The Semi-Numerical Evaluation of One-Loop Corrections

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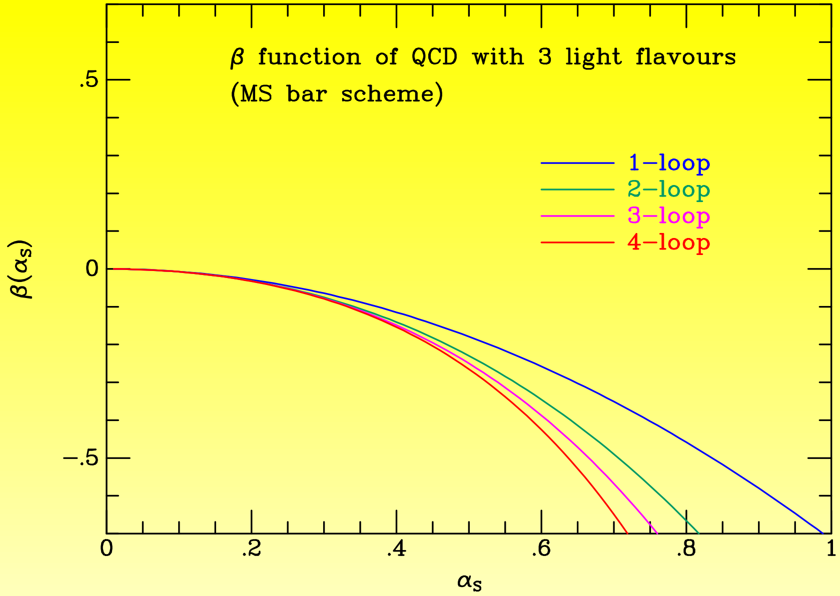
Slides available at http://theory.fnal.gov/people/ellis/Talks/

β function of QCD.

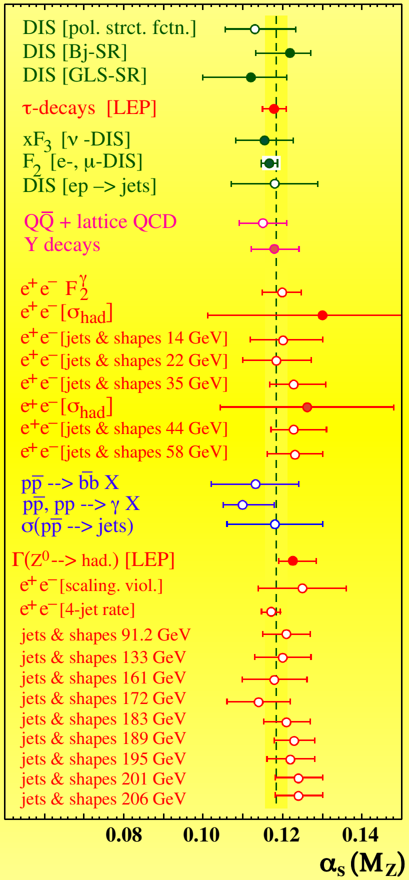
The β-function of QCD is negative. Terms up to $\mathcal{O}(\alpha_S^5)$ are known.

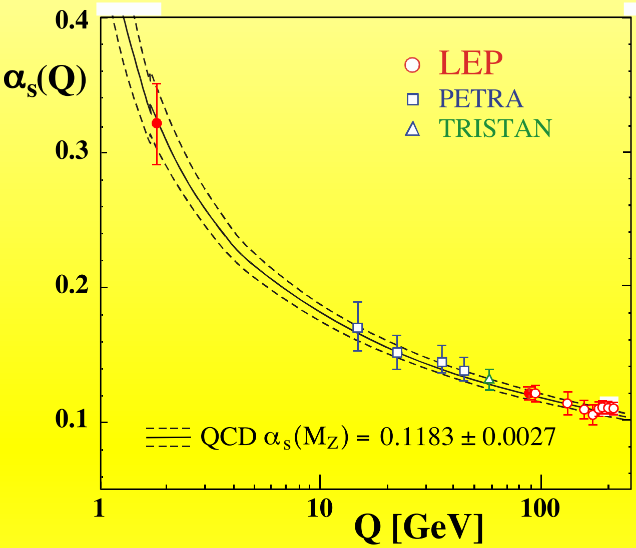
- α_S^2: Gross and Wilczek ; Politzer
- α_S^3: W. E. Caswell; D. R. T. Jones; E. Egorian and O. V. Tarasov
- α_S^4: O. V. Tarasov, A. A. Vladimirov and A. Y. Zharkov; S. A. Larin and J. A. M. Vermaseren
- α_S^5: T. van Ritbergen, J. A. M. Vermaseren and S. A. Larin



Current experimental results on α_S

Bethke,hep-ph/0407021





$$\alpha_S(M_Z) = 0.1182 \pm 0.0027, \overline{\text{MS}}, \text{NNLO}$$

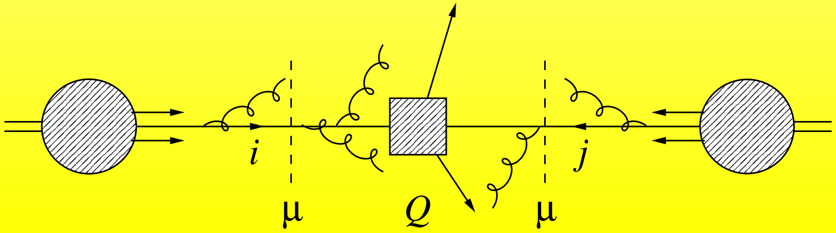
- The decrease of α_S is quite slow – as the inverse power of a logarithm.
- α_S is large at current scales.
- Higher order corrections are important.

The challenge

- The challenge is to provide the most accurate information possible to experimenters working at the Tevatron and the LHC.
- Proton (anti)proton collisions give rise to a rich event structure.
- Complexity of the events will increase as we pass from the Tevatron to the LHC.
- The goals
  - ⋆ To provide physics software tools which are both flexible and give the most accurate representations of the underlying theories.
  - ⋆ To discover new efficient ways of calculating in perturbative QCD.

