Progress and challenge in QCD

Bill Bardeen symposium, September 24, 2005.

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selected WAB contributions

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William A. Bardeen, H. Fritzsch, Murray Gell-Mann. 1972.
LIGHT CONE CURRENT ALGEBRA, PI0 DECAY, AND
E+ E- ANNIHILATION.
e-Print Archive: hep-ph/0211388

 William A. Bardeen, A.J. Buras, D.W. Duke, T. Muta,
  DEEP INELASTIC SCATTERING BEYOND THE LEADING ORDER IN
    ASYMPTOTICALLY FREE GAUGE THEORIES.
    Phys.Rev.D18:3998,1978.

 William A. Bardeen, Andrzej J. Buras, HIGHER ORDER
    ASYMPTOTIC FREEDOM CORRECTIONS TO PHOTON - PHOTON
    SCATTERING.
    Phys.Rev.D20:166,1979., Erratum-ibid.D21:2041,1980.
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Bill established the rules by which the game is played.

β function in perturbation theory

- Running of the QCD coupling α_S is determined by the β function,
- The β-function of QCD is negative.

$$\beta(\alpha_S) = -b\alpha_S^2(1 + b'\alpha_S) + \mathcal{O}(\alpha_S^4)$$

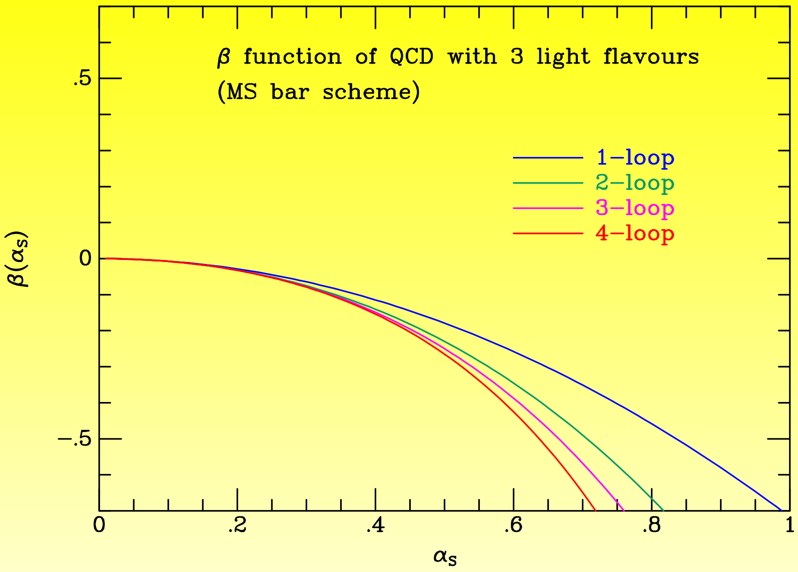
$$b = \frac{(11C_A - 2n_{lf})}{12\pi}, \quad b' = \frac{(17C_A^2 - 5C_A n_{lf} - 3C_F n_{lf})}{2\pi(11C_A - 2n_{lf})},$$

where n_{lf} is number of “active” light flavors. b'

Results of explicit calculation

Terms up to $\mathcal{O}(\alpha_S^5)$ are known.

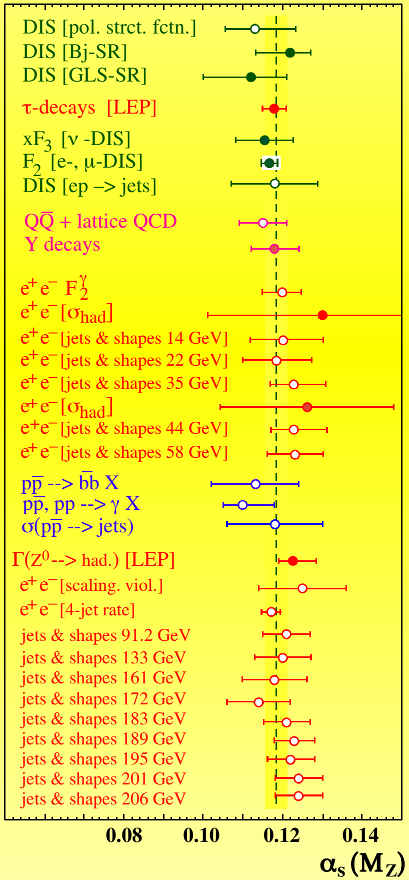
- (1) Gross and Wilczek ; Politzer
- (2) W. E. Caswell; D. R. T. Jones; E. Egorian and O. V. Tarasov
- (3) O. V. Tarasov, A. A. Vladimirov and A. Y. Zharkov;S. A. Larin and J. A. M. Vermaseren
- (4) T. van Ritbergen, J. A. M. Vermaseren and S. A. Larin



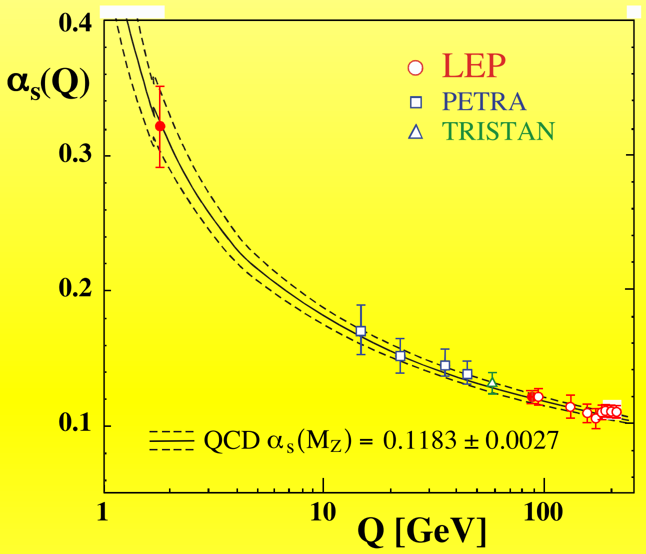
Current experimental results on α_S

Bethke,hep-ph/0407021

$$\alpha_S(M_Z) = 0.1182 \pm 0.0027, \overline{\text{MS}}, \text{NNLO}$$



Current experimental results on α_S



- The decrease of α_S is quite slow – as the inverse power of a logarithm.
- α_S is large at current scales.
- Higher order corrections are important.

A limited perspective

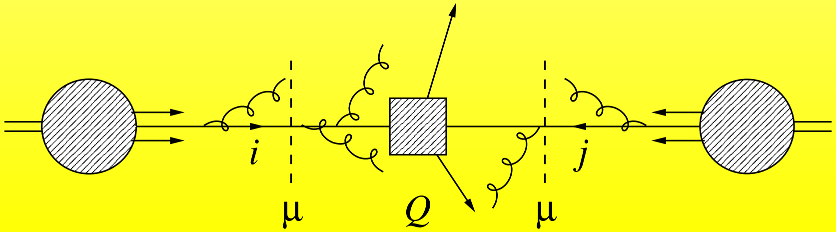
- QCD has many aspects, non-perturbative QCD, lattice QCD, quark-gluon plasma . . .
- For the purposes of this talk I shall limit the discussion to the calculation of short distance cross section in perturbative QCD, and to the evolution of the parton distribution functions.
- These features of QCD rely directly on the discovery of asymptotic freedom.
- They are of most interest for high mass physics at hadronic colliders.

The challenge

- The challenge is to provide the most accurate information possible to experimenters working at the Tevatron and the LHC.
- Proton (anti)proton collisions give rise to a rich event structure.
- Complexity of the events will increase as we pass from the Tevatron to the LHC.
- The goals
  - ⋆ To provide physics software tools which are both flexible and give the most accurate representations of the underlying theories.
  - ⋆ To discover new efficient ways of calculating in perturbative QCD.

