Marching orders for the Tevatron and the LHC

31 August 2005

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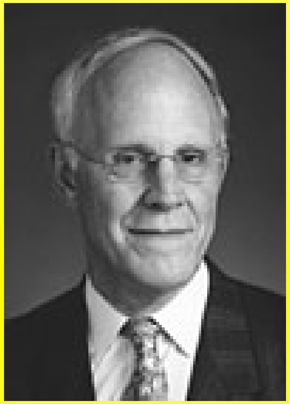
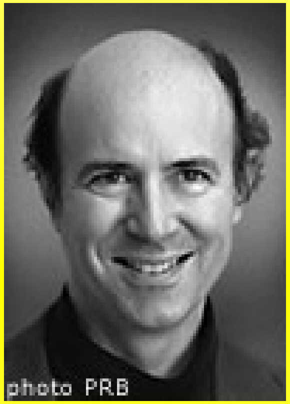
Fermilab/CERN

QCD after the prize (2004)

"for the discovery of asymptotic freedom in the theory of the strong interaction"





'a large body of significant advances . . . and are the work of not just three people but a great many scientists, . . . This is really a prize for that whole community', – David Politzer, Nobel Lecture.

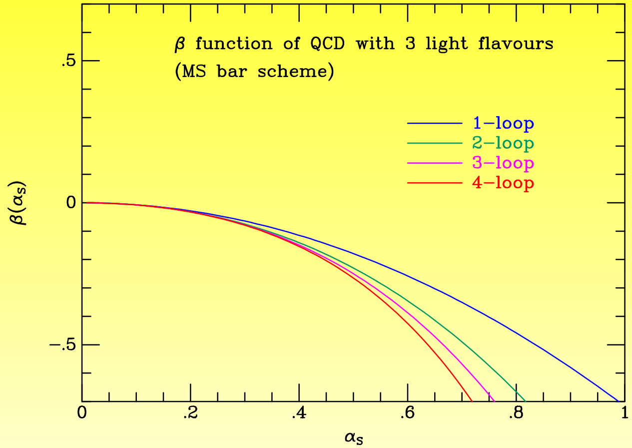
β function in perturbation theory

- Running of the QCD coupling α_S is determined by the β function,
- The β-function of QCD is negative.

$$\beta(\alpha_S) = -b\alpha_S^2(1 + b'\alpha_S) + \mathcal{O}(\alpha_S^4)$$

$$b = \frac{(11C_A - 2n_{lf})}{12\pi}, \quad b' = \frac{(17C_A^2 - 5C_A n_{lf} - 3C_F n_{lf})}{2\pi(11C_A - 2n_{lf})},$$

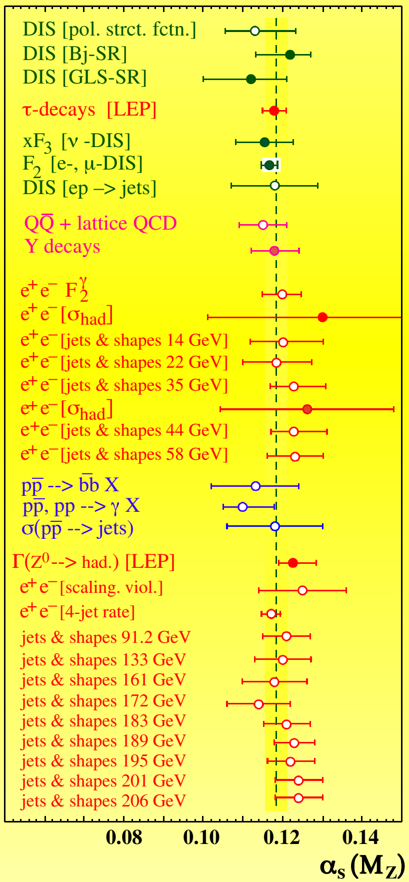
where n_{lf} is number of “active” light flavors. b', (Caswell, Jones)



Current experimental results on α_S

Bethke,hep-ph/0407021

$$\alpha_S(M_Z) = 0.1182 \pm 0.0027, \overline{\mathrm{MS}}, \mathrm{NNLO}$$



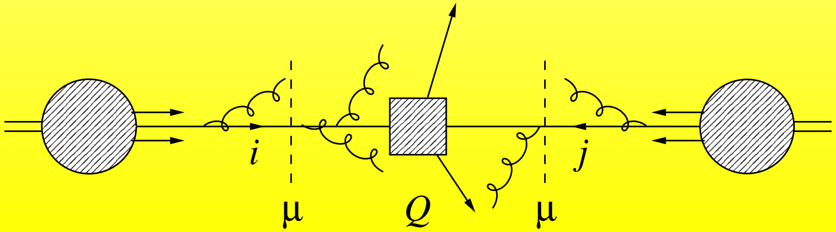
- α_S is large at current scales.
- The decrease of α_S is quite slow – as the inverse power of a logarithm.
- Higher order corrections are and will continue to be important.

The challenge

- The challenge is to provide the most accurate information possible to experimenters working at the Tevatron and the LHC.
- Proton (anti)proton collisions give rise to a rich event structure.
- Complexity of the events will increase as we pass from the Tevatron to the LHC.
- The goals
    - ⋆ To provide physics software tools which are both flexible and give the most accurate representations of the underlying theories.
    - ⋆ To discover new efficient ways of calculating in perturbative QCD, (e.g. MHV amplitudes as discussed by Khoze in his recent seminar).

Hadron-hadron processes

- In hard hadron-hadron scattering, constituent partons from each incoming hadron interact at short distance (large momentum transfer Q^2).



- Form of cross section is

$$\frac{d\sigma}{dX} = \sum_{i,j} \sum_{\tilde{X}} \int dx_1 dx_2 \, f_i(x_1,\mu^2) f_j(x_2,\mu^2)$$
$$\times \quad \hat{\sigma}_{ij}^{\tilde{X}}(\alpha_S(\mu^2), Q^2, \mu^2) \, F(\tilde{X} \to X, \mu^2)$$

where μ^2 is factorization scale and $\hat{\sigma}_{ij}$ is subprocess cross section for parton types i,j and X represents the hadronic final state.

Hadron-hadron processes II

- Short distance cross section $\hat{\sigma}_{ij}$ is calculable as a perturbation series in α_S.
- Notice that factorization scale is in principle arbitrary: affects only what we call part of subprocess or part of initial-state evolution (parton shower).
- There are also interactions between spectator partons, leading to *soft underlying event* and/or *multiple hard scattering*. This an important issue, but I will not talk further about it.

Short-distance cross section

- Tree graph level
  - ⋆ Automatic calculation of tree graphs (Madgraph/Helas, Alpgen, CompHEP, . . .)
- Combining tree graphs and parton showers
- NLO (MCFM, NLOJET++, DYRAD . . .)
- NLO + parton shower (CKKW, Sherpa)
  - ⋆ MC@NLO
- NNLO
  - ⋆ survey of observable results
  - ⋆ NNLO splitting functions
  - ⋆ Drell-Yan Luminosity monitor

I shall concentrate on NLO calculations, and talk about NNLO if time permits

The role of tree graphs

- Problems with tree graphs
  - ⋆ Overall normalization is uncertain.
    For example, W+4 jets is O(α_S^4). If scale uncertainty changes α_S by 10%, this leads to 40% uncertainty in cross section.
  - ⋆ If we wish talk about hadrons, we must apply fragmentation. To use universal fragmentation, we must evolve to a fixed scale. Tree graphs require a procedure to combine with parton showers.
  - ⋆ Sometimes a new parton process appears at NLO, leading to large change in shapes. (e.g., gluons at the LHC).
- For example, for W, Z + n jets at tree graph level.
  Madgraph II can generate processes with $\leq$ 9 external particles (madgraph.hep.uiuc.edu)
  Vecbos, W-boson plus up to 4 jets or a Z-boson plus up to 3 jets (theory.fnal.gov/people/giele/vecbos.html)
  Alpgen, W,Z + up to 6 jets etc, (mlm.home.cern.ch/mlm/alpgen/)

Why NLO?

The benefits of higher order calculations are:-

- Less sensitivity to unphysical input scales (eg. renormalization and factorization scales)
- First prediction of normalization of observables at NLO
- Hence more accurate estimates of backgrounds for new physics searches.
- Confidence that cross-sections are under control for precision measurements.
- It is a necessary prerequisite for other techniques matching with resummed calculations, (eg. MC@NLO).
- More physics
  - ⋆ Parton merging to give structure in jets.
  - ⋆ Initial state radiation.
  - ⋆ More species of incoming partons enter at NLO.

NLO calculation

- Ingredients in a NLO calculation are
  - ⋆ Born level amplitude
  - ⋆ Real contribution: Addition of one extra parton to Born level process
  - ⋆ Virtual contribution: Interference of one-loop amplitude with Born amplitude
- Real and virtual separately contain singularities from the soft and collinear regions which cancel in the sum.
- Calculation of one-loop amplitudes rapidly becomes complicated as number of partons increases.
- Especially true as we go beyond the most symmetric cases with all gluons.

MCFM overview

mcfm.fnal.gov

MCFM Overview

J.Campbell and RKE
(+F. Tramontano, +F. Maltoni, S. Willenbrock)

- Downloadable general purpose NLO code, "MCFM"

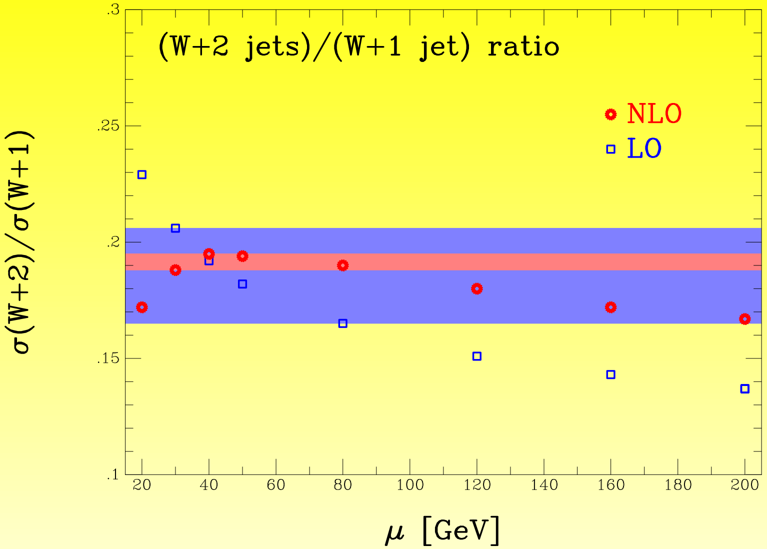
$p\bar{p} \to W^\pm/Z$	$p\bar{p} \to W^+ + W^-$
$p\bar{p} \to W^\pm + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^\pm + \gamma$	$p\bar{p} \to W^\pm/Z + H$
$p\bar{p} \to W^\pm + g^\star\,(\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W^\pm/Z +$ 1 jet	$p\bar{p} \to W^\pm/Z +$ 2 jets
$p\bar{p}(gg) \to H$	$p\bar{p}(gg) \to H +$ 1 jet
$p\bar{p}(VV) \to H +$ 2 jets	$p\bar{p} \to t + q$
$p\bar{p} \to H + b$	$p\bar{p} \to Z + b$