Hadron-hadron processes

- In hard hadron-hadron scattering, constituent partons from each incoming hadron interact at short distance (large momentum transfer Q^2).



- Form of cross section is

$$\frac{d\sigma}{dX} = \sum_{i,j} \sum_{\tilde{X}} \int dx_1 dx_2 \, f_i(x_1, \mu^2) f_j(x_2, \mu^2)$$
$$\times \quad \hat{\sigma}_{ij}^{\tilde{X}}(\alpha_S(\mu^2), Q^2, \mu^2) \, F(\tilde{X} \to X, \mu^2)$$

where μ^2 is factorization scale and $\hat{\sigma}_{ij}$ is subprocess cross section for parton types i, j and X represents the hadronic final state.

Hadron-hadron processes II

- Short distance cross section $\hat{\sigma}_{ij}$ is calculable as a perturbation series in α_S.
- Notice that factorization scale is in principle arbitrary: affects only what we call part of subprocess or part of initial-state evolution (parton shower).
- There are also interactions between spectator partons, leading to *soft underlying event* and/or *multiple hard scattering*. This an important issue, but I will not talk further about it.

Approaches to the calculation of $\hat{\sigma}$

- LO
  - ⋆ Automatic calculation of tree graphs (Madgraph/Helas, Alpgen, CompHEP, . . .)
  - ⋆ LO + parton shower
  - ⋆ New analytic techniques
- NLO
  - ⋆ Analytic techniques for loop diagrams
  - ⋆ Parton level Monte Carlo (MCFM, NLOJET++, . . .)
  - ⋆ Numerical techniques for loop diagrams
  - ⋆ NLO + parton shower (MC@NLO)
- NNLO
  - ⋆ a few (mostly) inclusive results are known

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  - ⋆ Numerical techniques for loop diagrams YES
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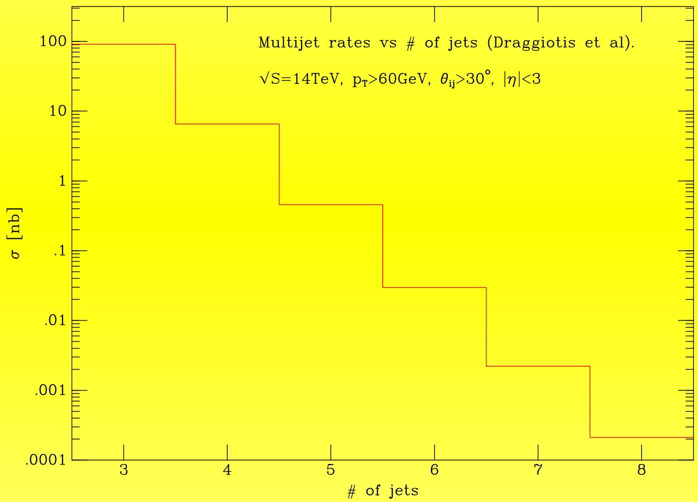
- NNLO
  - ⋆ a few (mostly) inclusive results are known NO

The role of tree graphs

- Problems with tree graphs
  - ⋆ Overall normalization is uncertain.
    For example, W+4 jets is O(α_S^4). If scale uncertainty changes α_S by 10%, this leads to 40% uncertainty in cross section.
  - ⋆ Sometimes a new parton process appears at NLO, leading to large change in shapes. (e.g., gluons at the LHC).
- For example, for W, Z + n jets at tree graph level.
  Madgraph II can generate processes with $\leq$ 9 external particles (madgraph.hep.uiuc.edu)
  Vecbos, W-boson plus up to 4 jets or a Z-boson plus up to 3 jets (theory.fnal.gov/people/giele/vecbos.html)
  Alpgen, W,Z + up to 6 jets etc, (mlm.home.cern.ch/mlm/alpgen/)

Multijet rates using tree graphs

- Calculation of tree graphs using off-shell recurrence relations is a solved problem Berends, Giele.



Draggiotis et al

- At 10^{33} cm^{-2}s^{-1}, left hand scale gives events per second
- $g = 1$
- Similar calculations are possible with other programs Madgraph, Alpgen, COMPHEP, ...

Why NLO?

The benefits of higher order calculations are:-

- Less sensitivity to unphysical input scales (eg. renormalization and factorization scales)
- First prediction of normalization of observables at NLO
  - ⋆ Hence more accurate estimates of backgrounds for new physics searches.
  - ⋆ Confidence that cross-sections are under control for precision measurements.
- It is a necessary prerequisite for other techniques matching with resummed calculations, (eg. MC@NLO).
- More physics
  - ⋆ Parton merging to give structure in jets.
  - ⋆ Initial state radiation.
  - ⋆ More species of incoming partons enter at NLO.

⋆ MCFM overview

mcfm.fnal.gov

MCFM Overview

J.Campbell and RKE
(+F. Tramontano, +F. Maltoni, S. Willenbrock)

- Downloadable general purpose NLO code, "MCFM"

