Collider Physics
Lecture IV: W and Z production at NLO

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Slides available from http://theory.fnal.gov/people/ellis/Talks/TASI06/

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Hard scattering cross sections

- Multiplicity growth with angular ordering
- Hadron-hadron processes and factorization
- W production
  - DY cross section
  - Subtraction method
- W + jet production
- Combining NLO corrections and parton showers

Multiplicity growth

$$D_i(x,t) = D_i(x,t_0) + \sum_j \int_x^1 \frac{dz}{z} \int_{t_0}^{z^2 t} \frac{dt'}{t'} \frac{\alpha_S}{2\pi} P_{ji}(z,\alpha_S) D_j(x/z,t') \ ,$$

or in differential form

$$t\frac{\partial}{\partial t} D_i(x,t) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_S}{2\pi} P_{ji}(z,\alpha_S) D_j(x/z,z^2 t) \ .$$

Notice that this differs from the conventional evolution equation only in the z-dependent change of scale on the right-hand side.
consider first the solution of taking α_S fixed and neglecting the sum over different branchings. Then we have

$$t\frac{\partial}{\partial t} \tilde{D}(j,t) = \frac{\alpha_S}{2\pi} \int_x^1 dz\, z^{j-1} P(z) \tilde{D}(j,z^2 t) \ .$$

- we try a solution of the form

$$D(j,t) \propto t^{\gamma(j,\alpha_S)}$$

Resummed anomalous dimension

- we find that the anomalous dimension $\gamma(j, \alpha_S)$ must satisfy the implicit equation

$$\gamma(j, \alpha_S) = \frac{\alpha_S}{2\pi} \int_0^1 dz\, z^{j-1+2\gamma(j,\alpha_S)} P(z)\ .$$

$$\gamma_{gg}(j, \alpha_S) = \frac{C_A \alpha_S}{\pi} \frac{1}{j - 1 + 2\gamma_{gg}(j, \alpha_S)}$$

$$\begin{aligned}\gamma_{gg}(j, \alpha_S) &= \frac{1}{4}\left[\sqrt{(j-1)^2 + \frac{8 C_A \alpha_S}{\pi}} - (j-1)\right] \\ &= \sqrt{\frac{C_A \alpha_S}{2\pi}} - \frac{1}{4}(j-1) + \ldots\ .\end{aligned}$$

- at any fixed $j \neq 1$ we can expand in a different way for sufficiently small α_S

$$\gamma_{gg}(j, \alpha_S) = \frac{C_A \alpha_S}{\pi} \frac{1}{(j-1)} - 2\left(\frac{C_A \alpha_S}{\pi}\right)^2 \frac{1}{(j-1)^3} + \ldots\ .$$

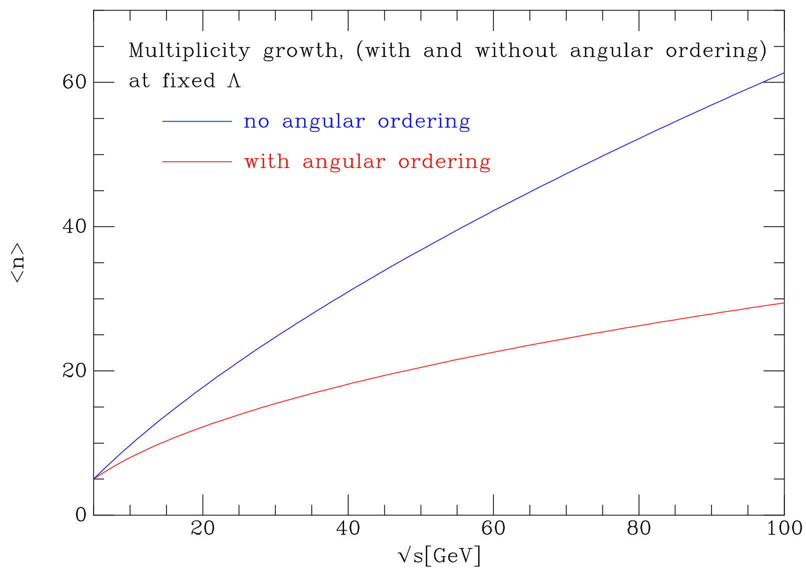
This series displays the terms that are most singular as $j \to 1$ in each order.

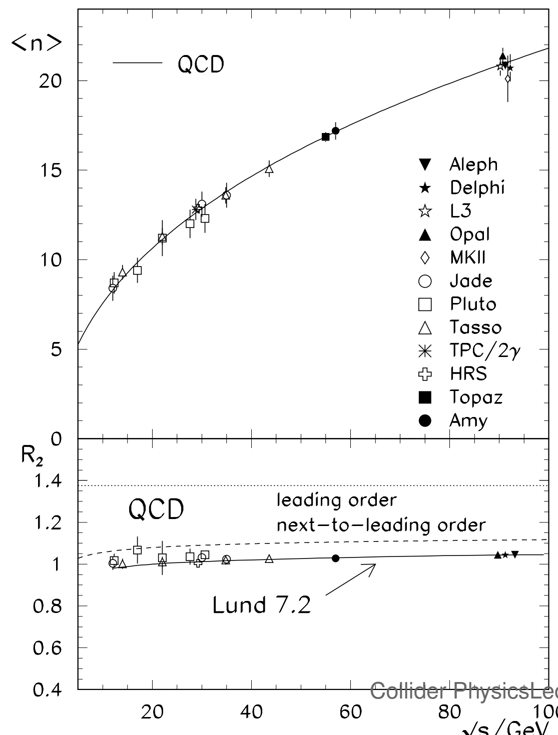
Multiplicity growth II

- we can introduce running α by writing
  $\tilde{D}(j,t) \sim t^{\gamma_{gg}(j,\alpha_S)} = \exp\left[\int^t \gamma_{gg}(j,\alpha_S)\frac{dt'}{t'}\right]$.
- we replace $\gamma_{gg}(j,\alpha_S)$ in the integrand by $\gamma_{gg}(j,\alpha_S(t'))$.

$$\int^t \gamma_{gg}(j,\alpha_S(t'))\frac{dt'}{t'} = \int^{\alpha_S(t)} \frac{\gamma_{gg}(j,\alpha_S)}{\beta(\alpha_S)}\,d\alpha_S, \quad \beta(\alpha_S) = -b\alpha_S^2 + \ldots,$$