Hadron-hadron processes

- In hard hadron-hadron scattering, constituent partons from each incoming hadron interact at short distance (large momentum transfer Q^2).



- Form of cross section is

$$\frac{d\sigma}{dX} = \sum_{i,j} \sum_{\tilde{X}} \int dx_1 dx_2 \, f_i(x_1,\mu^2) f_j(x_2,\mu^2)$$

$$\times \quad \hat{\sigma}_{ij}^{\tilde{X}}(\alpha_S(\mu^2), Q^2, \mu^2) \, F(\tilde{X} \to X, \mu^2)$$

where μ^2 is factorization scale and $\hat{\sigma}_{ij}$ is subprocess cross section for parton types i, j and X represents the hadronic final state.

Hadron-hadron processes II

- Short distance cross section $\hat{\sigma}_{ij}$ is calculable as a perturbation series in α_S.
- Notice that factorization scale is in principle arbitrary: affects only what we call part of subprocess or part of initial-state evolution (parton shower).
- Unlike e^+e^- or ep, we may have interaction between spectator partons, leading to *soft underlying event* and/or *multiple hard scattering*. This an important issue, but I will not talk further about it.

Short-distance cross section

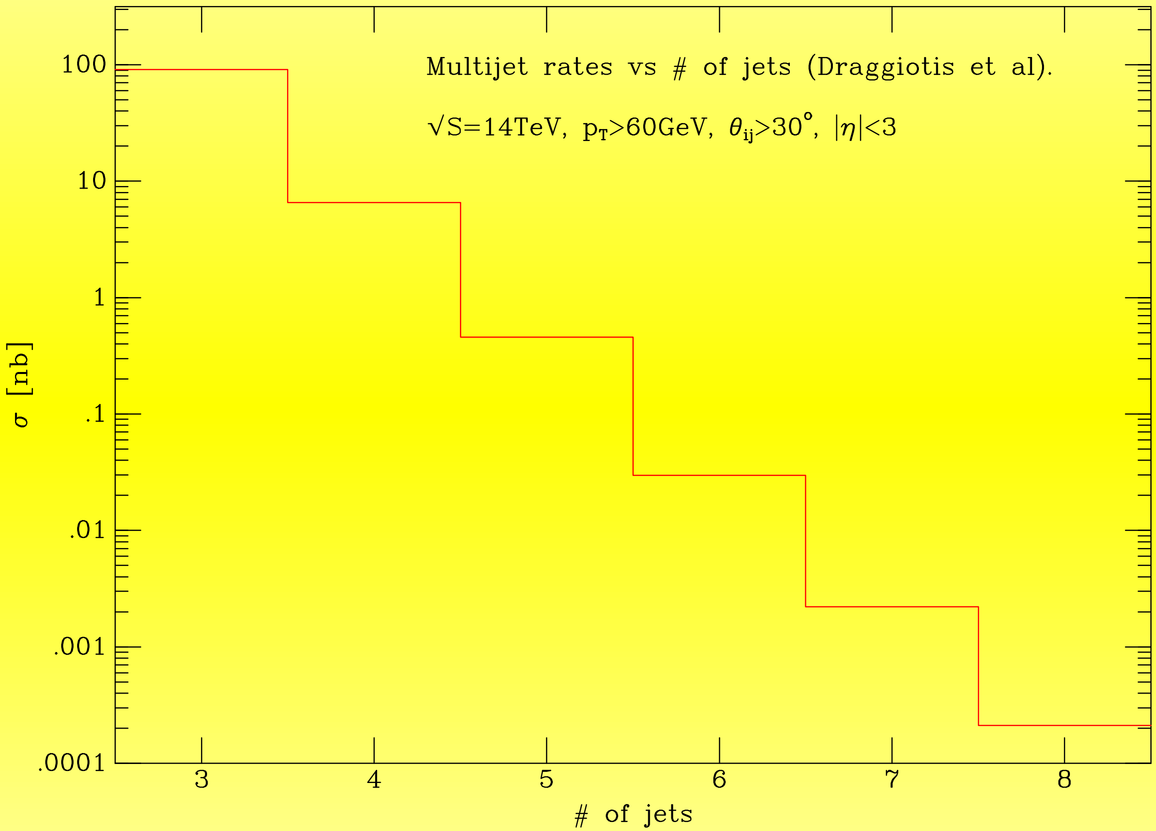
- Tree graph level
  - ⋆ Automatic calculation of tree graphs (Madgraph/Helas, Alpgen, CompHEP, . . .)
- Combining tree graphs and parton showers
- Techniques for efficient analytic calculation.
- NLO (MCFM, NLOJET++, DYRAD . . .)
- NLO + parton shower
  - ⋆ MC@NLO
- NNLO
  - ⋆ survey of observable results
  - ⋆ NNLO splitting functions
  - ⋆ Drell-Yan Luminosity monitor

The role of tree graphs

- Problems with tree graphs
  a) Overall normalization is uncertain.
  For example, W+4 jets is O(α_S^4). If scale uncertainty changes α_S by 10%, this leads to 40% uncertainty in cross section.
  b) If we wish talk about hadrons, we must apply fragmentation.
  To use universal fragmentation, we must evolve to a fixed scale.
  Tree graphs require a procedure to combine with parton showers.
  c) Sometimes a new parton process appears at NLO, leading to large change in shapes. (e.g., gluons at the LHC).
- For example, for W, Z + n jets at tree graph level.
  Madgraph II can generate processes with $\leq$ 9 external particles
  (madgraph.hep.uiuc.edu)
  Vecbos, W-boson plus up to 4 jets or a Z-boson plus up to 3 jets
  (theory.fnal.gov/people/giele/vecbos.html)
  Alpgen, W,Z + up to 6 jets

Multijet rates using tree graphs

Draggiotis et al



- At 10^{33} cm^{-2}s^{-1}, left hand scale gives events per second.
- $g = 1$.

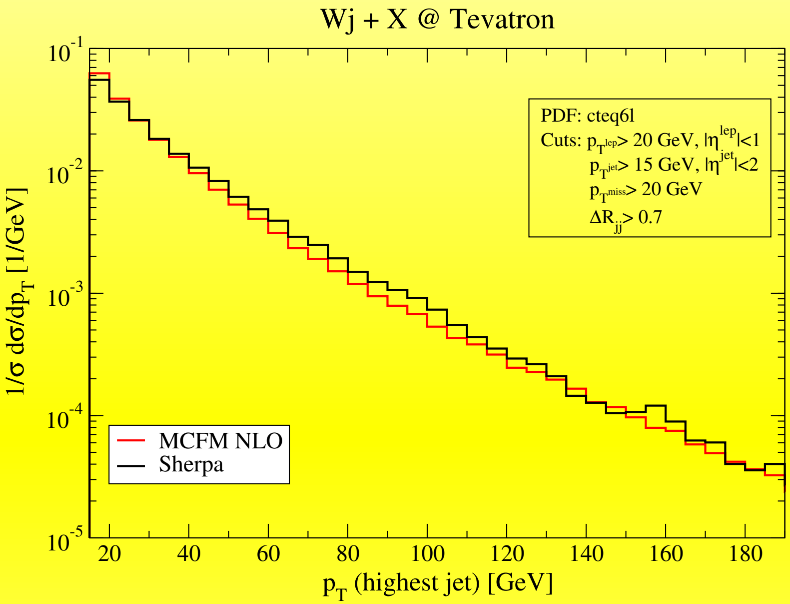
Combining Matrix elements and parton showers

Mangano et al, hep-ph/0108069
Mrenna, Richardson, hep-ph/0312274
CKKW, hep-ph/0109231
F. Krauss et al, hep-ph/0407365

- Shower Monte Carlo proceeds via Sudakov form factor $\Delta(Q^2, q^2)$, probability of parton transiting from scale Q^2 to q^2 without a branching.
- Divide phase space into two regions – region I for jet production modeled by the appropriate matrix element, region II for jet evolution modeled by the parton shower.
- region I, generate with exact matrix element and include Sudakov form factors to enforce no branching probabilities.
- region II, veto hard emission in the parton shower.
- Dependence on separation parameter cancels at NLL.