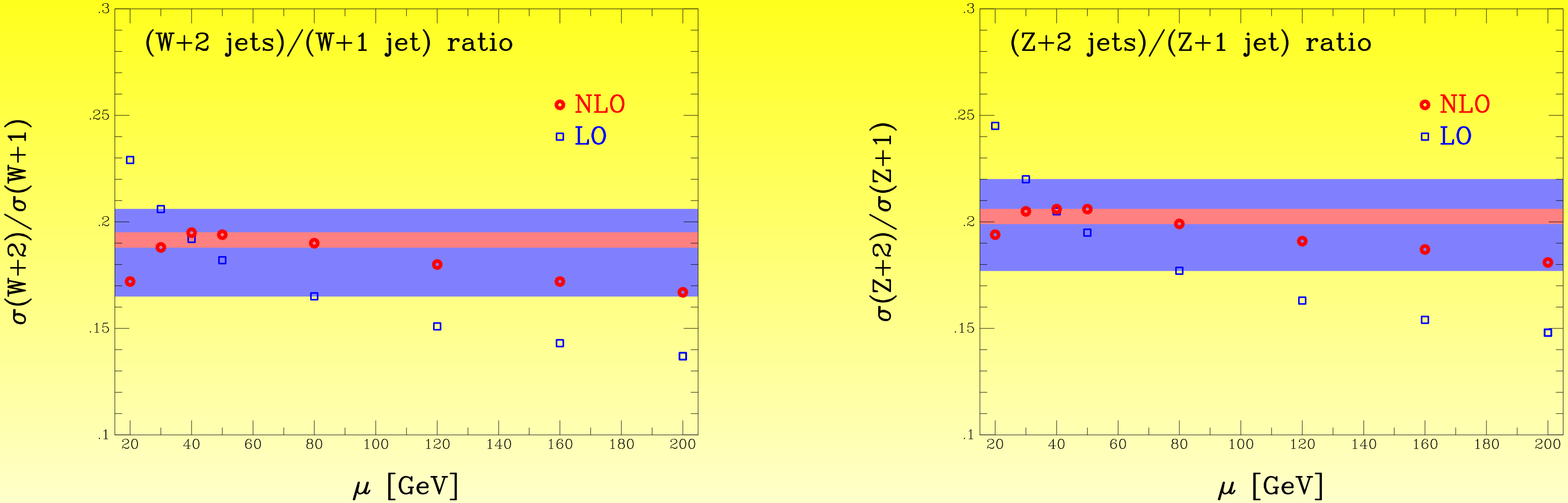
$p\bar{p} \to W^\pm/Z$	$p\bar{p} \to W^+ + W^-$
$p\bar{p} \to W^\pm + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^\pm + \gamma$	$p\bar{p} \to W^\pm/Z + H$
$p\bar{p} \to W^\pm + g^\star \ (\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W^\pm/Z +$ 1 jet	$p\bar{p} \to W^\pm/Z +$ 2 jets
$p\bar{p}(gg) \to H$	$p\bar{p}(gg) \to H +$ 1 jet
$p\bar{p}(VV) \to H +$ 2 jets	$p\bar{p} \to t + q$
$p\bar{p} \to H + b$	$p\bar{p} \to Z + b$

- Knowledge of these processes at NLO provides the first precise predictions of their event rates, which is used in various ways.
  - ⋆ production of pairs of W's and Z's: the structure of the weak interaction at high energy
  - ⋆ W and H production: possibly the first hint of a Higgs boson at the Tevatron
  - ⋆ $H + 2$ jets: an important discovery mode at the LHC

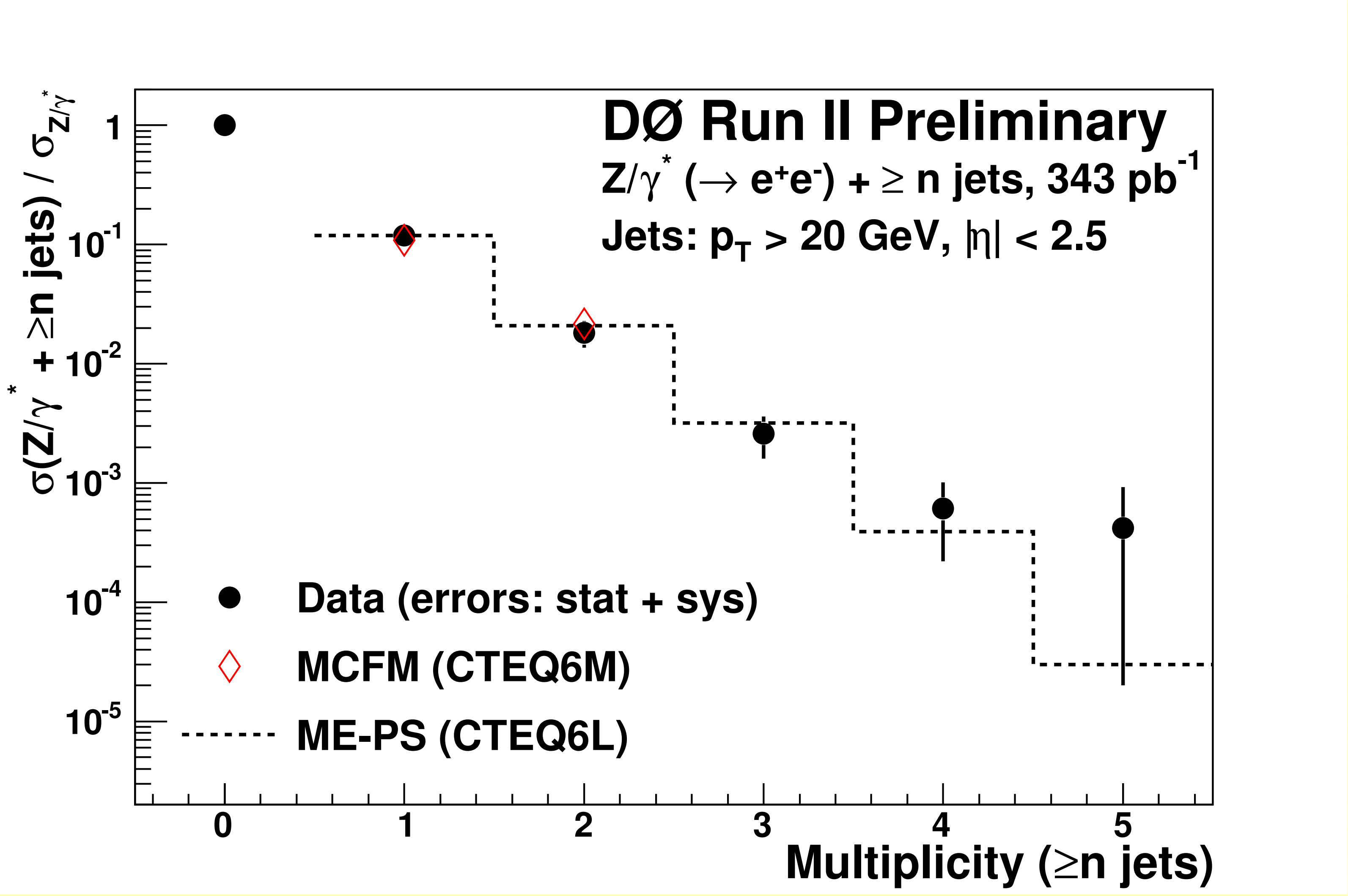
W/Z+jets cross-sections

Rates at the Tevatron

- The $W/Z+2$ jet NLO calculation is the most complicated (time-consuming) process currently implemented. This is due to both the lengthy virtual matrix elements (vector boson $+\,4$ partons) and the complicated structure of phase space.
- The usual features such as reduced scale dependence are observed, e.g. the theoretical prediction for the number of events containing 2 jets divided by the number with only 1 is improved.