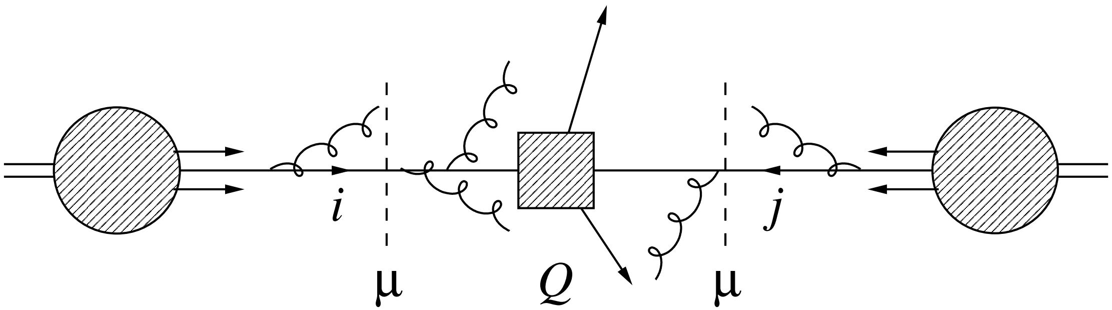
$$\langle n(s)\rangle \sim \exp\left[\frac{1}{b}\sqrt{\frac{2C_A}{\pi\alpha_S(s)}}\right] \sim \exp\sqrt{\frac{6}{\pi b}\ln\frac{s}{\Lambda^2}}$$





Hadron-hadron processes

- In hard hadron-hadron scattering, constituent partons from each incoming hadron interact at short distance (large momentum transfer Q^2).



- For hadron momenta P_1, P_2 ($S = 2P_1 \cdot P_2$), form of cross section is

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 D_i(x_1, \mu^2) D_j(x_2, \mu^2) \hat{\sigma}_{ij}(\hat{s} = x_1 x_2 S, \alpha_S(\mu^2), Q^2/\mu^2)$$

where μ^2 is factorization scale and $\hat{\sigma}_{ij}$ is subprocess cross section for parton types i, j.

  - ⋆ Notice that factorization scale is in principle arbitrary: affects only what we call part of subprocess or part of initial-state evolution (parton shower).
  - ⋆ Unlike e^+e^- or ep, we may have interaction between spectator partons, leading to soft underlying event and/or multiple hard scattering.

Factorization of the cross section

- Why does the factorization property hold and when it should fail?
- For a heuristic argument Consider the simplest hard process involving two hadrons

$$H_1(P_1) + H_2(P_2) \to V + X.$$

- Do the partons in hadron H_1, through the influence of their colour fields, change the distribution of partons in hadron H_2 before the vector boson is produced? Soft gluons which are emitted long before the collision are potentially troublesome.
- A simple model from classical electrodynamics. The vector potential due to an electromagnetic current density J is given by

$$A^\mu(t,\vec{x}) = \int dt' d\vec{x}' \; \frac{J^\mu(t',\vec{x}')}{|\vec{x}-\vec{x}'|} \; \delta(t' + |\vec{x}-\vec{x}'| - t) \; ,$$

where the delta function provides the retarded behaviour required by causality.

- Consider a particle with charge e travelling in the positive z direction with constant velocity β. The non-zero components of the current density are

$$
\begin{aligned}
J^t(t',\vec{x}') &= e\delta(\vec{x}' - \vec{r}(t'))\ , \\
J^z(t',\vec{x}') &= e\beta\delta(\vec{x}' - \vec{r}(t')), \quad \vec{r}(t') = \beta t'\hat{z},
\end{aligned}
$$

$\hat{z}$ is a unit vector in the z direction. At an observation point (the supposed position of hadron H_2) described by coordinates x, y and z, the vector potential (either performing the integrations using the current density given above, or by Lorentz transformation of the scalar potential in the rest frame of the particle) is

$$
\begin{aligned}
A^t(t,\vec{x}) &= \frac{e\gamma}{\sqrt{[x^2+y^2+\gamma^2(\beta t - z)^2]}} \\
A^x(t,\vec{x}) &= 0 \\
A^y(t,\vec{x}) &= 0 \\
A^z(t,\vec{x}) &= \frac{e\gamma\beta}{\sqrt{[x^2+y^2+\gamma^2(\beta t - z)^2]}},
\end{aligned}
$$

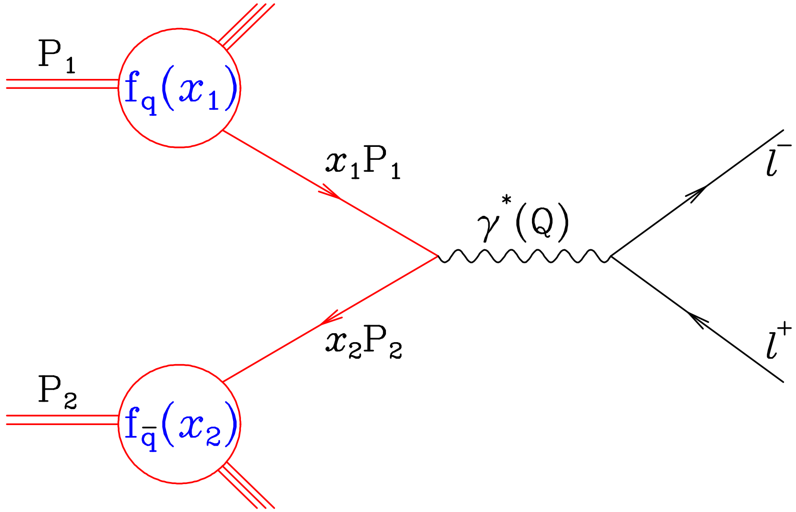
where $\gamma^2 = 1/(1-\beta^2)$. Target hadron H_2 is at rest near the origin, so that $\gamma \approx s/m^2$.

