

*Collider Physics*  
*Lecture III: Shower Monte Carlo*

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Slides available from <http://theory.fnal.gov/people/ellis/Talks/>

# *Bibliography*

QCD and Collider Physics

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by R. K. Ellis, W.J. Stirling and B.R. Webber

Monte Carlo Simulation Of Hard Hadronic Processes

B.R. Webber,

Ann.Rev.Nucl.Part.Sci.36:253-286,1986

Les Houches guidebook to Monte Carlo generators for hadron collider physics.

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e-Print Archive: hep-ph/0403045

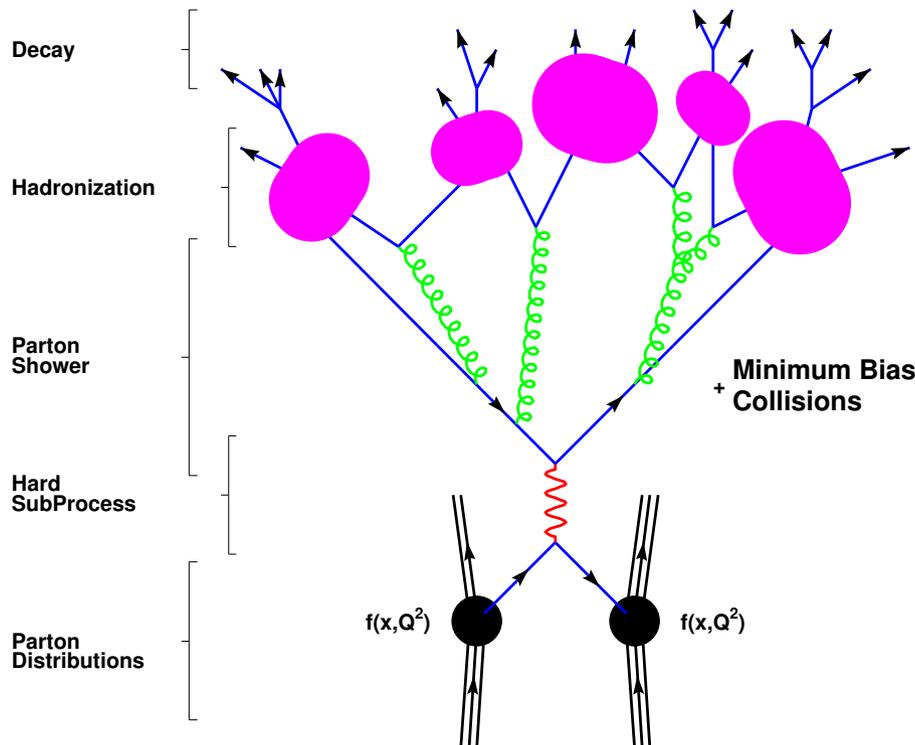
# *Monte Carlo and Jets*

- Shower Monte Carlos
- Sudakov form factor
  - ★ Infrared cutoff
- Monte Carlo method
- Soft gluon emission
  - ★ Angular ordering
  - ★ Coherent branching

# Shower Monte Carlos

Attempt to give a complete description of a hadron scattering event. Analysis of events at the Tevatron and the LHC will be performed using these programs.

- Large number of standard model hard scattering processes
- Inclusion of some BSM processes.
- Inclusion of real (and virtual) radiation using QCD-based parton shower approximation, both in final state and initial state (not shown).
- Fragmentation of partons into the observed hadrons.
- Model for resonance decay included



# *Leading general purpose programs*

- Pythia, T. Sjöstrand, L. Lönnblad, S. Mrenna and P.Z. Skands,  
<http://www.thep.lu.se/~torbjorn/Pythia.html>
- Pythia 8, T. Sjöstrand C++  
<http://www.thep.lu.se/~torbjorn/Pythia.html>
- HERWIG, G. Corcella, I.G. Knowles, G. Marchesini, S. Moretti, K. Odagiri, P. Richardson, M.H. Seymour, B.R. Webber,  
<http://hepwww.rl.ac.uk/theory/seymour/herwig/>
- Herwig++  
S. Gieseke, A. Ribon, P. Richardson, M.H. Seymour, P. Stephens, B.R. Webber  
<http://www.hep.phy.cam.ac.uk/theory/Herwig++/>
- SHERPA,  
Tanju Gleisberg, Frank Krauss, Andreas Schälicke, Steffen Schumann, Jan Winter
- ISAJET,  
F. Paige, S. Protopopescu, H. Baer and X. Tata  
<http://www.phy.bnl.gov/~isajet>

# Sudakov form factor

- DGLAP equations are convenient for evolution of parton distributions. Expressed in terms of the unregulated branching probability we have,

$$t \frac{\partial}{\partial t} D(x, t) = \int_x^1 \frac{dz}{z} \frac{\alpha_S}{2\pi} \hat{P}(z) D(x/z, t) - \int_0^1 dz \frac{\alpha_S}{2\pi} \hat{P}(z) D(x, t)$$