Since fixed order ME's are known, this should be quick to implement.

Results for inclusive W+1 jet rate

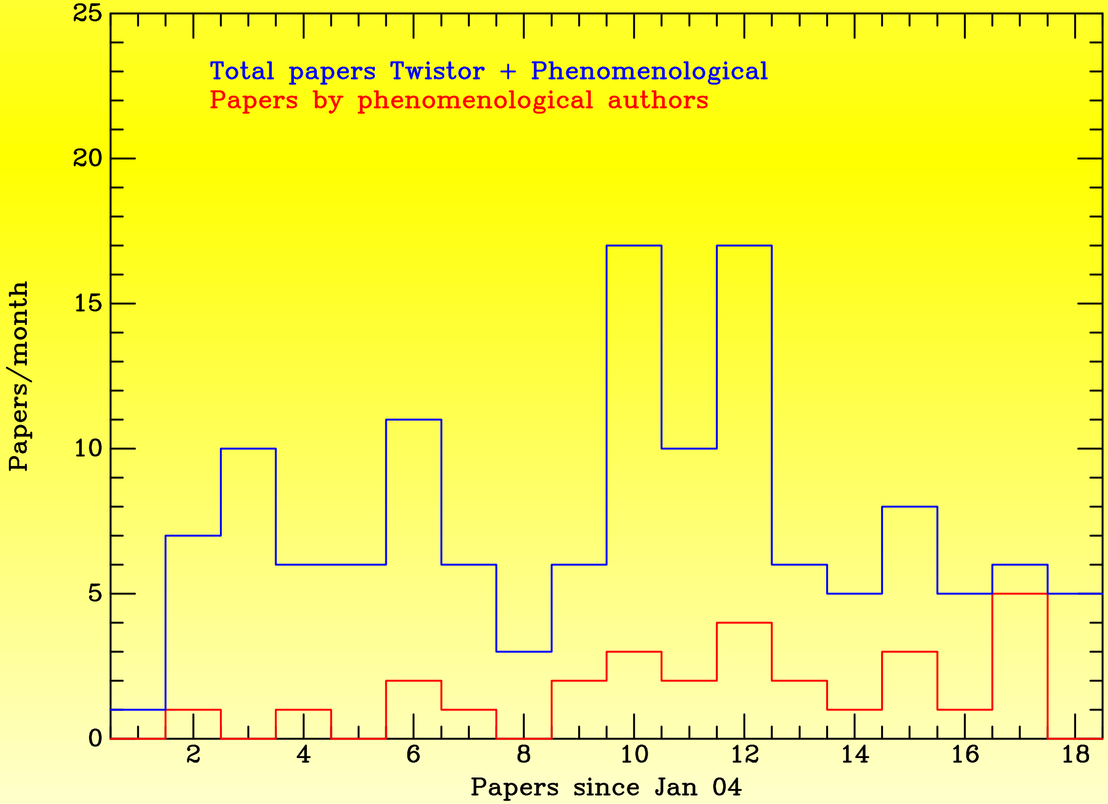


F. Krauss et al, hep-ph/0409106

- p_T spectrum of the hardest jet in inclusive W+1 jet, using Matrix element improved showering scheme.
- Agreement in shape between exact NLO calculation and ME improved shower (SHERPA).

New techniques for trees and loops

- Dramatic increase in interest in 'Twistor inspired' techniques for gauges theories.



Spinor notation

- Denote spinor for lightlike vectors as follows:-

$$\begin{aligned} |k+\rangle &= \text{right-handed spinor for massless vector } k \\ |k-\rangle &= \text{left-handed spinor for massless vector } k \end{aligned}$$

- Polarization vectors are given by ($q \equiv$ gauge choice)

$$\varepsilon_\mu^+(k) = \frac{\langle q^-|\gamma_\mu|k^-\rangle}{\sqrt{2}\langle qk\rangle}, \quad \varepsilon_\mu^-(k) = \frac{\langle q^+|\gamma_\mu|k^+\rangle}{\sqrt{2}[kq]}$$

- Obeys all the requirements of a polarization vector

$$\varepsilon_i^2 = 0, \quad k\cdot\varepsilon(k) = 0, \quad q\cdot\varepsilon(k) = 0, \quad \varepsilon^+\cdot\varepsilon^- = -1$$

- Equivalent notations

$$\epsilon^{ab}\lambda_{ja}\lambda_{lb} \equiv \langle jl\rangle \equiv \langle k_j^-|k_l^+\rangle = \sqrt{2k_j\cdot k_l}e^{i\phi}$$

$$\epsilon^{\dot{a}\dot{b}}\tilde{\lambda}_{j\dot{a}}\tilde{\lambda}_{l\dot{b}} \equiv [jl] \equiv \langle k_j^+|k_l^-\rangle = -\sqrt{2k_j\cdot k_l}e^{-i\phi}$$

MHV amplitudes – 5 gluon amplitude

- Decompose gluonic amplitude into color-ordered sub-amplitudes

$$A = \text{Tr}\{t^{a1}t^{a2}t^{a3}t^{a4}t^{a5}\}m(1,2,3,4,5) + 23 \text{ permutations}$$

- Two of the color stripped amplitudes vanish

$$\begin{aligned} m(g_1^+, g_2^+, g_3^+, g_4^+, g_5^+) &= 0 \\ m(g_1^-, g_2^+, g_3^+, g_4^+, g_5^+) &= 0 \end{aligned}$$

- The maximal helicity violating 5 gluon amplitude

$$m(g_1^-, g_2^-, g_3^+, g_4^+, g_5^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

$\langle ij \rangle, [ij]$ useful because QCD amplitudes have square root singularities.

MHV amplitudes

Parke and Taylor, Berends and Giele

- The generalization to the case with two contiguous positive helicity gluons and $n-2$ negative gluons is

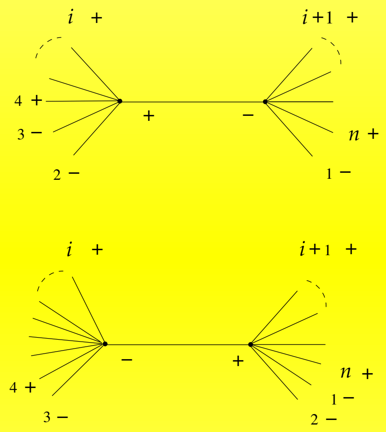
$$m(g_1^-, g_2^-, g_3^+, \ldots g_n^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle}$$

- Remember $\langle ij \rangle$ are the spinor products $\sim \sqrt{(2p_i \cdot p_j)}$

MHV calculus

Cachazo,Svrcek,Witten

- Use MHV amplitudes as effective vertices to build more complicated amplitudes



- Obtain simple expressions for tree amplitudes in terms of spinor products
- Individual terms in the expressions for tree amplitudes contain spurious poles which cancel in the sum. These may compromise the utility of the expressions for numerical evaluation.
- Extension to loops?

MHV calculus II

- Define an offshell MHV vertex using the QCD Parke-Taylor amplitude.