- Note that for large γ and fixed non-zero $(\beta t - z)$ some components of the potential tend to a constant independent of γ, suggesting that there will be non-zero fields which are not in coincidence with the arrival of the particle, even at high energy.
- However at large γ the potential is a pure gauge piece, $A^\mu = \partial^\mu \chi$ where χ is a scalar function
- Covariant formulation using the vector potential A has large fields which have no effect.
- For example, the electric field along the z direction is

$$E^z(t,\vec{x}) = F^{tz} \equiv \frac{\partial A^z}{\partial t} + \frac{\partial A^t}{\partial z} = \frac{e\gamma(\beta t - z)}{[x^2 + y^2 + \gamma^2(\beta t - z)^2]^{\frac{3}{2}}}.$$

The leading terms in γ cancel and the field strengths are of order $1/\gamma^2$ and hence of order m^4/s^2. The model suggests the force experienced by a charge in the hadron H_2, at any fixed time before the arrival of the quark, decreases as m^4/s^2.

Lepton-pair production



- Mechanism for Lepton pair production, W-production, Z-production, Vector-boson pairs, ...
- Collectively known as the Drell-Yan process.
- Colour average $1/N$.

$$\frac{d\hat{\sigma}}{dQ^2} = \frac{\sigma_0}{N} Q_q^2 \, \delta(\hat{s} - Q^2), \qquad \sigma_0 = \frac{4\pi\alpha^2}{3Q^2}, \ \text{cf } e^+e^- \text{ annihilation.}$$

In the CM frame of the two hadrons, the momenta of the incoming partons are

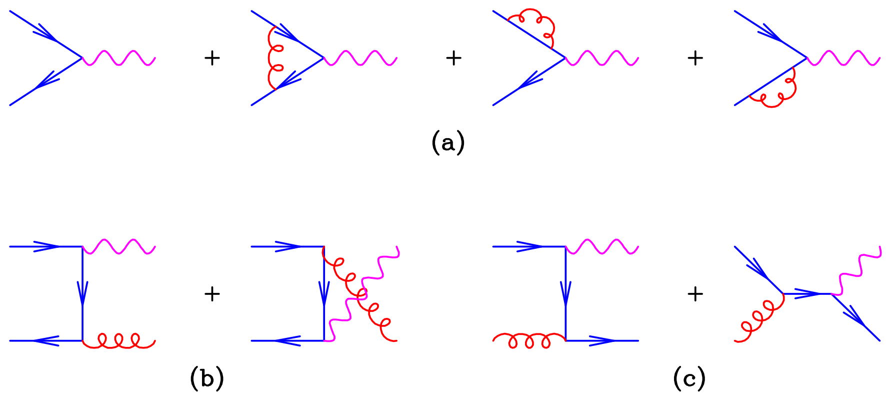
$$p_1 = \frac{\sqrt{s}}{2}(x_1, 0, 0, x_1), \quad p_2 = \frac{\sqrt{s}}{2}(x_2, 0, 0, -x_2) \; .$$

The square of the $q\bar{q}$ collision energy $\hat{s}$ is related to the overall hadron-hadron collision energy by $\hat{s} = (p_1 + p_2)^2 = x_1 x_2 s$. The parton-model cross section for this process is:

$$\begin{aligned} \frac{d\sigma}{dM^2} &= \int_0^1 dx_1 dx_2 \sum_q \left\{ f_q(x_1) f_{\bar{q}}(x_2) + (q \leftrightarrow \bar{q}) \right\} \frac{d\hat{\sigma}}{dM^2}(q\bar{q} \to l^+ l^-) \\ &= \frac{\sigma_0}{Ns} \int_0^1 \frac{dx_1}{x_1} \frac{dx_2}{x_2} \, \delta(1-z) \left[\sum_q Q_q^2 \left\{ f_q(x_1) f_{\bar{q}}(x_2) + (q \leftrightarrow \bar{q}) \right\} \right] . \end{aligned}$$

- For later convenience we have introduced the variable $z = \frac{Q^2}{\hat{s}} = \frac{Q^2}{x_1 x_2 s}$.
- The sum here is over quarks only and the $\bar{q}q$ contributions are indicated explicitly.

Next-to-leading order



- The contribution of the real diagrams (in four dimensions) is

$$|M|^2 \sim g^2 C_F \left[\frac{u}{t} + \frac{t}{u} + \frac{2Q^2 s}{ut} \right] = g^2 C_F \left[\Big(\frac{1+z^2}{1-z} \Big) \Big(\frac{-s}{t} + \frac{-s}{u} \Big) - 2 \right]$$

  where $z = Q^2/s, s+t+u = Q^2$.

- Note that the real diagrams contain collinear singularities, $u \to 0, t \to 0$ and soft singularities, $z \to 1$.
- The coefficient of the divergence is the unregulated branching probability $\hat{P}_{qq}(z)$.
- Ignore for simplicity the diagrams with incoming gluons.

- Control the divergences by continuing the dimensionality of space-time, $d = 4 - 2\epsilon$, (technically this is dimensional reduction). Performing the phase space integration, the total contribution of the real diagrams is

$$\begin{aligned}\sigma_R &= \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^\epsilon c_\Gamma \Bigg[\Big(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{\pi^2}{3}\Big)\delta(1-z) - \frac{2}{\epsilon} P_{qq}(z) \\ &- 2(1-z) + 4(1+z^2)\Big[\frac{\ln(1-z)}{1-z}\Big]_+ - 2\frac{1+z^2}{(1-z)} \ln z \Bigg]\end{aligned}$$

  with $c_\Gamma = (4\pi)^\epsilon / \Gamma(1-\epsilon)$.
- The contribution of the virtual diagrams is

$$\sigma_V = \delta(1-z)\left[1 + \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^\epsilon c'_\Gamma \Big(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 6 + \pi^2\Big)\right]$$

  $c'_\Gamma = c_\Gamma + O(\epsilon^3)$

- Adding it up we get in dim-reduction

$$\sigma_{R+V} = \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} c_\Gamma \left[\Big(\frac{2\pi^2}{3} - 6\Big)\delta(1-z) - \frac{2}{\epsilon} P_{qq}(z) - 2(1-z) \right.$$
$$\left. + \quad 4(1+z^2)\Big[\frac{\ln(1-z)}{1-z}\Big]_+ - 2\frac{1+z^2}{(1-z)}\ln z \right]$$

- The divergences, proportional to the branching probability , are universal.
- We will factorize them into the parton distributions. We perform the mass factorization by subtracting the counterterm