- To study structure of final states, slightly different form is useful. Consider again simplified treatment with only one type of branching. Introduce Sudakov form factor:

$$\Delta(t) \equiv \exp \left[ - \int_{t_0}^t \frac{dt'}{t'} \int dz \frac{\alpha_S}{2\pi} \hat{P}(z) \right] ,$$

$$t \frac{\partial}{\partial t} D(x, t) = \int \frac{dz}{z} \frac{\alpha_S}{2\pi} \hat{P}(z) D(x/z, t) + \frac{D(x, t)}{\Delta(t)} t \frac{\partial}{\partial t} \Delta(t) ,$$

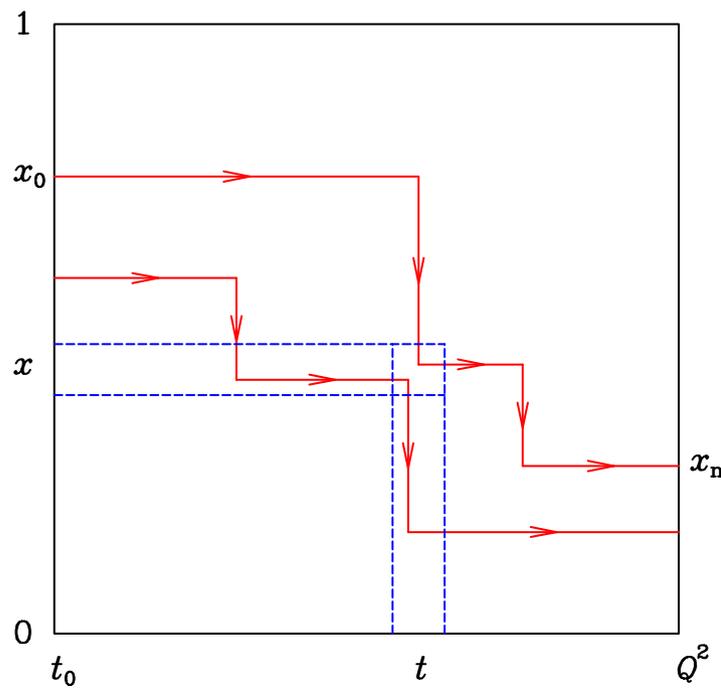
$$t \frac{\partial}{\partial t} \left( \frac{D}{\Delta} \right) = \frac{1}{\Delta} \int \frac{dz}{z} \frac{\alpha_S}{2\pi} \hat{P}(z) D(x/z, t) .$$

# Sudakov form factor

- This is similar to DGLAP, except  $D$  replaced by  $D/\Delta$  and regularized splitting function  $P$  replaced by unregularized  $\hat{P}$ . Integrating,

$$D(x, t) = \Delta(t)D(x, t_0) + \int_{t_0}^t \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \frac{\alpha_S}{2\pi} \hat{P}(z) D(x/z, t').$$

- This has simple interpretation. First term is contribution from paths that do not branch between scales  $t_0$  and  $t$ . Thus Sudakov form factor  $\Delta(t)$  is probability of evolving from  $t_0$  to  $t$  without branching. Second term is contribution from paths which have their last branching at scale  $t'$ . Factor of  $\Delta(t)/\Delta(t')$  is probability of evolving from  $t'$  to  $t$  without branching.



# Sudakov form factor

- Generalization to several species of partons straightforward. Species  $i$  has Sudakov form factor

$$\Delta_i(t) \equiv \exp \left[ - \sum_j \int_{t_0}^t \frac{dt'}{t'} \int dz \frac{\alpha_S}{2\pi} \hat{P}_{ji}(z) \right] ,$$

which is probability of it evolving from  $t_0$  to  $t$  without branching. Then

$$t \frac{\partial}{\partial t} \left( \frac{D_i}{\Delta_i} \right) = \frac{1}{\Delta_i} \sum_j \int \frac{dz}{z} \frac{\alpha_S}{2\pi} \hat{P}_{ij}(z) D_j(x/z, t) .$$

# Infrared cutoff

- In DGLAP equation, infrared singularities of splitting functions at  $z = 1$  are regularized by plus-prescription. However, in above form we must introduce an explicit infrared cutoff,  $z < 1 - \epsilon(t)$ . Branchings with  $z$  above this range are unresolvable: emitted parton is too soft to detect. Sudakov form factor with this cutoff is probability of evolving from  $t_0$  to  $t$  without any resolvable branching.
- Sudakov form factor sums enhanced virtual (parton loop) as well as real (parton emission) contributions. No-branching probability is the sum of virtual and unresolvable real contributions: both are divergent but their sum is finite.
- Infrared cutoff  $\epsilon(t)$  depends on what we classify as resolvable emission. For timelike branching, natural resolution limit is given by cutoff on parton virtual mass-squared,  $t > t_0$ . When parton energies are much larger than virtual masses, we may write, ( $n^2 = p^2 = p \cdot p_T = p \cdot p_T = 0, n \cdot p = 1$ )

$$p_a = p^\mu + \frac{p_a^2}{2} n^\mu$$

$$p_b = zp^\mu + \frac{p_T^2 + p_b^2}{2z} n^\nu + p_T^\mu$$

$$p_c = (1 - z)p^\mu + \frac{p_T^2 + p_c^2}{2(1 - z)} n^\mu - p_T^\mu$$

- Solving for the transverse momentum in  $a \rightarrow bc$  is

$$p_T^2 = z(1-z)p_a^2 - (1-z)p_b^2 - zp_c^2 > 0 .$$

- Hence for  $p_a^2 = t$  and  $p_b^2, p_c^2 > t_0$  we require

$$z(1-z) > t_0/t ,$$

that is,

$$z, 1-z > \epsilon(t) = \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4t_0/t} \simeq t_0/t .$$