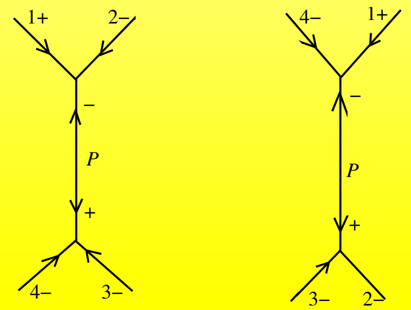
$$V(1^-,2^-,3^+,\ldots,n^+,P^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\ldots\langle n-1,n\rangle\langle n,P\rangle\langle P1\rangle}$$

- Continue the spinor off-shell $\langle iP\rangle = \eta\sum_{j=1}^{n}\langle i^-|\not{k}_j|q^-\rangle$ where $P = k_1 + k_2 + \ldots k_n$, with lightlike auxiliary q
- Final result independent of η and q
- Easy to sew MHV vertices together to obtain more complicated amplitudes
- n gluon $---+++\ldots++$ amplitude is the sum of $2(n-3)$ MHV diagrams

MHV example, (n=4)

- Consider the two MHV vertex diagrams which give $+---$ gluon amplitude (it vanishes in Yang-Mills theory)

  First diagram



$$m_1(1,2,3,4) = \frac{\langle 2P\rangle^4}{\langle 12\rangle\langle 2P\rangle\langle P1\rangle}\frac{1}{P^2}\frac{\langle 34\rangle^4}{\langle 34\rangle\langle 4P\rangle\langle P3\rangle}$$

- According to our continuation this is

$$\frac{\langle 2|(\not{1}+\not{2}|q\rangle^3}{\langle 12\rangle\langle 1|(\not{1}+\not{2}|q\rangle}\frac{1}{\langle 12\rangle[21]}\frac{\langle 34\rangle^3}{\langle 4|\not{3}+\not{4}\rangle|q\rangle\langle 3|\not{3}+\not{4}\rangle|q\rangle} = \frac{[1q]}{[2q][3q][4q]}\frac{\langle 34\rangle}{[21]}$$

- Adding the second diagram ($2 \leftrightarrow 4$), (NB $\langle ij\rangle[jk] = \langle i|\not{j}|k\rangle$)

$$m_1(1,2,3,4)+m_1(1,4,3,2) = \frac{[1q]}{[2q][3q][4q][21][41]}(\langle 34\rangle[41]+\langle 32\rangle[21]) = 0$$

MHV outlook

- Lead to beautiful results for gauge theory amplitudes; however the evaluation of pure gluon tree graphs is a numerically solved problem, (Berends-Giele recursion).
- So far impact on real phenomenology limited; simple tree graph results for Higgs+5 parton amplitudes Dixon et al, Badger et al
- Extension to loops is the next frontier; the new techniques solve the problem of computing one-loop amplitudes of gluons in $\mathcal{N} = 4$ super Yang-Mills. Will this lead to a comparable simplification of standard model one loop amplitudes?

Why NLO?

The benefits of higher order calculations are:-

- Less sensitivity to unphysical input scales (eg. renormalization scale)
- First prediction of normalization of observables at NLO
- More accurate estimates of backgrounds for new physics searches.
- Confidence that cross-sections are under control for precision measurements
- More physics
  - ⋆ Jet merging
  - ⋆ Initial state radiation
  - ⋆ More species of incoming partons enter at NLO
  - ⋆ It represents the first step for other techniques matching with resummed calculations, eg. NLO parton showers

NLO calculation

- Ingredients in a NLO calculation are
  - ⋆ Born level amplitude
  - ⋆ Real contribution: Addition of one extra parton to Born level process
  - ⋆ Virtual contribution: Interference of one-loop amplitude with Born amplitude
- Real and virtual separately contain singularities from the soft and collinear regions which cancel in the sum.
- Calculation of one loop amplitude rapidly becomes complicated as number of partons increases.
- Especially true as we go beyond the most symmetric cases with all gluons.

MCFM overview

John Campbell and R.K. Ellis

- Parton level cross-sections predicted to NLO in α_S

$p\bar{p} \to W^\pm/Z$	$p\bar{p} \to W^+ + W^-$
$p\bar{p} \to W^\pm + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^\pm + \gamma$	$p\bar{p} \to W^\pm/Z + H$
$p\bar{p} \to W^\pm + g^\star\,(\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W^\pm/Z +$ 1 jet	$p\bar{p} \to W^\pm/Z +$ 2 jets
$p\bar{p}(gg) \to H$	$p\bar{p}(gg) \to H +$ 1 jet
$p\bar{p}(VV) \to H +$ 2 jets	$p\bar{p} \to t + X$
$pp \to t + W$	

$\oplus$ less sensitivity to μ_R, μ_F, rates are better normalized, fully differential distributions.

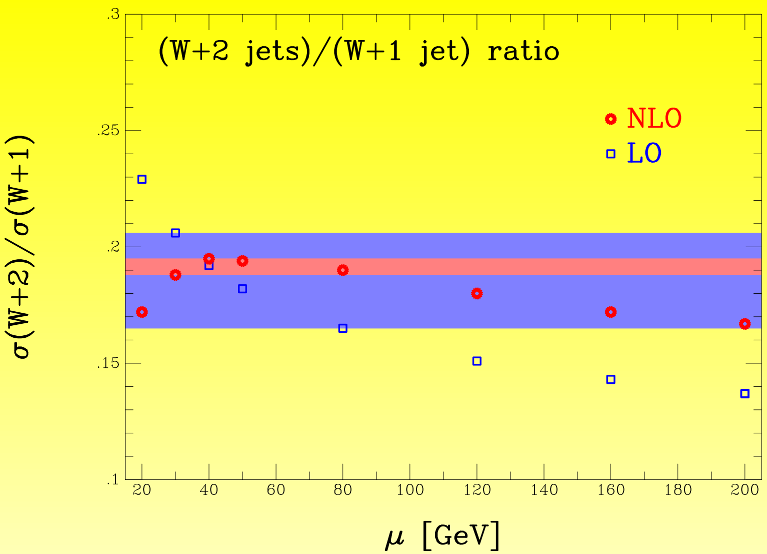
$\ominus$ low particle multiplicity (no showering), no hadronization, hard to model detector effects

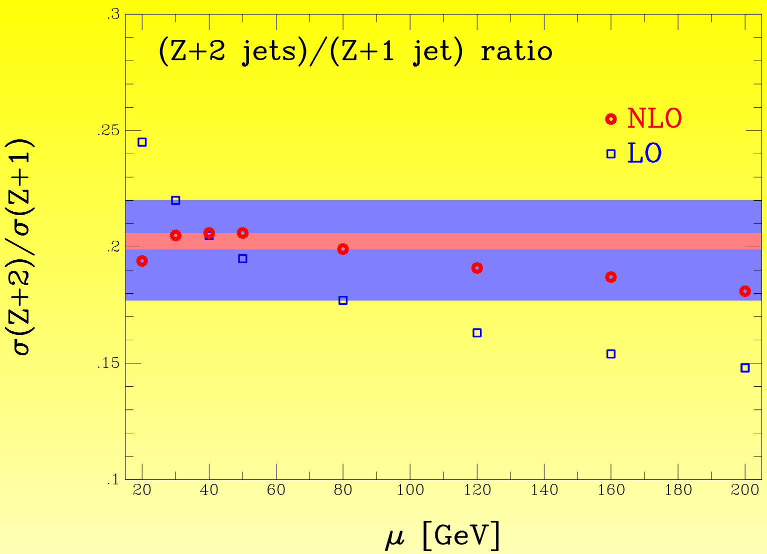
MCFM Information

- Version 4.1 (January 05) available at:
  http://mcfm.fnal.gov
- Improvements over previous releases:
  - ⋆ more processes ($Z + b$, single top, $\ldots$)
  - ⋆ better user interface
  - ⋆ support for PDFLIB, Les Houches PDF accord
    ($\longrightarrow$ PDF uncertainties)
  - ⋆ ntuples as well as histograms
  - ⋆ unweighted events
  - ⋆ Pythia/Les Houches generator interface (LO)
  - ⋆ separate variation of factorization and renormalization scales
  - ⋆ 'Behind-the-scenes' efficiency