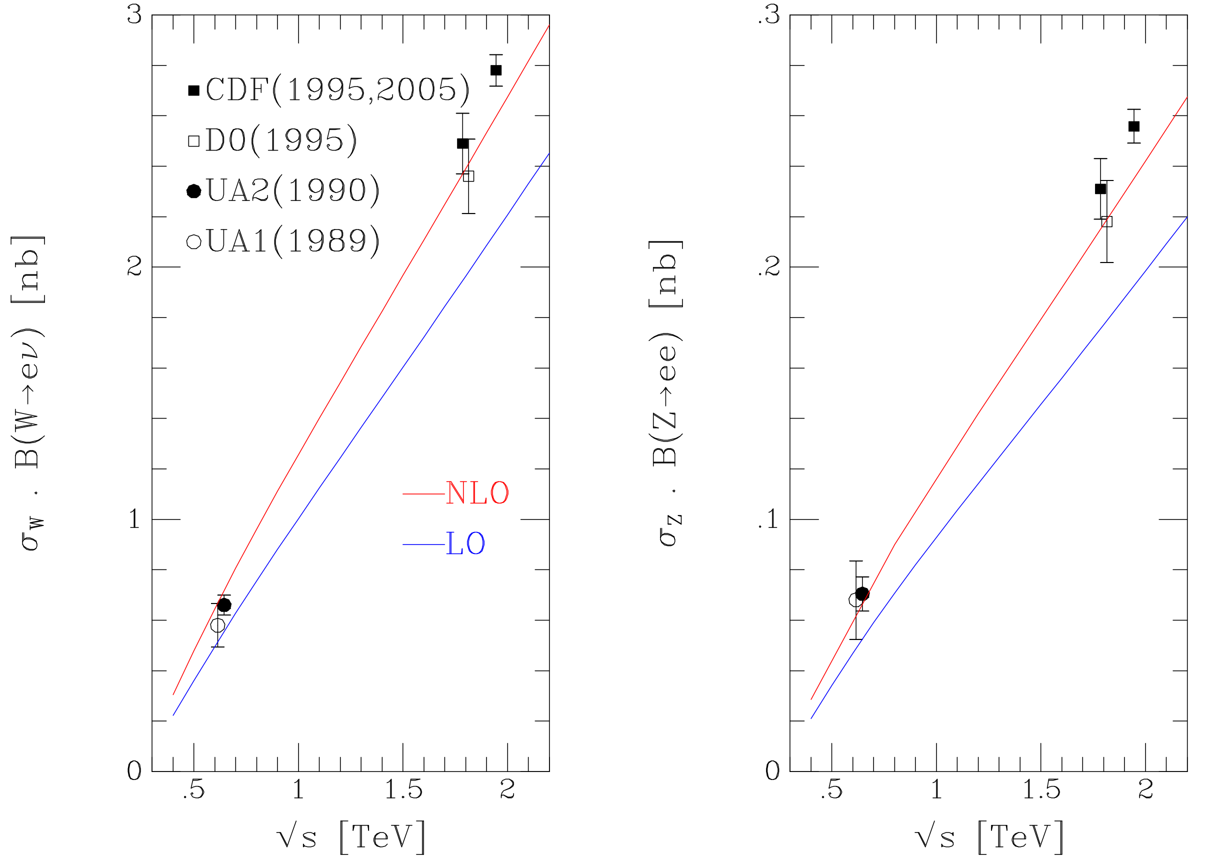
$$2\frac{\alpha_S}{2\pi} C_F \left[\frac{-c_\Gamma}{\epsilon} P_{qq}(z) - (1-z) + \delta(1-z) \right]$$

(The finite terms are necessary to get us to the $\overline{MS}$-scheme).

$$\hat{\sigma} = \frac{\alpha_S}{2\pi} C_F \left[\Big(\frac{2\pi^2}{3} - 8\Big)\delta(1-z) + 4(1+z^2)\Big[\frac{\ln(1-z)}{1-z}\Big]_+ - 2\frac{1+z^2}{(1-z)}\ln z + 2P_{qq}(z)\ln\frac{Q^2}{\mu^2} \right]$$

- Similar correction for incoming gluons.

Application to W, Z production



- Agreement with NLO theory is good.
- LO curves lie about 25% too low.
- NNLO results are also known and lead to a further modest (4%) ncrease at the Tevatron.

General calculational method for NLO

- Direct integration is good for the total cross section, but for differential distributions, (to which we want to apply cuts), we need a Monte Carlo method.
- We use a general subtraction procedure at NLO.
- at NLO the cross section for two initial partons a and b and for m outgoing partons, is given by

$$\sigma_{ab} = \sigma_{ab}^{LO} + \sigma_{ab}^{NLO}$$

where

$$\begin{aligned} \sigma_{ab}^{LO} &= \int_m d\sigma_{ab}^B \\ \sigma_{ab}^{NLO} &= \int_{m+1} d\sigma_{ab}^R + \int_m d\sigma_{ab}^V \end{aligned}$$

the singular parts of the QCD matrix elements for real emission, corresponding to soft and collinear emission can be isolated in a process independent manner

Calculational method (cont)

- One can use this the construct a set of counterterms

$$d\sigma^{ct} = \sum_{ct} \int_m d\sigma^B \otimes \int_1 dV_{ct}$$

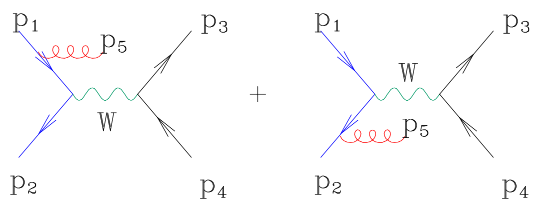
  where $d\sigma^B$ denotes the appropriate colour and spin projection of the Born-level cross section, and the counter-terms are independent of the details of the process under consideration.

- these counterterm cancel all non-integrable singularities in $d\sigma^R$, so that one can write

$$\sigma_{ab}^{NLO} = \int_{m+1} [d\sigma_{ab}^R - d\sigma_{ab}^{ct}] + \int_{m+1} d\sigma_{ab}^{ct} + \int_m d\sigma_{ab}^V$$

  The phase space integration in the first term can be performed numerically in four dimensions.