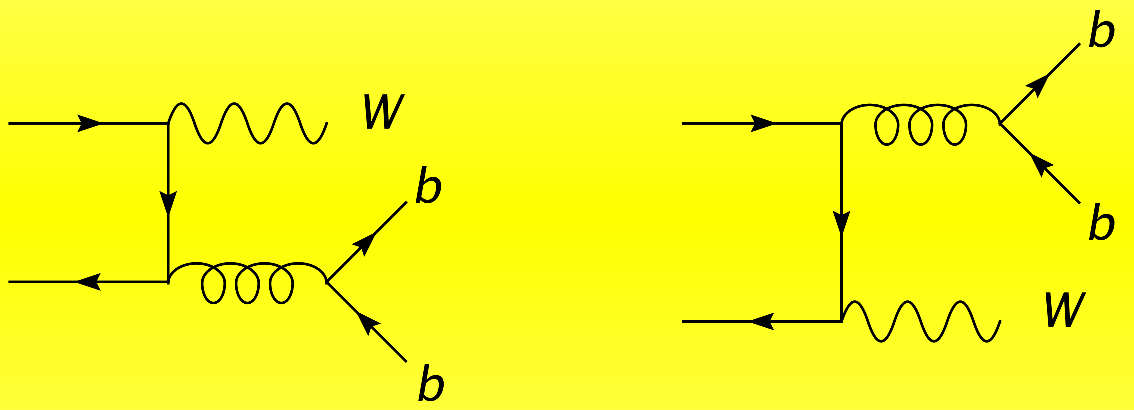


Preliminary data



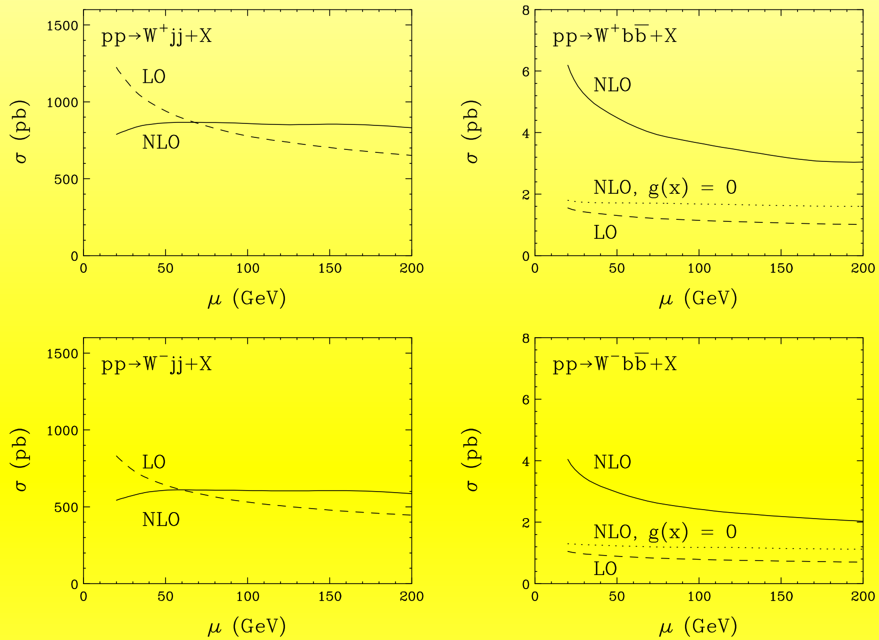
Vector boson + heavy flavour

- In lowest order bottom quark pairs are produced in association with W's by gluon splitting alone:

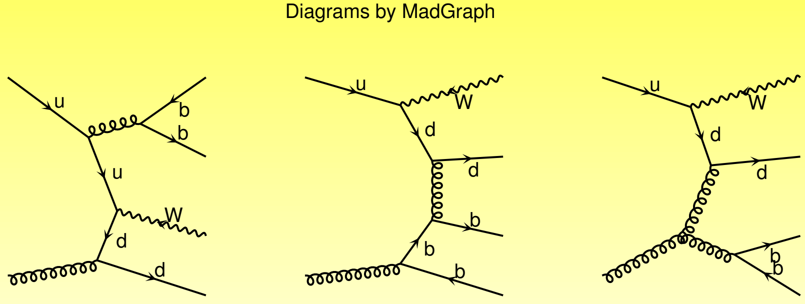


- Beyond LO, the b-quark is treated as a massless particle in MCFM
  - ⋆ a finite cross-section requires a cut on the b-quark p_T
  - ⋆ this means that this calculation is not suitable for estimating the rate with only a single b tag

Jets and heavy flavour at the LHC



- The large gluonic contribution, appearing in $Wb\bar{b}$ for the first time at NLO, results in a huge correction and poor scale dependence.



NLO calculation

- Ingredients in a NLO calculation are
  - ⋆ Born level amplitude
  - ⋆ Real contribution: Addition of one extra parton to Born level process
  - ⋆ Virtual contribution: Interference of one-loop amplitude with Born amplitude
- Real and virtual separately contain singularities from the soft and collinear regions which cancel in the sum.
- Calculation of one-loop amplitudes rapidly becomes complicated as number of partons increases.
- Especially true as we go beyond the most symmetric cases with all gluons.