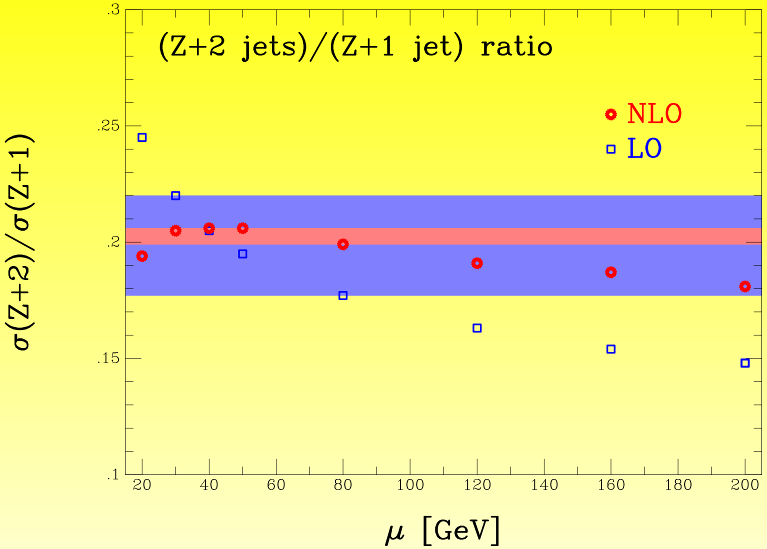
- Knowledge of these processes at NLO provides the first precise predictions of their event rates, which is used in various ways.
  - ⋆ production of pairs of W's and Z's: the structure of the weak interaction at high energy
  - ⋆ W and H production: possibly the first hint of a Higgs boson at the Tevatron
  - ⋆ $H + 2$ jets: an important discovery mode at the LHC

W/Z+jets cross-sections

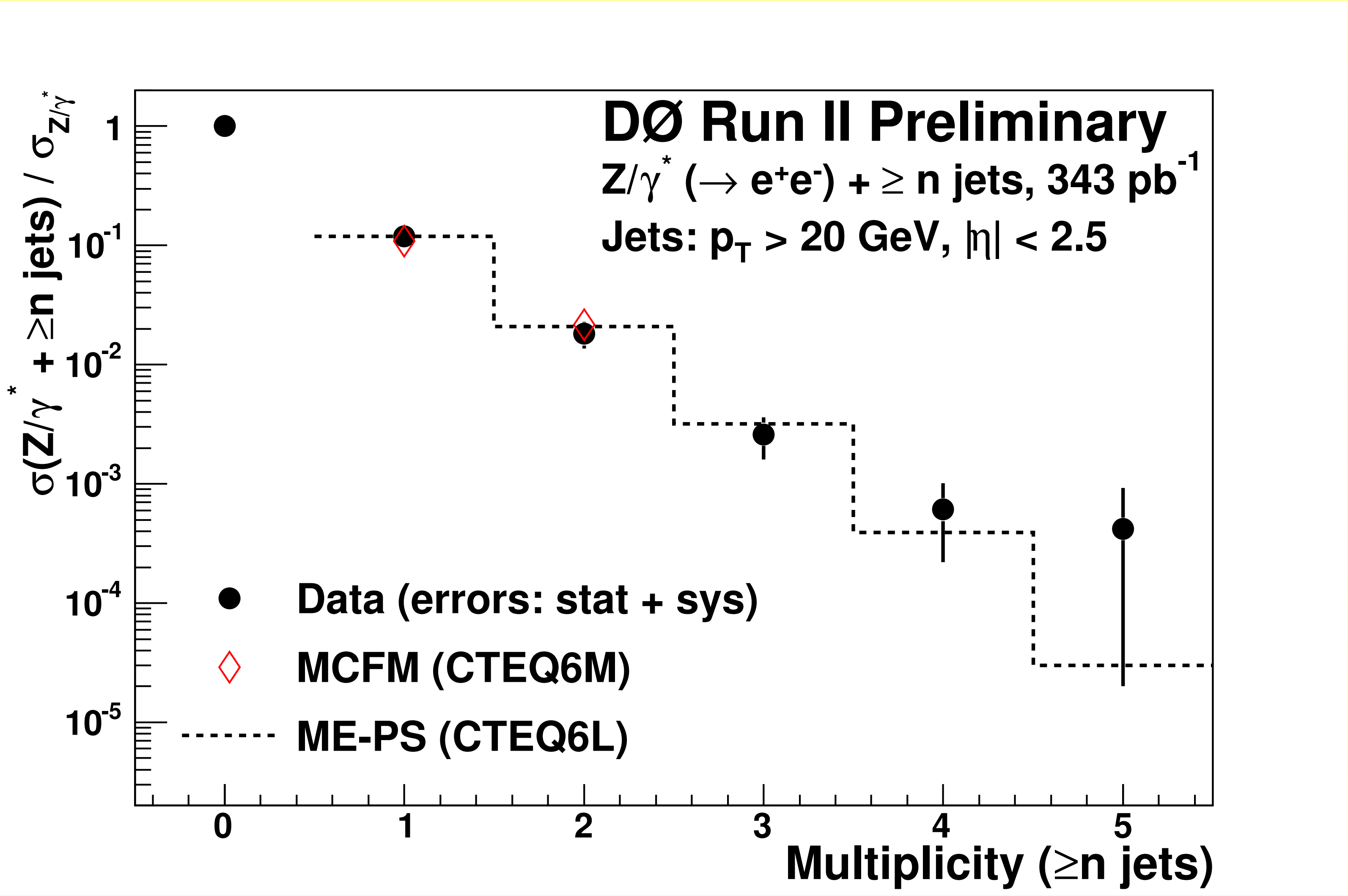
Rates at the Tevatron

- The $W/Z+2$ jet NLO calculation is the most complicated (time-consuming) process currently implemented. This is due to both the lengthy virtual matrix elements (vector boson $+\,4$ partons) and the complicated structure of phase space.
- The usual features such as reduced scale dependence are observed, e.g. the theoretical prediction for the number of events containing 2 jets divided by the number with only 1 is improved.



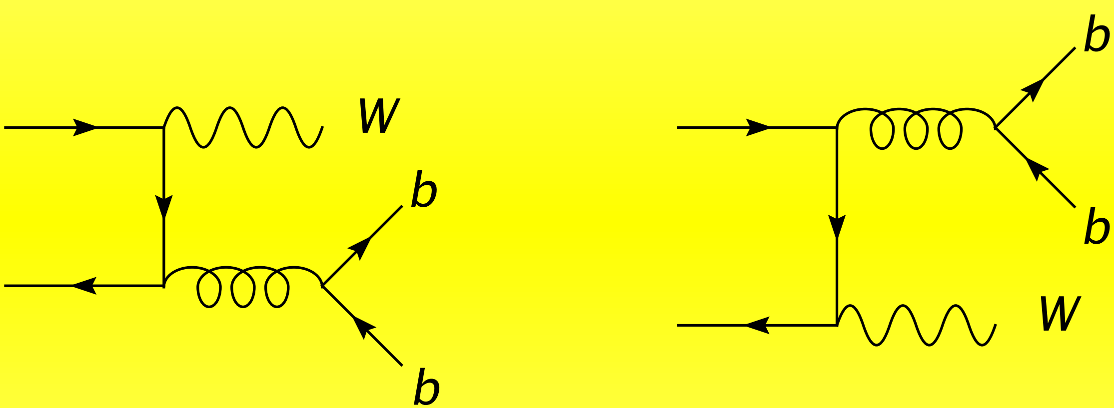


Preliminary data



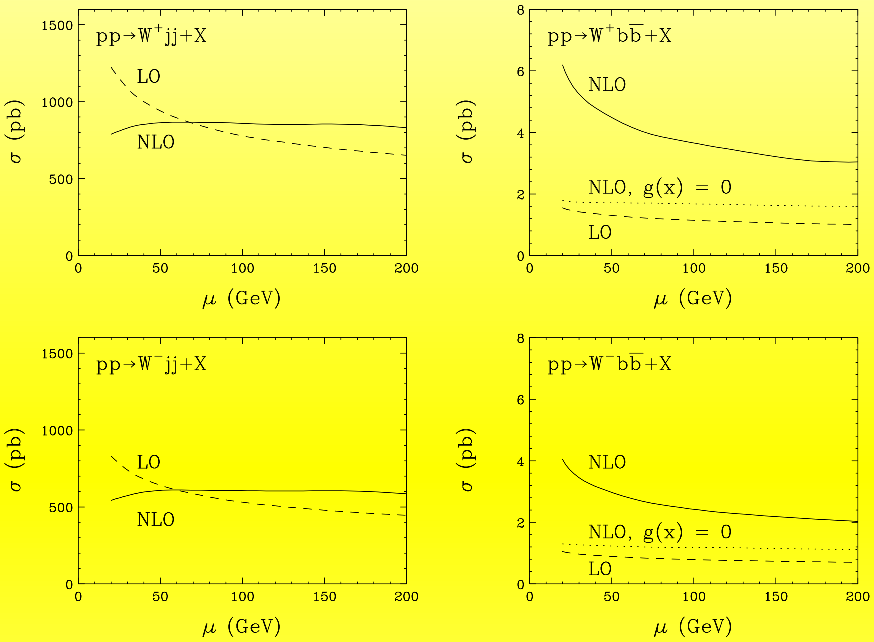
Vector boson + heavy flavour

- In lowest order bottom quark pairs are produced in association with W's by gluon splitting alone:



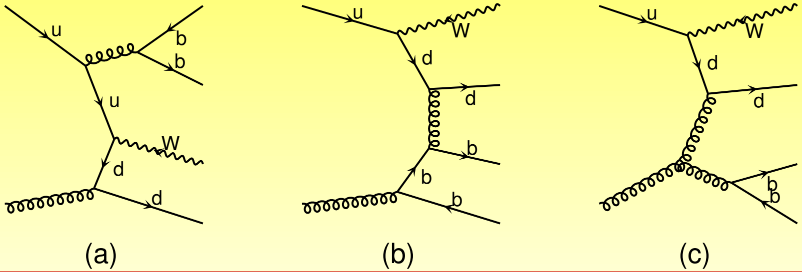
- Beyond LO, the b-quark is treated as a massless particle in MCFM
  - ⋆ a finite cross-section requires a cut on the b-quark p_T
  - ⋆ this means that this calculation is not suitable for estimating the rate with only a single b tag

Jets and heavy flavour at the LHC



- The large gluonic contribution appearing in $Wb\bar{b}$ for the first time at NLO results in a huge correction and poor scale dependence.

Diagrams by MadGraph

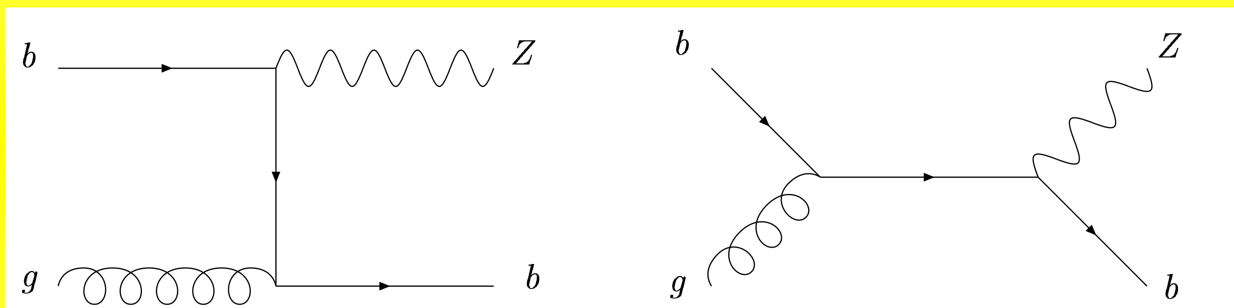


Single-tagged heavy flavour

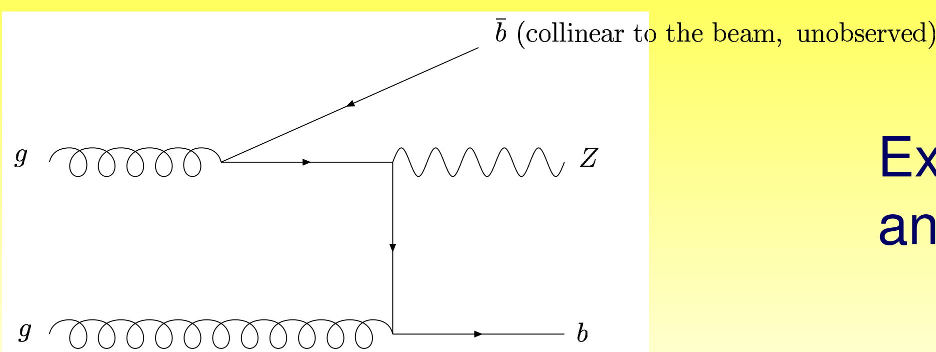
Campbell, Ellis, Maltoni, Willenbrock

Heavy flavour fraction revisited

- Often the presence of two b-quarks in the final state is actually only inferred from a single b-tag
- In this case, there is another way of computing the theoretical cross-section. For instance, in the case of $Z+$ heavy flavour:



- Requires knowledge of b-quark pdf's, but compare to:



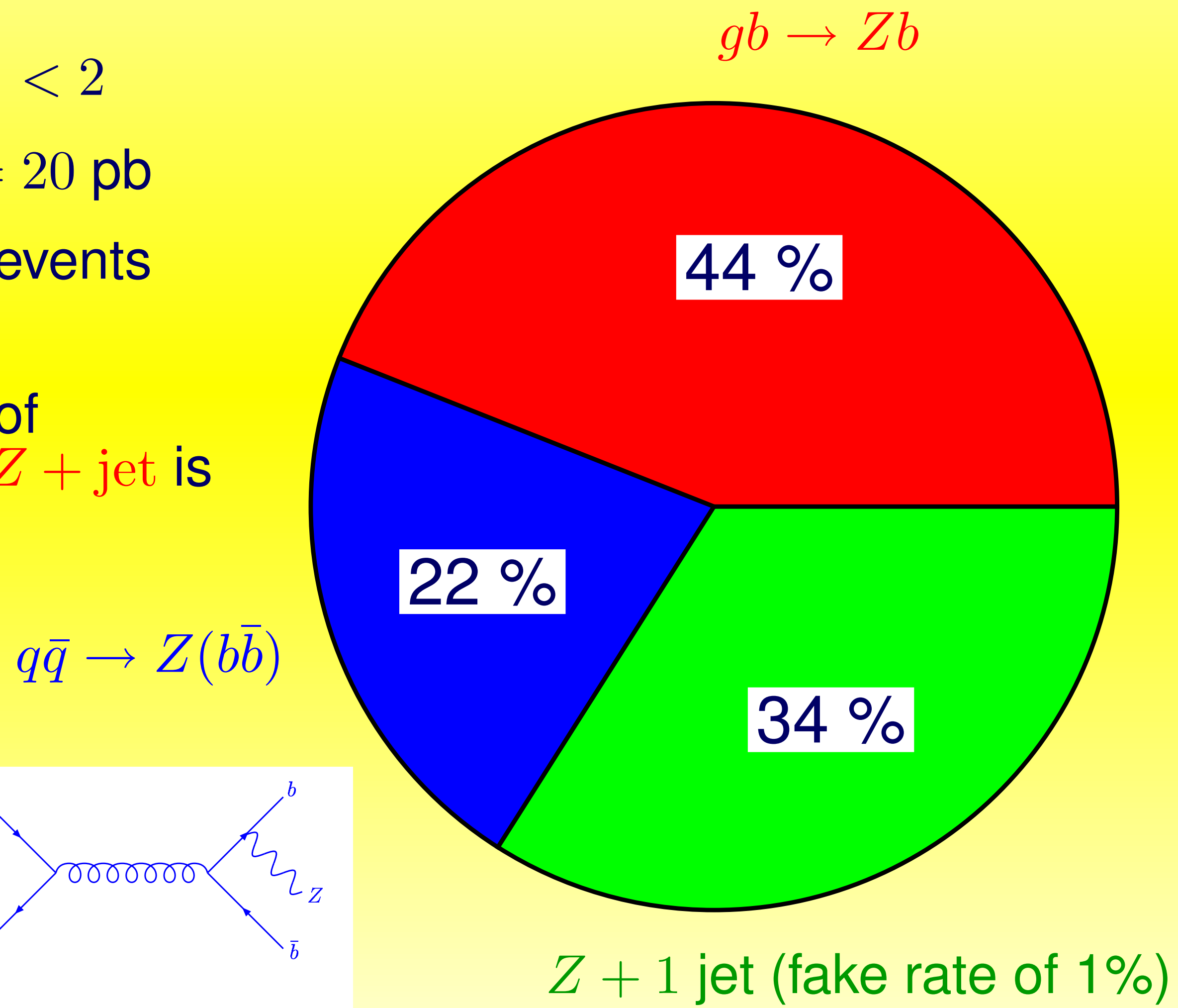
Expansion in $\alpha_s \ln(M_Z/m_b)$ and NLO calculation difficult

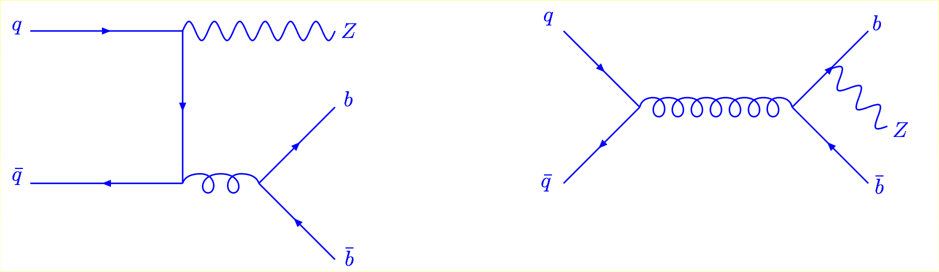
$Z + b$ at NLO - Run II

Campbell, RKE, F. Maltoni and S. Willenbrock, hep-ph/0312024

- $p_T^{\rm jet} > 15$ GeV, $|\eta^{\rm jet}| < 2$
- $\sigma(Z + \text{ one } b \text{ tag}) = 20$ pb
- Fakes from $Z+$ jet events are significant
- Prediction for ratio of $Z + b$ to untagged $Z + \text{jet}$ is

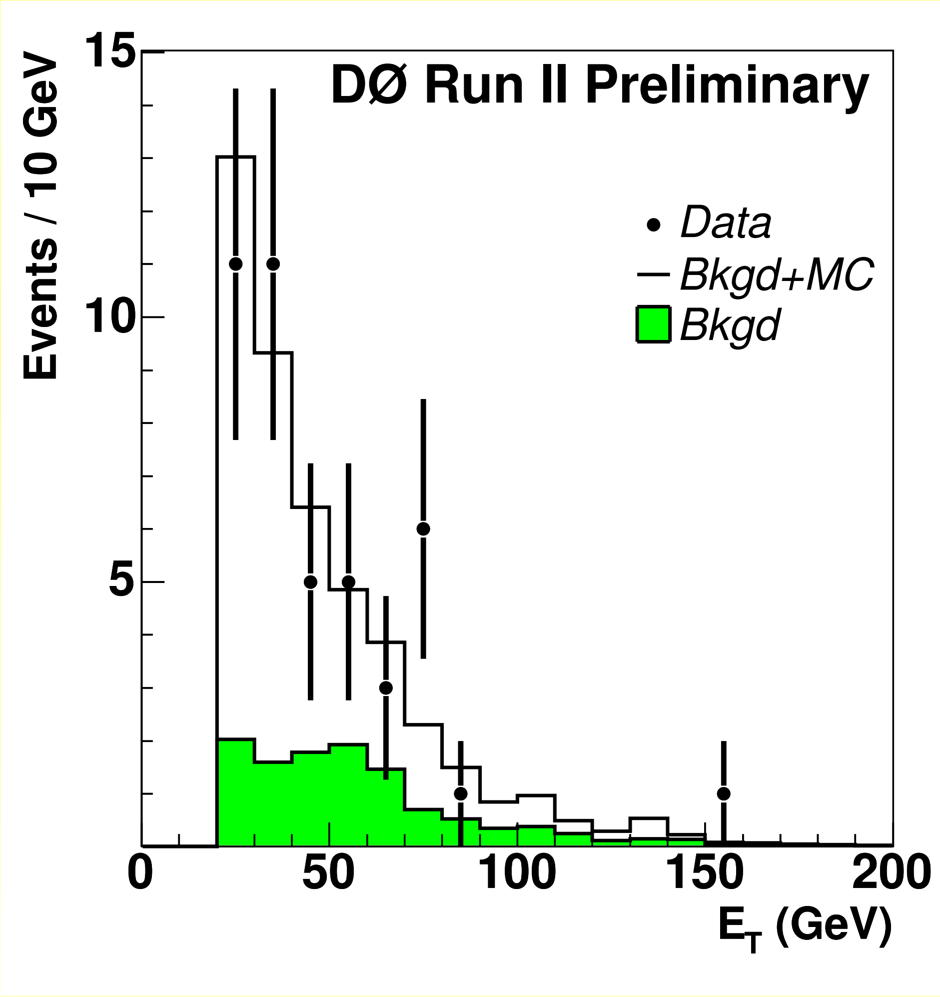
  0.02 ± 0.004





Experimental result

- Based on $189\ \mathrm{pb}^{-1}$ of data from Run II



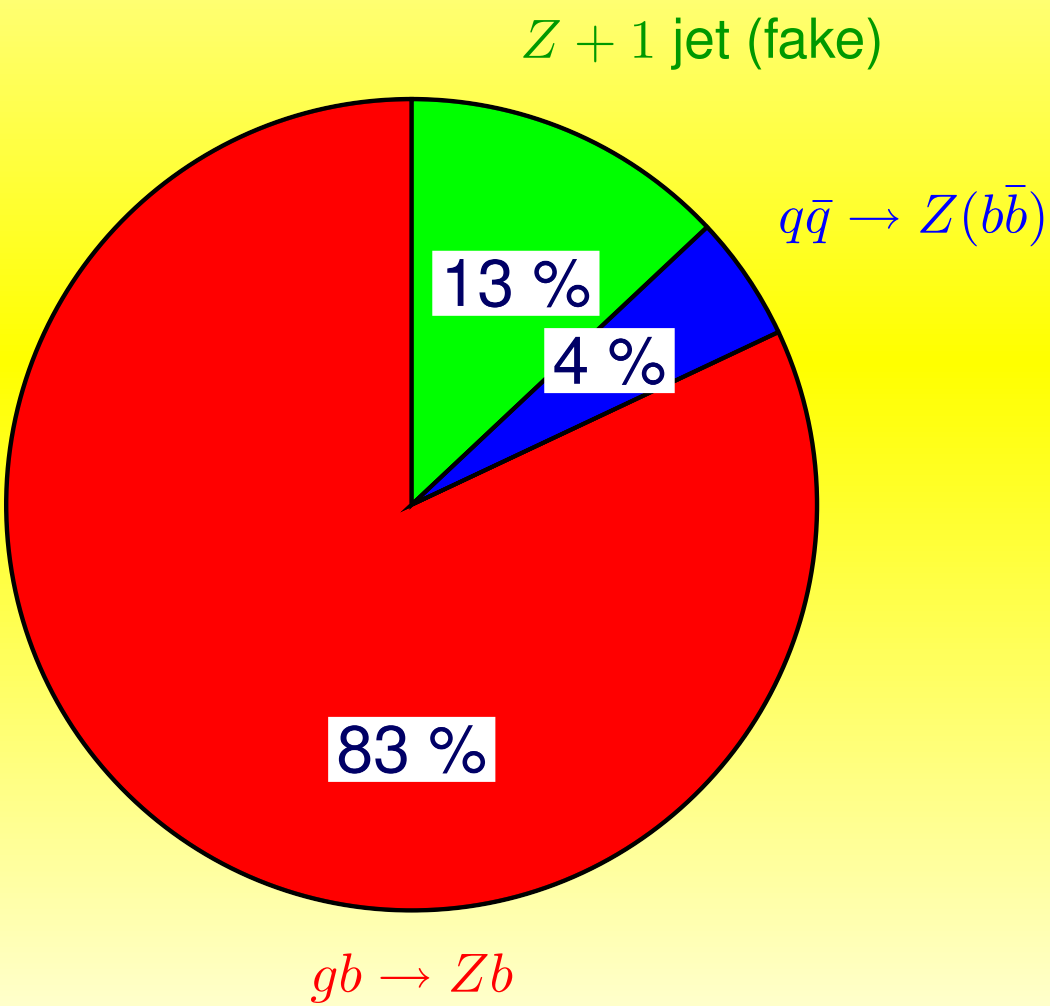
Ratio of cross-sections:

$$\frac{\sigma(Z+b)}{\sigma(Z+j)} = 0.024 \pm 0.007$$

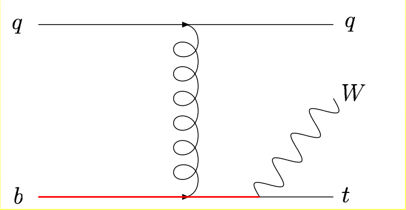
compatible with the NLO prediction from MCFM