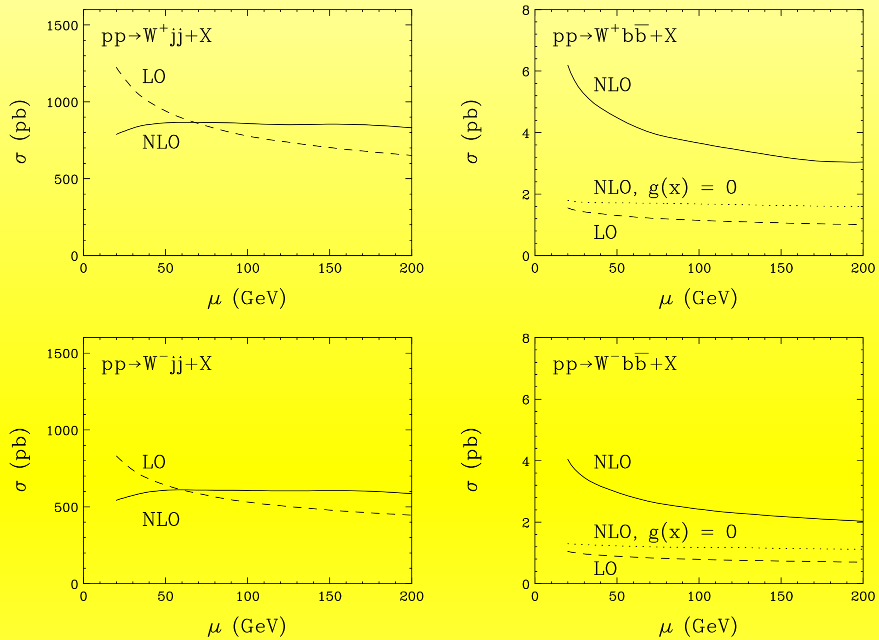
$W/Z+$ jet cross-sections

- The $W/Z+2$ jet cross-section has been calculated at NLO and should provide an interesting test of QCD (cf. many Run I studies using the $W/Z+1$ jet calculation in DYRAD)
- For instance, the theoretical prediction for the number of events containing 2 jets divided by the number containing only 1 is greatly improved.



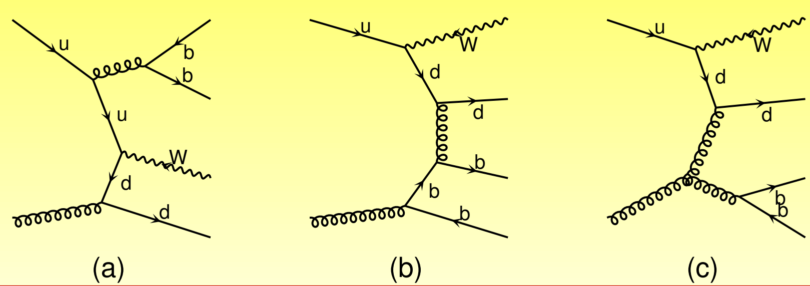


Jets and heavy flavour at the LHC



- The large gluonic contribution appearing in $Wb\bar{b}$ for the first time at NLO results in a large correction and poor scale dependence.

Diagrams by MadGraph



An experimenter's wishlist

Run II Monte Carlo Workshop

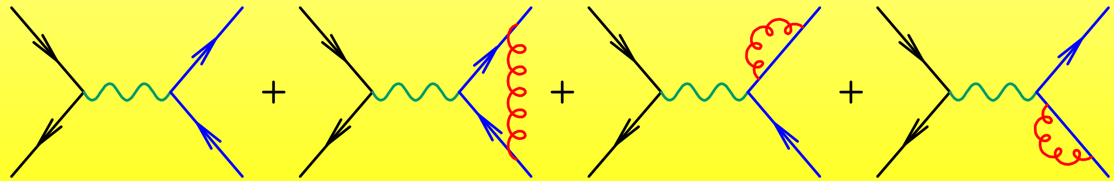
Single Boson	Diboson	Triboson	Heavy Flavour
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}\leq 3j$	$W+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}\leq 3j$	$W+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$Z+b\bar{b}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{b}\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$	$ZZZ+\leq 3j$	$b\bar{b}+\leq 3j$
$\gamma+b\bar{b}\leq 3j$	$\gamma\gamma+b\bar{b}\leq 3j$		
$\gamma+c\bar{c}\leq 3j$	$\gamma\gamma+c\bar{c}\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}\leq 3j$		
	$WZ+c\bar{c}\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

Automatic NLO corrections

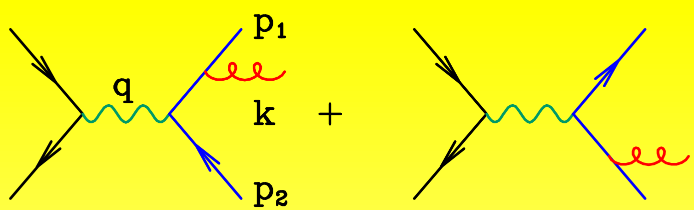
- What is needed is an automatic procedure to calculate NLO corrections.
- Current stumbling block is the calculation of virtual corrections.
- The virtual corrections contain singularities from the regions of collinear and soft gluon emission, (and in general also UV divergences).
- Divergences are normally controlled by dimensional regularization. A completely numerical procedure using, say, a gluon mass could cause problems with gauge invariance and is hence deprecated.

Example: e^+e^- total rate

- Consider the corrections to total $e^+e^- \to q\bar{q}$ rate.



(a)



(b)

$$\sigma^{q\bar{q}g} \; = \; 2\sigma_0 \frac{\alpha_S}{\pi} \; H(\epsilon) \; \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + \mathcal{O}(\epsilon) \right] \; .$$

- Soft and collinear singularities are regulated, appearing instead as poles at $D = 4$.

Virtual gluon contributions

- Virtual gluon contributions (a): using dimensional regularization again

$$\sigma^{q\bar{q}} \;=\; 3\sigma_0 \left\{1 + \frac{2\alpha_S}{3\pi}\, H(\epsilon)\, \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon)\right]\right\}\;.$$

- Adding real and virtual contributions, poles cancel and result is finite as $\epsilon \to 0$. R is an infrared safe quantity.

$$R \;=\; 3 \sum_q Q_q^2 \;\left\{1 + \frac{\alpha_S}{\pi} + \mathcal{O}(\alpha_S^2)\right\}.$$

- However the virtual corrections to $W^+ \to u\bar{d}gggg$ (relevant for W +4 jets calculation) are not so easily calculated.

Seminumerical approach

van Hameren et al., Ellis, Giele, Zanderighi

- Calculate integrals numerically by reducing to a simple basis set which are known as a Laurent series in ϵ.
- Proof of principle for a specific process for which analytical result can be calculated
- Result for the process $H \to q\bar{q}q'\bar{q}'$ with effective Lagrangian $HG^{\mu\nu}G_{\mu\nu}$.
- Choose a particular point in phase space

$$
\begin{aligned}
\text{Analytic} &= (-46.7813035247351, 0.00000000000000)/\epsilon^2 \\
&+ (111.948110122775, 18.3709749348328)/\epsilon \\
&+ (120.012242523826, -335.917283834563) \\
\text{Numerical} &= (-46.7813035247350, -0.00000000000000)/\epsilon^2 \\
&+ (111.948110122775, 18.3709749348302)/\epsilon \\
&+ (120.012242523817, -335.917283834578)
\end{aligned}
$$

Can one improve on NLO?