The future of NLO calculations

An experimenter's wishlist

Run II Monte Carlo Workshop

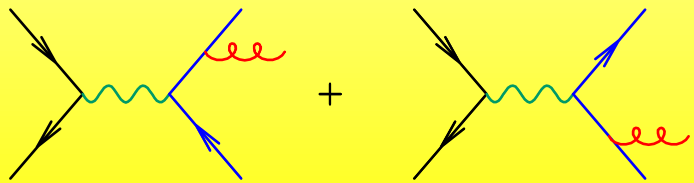
Single Boson	Diboson	Triboson	Heavy Flavour
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}\leq 3j$	$W+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}\leq 3j$	$W+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$Z+b\bar{b}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{b}\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$	$ZZZ+\leq 3j$	$b\bar{b}+\leq 3j$
$\gamma+b\bar{b}\leq 3j$	$\gamma\gamma+b\bar{b}\leq 3j$		single top
$\gamma+c\bar{c}\leq 3j$	$\gamma\gamma+c\bar{c}\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}\leq 3j$		
	$WZ+c\bar{c}\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

Automatic NLO corrections

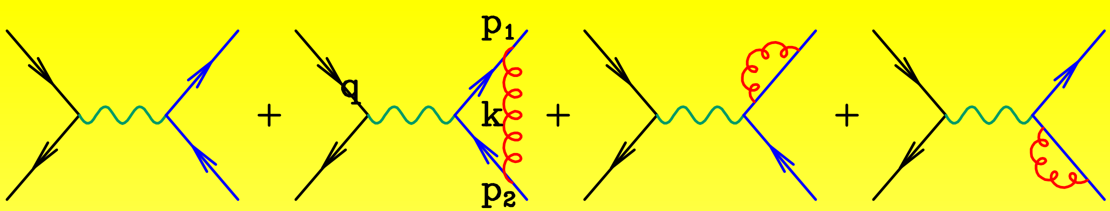
- What is needed is an automatic procedure to calculate NLO corrections (MadLoop?).
- Current stumbling block is the calculation of virtual corrections.
- The virtual corrections contain singularities from the regions of collinear and soft gluon emission, (and in general also UV divergences).
- Divergences are normally controlled by dimensional regularization. A completely numerical procedure using, say, a gluon mass could cause problems with gauge invariance and is hence deprecated.

Example: e^+e^- *total rate*

- Consider the corrections to total $e^+e^- \to q\bar{q}$ rate.



(a)



(b)

$$\sigma^{q\bar{q}g} \; = \; 2\sigma_0 \frac{\alpha_S}{\pi} \, H(\epsilon) \, \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + \mathcal{O}(\epsilon) \right] \; .$$

- Soft and collinear singularities in real emisssion amplitudes (a) are regulated, appearing instead as poles at $D = 4$.

Virtual gluon contributions

- Virtual gluon contributions (b): using dimensional regularization again

$$\sigma^{q\bar{q}} \;=\; 3\sigma_0 \left\{ 1 + \frac{2\alpha_S}{3\pi}\, H(\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon) \right] \right\} .$$

- Adding real and virtual contributions, poles cancel and result is finite as $\epsilon \to 0$. R is an infrared safe quantity.

$$R \;=\; 3 \sum_q Q_q^2 \left\{ 1 + \frac{\alpha_S}{\pi} + \mathcal{O}(\alpha_S^2) \right\} .$$

- However the virtual corrections to $W^+ \to u\bar{d}gggg$ are not so easily calculated.

Tensor one-loop diagrams

We want to consider tensor integrals of the form

$$I^{\mu_1\cdots\mu_M} = \int \frac{d^Dl}{i\pi^{D/2}} \; \frac{l^{\mu_1}\ldots l^{\mu_M}}{d_1 d_2 \ldots d_N}$$

where $d_i = (l + \sum_{j=1}^{j=i} p_j)^2$ are the standard propagator factors.
Passarino and Veltman (1979) wrote a form factor expansion for one-loop integrals, with $M \leq N, N \leq 4$. For example,

$$\int \frac{d^Dl}{i\pi^{D/2}} \frac{l^\mu}{l^2(l+p_1)^2(l+p_1+p_2)^2} = C_1(p_1,p_2)p_1^\mu + C_2(p_1,p_2)p_2^\mu$$

Contracting with p_1 and p_2 and using the identities

$$l\cdot p_1 = \tfrac{1}{2}[(l+p_1)^2 - l^2 - p_1^2], l\cdot p_2 = \tfrac{1}{2}[(l+p_1+p_2)^2 - (l+p_1)^2 - p_2^2 - 2p_1\cdot p_2]$$

Historical perspective II

We derive a linear equation expressing C_1, C_2 in terms of scalar integrals

$$\begin{pmatrix} 2p_1 \cdot p_1 & 2p_1 \cdot p_2 \\ 2p_2 \cdot p_1 & 2p_2 \cdot p_2 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} R_1 \\ R_2 \end{pmatrix}$$

where $R_1 = [B_0(p_1+p_2) - B_0(p_2) - p_1^2\, C_0(p_1, p_2)]$

and $R_2 = [B_0(p_1) - B_0(p_1+p_2) - (p_2^2 + 2p_1 \cdot p_2)\, C_0(p_1, p_2)]$

$$C_0(p_1, p_2) = \int [dl] \frac{1}{l^2 (l+p_1)^2 (l+p_1+p_2)^2}, B_0(p_1) = \int [dl] \frac{1}{l^2 (l+p_1)^2}$$

Solution involves the inverse of the Gram matrix, $G_{ij} \equiv 2p_i \cdot p_j$

$$G^{-1} = \begin{pmatrix} +p_2 \cdot p_2 & -p_1 \cdot p_2 \\ -p_1 \cdot p_2 & +p_1 \cdot p_1 \end{pmatrix} / [2(p_1 \cdot p_1\, p_2 \cdot p_2 - (p_1 \cdot p_2)^2)]$$

Historical perspective III

- M. Veltman wrote a CDC program for numerical evaluation of the formfactors in processes with only UV divergences, Utrecht (1979).
- He dealt with exceptional regions, (e.g. regions where the Gram determinant vanishes), by implementing parts of the program in quadruple precision.
- Translation and improvement by Van Oldenborgh (1990) and further work on interface by T. Hahn and M. Perez-Victoria (1998).

However this is not sufficient for our needs.