LHC expectations

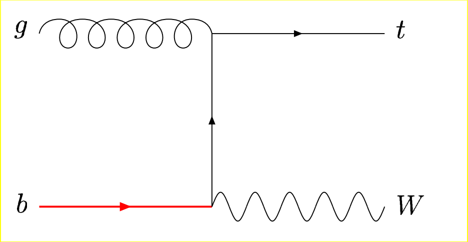
- $p_T^{\rm jet} > 15$ GeV, $|\eta^{\rm jet}| < 2.5$
- $\sigma(Z + \text{ one } b \text{ tag}) = 1$ nb
- Fakes from $Z+$ jet events are much less significant and $q\bar{q}$ contribution is tiny
- This should allow a fairly clean measurement of heavy quark PDF's (currently, only derived perturbatively)



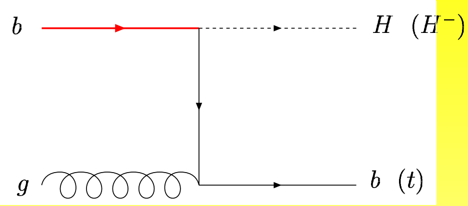
b-PDF uses



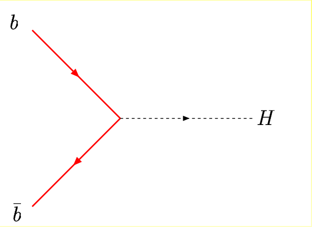
single-top $qb \to qWb$



single-top $gb \to tW$



(charged) Higgs+b



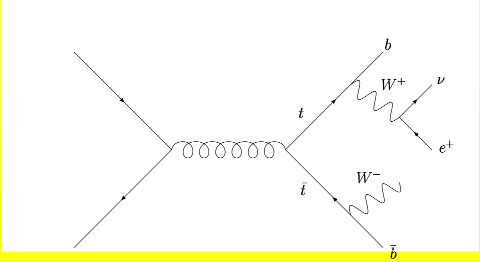
inclusive Higgs

Single top production and decay

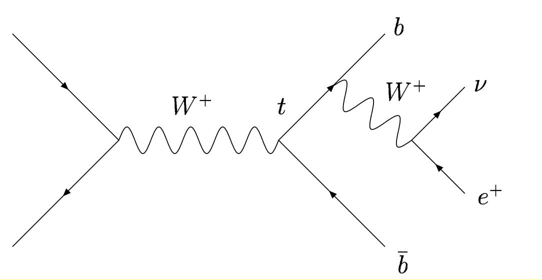
Campbell, Ellis, Tramontano

Producing the top quark

- The top quark was discovered in Run I of the Tevatron by producing it in pairs:



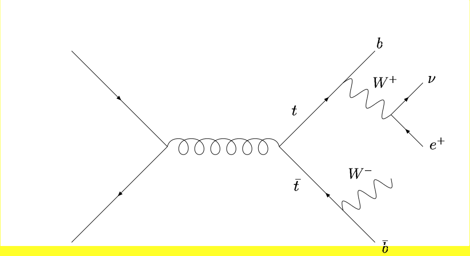
- However, it should also be possible to produce it singly in Run II, for example:



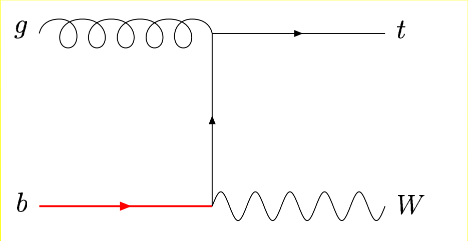
- This is especially interesting since it would yield information about the weak interaction of top quarks (V_{tb}).

Top production rates

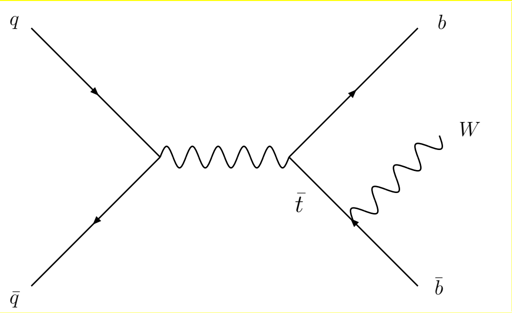
Campbell, Tramontano for Wt



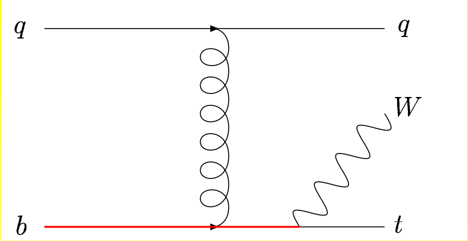
6 pb
720 pb



1.8 pb
240 pb



0.8 pb
10 pb

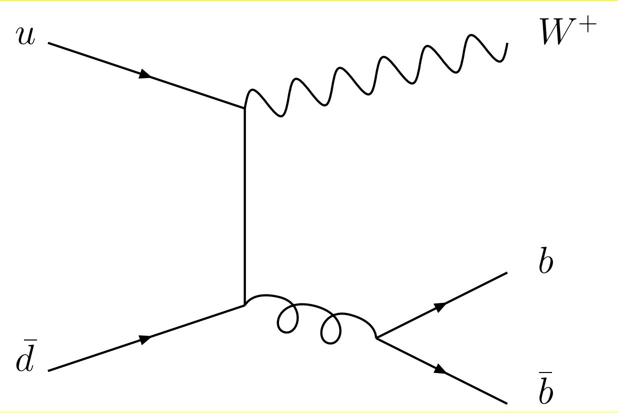


0.08 pb
50 pb

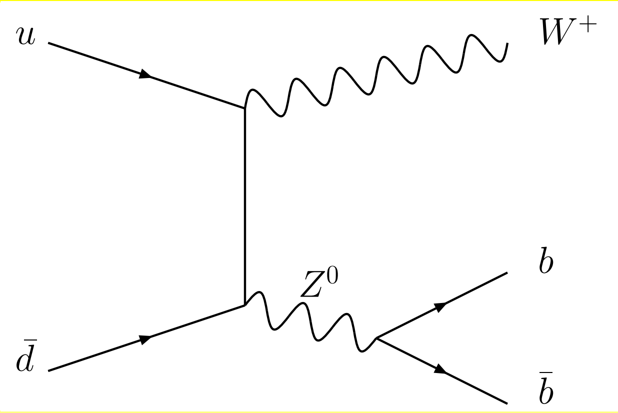
- All cross-sections are known to NLO (Tevatron / LHC)
- The total single top cross-section $(t + \bar{t})$ is smaller than the $t\bar{t}$ rate by about a factor of two, at both machines

Experimental signature

- The experimental “signature” is an event which contains a top quark – identified by the combined mass of its decay products – and which also has two jets containing b-quarks. These can be distinguished from other jets around 50% of the time.
- Observed events such as these can also be the result of other basic processes. These backgrounds include, for example:



$Wb\bar{b}$

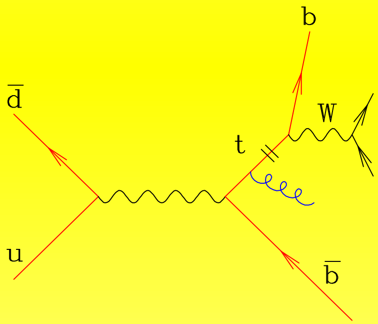


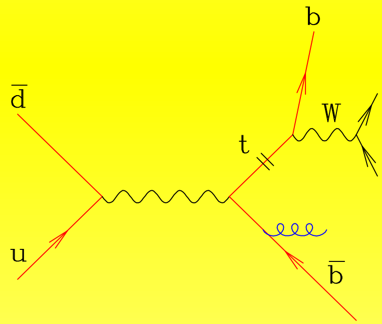
$WZ(\rightarrow b\bar{b})$

- MCFM can calculate the signal and backgrounds at NLO.

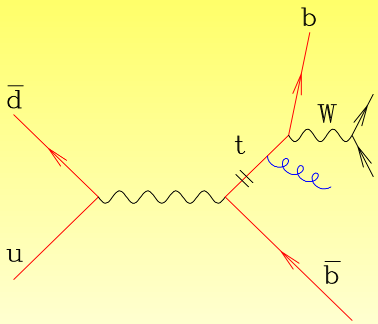
Inclusion of decay

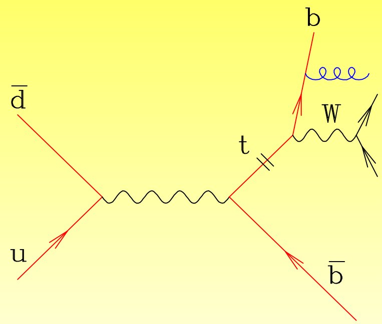
- Results had previously been presented without including the decay of the top quark. Without it, predictions for some quantities used in Tevatron search strategies are impossible
- Final state radiation that enters at next-to-leading order is possible in either the production or decay phase:





production





decay

Results

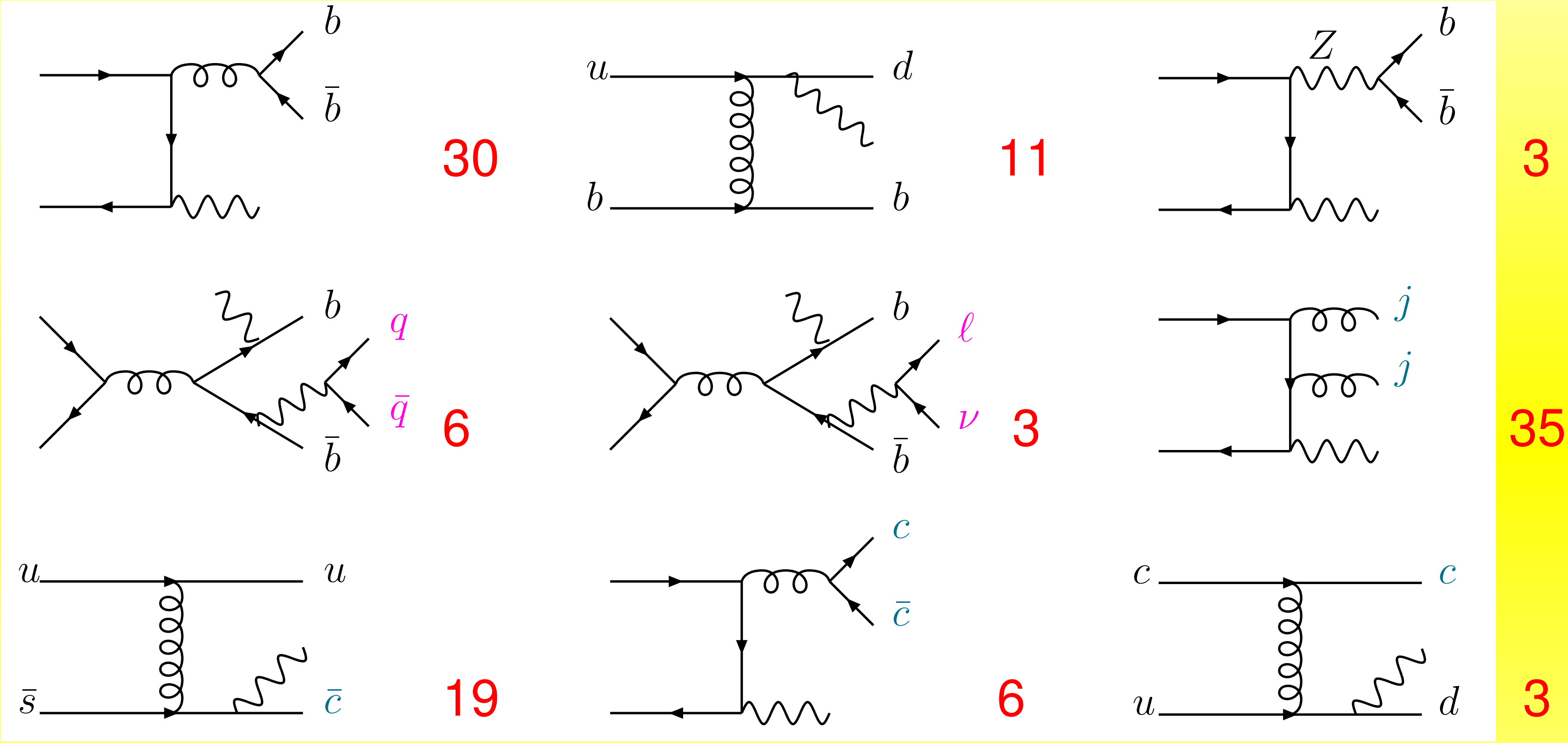
- Parton level study of the Tevatron single top analysis performed by CDF