Matrix element counter-event for W production



In the soft limit $p_5 \to 0$ we have

$$|M_1(p_1,p_2,p_3,p_4,p_5)|^2 = g^2 C_F \frac{p_1 \cdot p_2}{p_1 \cdot p_5 \; p_2 \cdot p_5} |M_0(p_1,p_2,p_3,p_4)|^2$$

- Eikonal factor can be associated with radiation from a given leg by partial fractioning

$$\frac{p_1 \cdot p_2}{p_1 \cdot p_5 \; p_2 \cdot p_5} = [\frac{p_1 \cdot p_2}{p_1 \cdot p_5 + p_2 \cdot p_5}][\frac{1}{p_1 \cdot p_5} + \frac{1}{p_2 \cdot p_5}]$$

- including the collinear contributions, singular as $p_1 \cdot p_5 \to 0$, the matrix element for the counter event has the structure

$$|M_1(p_1,p_2,p_3,p_4,p_5)|^2 = \frac{g^2}{x_a p_1 \cdot p_5} \hat{P}_{qq}(x_a) |M_0(x_a p_1, p_2, \tilde{p}_3, \tilde{p}_4)|^2$$

where $1 - x_a = (p_1 \cdot p_5 + p_2 \cdot p_5)/p_1 \cdot p_2$ and $\hat{P}_{qq}(x_a) = C_F(1+x^2)/(1-x)$

Subtraction method for NLO

- For event $q(p_1) + \bar{q}(p_2) \to W^+(\nu(p_3) + e^+(p_4)) + g(p_5)$ with $p_1 + p_2 = \sum_{i=3}^{5} p_i$
- generate a counter event $q(x_a p_1) + \bar{q}(p_2) \to W^+(\nu(\tilde{p}_3) + e^+(\tilde{p}_4))$ and $x_a p_1 + p_2 = \sum_{i=3}^{4} \tilde{p}_i$ with $1 - x_a = (p_1 \cdot p_5 + p_2 \cdot p_5)/p_1 \cdot p_2$.
- A Lorentz transformation is performed on all j final state momenta $\tilde{p}_j = \Lambda^\mu_\nu p^\nu_j, j = 3, 4$ such that $\tilde{p}^\mu_j \to p^\mu_j$ for p_5 collinear or soft.
- The longitudinal momentum of p_5 is absorbed by rescaling with x.
- The other components of the momentum, p_5 are absorbed by the Lorentz transformation.
- In terms of these variables the phase space has a convolution structure,

$$d\phi^{(3)}(p_1, p_2; p_3, p_4, p_5) = \int_0^1 dx \; d\phi^{(2)}(p_2, x p_1; \tilde{p_3}, \tilde{p_4})[dp_5(p_1, p_2, x)]$$

where

$$[dp_5(p_1, p_2, x_a)] = \frac{d^d p_5}{(2\pi)^3} \delta^+(p_5^2)\Theta(x)\Theta(1-x)\delta(x - x_a)$$

- If k_i is the emitted parton, and p_a, p_b are the incoming momenta, define the shifted momenta

$$\widetilde{k}_j^\mu = k_j^\mu - \frac{2k_j \cdot (K + \widetilde{K})}{(K + \widetilde{K})^2} \, (K + \widetilde{K})^\mu + \frac{2k_j \cdot K}{K^2} \, \widetilde{K}^\mu \ ,$$

where the momenta K^μ and $\widetilde{K}^\mu$ are,

$$K^\mu = p_a^\mu + p_b^\mu - p_i^\mu \ \ , \widetilde{K}^\mu = \widetilde{p}_{ai}^\mu + p_b^\mu \ \ .$$

- Since $2\sum_j k_j \cdot K = 2K^2$ and $2\sum_j k_j \cdot (K + \widetilde{K}) = 2K^2 + 2K \cdot \widetilde{K} = (K + \widetilde{K})^2$ the momentum conservation constraint in the $m+1$-parton matrix

$$p_a^\mu + p_b^\mu - \sum_j k_j^\mu - p_i^\mu = 0 \ \ .$$

implies

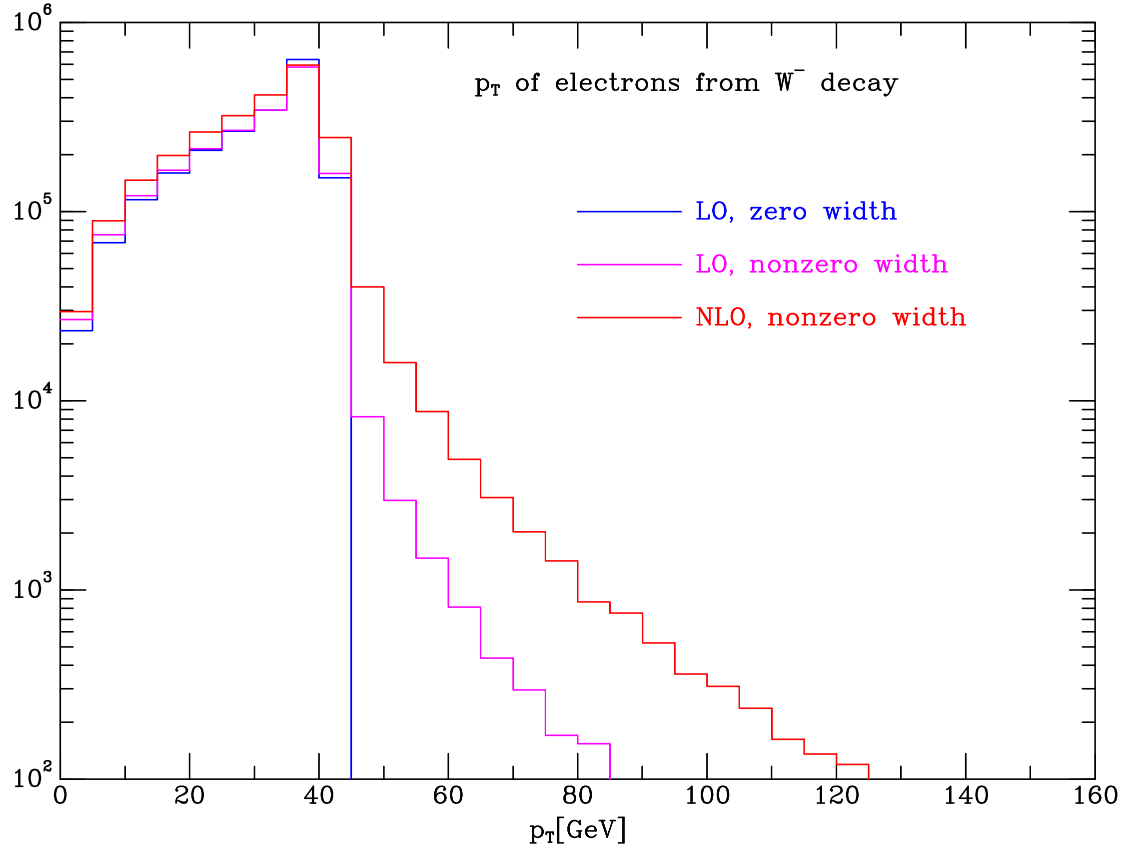
$$\widetilde{p}_{ai}^\mu + p_b^\mu - \sum_j \widetilde{k}_j^\mu = 0 \ \ .$$