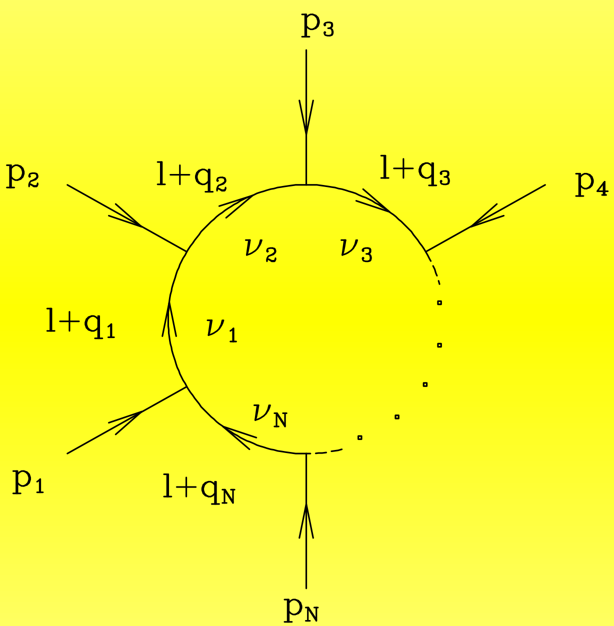
- We are interested in processes with more than 4 external legs.
- We are often interested in loop processes with collinear and soft singularities due to the presence of massless particles. These are most commonly (and elegantly) controlled by dimensional regularization.

Bibliography, Tensor reduction

- D. B. Melrose, Nuovo Cimento, 1965
  In a d dimensional space, a scalar diagram with $n > d$ external legs can be reduced to a sum of diagrams with d external legs.
- Passarino and Veltman, Nucl. Phys. 1979
- Binoth et al., hep-ph/0504267, hep-ph/9911342
- Denner and Dittmaier, hep-ph/0509141
- Giele and Glover, hep-ph/0402152, Giele and Glover and Zanderighi hep-ph/0407016
- Anastasiou and Daleo, hep-ph/0511176

Recursion relations I

Define generalized scalar integrals



$$d_i \equiv (l + q_i)^2$$

$$q_i \equiv \sum_{j=1}^{i} p_j$$

$$q_N \equiv \sum_{j=1}^{N} p_j = 0,$$

$$I(D;\nu_1,\nu_2,\ldots,\nu_N) = I(D;\{\nu_k\}_{k=1}^{N}) \equiv \int \frac{d^D\, l}{i\pi^{D/2}}\, \frac{1}{d_1^{\nu_1} d_2^{\nu_2} \cdots d_N^{\nu_N}}\,,$$

Form-factor expansion

Davydchev

- For form factor expansion in terms of the q's the coefficients are generalized scalar integrals in shifted dimensionalities
- e.g., the rank-1 and rank-2 tensor integrals with N external legs can be decomposed as

$$
\begin{aligned}
& I^{\mu_1}(D; q_1, \ldots, q_N) = \sum_{i_1=1}^{N} I(D+2; \{1+\delta_{i_1 k}\}_{k=1}^{N})\; q_{i_1}^{\mu_1} \\
= \quad & I(D+2; 2,1,1,\ldots,1)\; q_1^{\mu_1} + I(D+2; 1,2,1,\ldots,1)\; q_2^{\mu_1} \\
+ \quad & \cdots + I(D+2; 1,1,1,\ldots,2)\; q_N^{\mu_1}\;. \\
& I^{\mu_1\mu_2}(D; q_1, \ldots, q_N) = -\frac{1}{2} I(D+2; 1,1,1,\ldots,1)\; g^{\mu_1\mu_2} \\
& +2\; I(D+4; 3,1,1,\ldots,1)\; q_1^{\mu_1} q_1^{\mu_2} \\
+ \quad & I(D+4; 2,2,1,\ldots,1)\; (q_1^{\mu_1} q_2^{\mu_2} + q_1^{\mu_2} q_2^{\mu_1}) + \cdots
\end{aligned}
$$

Basic identity

Tkachev,Chetyrkin,Tarasov,Duplancic,Nizic

$$\int \frac{d^D l}{i\pi^{D/2}} \frac{\partial}{\partial l^\mu} \left(\frac{\left(\sum_{i=1}^N y_i\right) l^\mu + \left(\sum_{i=1}^N y_i q_i^\mu\right)}{d_1^{\nu_1} d_2^{\nu_2} \cdots d_N^{\nu_N}} \right) = 0\,.$$

valid for arbitrary y_i. Differentiating we obtain the base identity

$$\sum_{j=1}^N \left(\sum_{i=1}^N S_{ji} y_i \right) \nu_j I(D; \{\nu_k + \delta_{kj}\}_{k=1}^N) = -\sum_{i=1}^N y_i I(D-2; \{\nu_k - \delta_{ki}\}_{k=1}^N)$$

$$- \quad \left(D - 1 - \sum_{j=1}^N \nu_j \right) \left(\sum_{i=1}^N y_i \right) I(D; \{\nu_k\}_{k=1}^N)\,,$$

where S is a kinematic matrix which, for massless internal particles, takes the form

$$S_{ij} \equiv (q_i - q_j)^2\,.$$

Recursion relations II

Solving $\sum_i S_{ji} y_i = \delta_{lj}$ (assuming that the inverse of the matrix S exists), we derive the basic recursion relation