- Quark Sudakov form factor is then

$$\Delta_q(t) \simeq \exp \left[ - \int_{2t_0}^t \frac{dt'}{t'} \int_{t_0/t'}^{1-t_0/t'} dz \frac{\alpha_S}{2\pi} \hat{P}_{qq}(z) \right] .$$

- Taking the argument of the running coupling to be  $t'$  at large  $t$  we have

$$\Delta_q(t) \sim \left( \frac{t_0}{t} \right)^{p_1} \left( \frac{\alpha_S(t)}{\alpha_S(t_0)} \right)^{p_2} ,$$

where  $p_1 = a, p_2 = a \ln(t_0/\Lambda), a = 12C_F/(33 - 2n_f) \sim 16/25$ .

- Careful treatment of running coupling suggests its argument should be  $p_T^2 \sim z(1-z)t'$ .

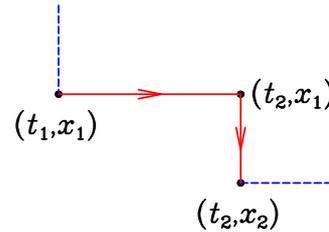
$$\Delta_q(t) \sim \left( \frac{\alpha_S(t)}{\alpha_S(t_0)} \right)^{p \ln t} ,$$

( $p = a$  constant), which tends to zero faster than any negative power of  $t$ .

- Infrared cutoff discussed here follows from kinematics. We shall see later that QCD dynamics effectively reduces phase space for parton branching, leading to a more restrictive effective cutoff.
- Formulation in terms of Sudakov form factor is well suited to computer implementation, and is basis of “parton shower” Monte Carlo programs.

# Monte Carlo method

- Monte Carlo branching algorithm operates as follows: given virtual mass scale and momentum fraction  $(t_1, x_1)$  after some step of the evolution, or as initial conditions, it generates values  $(t_2, x_2)$  after the next step.



- ★ Since probability of evolving from  $t_1$  to  $t_2$  without branching is  $\Delta(t_2)/\Delta(t_1)$ ,  $t_2$  can be generated with the correct distribution by solving

$$\frac{\Delta(t_2)}{\Delta(t_1)} = \mathcal{R}$$

where  $\mathcal{R}$  is random number (uniform on  $[0, 1]$ ).

- ★ If  $t_2$  is higher than hard process scale  $Q^2$ , this means branching has finished.
- ★ Otherwise, generate  $z = x_2/x_1$  with distribution proportional to  $(\alpha_S/2\pi)P(z)$ , where  $P(z)$  is appropriate splitting function, by solving

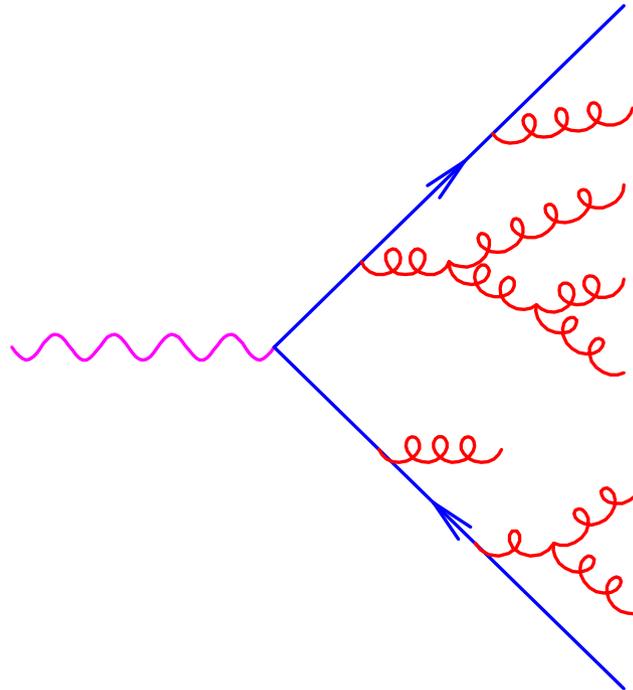
$$\int_{\epsilon}^{x_2/x_1} dz \frac{\alpha_S}{2\pi} P(z) = \mathcal{R}' \int_{\epsilon}^{1-\epsilon} dz \frac{\alpha_S}{2\pi} P(z)$$

- In DIS,  $(t_i, x_i)$  values generated define virtual masses and momentum fractions of exchanged quark, from which momenta of emitted gluons can be computed. Azimuthal emission angles are then generated uniformly in the range  $[0, 2\pi]$ . More generally, e.g. when exchanged parton is a gluon, azimuths must be generated with polarization angular correlations.
- Each emitted (timelike) parton can itself branch. In that case  $t$  evolves downwards towards cutoff value  $t_0$ , rather than upwards towards hard process scale  $Q^2$ . Probability of evolving downwards without branching between  $t_1$  and  $t_2$  is now given by

$$\frac{\Delta(t_1)}{\Delta(t_2)} = \mathcal{R} .$$

Thus branching stops when  $\mathcal{R} < \Delta(t_1)$ .