- Note also that the shifted momenta can be rewritten in the following way:

$$\begin{aligned}\widetilde{k}_j^\mu &= \Lambda^\mu{}_\nu(K,\widetilde{K})\; k_j^\nu \quad , \\ \Lambda^\mu{}_\nu(K,\widetilde{K}) &= g^\mu{}_\nu - \frac{2(K+\widetilde{K})^\mu (K+\widetilde{K})_\nu}{(K+\widetilde{K})^2} + \frac{2\widetilde{K}^\mu K_\nu}{K^2} \quad ,\end{aligned}$$

- the matrix $\Lambda^\mu{}_\nu(K,\widetilde{K})$ generates a proper Lorentz transformation on the final-state momenta.
- If the emitted parton has zero transverse momenta, the Lorentz transformation reduces to the identity.

Why NLO?



- Calculation of NLO corrections, give a better prediction for the rate.
- Extra radiation can modify kinematic distributions.

MCFM overview

John Campbell and R.K. Ellis

- Parton level cross-sections predicted to NLO in α_S

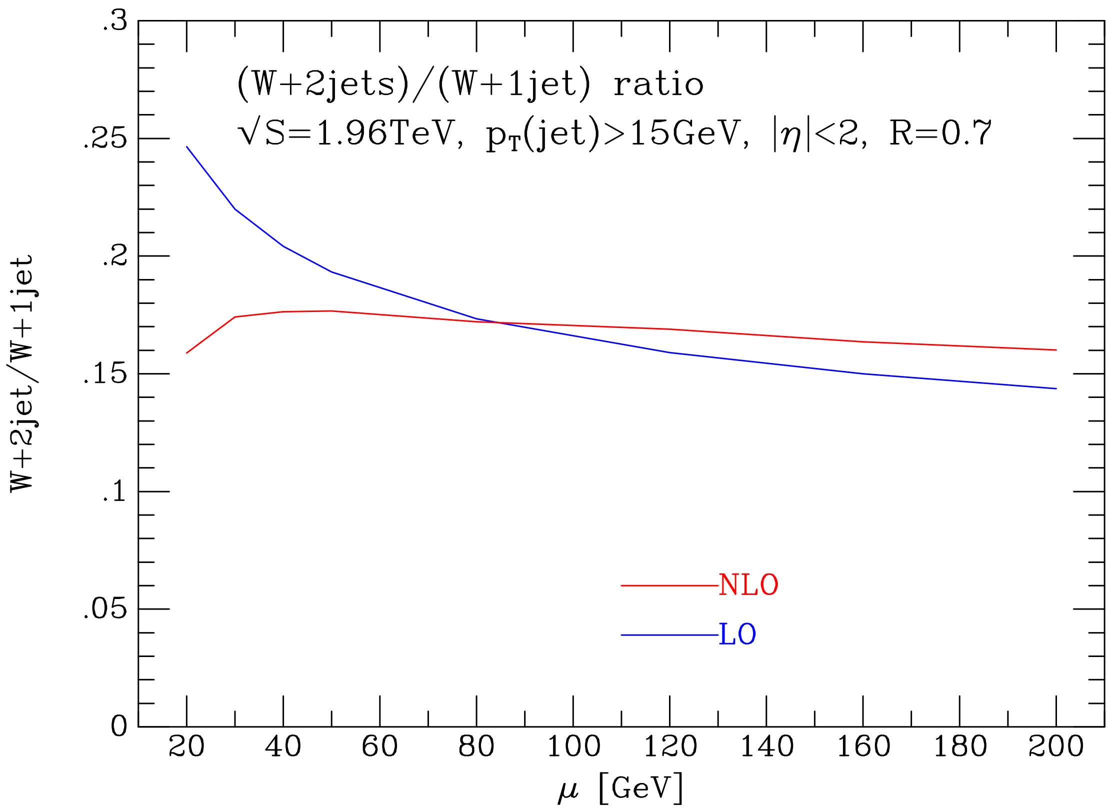
$p\bar{p} \to W^\pm/Z$	$p\bar{p} \to W^+ + W^-$
$p\bar{p} \to W^\pm + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^\pm + \gamma$	$p\bar{p} \to W^\pm/Z + H$
$p\bar{p} \to W^\pm + g^\star\ (\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W^\pm/Z +$ 1 jet	$p\bar{p} \to W^\pm/Z +$ 2 jets
$p\bar{p}(gg) \to H$	$p\bar{p}(gg) \to H +$ 1 jet
$p\bar{p}(VV) \to H +$ 2 jets	$p\bar{p} \to t + X$
$pp \to t + W$	

$\oplus$ less sensitivity to μ_R, μ_F, rates are better normalized, fully differential distributions.