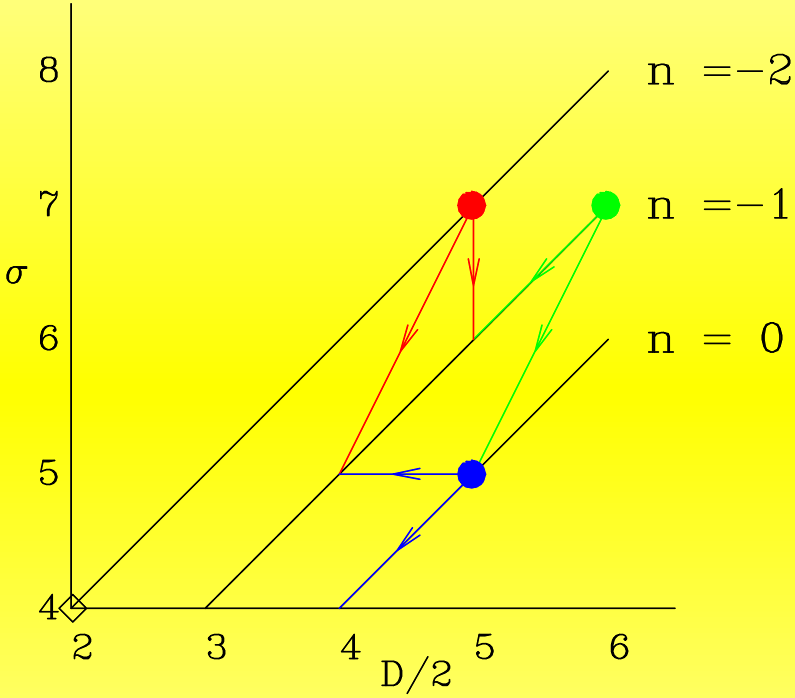
$$
\begin{aligned}
&(\nu_l - 1) I(D; \{\nu_k\}_{k=1}^N) \\
&\quad = \; -\sum_{i=1}^N S_{li}^{-1} I(D-2; \{\nu_k - \delta_{ik} - \delta_{lk}\}_{k=1}^N) \\
&\quad - \; b_l \, (D - \sigma) \, I(D; \{\nu_k - \delta_{lk}\}_{k=1}^N).
\end{aligned}
$$

$$
\sigma \equiv \sum_{i=1}^N \nu_i; \quad b_i \equiv \sum_{j=1}^N S_{ij}^{-1}; \quad B \equiv \sum_{i=1}^N b_i = \sum_{i,j=1}^N S_{ij}^{-1} \, .
$$

The strategy is to reduce more complicated integrals to a set of simpler basis integrals which are known analytically.
Hence the method is seminumerical.

Recursion relations III

- Example: reduction of boxes



- Using the basic identity (red lines) and other subsidiary identities (blue and green lines) one can always arrive at the basis integral, (four-dimensional box), denoted by a diamond, (or integrals with fewer external legs).

Two mass triangles

$$S = \begin{pmatrix} 0 & p_2^2 & p_1^2 \\ p_2^2 & 0 & 0 \\ p_1^2 & 0 & 0 \end{pmatrix},$$

is singular. By choosing y_i which satisfy the relation, $\sum_i S_{ji} y_i = a_j$

$$a = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad y = \begin{pmatrix} 0 \\ \frac{\alpha}{p_2^2} \\ \frac{1-\alpha}{p_1^2} \end{pmatrix},$$

and choosing the parameter α such that $\sum_i^3 y_i = 0$, we derive the recursion relation valid for $\nu_1 > 1$ and $p_1^2 \neq p_2^2$

Two mass triangles II

- Basic identity

$$\sum_{j=1}^{N}\left(\sum_{i=1}^{N} S_{ji} y_i\right) \nu_j I(D; \{\nu_k + \delta_{kj}\}_{k=1}^{N}) =$$

$$-\sum_{i=1}^{N} y_i I(D-2; \{\nu_k - \delta_{ki}\}_{k=1}^{N}) - (D-1-\sigma)\left(\sum_{i=1}^{N} y_i\right) I(D; \{\nu_k\}_{k=1}^{N})$$

With the choices of previous page become

$$I(D; \nu_1, \nu_2, \nu_3) = \frac{1}{p_1^2 - p_2^2} \frac{1}{(\nu_1 - 1)} \Big[I(D-2; \nu_1 - 1, \nu_2 - 1, \nu_3)$$

$$- \quad I(D-2; \nu_1 - 1, \nu_2, \nu_3 - 1) \Big] .$$

This relation lowers $D/2$ by one unit and σ by two units.

Exceptional regions

Ellis, Giele, Zanderighi, hep-ph/0508308

$$\sum_{j=1}^{N}\left(\sum_{i=1}^{N} S_{ji} y_i\right) \nu_j I(D;\{\nu_k+\delta_{kj}\}_{k=1}^{N}) =$$

$$-\sum_{i=1}^{N} y_i I(D-2;\{\nu_k-\delta_{ki}\}_{k=1}^{N}) - (D-1-\sigma)\left(\sum_{i=1}^{N} y_i\right) I(D;\{\nu_k\}_{k=1}^{N}),$$

If S has a zero eigenvalue

$$0 = -\sum_{i=1}^{N} y_i I(D-2;\{\nu_k-\delta_{ik}\}_{k=1}^{N}) - (D-1-\sigma)\left(\sum_{i=1}^{N} y_i\right) I(D;\{\nu_k\}_{k=1}^{N}).$$

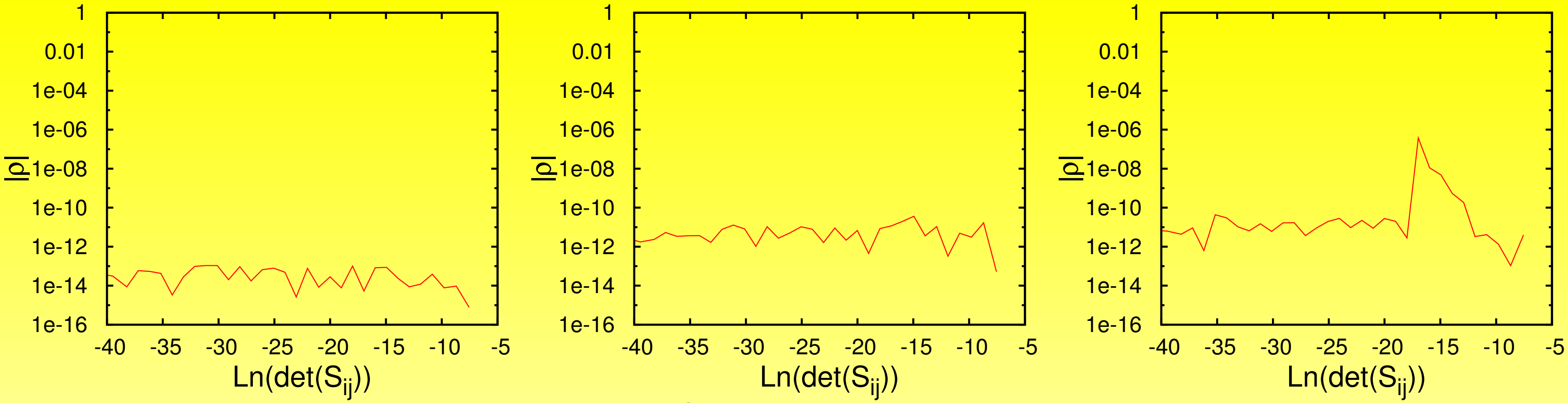
If $\sum_{i=1}^{N} y_i \neq 0$ one obtains the relation

