs, using effective Lagrangian $HG^{\mu\nu}G_{\mu\nu}$, obtained in heavy top limit.
- Cross sections for Higgs+anything or Higgs+1 jet+anything are the same.
- Radiation probability is one, and NLO is clearly inadequate.
- what is needed is a combination of NLO and shower Monte-Carlo, (MC@NLO)





NLO: Schematic description

- A schematic description of a NLO calculation is as follows.

$$\left(\frac{d\sigma}{dx}\right)_B = B\delta(x)$$

$$\left(\frac{d\sigma}{dx}\right)_V = a\left(\frac{B}{2\epsilon}+V\right)\delta(x)$$

$$\left(\frac{d\sigma}{dx}\right)_R = a\frac{R(x)}{x}$$

- In terms of the above the calculation of any observable O can written as

$$\langle O\rangle = \lim_{\epsilon\longrightarrow 0}\int_0^1 dx x^{-2\epsilon} O(x)\left[\left(\frac{d\sigma}{dx}\right)_B + \left(\frac{d\sigma}{dx}\right)_V + \left(\frac{d\sigma}{dx}\right)_R\right]$$

Subtraction method

We can isolate the divergent part of the real radiation contribution

$$\langle O \rangle_{\mathrm{R}} = aBO(0) \int_0^1 dx\, \frac{x^{-2\epsilon}}{x} + a \int_0^1 dx\, \frac{O(x)R(x) - BO(0)}{x^{1+2\epsilon}} \ .$$

The second term does not contain singularities so we can set $\epsilon = 0$

$$\langle O \rangle_{\mathrm{R}} = -a\frac{B}{2\epsilon} O(0) + a \int_0^1 dx\, \frac{O(x)R(x) - BO(0)}{x} \ .$$

The NLO prediction using the subtraction method is

$$\langle O \rangle_{\mathrm{sub}} = BO(0) + a \left[VO(0) + \int_0^1 dx\, \frac{O(x)R(x) - BO(0)}{x} \right] .$$

Toy Monte Carlo

- Rewrite the basic NLO formula in a different which allows simpler matching with the Monte Carlo:

$$\langle O \rangle_{\rm sub} = \int_0^1 dx \left[O(x) \frac{aR(x)}{x} + O(0) \left(B + aV - \frac{aB}{x} \right) \right].$$

- Introduce Sudakov form factor for the toy model

$$\Delta(x_1, x_2) = \exp\left[-a \int_{x_1}^{x_2} dz \frac{Q(z)}{z} \right],$$

where $Q(z)$ is a radiation function with the following general properties:

$$0 \leq Q(z) \leq 1, \quad \lim_{z \to 0} Q(z) = 1, \quad \lim_{z \to 1} Q(z) = 0.$$

If x_s is the energy of the system before the first branching occurs, then $\Delta(x, x_s)$ is the probability that no photon be emitted with energy z such that $x \leq z \leq x_s$.

Matching NLO and MC

$$\left(\frac{d\sigma}{dO}\right)_{\rm MC@LO} = B I_{\rm MC}(O,1).$$

$$\left(\frac{d\sigma}{dO}\right)_{\rm naive} = \int_0^1 dx \left[I_{\rm MC}(O, x_{\rm M}(x)) \frac{aR(x)}{x} + I_{\rm MC}(O,1)\left(B + aV - \frac{aB}{x}\right)\right].$$

This equation suggests the following procedure:

- Pick at random $0 \le x \le 1$.
- Generate an MC event with $x_{\rm M}(x)$ as maximum energy available to the photon in the first branching; attach to this event the weight $w_{\rm EV} = aR(x)/x$.
- Generate another MC event (a “counter-event”) with $x_{\rm M} = 1$; attach to this event the weight $w_{\rm CT} = B + aV - aB/x$.
- Repeat the first three steps N times, and normalize with $1/N$.

This procedure fails, since the weights $w_{\rm EV}$ and $w_{\rm CT}$ diverge as $x \to 0$.

Modified subtraction method

$$\left(\frac{d\sigma}{dO}\right)_{\rm msub} = \int_0^1 dx \Bigg[I_{\rm MC}(O, x_{\rm M}(x))\,\frac{a[R(x) - BQ(x)]}{x}$$

$$+ I_{\rm MC}(O,1)\left(B + aV + \frac{aB[Q(x)-1]}{x}\right)\Bigg] .$$

- We subtract and add the quantities

$$I_{\rm MC}(O,1)\,\frac{aBQ(x)}{x}, \quad I_{\rm MC}(O,x_{\rm M})\,\frac{aBQ(x)}{x}$$

- The two terms involving $Q(x)$ are not identical, so this is not a subtraction in the usual sense of an NLO computation.
- The two terms do not contribute to the observable O at $\mathcal{O}(a)$, because they are compensated by terms in the parton shower $BI_{\rm MC}(O,1)$

Expansion to $O(\alpha_S)$

- Expansion of Monte Carlo piece is

$$I_{\rm MC} = (1 - a \int_{x_0}^{1} \; dt \frac{Q(t)}{t} \; \delta(O - O(0)) + a \int_{x_0}^{1} \; dt \frac{Q(t)}{t} \; \delta(O - O(t)) + O(a^2)$$