Lepton p_T	$p_T^e > 20$ GeV
Lepton pseudorapidity	$\lvert\eta^e\rvert < 1.1$
Missing E_T	$\not{E}_T > 20$ GeV
Jet p_T	$p_T^{\text{jet}} > 15$ GeV
Jet pseudorapidity	$\lvert\eta^{\text{jet}}\rvert < 2.8$
Mass of $b + l + \nu$	$140 < m_{bl\nu} < 210$ GeV

- The inclusion of radiation in the decay lowers the (exclusive two-jet) cross-section slightly:

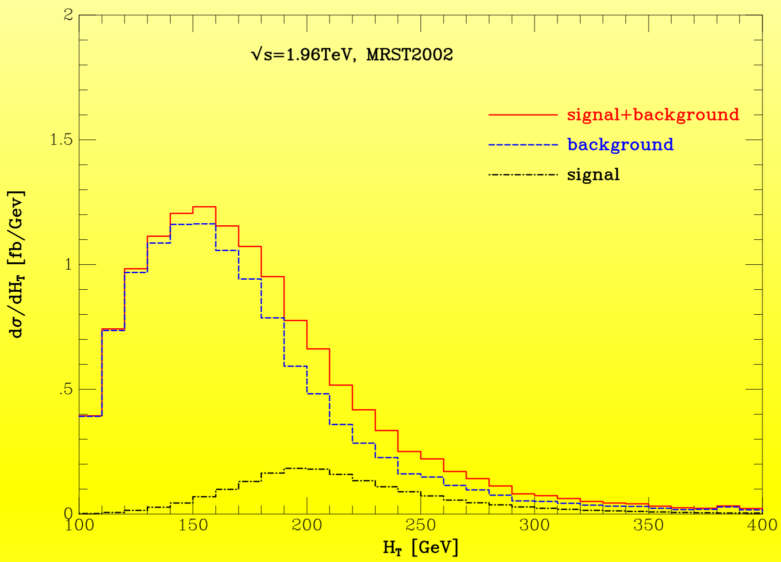
Process	σ_{LO} [fb]	σ_{NLO} [fb]
s-channel single top	10.3	11.7
s-channel (with decay radiation)	10.3	11.3
t-channel single top	38.8	29.4
t-channel (with decay radiation)	38.8	26.6

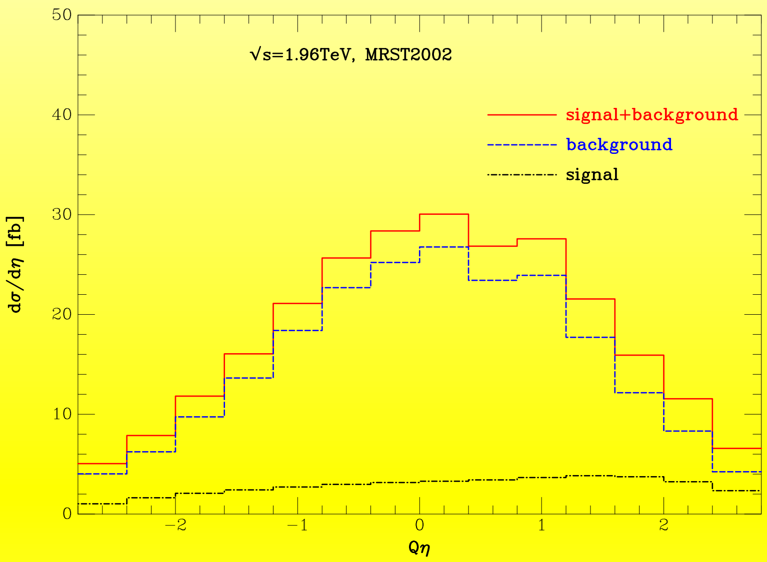
Backgrounds



- Cross-sections in fb include nominal tagging efficiences and mis-tagging/fake rates. Calculated with MCFM, most at NLO
- Rates are 7 fb and 11 fb for s- and $t-$channel signal

Single top signal vs. backgrounds





- $H_T =$ scalar sum of jet, lepton and missing E_T
- $Q\eta$ is the product of the lepton charge and the rapidity of the untagged jet, useful for picking out the t-channel process
- Signal:Background (with our nominal efficiencies) is about $1:6$ – a very challenging measurement indeed. Production in this mode has not yet been observed at Fermilab.
- Currently D0 estimate that a luminosity of 7 fb^{-1} is required for a 5σ observation.

Anyes Taffard, HCP 2005.

Shortcomings

The approach in MCFM involves a number of approximations:

- The b-quark is massless
  LO calculation with $m_b = 4.75$ GeV $\longrightarrow < 1\%$ effect
- The top quark is put on its mass-shell
  LO calculation with a Breit-Wigner $\longrightarrow 1\%$ effect
- We neglect interference between radiation in production/decay
  qualitative argument for $\mathcal{O}(\alpha_s \Gamma_t / m_t) \sim$ less than a percent
- We assume p_T-independent heavy flavour tagging efficiencies, as well as stable b and c quarks
  easily addressed by a more detailed experimental analysis
  with the publicly-available code
- No showering or hadronization is performed
  no NLO/PS prediction yet available; however the large cone
  size $\Delta R = 1$ should help minimize these effects

The future of NLO calculations

An experimenter's wishlist

Run II Monte Carlo Workshop

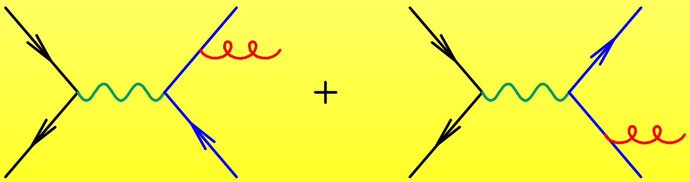
Single Boson	Diboson	Triboson	Heavy Flavour
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}\leq 3j$	$W+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}\leq 3j$	$W+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$Z+b\bar{b}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{b}\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$	$ZZZ+\leq 3j$	$b\bar{b}+\leq 3j$
$\gamma+b\bar{b}\leq 3j$	$\gamma\gamma+b\bar{b}\leq 3j$		single top
$\gamma+c\bar{c}\leq 3j$	$\gamma\gamma+c\bar{c}\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}\leq 3j$		
	$WZ+c\bar{c}\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

Automatic NLO corrections

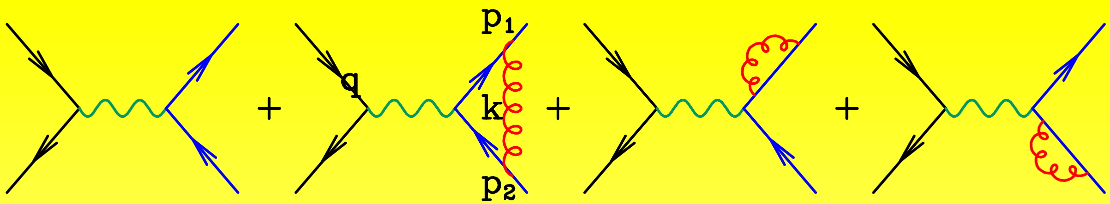
- What is needed is an automatic procedure to calculate NLO corrections (MadLoop?).
- Current stumbling block is the calculation of virtual corrections.
- The virtual corrections contain singularities from the regions of collinear and soft gluon emission, (and in general also UV divergences).
- Divergences are normally controlled by dimensional regularization. A completely numerical procedure using, say, a gluon mass could cause problems with gauge invariance and is hence deprecated.

Example: e^+e^- total rate

- Consider the corrections to total $e^+e^- \to q\bar{q}$ rate.



(a)



(b)

$$\sigma^{q\bar{q}g} \;=\; 2\sigma_0 \frac{\alpha_S}{\pi}\, H(\epsilon)\,\left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + \mathcal{O}(\epsilon)\right]\,.$$

- Soft and collinear singularities in real emisssion amplitudes (a) are regulated, appearing instead as poles at $D = 4$.

Virtual gluon contributions

- Virtual gluon contributions (b): using dimensional regularization again

$$\sigma^{q\bar{q}} \;=\; 3\sigma_0 \left\{1 + \frac{2\alpha_S}{3\pi}\, H(\epsilon)\, \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon)\right]\right\} \; .$$

- Adding real and virtual contributions, poles cancel and result is finite as $\epsilon \to 0$. R is an infrared safe quantity.

$$R \;=\; 3\sum_q Q_q^2\, \left\{1 + \frac{\alpha_S}{\pi} + \mathcal{O}(\alpha_S^2)\right\}.$$

- However the virtual corrections to $W^+ \to u\bar{d}gggg$ are not so easily calculated.

Historical perspective

We want to consider tensor integrals of the form

$$I^{\mu_1 \cdots \mu_M} = \int \frac{d^D l}{i\pi^{D/2}} \; \frac{l^{\mu_1} \ldots l^{\mu_M}}{d_1 d_2 \ldots d_N}$$

where $d_i = (l + \sum_{j=1}^{j=i} p_j)^2$ are the standard propagator factors.
Passarino and Veltman (1979) wrote a form factor expansion for one-loop integrals, with $M \leq N, N \leq 4$. For example,

$$\int \frac{d^D l}{i\pi^{D/2}} \frac{l^\mu}{l^2 (l+p_1)^2 (l+p_1+p_2)^2} = C_1(p_1,p_2) p_1^\mu + C_2(p_1,p_2) p_2^\mu$$

Contracting with p_1 and p_2 and using the identities

$$l \cdot p_1 = \frac{1}{2}[(l+p_1)^2 - l^2 - p_1^2], l \cdot p_2 = \frac{1}{2}[(l+p_1+p_2)^2 - (l+p_1)^2 - p_2^2 - 2 p_1 \cdot p_2]$$

Historical perspective II

We derive a linear equation expressing C_1, C_2 in terms of scalar integrals

$$\left(\begin{array}{cc} 2p_1\cdot p_1 & 2p_1\cdot p_2 \\ 2p_2\cdot p_1 & 2p_2\cdot p_2 \end{array}\right)\left(\begin{array}{c} C_1 \\ C_2 \end{array}\right) = \left(\begin{array}{c} R_1 \\ R_2 \end{array}\right)$$

where $R_1 = [B_0(p_1+p_2) - B_0(p_2) - p_1^2\, C_0(p_1,p_2)]$

and $R_2 = [B_0(p_1) - B_0(p_1+p_2) - (p_2^2 + 2p_1\cdot p_2)\, C_0(p_1,p_2)]$

$$C_0(p_1,p_2) = \int [dl] \frac{1}{l^2(l+p_1)^2(l+p_1+p_2)^2}, B_0(p_1) = \int [dl] \frac{1}{l^2(l+p_1)^2}$$