# Parton Cascade



- Due to successive branching, parton cascade or shower develops. Each outgoing line is source of new cascade, until all outgoing lines have stopped branching. At this stage, which depends on cutoff scale  $t_0$ , outgoing partons have to be converted into hadrons via a hadronization model.

# Backwards evolution

- The probability of evolving backwards from  $(t_2, x)$  to  $(t_1, x)$  without branching is given by the function

$$\Pi(t_1, t_2; x) = \frac{f(x, t_1)\Delta(t_2)}{f(x, t_2)\Delta(t_1)} = \frac{\Delta(t_2)}{f(x, t_2)} \frac{f(x, t_1)}{\Delta(t_1)}.$$

In other words, in place of the Sudakov form factor  $\Delta(t_i)$  we should now use  $\Delta(t_i)/f(x, t_i)$ . The correct distribution of  $t_1$  in the elementary backward step is obtained by solving the equation

$$\Pi(t_1, t_2; x_2) = \mathcal{R}$$

where  $\mathcal{R}$  is a random number distributed uniformly in the interval  $[0, 1]$ .

Thus we have to generate  $z = x_2/x_1$  with a probability distribution proportional to

$$\frac{\alpha_S}{2\pi} \frac{P(z)}{z} f(x_2/z, t_1)$$

where  $P(z)$  is the appropriate splitting function.

- define  $dF$  as the fraction of partons at  $(t_2, x)$  that came from branching between  $t$  and  $t + dt$ . Then the fraction that did not branch between  $t_1$  and  $t_2$  is

$$\Pi(t_1, t_2; x) = 1 - \int_{t_1}^{t_2} dF .$$

- Now it follows from the evolution equation

$$f(x, t) = \Delta(t)f(x, t_0) + \int_{t_0}^t \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \frac{\alpha_S}{2\pi} \hat{P}(z) f(x/z, t') .$$

that

$$\begin{aligned} f(x, t_2)dF &= \frac{dt}{t} \frac{\Delta(t_2)}{\Delta(t)} \int \frac{dz}{z} \frac{\alpha_S}{2\pi} \hat{P}(z) f(x/z, t) \\ &= dt \frac{\partial}{\partial t} \left[ \frac{\Delta(t_2)}{\Delta(t)} f(x, t) \right] . \end{aligned}$$

- Performing the integration we obtain the

$$\Pi(t_1, t_2; x) = \frac{f(x, t_1)\Delta(t_2)}{f(x, t_2)\Delta(t_1)} = \frac{\Delta(t_2)}{f(x, t_2)} \frac{f(x, t_1)}{\Delta(t_1)} .$$

# Soft gluon emission

- Parton branching formalism discussed so far takes account of collinear enhancements to all orders in PT. There are also soft enhancements: When external line with momentum  $p$  and mass  $m$  (not necessarily small) emits gluon with momentum  $q$ , propagator factor is

$$\frac{1}{(p \pm q)^2 - m^2} = \frac{\pm 1}{2p \cdot q} = \frac{\pm 1}{2\omega E(1 - v \cos \theta)}$$

where  $\omega$  is emitted gluon energy,  $E$  and  $v$  are energy and velocity of parton emitting it, and  $\theta$  is angle of emission. This diverges as  $\omega \rightarrow 0$ , for any velocity and emission angle.

- Including numerator, soft gluon emission gives a colour factor times universal, spin-independent factor in amplitude

$$F_{\text{soft}} = \frac{p \cdot \epsilon}{p \cdot q}$$

where  $\epsilon$  is polarization of emitted gluon.

- For example, emission from quark gives numerator factor  $N \cdot \epsilon$ , where

$$\begin{aligned}
 N^\mu &= (\not{p} + \not{q} + m)\gamma^\mu u(p) \xrightarrow{\omega \rightarrow 0} (\gamma^\nu \gamma^\mu p_\nu + \gamma^\mu m)u(p) \\
 &= (2p^\mu - \gamma^\mu \not{p} + \gamma^\mu m) u(p) = 2p^\mu u(p) .
 \end{aligned}$$

(using Dirac equation for on-mass-shell spinor  $u(p)$ ).

- Universal factor  $F_{\text{Soft}}$  coincides with classical eikonal formula for radiation from current  $p^\mu$ , valid in long-wavelength limit.
- No soft enhancement of radiation from off-mass-shell internal lines, since associated denominator factor  $(p + q)^2 - m^2 \rightarrow p^2 - m^2 \neq 0$  as  $\omega \rightarrow 0$ .