$\ominus$ low particle multiplicity (no showering), no hadronization, hard to model detector effects

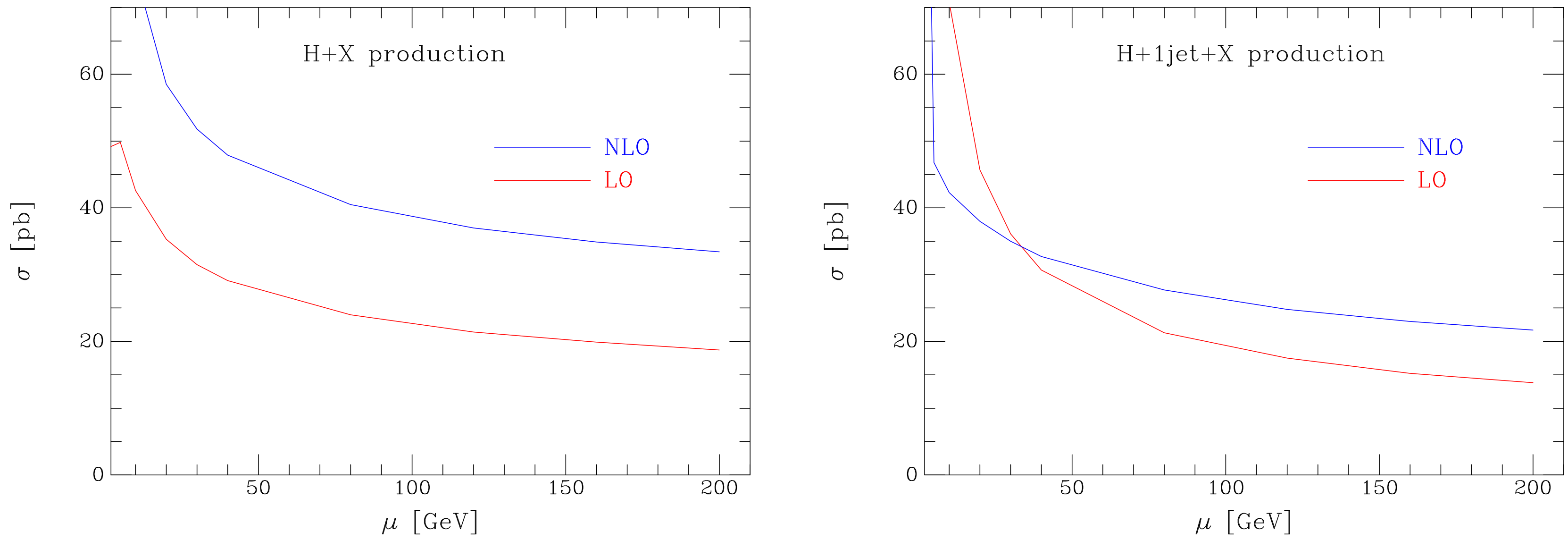
MCFM:examples

- $(W$+2 jet)/$(W$+1 jet)



MCFM examples

- Production of a $m_H = 120$ GeV Higgs, using effective Lagrangian $HG^{\mu\nu}G_{\mu\nu}$, obtained in heavy top limit.
- Cross sections for Higgs+anything or Higgs+1 jet+anything are the same.
- Radiation probability is one, and NLO is clearly inadequate.
- what is needed is a combination of NLO and shower Monte-Carlo, (MC@NLO)



NLO: Schematic description

- A schematic description of a NLO calculation is as follows.

$$\begin{aligned}\left(\frac{d\sigma}{dx}\right)_B &= B\delta(x)\\ \left(\frac{d\sigma}{dx}\right)_V &= a\left(\frac{B}{2\epsilon}+V\right)\delta(x)\\ \left(\frac{d\sigma}{dx}\right)_R &= a\frac{R(x)}{x}\end{aligned}$$

- In terms of the above the calculation of any observable O can written as

$$\langle O\rangle = \lim_{\epsilon\to 0}\int_0^1 dx x^{-2\epsilon} O(x)\left[\left(\frac{d\sigma}{dx}\right)_B + \left(\frac{d\sigma}{dx}\right)_V + \left(\frac{d\sigma}{dx}\right)_R\right]$$

Subtraction method

We can isolate the divergent part of the real radiation contribution

$$\langle O \rangle_{\mathrm{R}} = aBO(0) \int_0^1 dx\, \frac{x^{-2\epsilon}}{x} + a \int_0^1 dx\, \frac{O(x)R(x) - BO(0)}{x^{1+2\epsilon}} \ .$$

The second term does not contain singularities so we can set $\epsilon = 0$

$$\langle O \rangle_{\mathrm{R}} = -a\frac{B}{2\epsilon}O(0) + a \int_0^1 dx\, \frac{O(x)R(x) - BO(0)}{x} \ .$$

The NLO prediction using the subtraction method is

$$\langle O \rangle_{\mathrm{sub}} = BO(0) + a \left[VO(0) + \int_0^1 dx\, \frac{O(x)R(x) - BO(0)}{x} \right] .$$

Toy Monte Carlo

Frixione-Webber

- Rewrite the basic NLO formula in a different which allows simpler matching with the Monte Carlo:

$$\langle O \rangle_{\text{sub}} = \int_0^1 dx \left[O(x) \frac{aR(x)}{x} + O(0) \left(B + aV - \frac{aB}{x} \right) \right].$$

- Introduce Sudakov form factor for the toy model

$$\Delta(x_1, x_2) = \exp \left[-a \int_{x_1}^{x_2} dz \frac{Q(z)}{z} \right],$$

where $Q(z)$ is a radiation function with the following general properties:

$$0 \leq Q(z) \leq 1, \qquad \lim_{z \to 0} Q(z) = 1, \qquad \lim_{z \to 1} Q(z) = 0.$$

If x_s is the energy of the system before the first branching occurs, then $\Delta(x, x_s)$ is the probability that no photon be emitted with energy z such that $x \leq z \leq x_s$.

Matching NLO and MC

$$\left(\frac{d\sigma}{dO}\right)_{\rm MC@LO} = B I_{\rm MC}(O,1).$$

$$\left(\frac{d\sigma}{dO}\right)_{\rm naive} = \int_0^1 dx \left[I_{\rm MC}(O, x_{\rm M}(x)) \frac{aR(x)}{x} + I_{\rm MC}(O,1) \left(B + aV - \frac{aB}{x} \right) \right].$$

This equation suggests the following procedure:

- Pick at random $0 \le x \le 1$.
- Generate an MC event with $x_{\rm M}(x)$ as maximum energy available to the photon in the first branching; attach to this event the weight $w_{\rm EV} = aR(x)/x$.
- Generate another MC event (a “counter-event”) with $x_{\rm M} = 1$; attach to this event the weight $w_{\rm CT} = B + aV - aB/x$.
- Repeat the first three steps N times, and normalize with $1/N$.

This procedure fails, since the weights $w_{\rm EV}$ and $w_{\rm CT}$ diverge as $x \to 0$.