Solution involves the inverse of the Gram matrix, $G_{ij} \equiv 2p_i\cdot p_j$

$$G^{-1} = \left(\begin{array}{cc} +p_2\cdot p_2 & -p_1\cdot p_2 \\ -p_1\cdot p_2 & +p_1\cdot p_1 \end{array}\right) /[2(p_1\cdot p_1\, p_2\cdot p_2 - (p_1\cdot p_2)^2)]$$

Historical perspective III

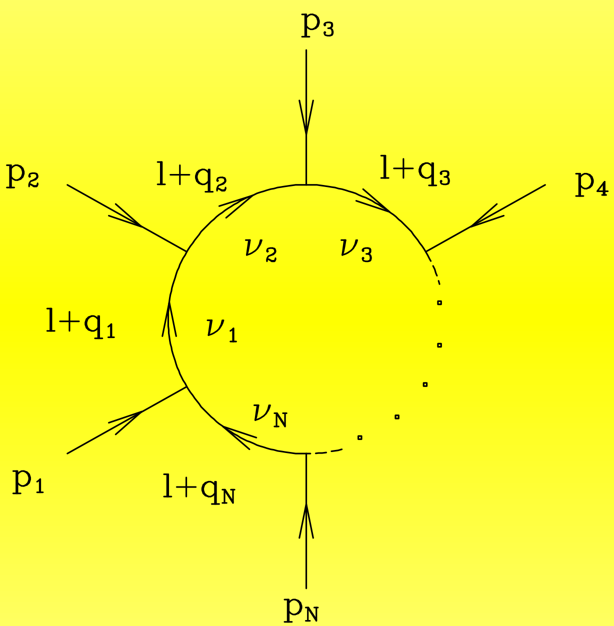
- M. Veltman wrote a CDC program for numerical evaluation of the formfactors in processes with only UV divergences, Utrecht (1979).
- He dealt with exceptional regions, (e.g. regions where the Gram determinant vanishes), by implementing parts of the program in quadruple precision.
- Translation and improvement by Van Oldenborgh (1990) and further work on interface by T. Hahn and M. Perez-Victoria (1998).

However this is not sufficient for our needs.

- We are interested in processes with more than 4 external legs.
- We are often interested in loop processes with collinear and soft singularities due to the presence of massless particles. These are most commonly (and elegantly) controlled by dimensional regularization.

Recursion relations I

Define generalized scalar integrals



$$d_i \equiv (l+q_i)^2$$

$$q_i \equiv \sum_{j=1}^{i} p_j$$

$$q_N \equiv \sum_{j=1}^{N} p_j = 0,$$

$$I(D;\nu_1,\nu_2,\ldots,\nu_N) = I(D;\{\nu_k\}_{k=1}^N) \equiv \int \frac{d^D\, l}{i\pi^{D/2}}\, \frac{1}{d_1^{\nu_1} d_2^{\nu_2}\cdots d_N^{\nu_N}}\,,$$

Form-factor expansion

Davydchev

- For form factor expansion in terms of the q's the coefficients are generalized scalar integrals in shifted dimensionalities
- e.g., the rank-1 and rank-2 tensor integrals with N external legs can be decomposed as

$$
\begin{aligned}
& I^{\mu_1}(D; q_1, \ldots, q_N) = \sum_{i_1=1}^{N} I(D+2; \{1+\delta_{i_1 k}\}_{k=1}^{N})\, q_{i_1}^{\mu_1} \\
= \quad & I(D+2; 2, 1, 1, \ldots, 1)\, q_1^{\mu_1} + I(D+2; 1, 2, 1, \ldots, 1)\, q_2^{\mu_1} \\
+ \quad & \cdots + I(D+2; 1, 1, 1, \ldots, 2)\, q_N^{\mu_1}\, . \\
& I^{\mu_1 \mu_2}(D; q_1, \ldots, q_N) = -\frac{1}{2} I(D+2; 1, 1, 1, \ldots, 1)\, g^{\mu_1 \mu_2} \\
& +2\, I(D+4; 3, 1, 1, \ldots, 1)\, q_1^{\mu_1} q_1^{\mu_2} \\
+ \quad & I(D+4; 2, 2, 1, \ldots, 1)\, (q_1^{\mu_1} q_2^{\mu_2} + q_1^{\mu_2} q_2^{\mu_1}) + \cdots
\end{aligned}
$$

Basic identity

Tkachev,Cetyrkin,Tarasov,Duplancic,Nizic

$$\int \frac{d^D l}{i\pi^{D/2}} \frac{\partial}{\partial l^\mu} \left(\frac{\left(\sum_{i=1}^N y_i\right) l^\mu + \left(\sum_{i=1}^N y_i q_i^\mu\right)}{d_1^{\nu_1} d_2^{\nu_2} \cdots d_N^{\nu_N}} \right) = 0\,.$$

valid for arbitrary y_i. Differentiating we obtain the base identity

$$\sum_{j=1}^N \left(\sum_{i=1}^N S_{ji} y_i\right) \nu_j I(D; \{\nu_k + \delta_{kj}\}_{k=1}^N) = -\sum_{i=1}^N y_i I(D-2; \{\nu_k - \delta_{ki}\}_{k=1}^N)$$

$$- \quad \left(D - 1 - \sum_{j=1}^N \nu_j\right) \left(\sum_{i=1}^N y_i\right) I(D; \{\nu_k\}_{k=1}^N)\,,$$

where S is a kinematic matrix which, for massless internal particles, takes the form

$$S_{ij} \equiv (q_i - q_j)^2\,.$$

Recursion relations II

Solving $\sum_i S_{ji} y_i = \delta_{lj}$ (assuming that the inverse of the matrix S exists), we derive the basic recursion relation

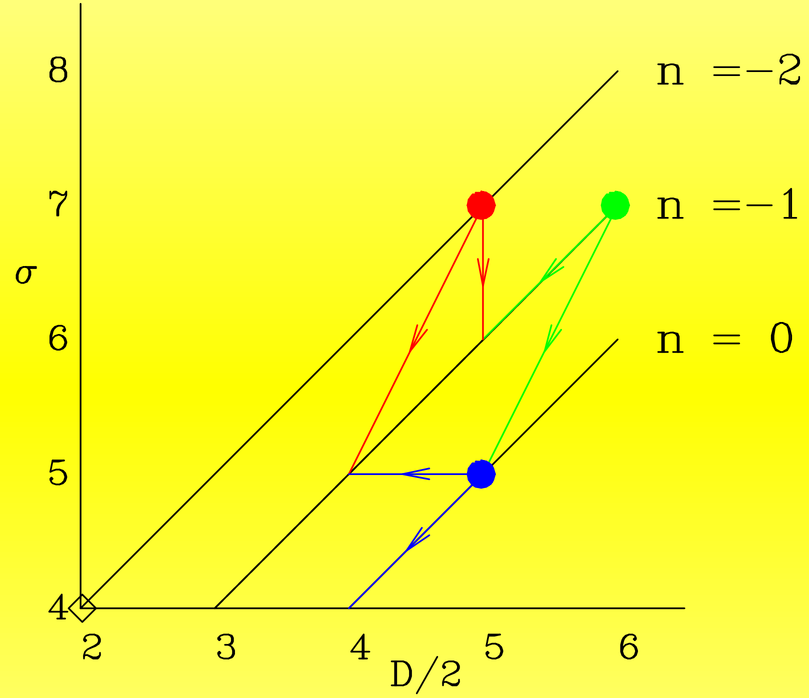
$$\begin{aligned} & (\nu_l - 1) I(D; \{\nu_k\}_{k=1}^N) \\ = & \quad -\sum_{i=1}^N S_{li}^{-1} I(D-2; \{\nu_k - \delta_{ik} - \delta_{lk}\}_{k=1}^N) \\ - & \quad b_l \, (D - \sigma) \, I(D; \{\nu_k - \delta_{lk}\}_{k=1}^N). \end{aligned}$$

$$\sigma \equiv \sum_{i=1}^N \nu_i; \qquad b_i \equiv \sum_{j=1}^N S_{ij}^{-1}; \qquad B \equiv \sum_{i=1}^N b_i = \sum_{i,j=1}^N S_{ij}^{-1} \,.$$

The strategy is to reduce more complicated integrals to a set of simpler basis integrals which are known analytically.
Hence the method is seminumerical.

Recursion relations III

- Example: reduction of boxes



- Using the basic identity (red lines) and other subsidiary identities (blue and green lines) one can always arrive at the basis integral, (four-dimensional box), denoted by a diamond, (or integrals with fewer external legs).

Proof of principle

Ellis, Giele, Zanderighi

Use the effective theory ($m_t \to \infty$) for Hgg coupling

$$\mathcal{L}_{\rm eff} = \frac{1}{4} A(1+\Delta) H G^a_{\mu\nu} G^{a\,\mu\nu} \,.$$

$G^a_{\mu\nu}$ is the field strength of the gluon field and H is the Higgs-boson field, $A = \frac{g^2}{12\pi^2 v}$ where g is the bare strong coupling and v is the vacuum expectation value parameter, $v^2 = (G_F \sqrt{2})^{-1} = (246 \text{ GeV})^2$.
Δ is a finite correction. Calculate virtual corrections to

$$\begin{array}{llcll}
A) & H & \to & q\bar{q}q'\bar{q}', & (30 \text{ diagrams}), \\
B) & H & \to & q\bar{q}q\bar{q}, & (60 \text{ diagrams}), \\
C) & H & \to & q\bar{q}gg, & (191 \text{ diagrams}), \\
D) & H & \to & gggg, & (739 \text{ diagrams}).
\end{array}$$

Comparison of numerical and analytic results for $H \rightarrow$ four partons

	$\frac{1}{\epsilon^2}$	$\frac{1}{\epsilon}$	1
A_B	0	0	12.9162958212387
$A_{V,N}$	-68.8869110466063	-114.642248172519	120.018444115458
$A_{V,A}$	-68.8869110466064	-114.642248172523	120.018444115429
B_B	0	0	858.856417157052
$B_{V,N}$	-4580.56755817094	-436.142317955208	26470.9608978350
$B_{V,A}$	-4580.56755817099	-436.142317955660	26470.9608978346
C_B	0	0	968.590160211857
$C_{V,N}$	-8394.44805516930	-19808.0396331354	-1287.90574949112
$C_{V,A}$	-8394.44805516942	-19808.0396331363	not known analytically
D_B	0	0	3576991.27960852
$D_{V,N}$	-4.29238953553022 $\cdot 10^7$	-1.04436372655580 $\cdot 10^8$	-6.79830911471604$\cdot 10^7$
$D_{V,A}$	-4.29238953553022$\cdot 10^7$	-1.04436372655580 $\cdot 10^8$	not known analytically

Exceptional regions

Ellis, Giele, Zanderighi, hep-ph/0508308

$$\sum_{j=1}^{N}\left(\sum_{i=1}^{N} S_{ji}y_i\right)\nu_j I(D;\{\nu_k+\delta_{kj}\}_{k=1}^{N}) =$$

$$-\sum_{i=1}^{N} y_i I(D-2;\{\nu_k-\delta_{ki}\}_{k=1}^{N}) - \left(D-1-\sum_{j=1}^{N}\nu_j\right)\left(\sum_{i=1}^{N} y_i\right) I(D;\{\nu_k\}_{k=1}^{N}$$

If S has a zero eigenvalue

$$0 = -\sum_{i=1}^{N} y_i I(D-2;\{\nu_k-\delta_{ik}\}_{k=1}^{N}) - (D-1-\sigma)\left(\sum_{i=1}^{N} y_i\right) I(D;\{\nu_k\}_{k=1}^{N}) .$$