- Enhancement factor in amplitude for each external line implies cross section enhancement is sum over all pairs of external lines  $\{i, j\}$ :

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega} \frac{d\Omega}{2\pi} \frac{\alpha_S}{2\pi} \sum_{i,j} C_{ij} W_{ij}$$

where  $d\Omega$  is element of solid angle for emitted gluon,  $C_{ij}$  is a colour factor, and radiation function  $W_{ij}$  is given by

$$W_{ij} = \frac{\omega^2 p_i \cdot p_j}{p_i \cdot q p_j \cdot q} = \frac{1 - v_i v_j \cos \theta_{ij}}{(1 - v_i \cos \theta_{iq})(1 - v_j \cos \theta_{jq})} .$$

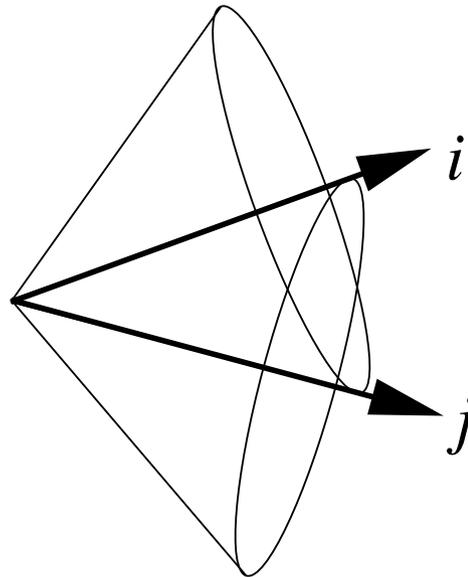
Colour-weighted sum of radiation functions  $C_{ij} W_{ij}$  is antenna pattern of hard process.

- Radiation function can be separated into two parts containing collinear singularities along lines  $i$  and  $j$ . Consider for simplicity massless particles,  $v_{i,j} = 1$ . Then  $W_{ij} = W_{ij}^i + W_{ij}^j$  where

$$W_{ij}^i = \frac{1}{2} \left( W_{ij} + \frac{1}{1 - \cos \theta_{iq}} - \frac{1}{1 - \cos \theta_{jq}} \right) \equiv \frac{1}{2(1 - \cos \theta_{iq})} \left( 1 + \frac{\cos \theta_{iq} - \cos \theta_{ij}}{1 - \cos \theta_{jq}} \right)$$

- This function has remarkable property of angular ordering. Write angular integration in polar coordinates w.r.t. direction of  $i$ ,  $d\Omega = d\cos\theta_{iq} d\phi_{iq}$ . Performing azimuthal integration, we find

$$\int_0^{2\pi} \frac{d\phi_{iq}}{2\pi} W_{ij}^i = \frac{1}{1 - \cos\theta_{iq}} \quad \text{if } \theta_{iq} < \theta_{ij}, \text{ otherwise } 0.$$



Thus, after azimuthal averaging, contribution from  $W_{ij}^i$  is confined to cone, centred on direction of  $i$ , extending in angle to direction of  $j$ . Similarly,  $W_{ij}^j$ , averaged over  $\phi_{jq}$ , is confined to cone centred on line  $j$  extending to direction of  $i$ .

# Angular ordering

- To prove angular ordering property, write  $n_i = (0, 0, 1)$ ,  $n_j = (0, \sin \theta_{ij}, \cos \theta_{ij})$ ,  $n_q = (\sin \theta_{iq} \sin \phi_{iq}, -\sin \theta_{iq} \cos \phi_{iq}, \cos \theta_{iq})$ , so that

$$1 - \cos \theta_{jq} = a - b \cos \phi_{iq}$$

where  $a = 1 - \cos \theta_{ij} \cos \theta_{iq}$ ,  $b = \sin \theta_{ij} \sin \theta_{iq}$ . Defining  $z = \exp(i\phi_{iq})$ , we have

$$I_{ij}^i \equiv \int_0^{2\pi} \frac{d\phi_{iq}}{2\pi} \frac{1}{1 - \cos \theta_{jq}} = \frac{1}{i\pi b} \oint \frac{dz}{(z_+ - z)(z - z_-)}$$

where  $z$ -integration contour is the unit circle and

$$z_{\pm} = \frac{a}{b} \pm \sqrt{\frac{a^2}{b^2} - 1}.$$

Now only pole at  $z = z_-$  can lie inside unit circle, so

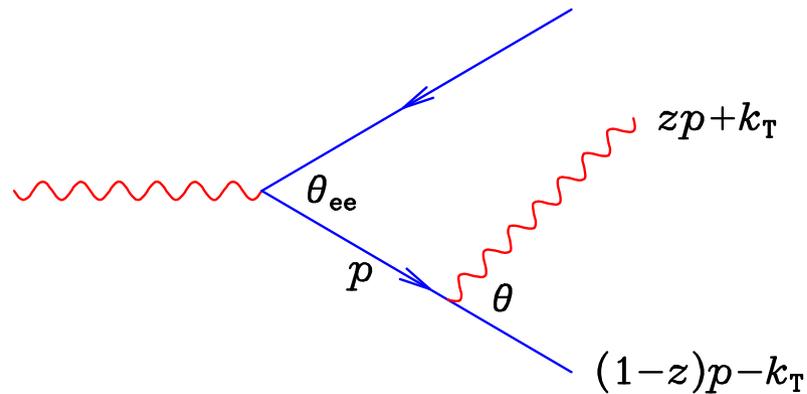
$$I_{ij}^i = \int_0^{2\pi} \frac{d\phi_{iq}}{2\pi} \frac{1}{a - b \cos \phi_{iq}} = \sqrt{\frac{1}{a^2 - b^2}} \equiv \frac{1}{|\cos \theta_{iq} - \cos \theta_{ij}|}.$$