- Insertion of this piece in the modified Monte-Carlo formula gives

$$\begin{aligned}
\left(\frac{d\sigma}{dO}\right)_{\rm msub} \quad = \quad & \int_0^1 dx \Bigg[\delta(O - O(x)) \frac{a[R(x) - BQ(x)]}{x} \\
& + \delta(O - O(0)) \left(B + aV - \frac{aB}{x} \right) \\
& + aB\delta(O - O(0)) \left(\frac{Q(x)}{x} - \int_{x_0}^{1} dt \frac{Q(t)}{t} \right) \\
& + aB \int_{x_0}^{1} dt \delta(O - O(t)) \frac{Q(t)}{t} \Bigg] + \mathcal{O}(a^2).
\end{aligned}$$

Expansion (continued)

- Collecting terms we obtain the starting formula for a NLO correction, plus power suppressed terms which are anyway not controlled in the Monte Carlo

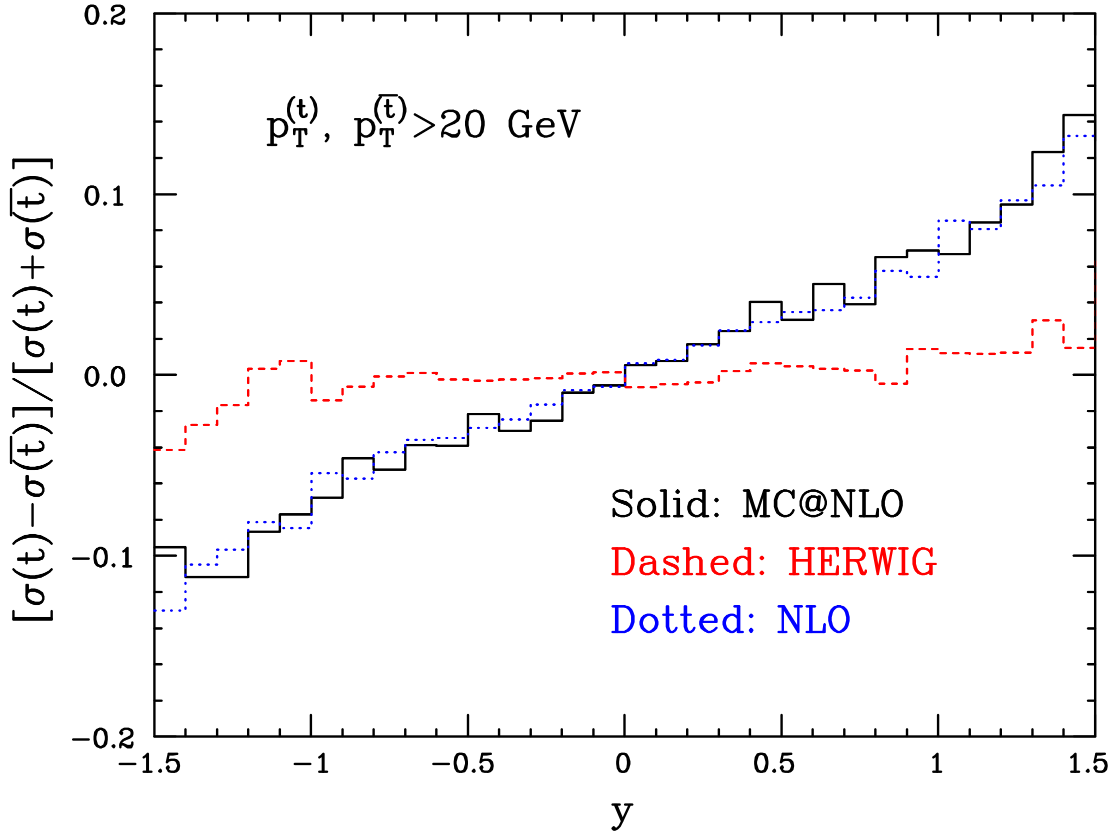
$$
\begin{aligned}
\left(\frac{d\sigma}{dO}\right)_{\rm msub} &= \int_0^1 dx \left[\delta(O-O(x))\frac{aR(x)}{x} + \delta(O-O(0))\left(B + aV - \frac{aB}{x}\right)\right] \\
&+ aB\int_0^{x_0} dx \frac{Q(x)}{x}\Big[\delta(O-O(0)) - \delta(O-O(x))\Big] + \mathcal{O}(a^2).
\end{aligned}
$$

- It can also be shown that the normal summation of branching logarithms is not compromised by this procedure.

Asymmetry in top production

Frixione,Nason,Webber

- Example of $t\bar{t}$-production using MC@NLO
- NLO curve (in blue, dotted).



Conclusions

- NLO formulation of QCD processes gives better information about normalization, and less dependence on unphysical scales.
- Matching with Monte Carlo can be implemented.
- Much remains to be done
  - ⋆ The NLO corrections which necessary for normalization are unknown for many of the most interesting processes. $2 \to 2$ processes are known, some $2 \to 3$ processes, one or two $2 \to 4$ processes.
  - ⋆ MC@NLO is known only for a very limited set of processes, namelty the hadroproduction of single vector and Higgs bosons, vector boson pairs, heavy quark pairs, singletop, lepton pairs, and Higgs bosons in association with a W or Z.