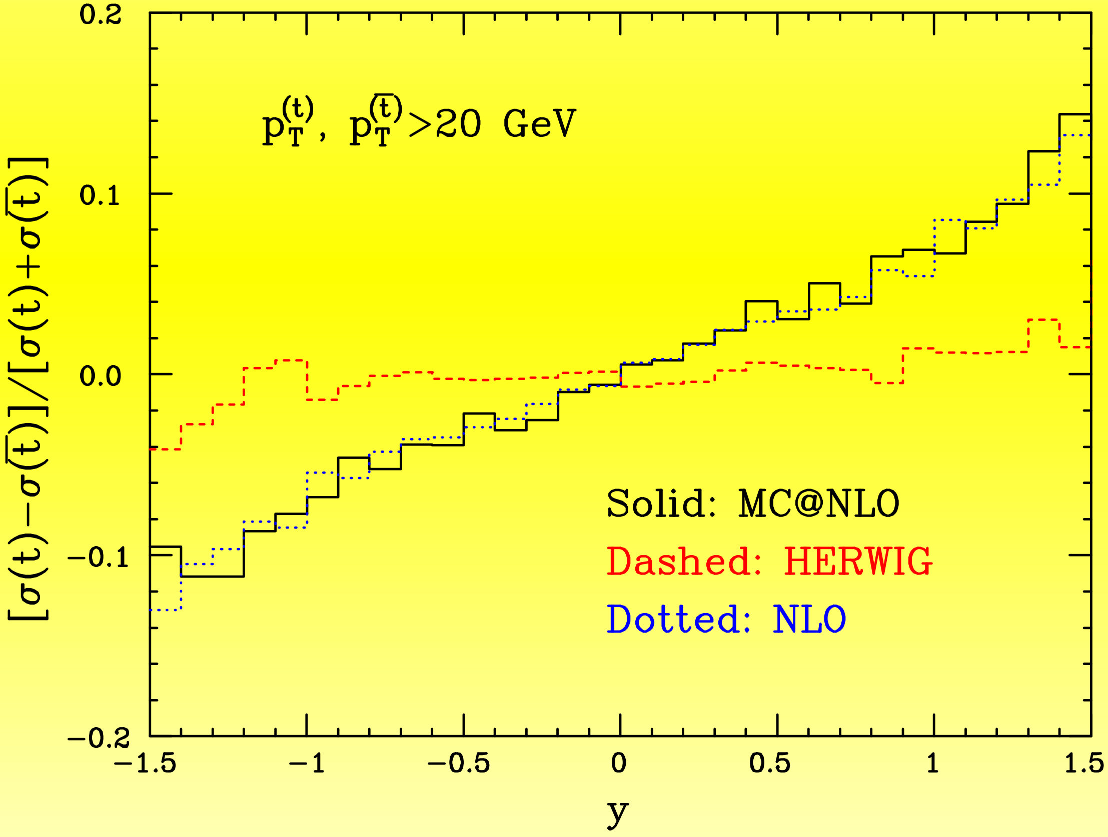
Frixione et al,hep-ph/0305252,hep-ph/0204244

- MC @ NLO
  www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/
- Relies on the appropriate NLO process having been calculated.
- Output is a set of events, which are fully inclusive
- Total rates are accurate to NLO
- NLO results for all observables are recovered upon expansion in α_S
- Currently a limited number of available processes, Higgs boson,
  single vector boson, W/Z,
  vector boson pair, WW,
  heavy quark pair, $Q\bar{Q}$
  lepton pair production, e^+e^-

Asymmetry in top production

Frixione,Nason,Webber

- Example of $t\bar{t}$-production using MC@NLO
- NLO curve (in blue, dotted).



Why NNLO?

- reduced scale dependence
- Event has more partons in the final state and hence closer to the real world
- Better description of transverse momentum of final state due to double radiation off initial states.
- NNLO is the first serious estimate of error.
- obvious application: Reduction of uncertainty in α_s at e^+e^- colliders. Currently: $\alpha_S = 0.121 \pm 0.001(\text{exp}) \pm 0.006(\text{theory})$ (resummed NLO). NNLO would reduce the uncertainty.
- Potent theoretical tool for investigating perturbation theory

The first few steps at NNLO

- Number of processes known at NNLO is rather small.
- Processes considered tend to be the most inclusive.
- For more exclusive processes there may be other theoretical uncertainties of the same order as the NNLO contributions.

Processes known at NNLO

Stirling

ep	DIS polarised and unpolarised structure function coefficient functions Sum Rules (GLS, Bj, ...) DGLAP splitting functions
e^+e^-	total hadronic cross section, and $Z \to$ hadrons, $\tau \to \nu +$ hadrons heavy quark pair production near threshold C_F^3 part of $\sigma(3\,\mathrm{jet})$
pp	inclusive W, Z, γ^* inclusive γ^* with longitudinally polarised beams W, Z, γ^* differential rapidity distribution H, A total and differential rapidity distribution WH, ZH
HQ	$Q\overline{Q}$–onium and $Q\bar{q}$ meson decay rates