$$I(D;\{\nu_k\}_{k=1}^{N}) = -\frac{1}{D-1-\sigma}\sum_{i=1}^{N}\frac{y_i}{\sum_{i=1}^{N} y_i} I(D-2;\{\nu_k-\delta_{ik}\}_{k=1}^{N}),$$

which reduces both D and σ (and possibly N), while keeping n fixed.

Exceptional regions II

$$
\begin{aligned}
I(D;\{\nu_k\}_{k=1}^N) = \quad & - \quad \frac{1}{D-1-\sigma}\sum_{j=1}^{N}\frac{y_j}{\sum_{i=1}^N y_i} I(D-2;\{\nu_k-\delta_{kj}\}_{k=1}^N) \\
& - \quad \frac{1}{D-1-\sigma}\sum_{j=1}^{N}\frac{\sum_{i=1}^N S_{ji}y_i}{\sum_{i=1}^N y_i}\nu_j I(D;\{\nu_k+\delta_{kj}\}_{k=1}^N)\,.
\end{aligned}
$$



Relative accuracy $|\rho|$ for the $1/\epsilon^2$ pole (left), the $1/\epsilon$ pole (center) and the constant part (right) of the one-loop amplitude squared for $H \to q\bar{q}q'\bar{q}'$

Proof of principle

Ellis, Giele, Zanderighi

Use the effective theory ($m_t \to \infty$) for Hgg coupling

$$\mathcal{L}_{\text{eff}} = \frac{1}{4} A(1+\Delta) H G^a_{\mu\nu} G^{a\,\mu\nu}\,.$$

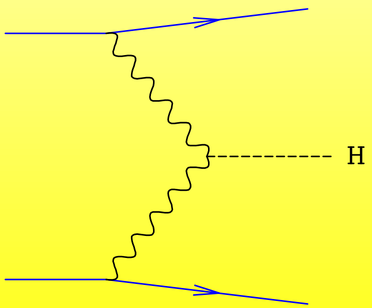
$G^a_{\mu\nu}$ is the field strength of the gluon field and H is the Higgs-boson field, $A = \frac{g^2}{12\pi^2 v}$ where g is the bare strong coupling and v is the vacuum expectation value parameter, $v^2 = (G_F\sqrt{2})^{-1} = (246\ \text{GeV})^2$. Δ is a finite correction. Calculate virtual corrections to

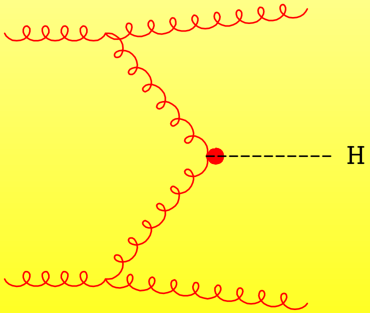
$$\begin{aligned}
A)\ \ H &\to q\bar{q}q'\bar{q}', \quad (30 \text{ diagrams}),\\
B)\ \ H &\to q\bar{q}q\bar{q}, \quad (60 \text{ diagrams}),\\
C)\ \ H &\to q\bar{q}gg, \quad (191 \text{ diagrams}),\\
D)\ \ H &\to gggg, \quad (739 \text{ diagrams}).
\end{aligned}$$

Comparison of numerical and analytic results for $H \to$ four partons

	$\frac{1}{\epsilon^2}$	$\frac{1}{\epsilon}$	1
A_B	0	0	12.9162958212387
$A_{V,N}$	-68.8869110466063	-114.642248172519	120.018444115458
$A_{V,A}$	-68.8869110466064	-114.642248172523	120.018444115429
B_B	0	0	858.856417157052
$B_{V,N}$	-4580.56755817094	-436.142317955208	26470.9608978350
$B_{V,A}$	-4580.56755817099	-436.142317955660	26470.9608978346
C_B	0	0	968.590160211857
$C_{V,N}$	-8394.44805516930	-19808.0396331354	-1287.90574949112
$C_{V,A}$	-8394.44805516942	-19808.0396331363	not known analytically
D_B	0	0	3576991.27960852
$D_{V,N}$	-4.29238953553022 $\cdot 10^7$	-1.04436372655580 $\cdot 10^8$	-6.79830911471604$\cdot 10^7$
$D_{V,A}$	-4.29238953553022$\cdot 10^7$	-1.04436372655580 $\cdot 10^8$	not known analytically

H+2 jet calculation





- NLO corrections to W-fusion mechanism already calculated by many authors.
- All the elements are in place for a full NLO Higgs + 2 jets calculation via gluon fusion mechanism
  - ⋆ Born level calculation Higgs + 4 partons
  - ⋆ Real calculation Higgs + 5 partons,
    Del Duca et al, Dixon et al, Badger et al
  - ⋆ Virtual calculation — Ellis, Giele and Zanderighi, presented above
  - ⋆ Subtraction terms — Campbell, Ellis and Zanderighi, in preparation

Summary

- Making an accurate assessment of particle rates and extracting detailed information from the data requires calculations that go beyond the simplest approximation.
- This is especially true at the LHC where, on average, many more particles (partons) will be produced per collision.
- Next-to-leading order calculations are the first step towards the precision needed.
- I have demonstrated that a semi-numerical approach can provide interesting results, although the verification in a specific physical process is not yet complete.
- Although MCFM is a tool which provides a step in this direction, it is certainly not enough.
- What is needed is a concerted effort to create an automatic program, which will return virtual corrections for a process of arbitrary complexity.