## Angular ordering (cont)

$$\begin{aligned}\int_0^{2\pi} \frac{d\phi_{iq}}{2\pi} W_{ij}^i &= \frac{1}{2(1 - \cos \theta_{iq})} [1 + (\cos \theta_{iq} - \cos \theta_{ij}) I_{ij}^i] \\ &= \frac{1}{1 - \cos \theta_{iq}} \quad \text{if } \theta_{iq} < \theta_{ij}, \text{ otherwise } 0.\end{aligned}$$

# Chudakov effect

- Angular ordering is coherence effect common to all gauge theories. In QED it causes Chudakov effect – suppression of soft bremsstrahlung from  $e^+e^-$  pairs, which has simple explanation in old-fashioned (time-ordered) perturbation theory.



- Consider emission of soft photon at angle  $\theta$  from electron in pair with opening angle  $\theta_{ee} < \theta$ . For simplicity assume  $\theta_{ee}, \theta \ll 1$ .
- Transverse momentum of photon is  $k_T \sim zp\theta$  and energy imbalance at  $e \rightarrow e\gamma$  vertex is

$$\Delta E \sim k_T^2 / zp \sim zp\theta^2 .$$

- Time available for emission is  $\Delta t \sim 1/\Delta E$ . In this time transverse separation of pair will be  $\Delta b \sim \theta_{ee}\Delta t$ .

# Chudakov effect

- For non-negligible probability of emission, photon must resolve this transverse separation of pair, so

$$\Delta b > \lambda/\theta \sim (zp\theta)^{-1}$$

where  $\lambda$  is photon wavelength.

- This implies that

$$\theta_{ee}(zp\theta^2)^{-1} > (zp\theta)^{-1} ,$$

and hence  $\theta_{ee} > \theta$ . Thus soft photon emission is suppressed at angles larger than opening angle of pair, which is angular ordering.

- Photons at larger angles cannot resolve electron and positron charges separately – they see only total charge of pair, which is zero, implying no emission.
- More generally, if  $i$  and  $j$  come from branching of parton  $k$ , with (colour) charge  $Q_k = Q_i + Q_j$ , then radiation outside angular-ordered cones is emitted coherently by  $i$  and  $j$  and can be treated as coming directly from (colour) charge of  $k$ .

# Coherent branching

- Angular ordering provides basis for coherent parton branching formalism, which includes leading soft gluon enhancements to all orders.
- In place of virtual mass-squared variable  $t$  in earlier treatment, use angular variable

$$\zeta = \frac{p_b \cdot p_c}{E_b E_c} \simeq 1 - \cos \theta$$

as evolution variable for branching  $a \rightarrow bc$ , and impose angular ordering  $\zeta' < \zeta$  for successive branchings. Iterative formula for  $n$ -parton emission becomes

$$d\sigma_{n+1} = d\sigma_n \frac{d\zeta}{\zeta} dz \frac{\alpha_S}{2\pi} \hat{P}_{ba}(z) .$$

# Coherent branching

- In place of virtual mass-squared cutoff  $t_0$ , must use angular cutoff  $\zeta_0$  for coherent branching. This is to some extent arbitrary, depending on how we classify emission as unresolvable. Simplest choice is

$$\zeta_0 = t_0/E^2$$

for parton of energy  $E$ .

- For radiation from particle  $i$  with finite mass-squared  $t_0$ , radiation function becomes

$$\omega^2 \left( \frac{p_i \cdot p_j}{p_i \cdot q p_j \cdot q} - \frac{p_i^2}{(p_i \cdot q)^2} \right) \simeq \frac{1}{\zeta} \left( 1 - \frac{t_0}{E^2 \zeta} \right),$$

so angular distribution of radiation is cut off at  $\zeta = t_0/E^2$ . Thus  $t_0$  can still be interpreted as minimum virtual mass-squared.

- With this cutoff, most convenient definition of evolution variable is not  $\zeta$  itself but rather

$$\tilde{t} = E^2 \zeta \geq t_0 .$$

# Coherent branching

Angular ordering condition  $\zeta_b, \zeta_c < \zeta_a$  for timelike branching  $a \rightarrow bc$  ( $a$  outgoing) becomes

$$\tilde{t}_b < z^2 \tilde{t}, \quad \tilde{t}_c < (1 - z)^2 \tilde{t}$$

where  $\tilde{t} = \tilde{t}_a$  and  $z = E_b/E_a$ . Thus cutoff on  $z$  becomes

$$\sqrt{t_0/\tilde{t}} < z < 1 - \sqrt{t_0/\tilde{t}}.$$

- Neglecting masses of  $b$  and  $c$ , virtual mass-squared of  $a$  and transverse momentum of branching are