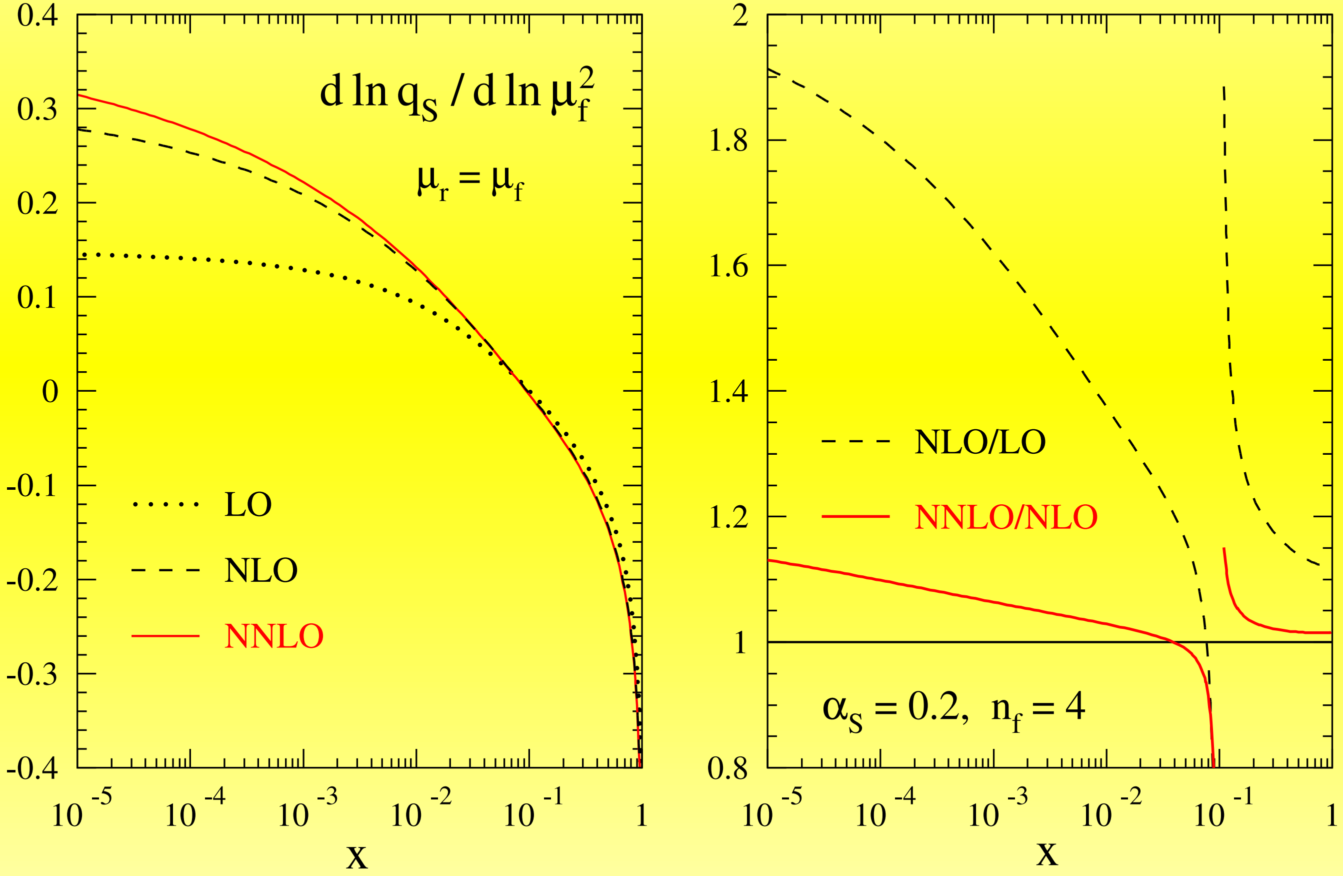
Deep Inelastic scattering at NNLO

Moch,Vogt,Vermaseren

- Current status is that splitting function is known to NNLO:
  $P(x, \alpha_S) = P^{(0)} + \alpha_S P^{(1)} + \alpha_S^2 P^{(2)} + \ldots$
- Coefficient function: $\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S \hat{\sigma}^{(1)} + \alpha_S^2 \hat{\sigma}^{(2)}$
- Need to know both the coefficient function and the splitting function to the same order for a valid prediction.
- We can now make consistent NNLO predictions for Tevatron and LHC quantities.
- New results on the coefficient function for the longitudinal structure function at appropriate order (2005)

Evolution of quarks

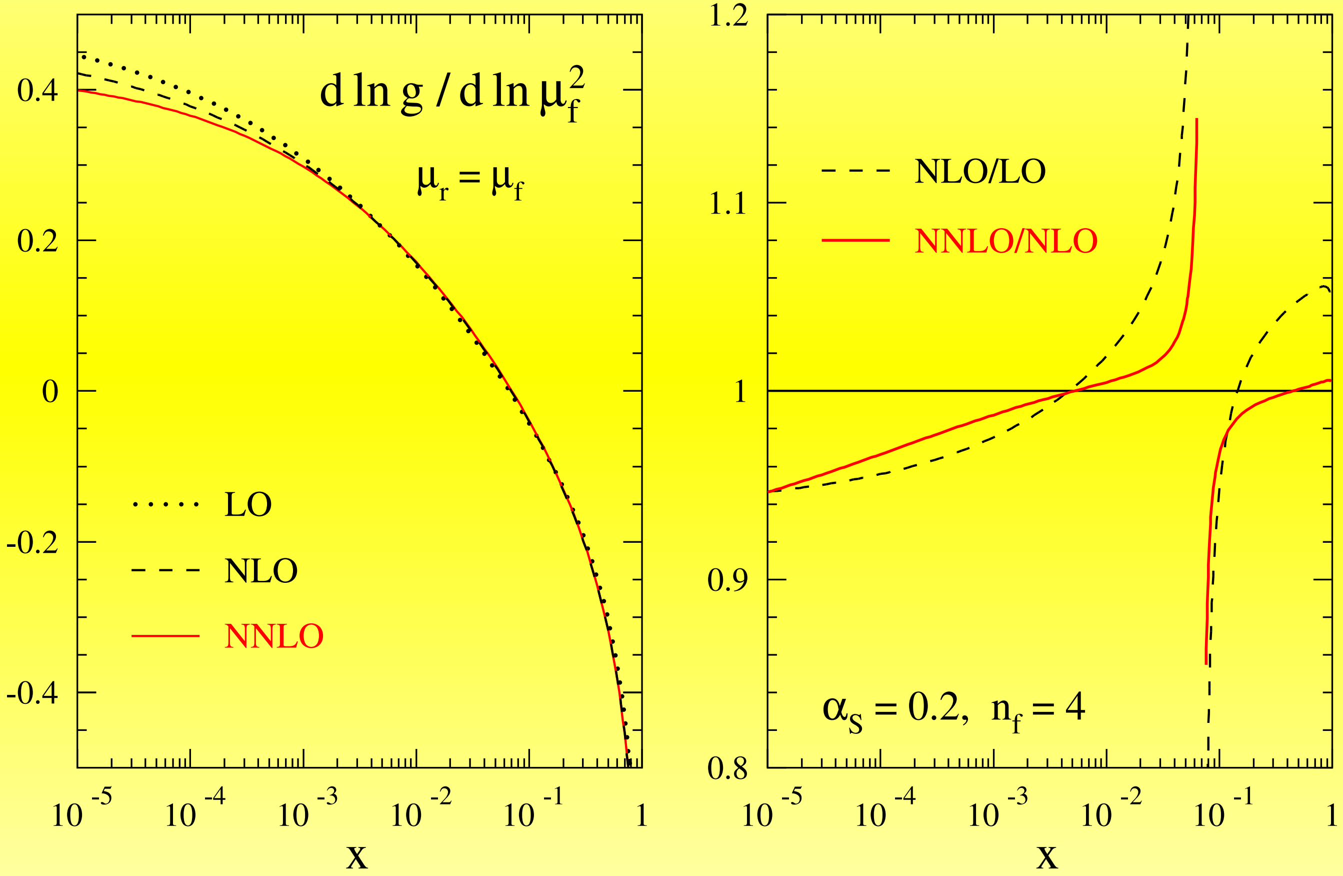
Moch,Vogt,Vermaseren



- Stability of perturbation series improved.

Evolution of gluons

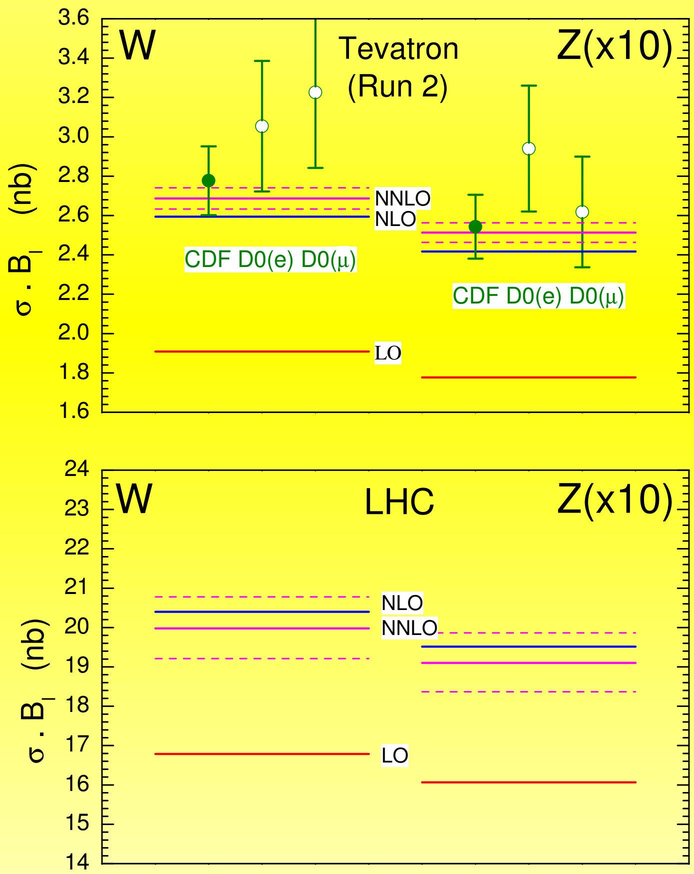
Moch,Vogt,Vermaseren



- Stability of perturbation series confirmed (small x) and improved (large x).