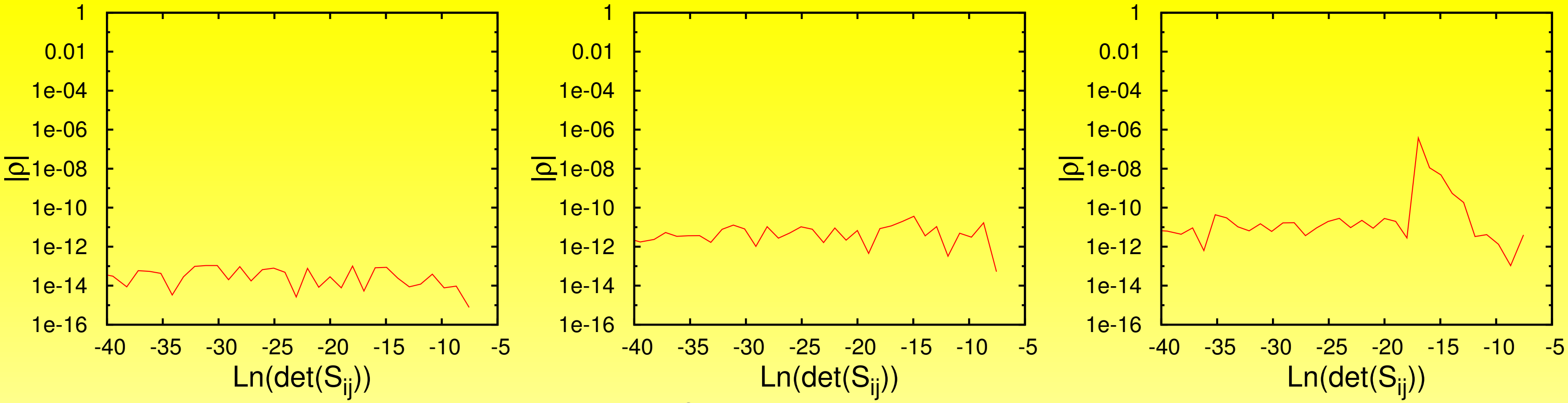
If $\sum_{i=1}^{N} y_i \neq 0$ one obtains the relation

$$I(D;\{\nu_k\}_{k=1}^{N}) = -\frac{1}{D-1-\sigma}\sum_{i=1}^{N}\frac{y_i}{\sum_{i=1}^{N} y_i} I(D-2;\{\nu_k-\delta_{ik}\}_{k=1}^{N}) ,$$

which reduces both D and σ (and possibly N), while keeping n fixed.

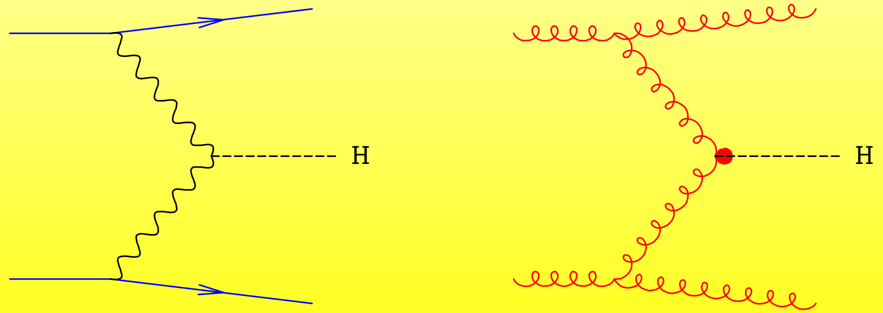
Exceptional regions II

$$
\begin{aligned}
I(D;\{\nu_k\}_{k=1}^N) = & - \frac{1}{D-1-\sigma}\sum_{j=1}^N \frac{y_j}{\sum_{i=1}^N y_i} I(D-2;\{\nu_k-\delta_{kj}\}_{k=1}^N) \\
& - \frac{1}{D-1-\sigma}\sum_{j=1}^N \frac{\sum_{i=1}^N S_{ji}y_i}{\sum_{i=1}^N y_i}\nu_j I(D;\{\nu_k+\delta_{kj}\}_{k=1}^N)\,.
\end{aligned}
$$



Relative accuracy $|\rho|$ for the $1/\epsilon^2$ pole (left), the $1/\epsilon$ pole (center) and the constant part (right) of the one-loop amplitude squared for $H \to q\bar{q}q'\bar{q}'$

H+2 jet calculation



- NLO corrections to W-fusion mechanism already calculated by many authors.
- All the elements are in place for a full NLO Higgs + 2 jets calculation via gluon fusion mechanism
  - ⋆ Born level calculation Higgs + 4 partons
  - ⋆ Real calculation Higgs + 5 partons,
    Del Duca et al, Dixon et al, Badger et al
  - ⋆ Virtual calculation — Ellis, Giele and Zanderighi, presented above
  - ⋆ Subtraction terms — Campbell, Ellis and Zanderighi, in preparation

Summary

- Making an accurate assessment of particle rates and extracting detailed information from the data requires calculations that go beyond the simplest approximation.
- This is highlighted at the LHC where, on average, many more particles are produced per collision.
- Next-to-leading order calculations are the first step towards the precision needed.
- I have demonstrated that a semi-numerical approach can provide interesting results, although the verification in a specific physical process is not yet complete.
- Although MCFM is a tool which provides a step in this direction, it is certainly not enough.
- What is needed is a concerted effort to create an automatic program, which will return virtual corrections for a process of arbitrary complexity.

Why NNLO?

- reduced scale dependence
- Event has more partons in the final state and hence closer to the real world
- Better description of transverse momentum of final state due to double radiation off initial states.
- NNLO is the first serious estimate of error.
- obvious application: Reduction of uncertainty in α_s at e^+e^- colliders. Currently: $\alpha_S = 0.121 \pm 0.001(\text{exp}) \pm 0.006(\text{theory})$ (resummed NLO). NNLO would reduce the uncertainty.
- Potent theoretical tool for investigating perturbation theory

The first few steps at NNLO

- Number of processes known at NNLO is rather small.
- Processes considered tend to be the most inclusive.
- For more exclusive processes there may be other theoretical uncertainties of the same order as the NNLO contributions.

Processes known at NNLO

Stirling

ep	DIS polarised and unpolarised structure function coefficient functions Sum Rules (GLS, Bj, ...) DGLAP splitting functions
e^+e^-	total hadronic cross section, and $Z \to$ hadrons, $\tau \to \nu +$ hadrons heavy quark pair production near threshold C_F^3 part of $\sigma(3\ \mathrm{jet})$
pp	inclusive W, Z, γ^* inclusive γ^* with longitudinally polarised beams W, Z, γ^* differential rapidity distribution H, A total and differential rapidity distribution WH, ZH
HQ	$Q\overline{Q}$–onium and $Q\bar{q}$ meson decay rates

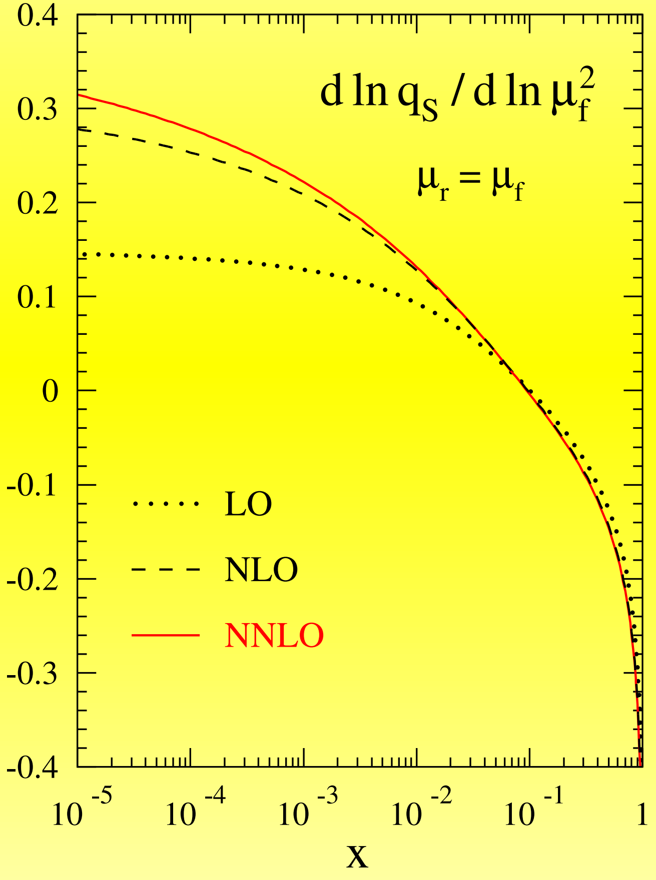
Deep Inelastic scattering at NNLO

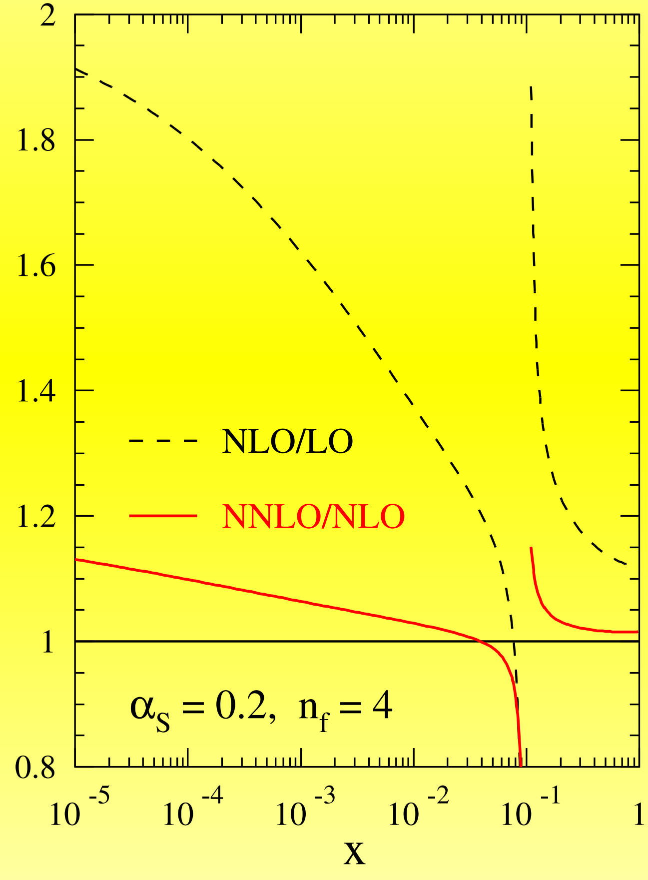
Moch,Vogt,Vermaseren

- Current status is that splitting function is known to NNLO:
  $P(x, \alpha_S) = P^{(0)} + \alpha_S P^{(1)} + \alpha_S^2 P^{(2)} + \dots$
- Coefficient function: $\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S \hat{\sigma}^{(1)} + \alpha_S^2 \hat{\sigma}^{(2)}$
- Need to know both the coefficient function and the splitting function to the same order for a valid prediction.
- We can now make consistent NNLO predictions for Tevatron and LHC quantities.
- New results on the coefficient function for the longitudinal structure function at appropriate order (2005)