Modified subtraction method

$$
\begin{aligned}
\left(\frac{d\sigma}{dO}\right)_{\rm msub} \quad = \quad & \int_0^1 dx \left[I_{\rm MC}(O, x_{\rm M}(x)) \frac{a[R(x) - BQ(x)]}{x} \right. \\
& \left. + I_{\rm MC}(O,1) \left(B + aV + \frac{aB[Q(x)-1]}{x} \right) \right] .
\end{aligned}
$$

- We subtract and add the quantities

$$
I_{\rm MC}(O,1) \frac{aBQ(x)}{x}, \quad I_{\rm MC}(O, x_{\rm M}) \frac{aBQ(x)}{x}
$$

- The two terms involving $Q(x)$ are not identical, so this is not a subtraction in the usual sense of an NLO computation.
- The two terms do not contribute to the observable O at $\mathcal{O}(a)$, because they are compensated by terms in the parton shower $BI_{\rm MC}(O, 1)$

Expansion to $O(\alpha_S)$

- Expansion of Monte Carlo piece is

$$I_{\rm MC} = (1 - a\int_{x_0}^{1} dt\frac{Q(t)}{t}\;\delta(O-O(0)) + a\int_{x_0}^{1} dt\frac{Q(t)}{t}\;\delta(O-O(t)) + O(a^2)$$

- Insertion of this piece in the modified Monte-Carlo formula gives

$$\begin{aligned}\left(\frac{d\sigma}{dO}\right)_{\rm msub} = & \int_0^1 dx\Bigg[\delta(O-O(x))\frac{a[R(x)-BQ(x)]}{x} \\ & +\delta(O-O(0))\left(B+aV-\frac{aB}{x}\right) \\ & +aB\delta(O-O(0))\left(\frac{Q(x)}{x}-\int_{x_0}^1 dt\frac{Q(t)}{t}\right) \\ & +aB\int_{x_0}^1 dt\delta(O-O(t))\frac{Q(t)}{t}\Bigg] + \mathcal{O}(a^2).\end{aligned}$$

Expansion (continued)

- Collecting terms we obtain the starting formula for a NLO correction, plus power suppressed terms which are anyway not controlled in the Monte Carlo

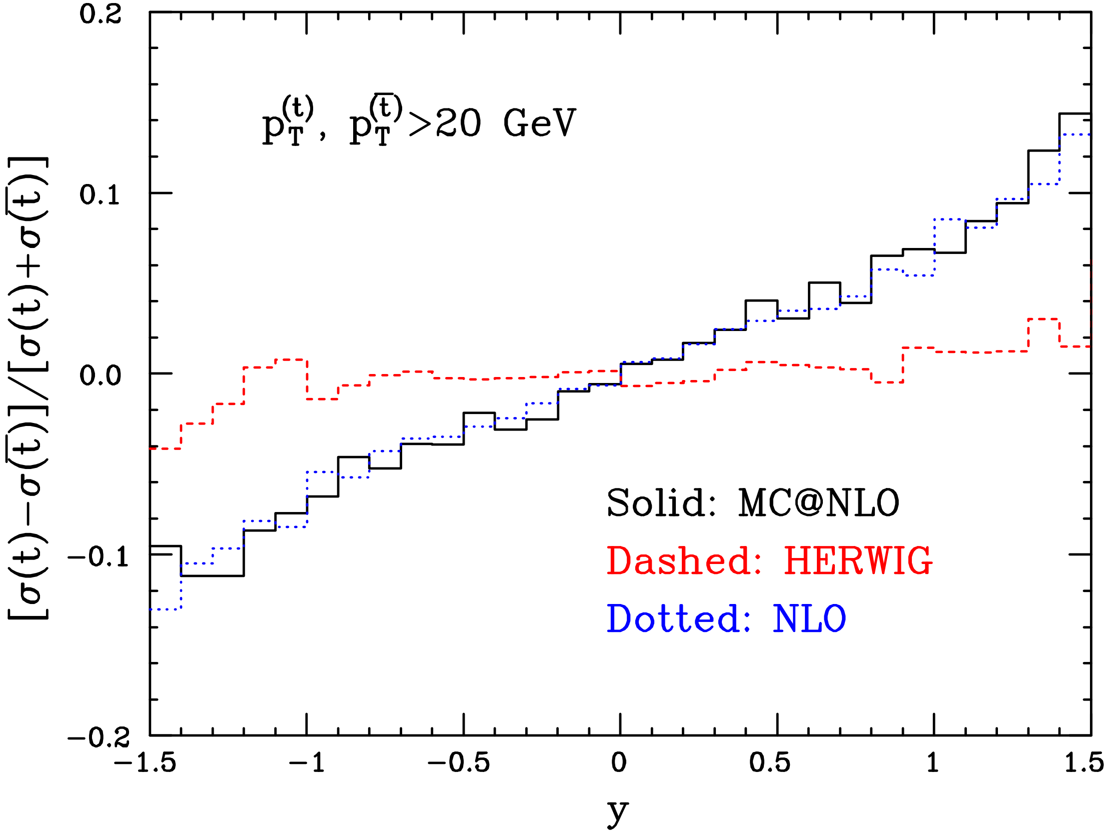
$$\left(\frac{d\sigma}{dO}\right)_{\mathrm{msub}} = \int_0^1 dx \left[\delta(O - O(x))\frac{aR(x)}{x} + \delta(O - O(0))\left(B + aV - \frac{aB}{x}\right)\right] + aB\int_0^{x_0} dx \frac{Q(x)}{x}\Big[\delta(O - O(0)) - \delta(O - O(x))\Big] + \mathcal{O}(a^2).$$

- It can also be shown that the normal summation of branching logarithms is not compromised by this procedure.

Asymmetry in top production

Frixione,Nason,Webber

- Example of $t\bar{t}$-production using MC@NLO
- NLO curve (in blue, dotted).



Conclusions

- NLO formulation of QCD processes gives better information about normalization, and less dependence on unphysical scales.
- Matching with Monte Carlo can be implemented.
- Much remains to be done
  - ⋆ The NLO corrections which necessary for normalization are unknown for many of the most interesting processes. $2 \to 2$ processes are known, some $2 \to 3$ processes, one or two $2 \to 4$ processes.
  - ⋆ MC@NLO is known only for a very limited set of processes, namely the hadroproduction of single vector and Higgs bosons, vector boson pairs, heavy quark pairs, singletop, lepton pairs, and Higgs bosons in association with a W or Z.