$$t = z(1 - z)\tilde{t}, \quad p_t^2 = z^2(1 - z)^2\tilde{t}.$$

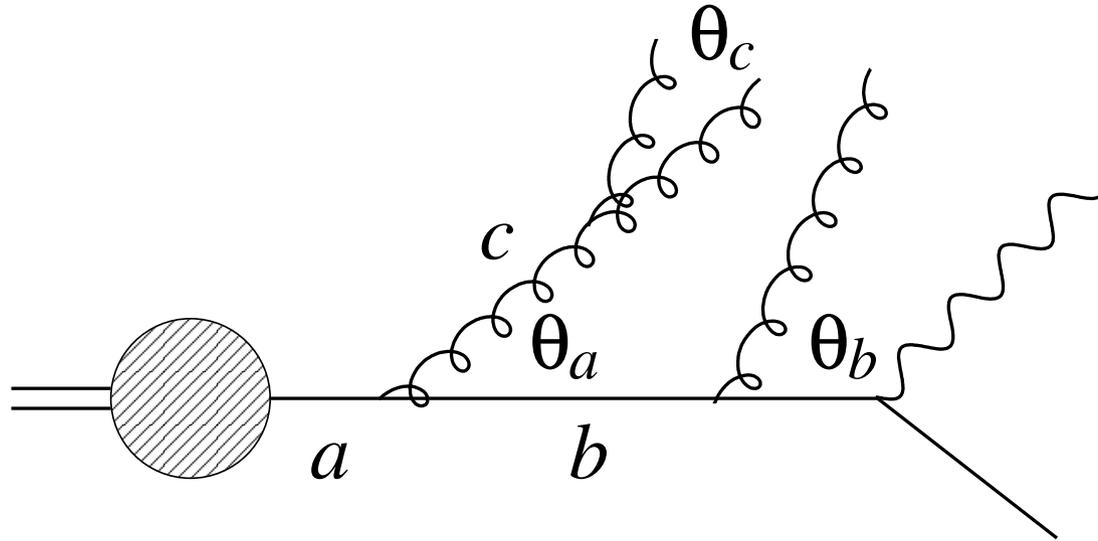
Thus for coherent branching Sudakov form factor of quark becomes

$$\tilde{\Delta}_q(\tilde{t}) = \exp \left[ - \int_{4t_0}^{\tilde{t}} \frac{dt'}{t'} \int_{\sqrt{t_0/t'}}^{1 - \sqrt{t_0/t'}} \frac{dz}{2\pi} \alpha_S(z^2(1 - z)^2 t') \hat{P}_{qq}(z) \right]$$

At large  $\tilde{t}$  this falls more slowly than form factor without coherence, due to the suppression of soft gluon emission by angular ordering.

# Coherent branching

- Note that for spacelike branching  $a \rightarrow bc$  ( $a$  incoming,  $b$  spacelike), angular ordering condition is