Evolution of quarks

Moch,Vogt,Vermaseren

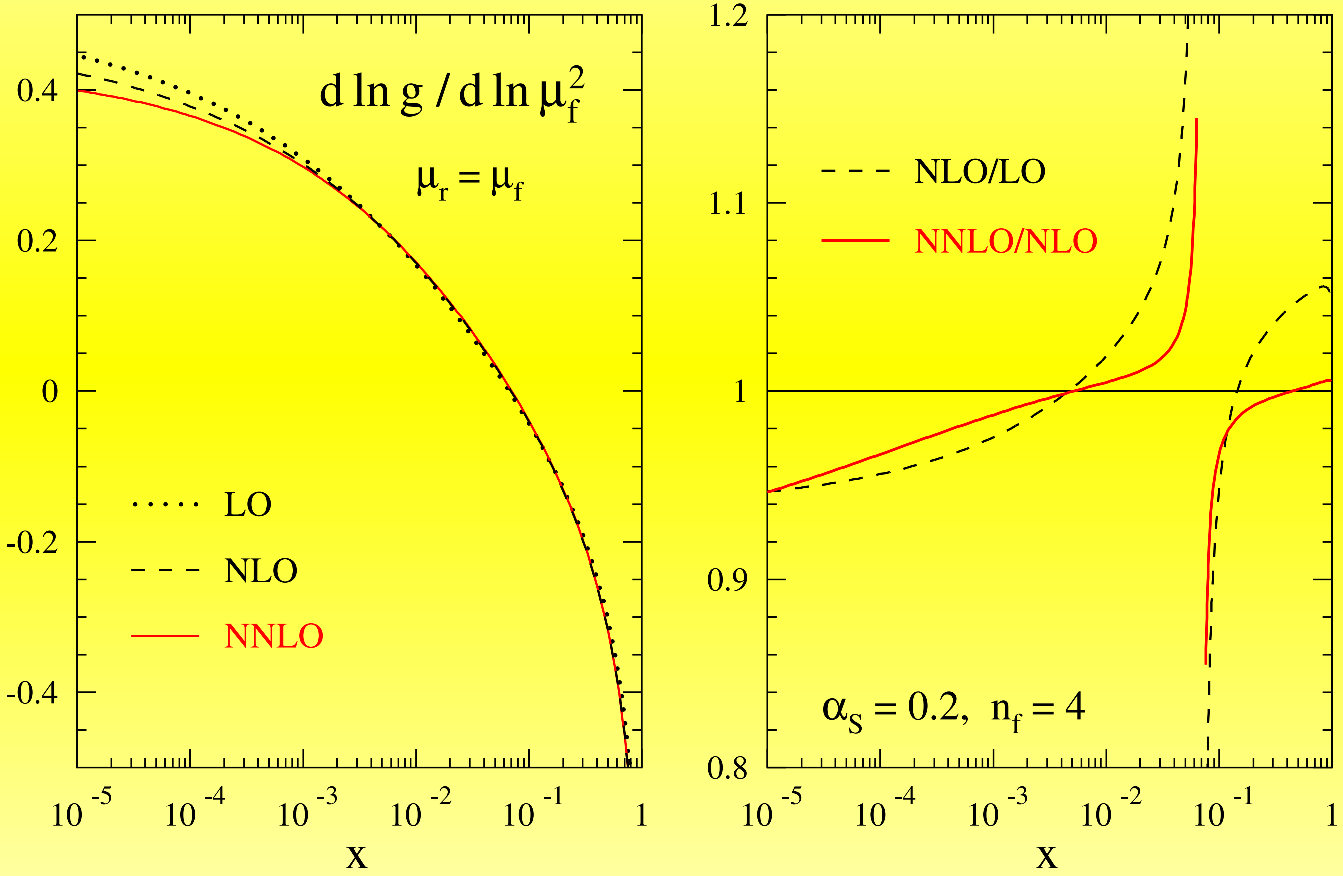




- Stability of perturbation series improved.

Evolution of gluons

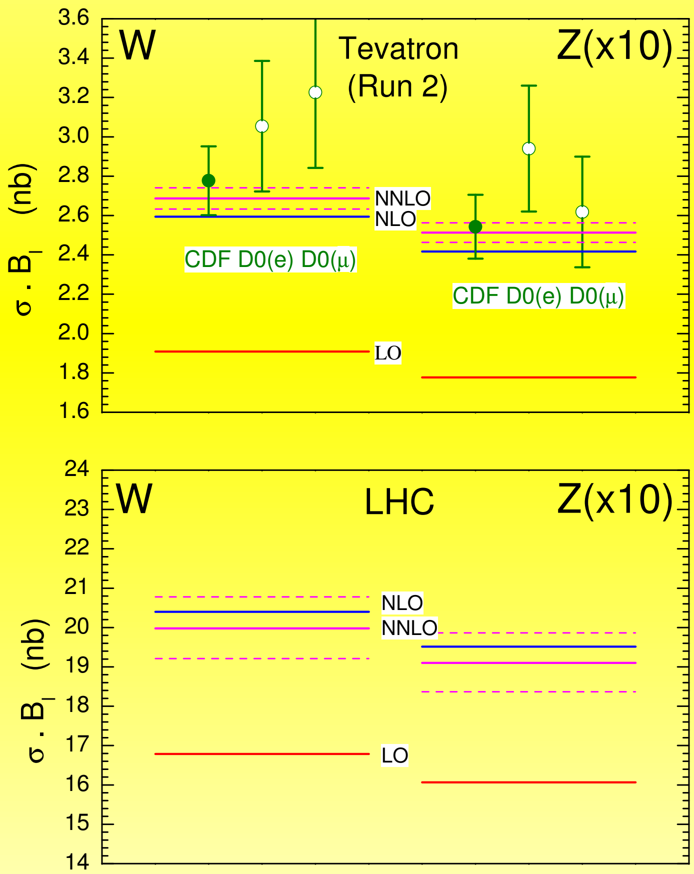
Moch,Vogt,Vermaseren



- Stability of perturbation series confirmed (small x) and improved (large x).

W and *Z* production at NNLO

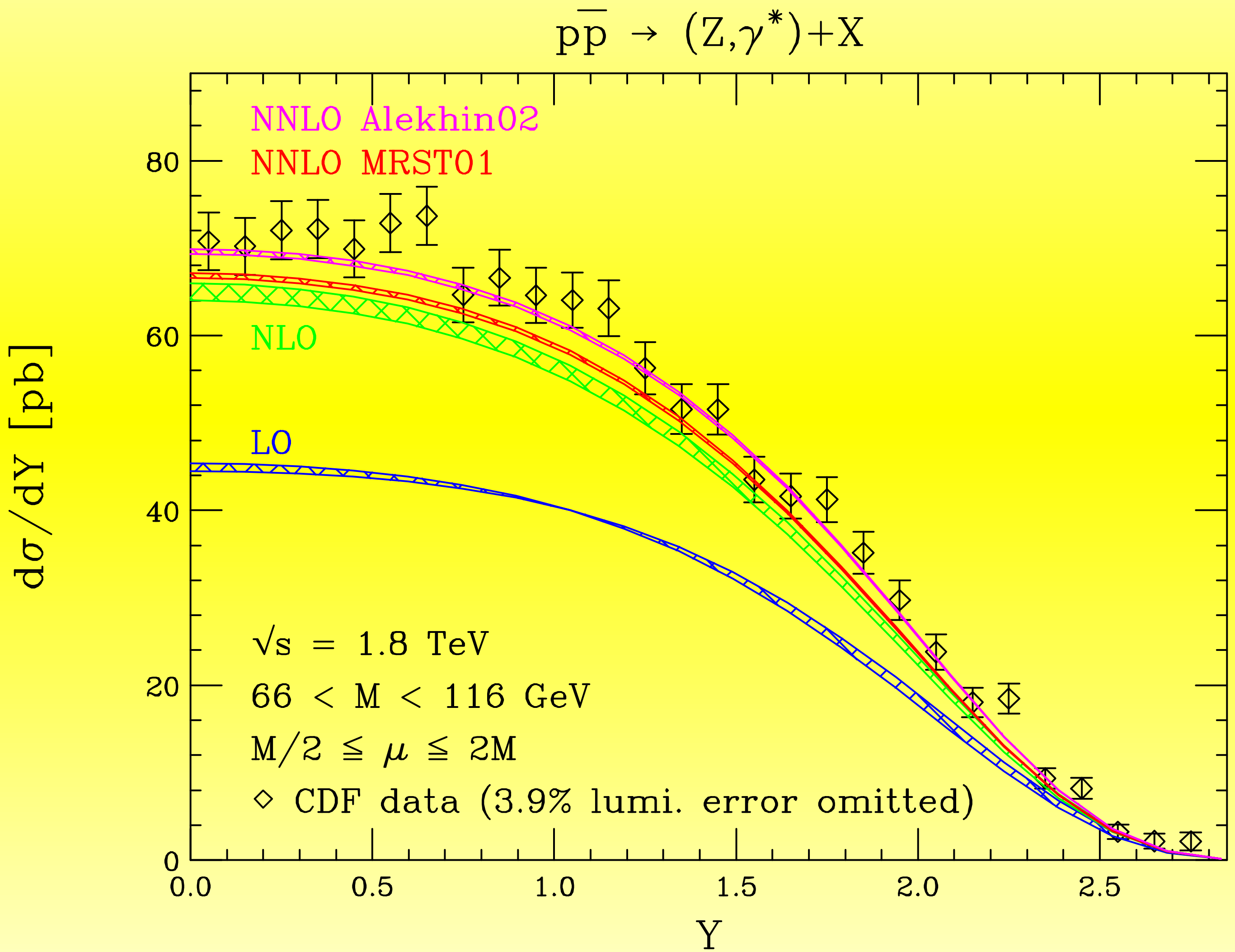
Martin et al, (MRST)



- Large correction at NLO, indicates that we need NNLO to inspire confidence in stability of prediction.
- Good agreement with Tevatron data.
- 4% theoretical uncertainty at LHC is comparable with estimate of error on luminosity measurement from elastic scattering
- W and Z cross sections can be used as luminosity monitor at LHC.

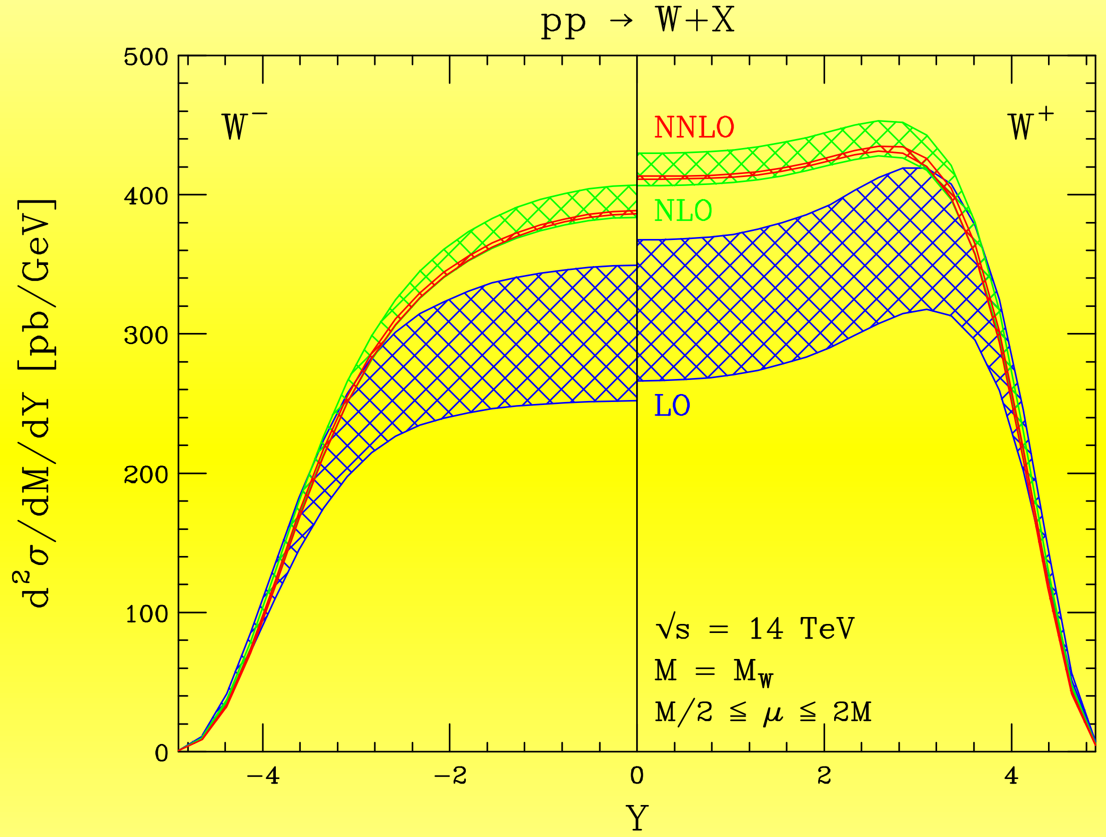
Drell-Yan processes at NNLO

Anastasiou et al.



Luminosity monitor for LHC

Anastasiou et al.



- Bands correspond to scale variation only.
- Reweighting NLO results by $\sigma_{NNLO}/\sigma_{NLO}$ is good to $\leq 1\%$.