W and *Z* production at NNLO

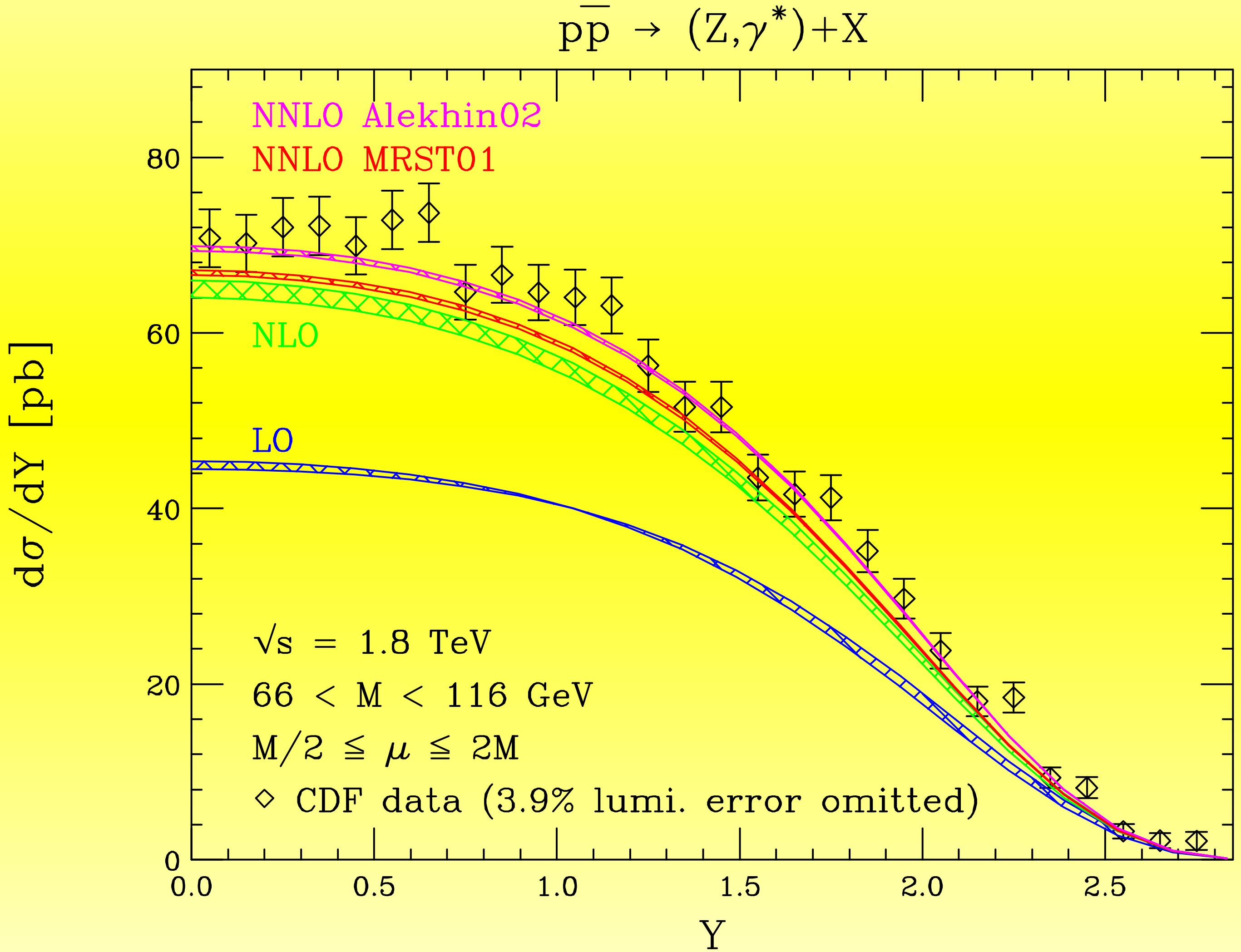
Martin et al, (MRST)



- Large correction at NLO, indicates that we need NNLO to inspire confidence in stability of prediction.
- Good agreement with Tevatron data.
- 4% theoretical uncertainty at LHC is comparable with estimate of error on luminosity measurement from elastic scattering
- W and Z cross sections can be used as luminosity monitor at LHC.

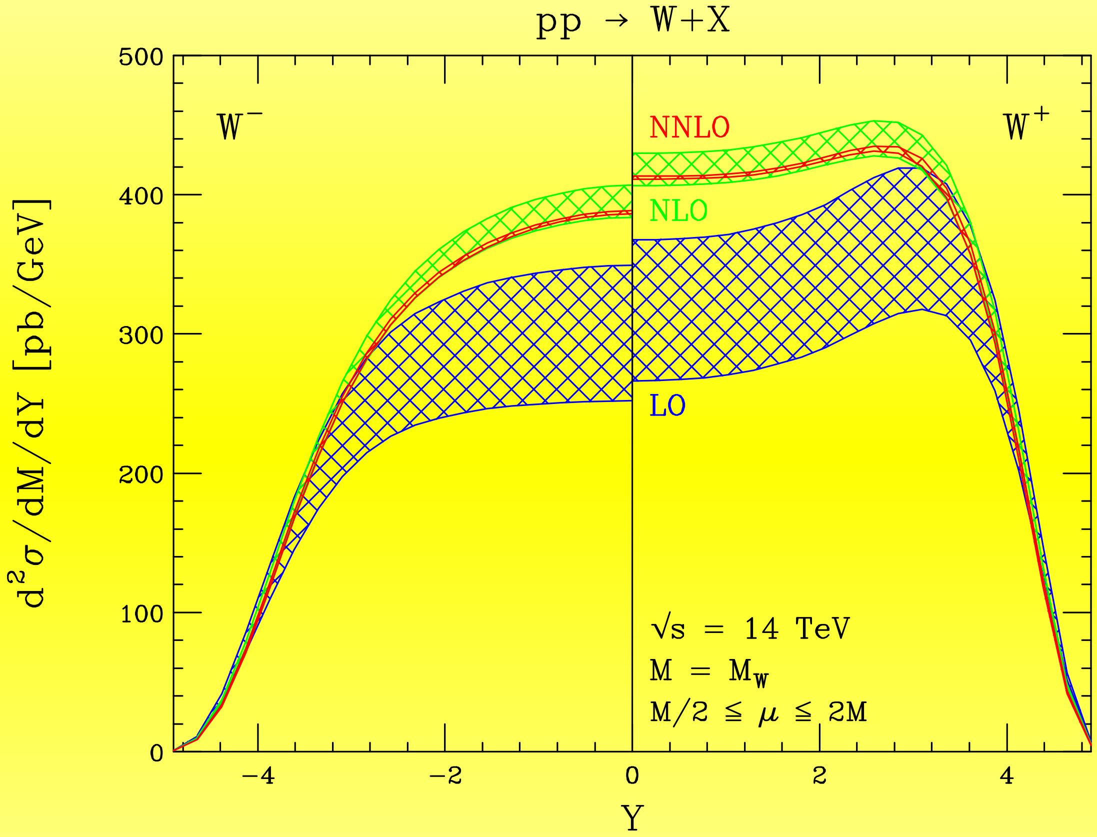
Drell-Yan processes at NNLO

Anastasiou et al.



Luminosity monitor for LHC

Anastasiou et al.



- Bands correspond to scale variation only.
- Reweighting NLO results by $\sigma_{NNLO}/\sigma_{NLO}$ is good to $\leq 1\%$.

Current research directions

- Further study of ideas regarding combining parton showers and matrix elements is most promising in the short term. Application to more processes needed.
- Jet cross-sections at NLO
  - ⋆ Stumbling block for higher leg processes: Virtual corrections
  - ⋆ New technology needed, (presumably semi-numerical)
- Merging of existing NLO calculations with a parton shower
  - ⋆ MC@NLO combination of NLO with existing shower Monte Carlo; has yet to be applied to $W/Z+$ jets
  - ⋆ Should we rather (re)-design shower Monte Carlos to allow easy introduction of NLO corrections?
- Comparisons of all the approaches amongst themselves and with data is crucial both for the Tevatron and the LHC.