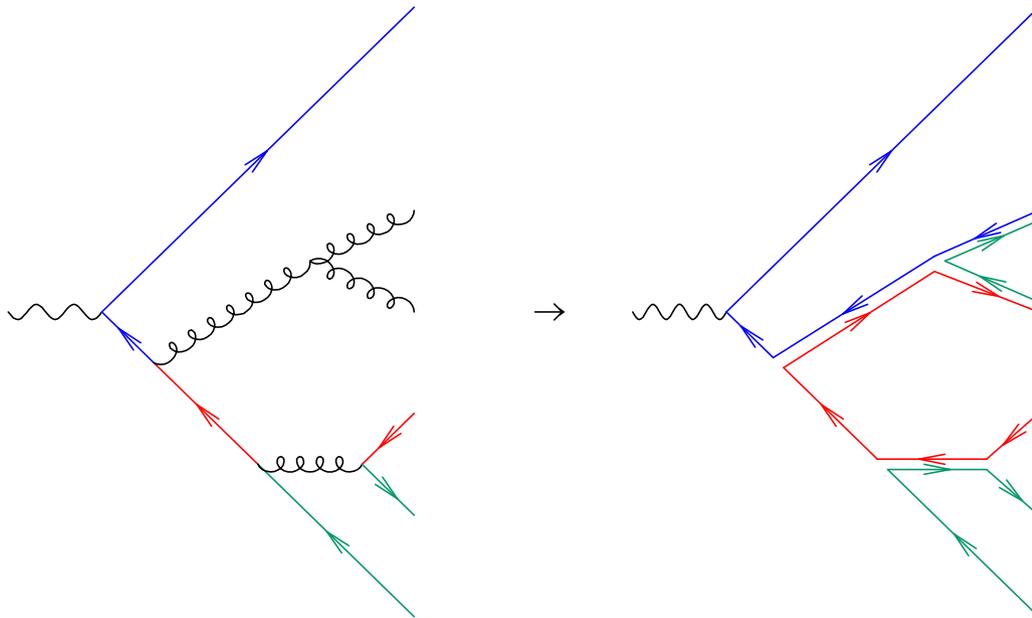
$$\theta_b > \theta_a > \theta_c ,$$

and so for  $z = E_b/E_a$  we now have

$$\tilde{t}_b > z^2 \tilde{t}_a , \quad \tilde{t}_c < (1 - z)^2 \tilde{t}_a .$$

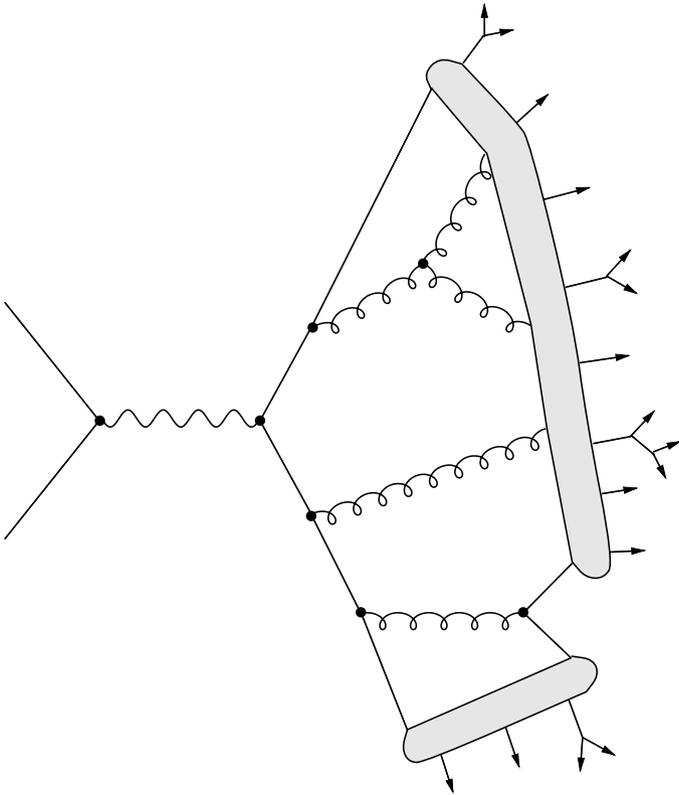
- Thus we can have either  $\tilde{t}_b > \tilde{t}_a$  or  $\tilde{t}_b < \tilde{t}_a$ , especially at small  $z$  — spacelike branching becomes disordered at small  $x$ .

# Color



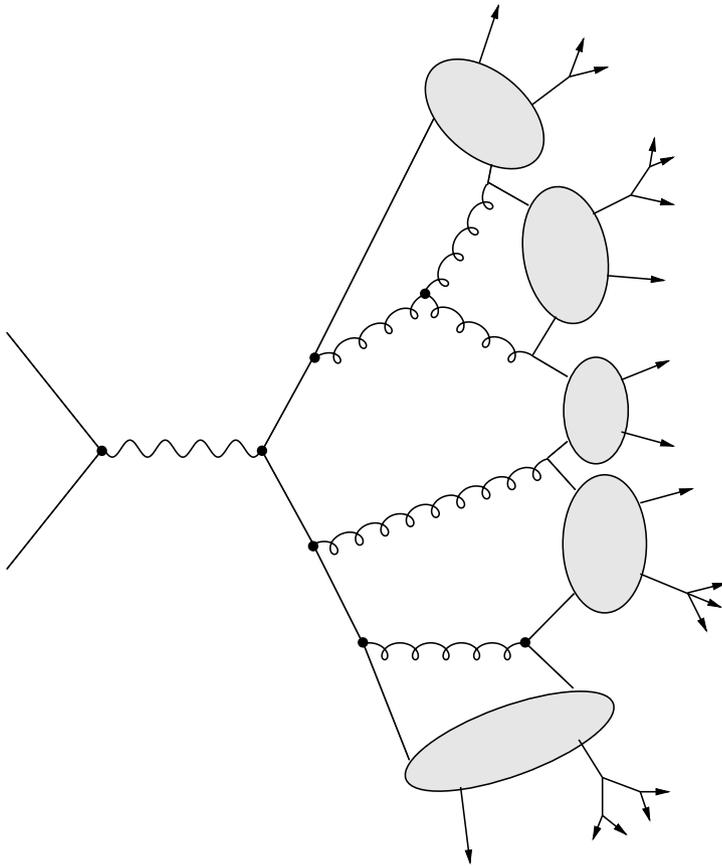
- Colour set up according to the cross section for each colour configuration.
- Gluons treated in the large  $N$  limit.
- Colour information used in the fragmentation.

# String Fragmentation



- As quark and antiquark separate, colour field has a string like configuration, with uniform energy per unit length.
- String breaks up into hadron sized pieces, through  $q\bar{q}$  creation in the intense colour field

# Cluster Fragmentation



- Colour-singlet clusters of partons form after the perturbative phase of jet development and then decay into the observed hadrons.
- non-perturbative splitting of gluons into  $q\bar{q}$  pairs, followed by combination of neighbouring quarks and antiquarks into color singlet clusters.
- if  $t_0$  is low, clusters have a mass of a few GeV.
- Clusters decay isotropically in their rest frames into pairs of hadrons.

# Recap

- Parton evolution can be represented as a branching process from higher values of  $x$
- DGLAP equation predicts growth at small  $x$  and shrinkage at large  $x$  with increasing  $Q^2$ .
- The Sudakov form factor  $\Delta(t)$  is the probability of evolving from  $t_0$  to  $t$  without branching.
- branching from  $(t_1, x_1)$  to  $(t_2, x_2)$  with the right probability can be performed with by choosing three random numbers,  $(t, x, \phi)$
- Branching is subject to an angular ordering constraint. Large angle emission is dynamically suppressed.
- Shower Monte Carlo programs contain, hard scattering cross section, timelike and spacelike parton showers, models for parton hadronization and resonance decay.