Extracting Hadronic Resonances using Jet Ensemble Correlations

Fermilab Workshop

Beyond the Standard Model: from the Tevatron to the LHC

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Searches for New Physics

- Solution to hierarchy problem suggests new physics below the TeV scale

- Many possibilities: must be prepared for "anything"

- Assume you strongly produce new particle Q: e.g. pp → QQ

New particles (Q) usually decay into SM particles

- Leptons
- Photons
- MET

Electroweak Decays: S/B *large* — "EASY"

- Jets → Strongly Interacting Decays: S/B *small* — "HARD"

Searches for New Physics in Multi-Jets

- Look for new physics in *multi-jets without leptons and Missing Transverse Energy* *(in particular focus on six-jet signal)*
- Challenging!
  - Large backgrounds
  - Magnitude of Multi-Jet Backgrounds from High Order Processes Difficult to Calculate a priori ($O(\alpha_s^n)$)
- But very important!
  - New physics may be hidden in jets (at Tevatron & LHC)
  - Other studies focus on electroweak decays (usually)
  - Large production cross-section - could be detected early at LHC
- Use kinematic cuts and correlations + kinematic features
- Use an ensemble of jet combinations
- Techniques also useful for multi-jet signals *with* leptons and MET

Several possibilities for Multi-Jet Production:

- Additional heavy fermions/scalars (3, 6, 8, … of $SU(3)_C$) can produce multi-jet resonances

Many models:
e.g. Farhi & Susskind (1979), Marciano (1980), Konishi & Tripiccione (1983), Zoupanos (1983), Lust et. al. (1986), Frampton & Glashow (1987), Kang & White (1987), Fukazawa et. al. (1991), Martin (1992)
more recently:
Dobrescu, Kong, Mahbubani (2007), Kilic, Okui, Sundrum (2008)

- First multi-jet study for LHC (& SSC): Chivukula, Golden, Simmons (1991)

--- In this talk will focus on:

$pp \rightarrow QQ \rightarrow 3j+3j = 6j$

$Q = SU(3)_C$ octet fermion

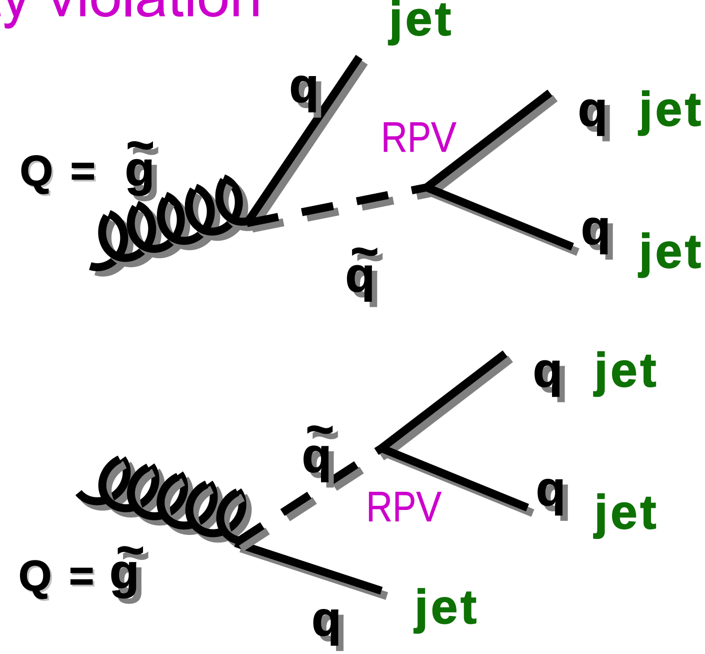
(e.g. gluino in MSSM)

- But techniques useful also for other cases

- SM background is pure QCD and all-hadronic decay of $t\text{-}\bar{t}$

How did we generate the events?

- Signal: PYTHIA - use MSSM w/ R-parity violation
  - gluino pair production → 6 jets
  - other sparticles heavy



- Six-jet pure QCD background:
  - ALPGEN (ΔR_{ij} > 0.4) → PYTHIA
- All-hadronic top background:
  - PYTHIA
- Detector Simulation:
  - PGS 4 --- Can compare to full CMS Software (CMSSW) Detector Simulation (conclusion: PGS 4 reliable)
- Analysis: "Bump hunt" - aim to extract 3-jet Resonance (Q → 3 jets)

Summary of data generated

Focus on LHC
(comment about
Tevatron later)

		σ (pb)	Events	$\mathcal{L}$ (fb^{-1})
Signal: pp → QQ → 6j	m_Q=290 GeV	344	100000	0.3
	m_Q=420 GeV	50	100000	2.0
	m_Q=660 GeV	3.4	100000	29.6
	m_Q=890 GeV	0.5	100000	215.5
Pure QCD 6-jet background	6j, $p_T > 20$	407000	315069	0.00077
	6j, $p_T > 50$	2400	434011	0.2
	6j, $p_T > 80$	120	507428	4.2
	6j, $p_T > 110$	14	448520	32.3
background from all-hadronic decay of top quarks	$t\bar{t}$	490	490000	1.0

Background >> Signal !!!

pp → QQ → 6 jets: Basic Trigger-Level Cuts

- Five possible triggers:

1) $p_{T,1} > 400\ \mathrm{GeV} \quad \cup$
2) $p_{T,2} > 350\ \mathrm{GeV} \quad \cup$
3) $p_{T,3} > 195\ \mathrm{GeV} \quad \cup$
4) $p_{T,4} > 80\ \mathrm{GeV} \quad \cup$ ← dominates
5) $\sum p_{T,j} > 1000\ \mathrm{GeV}$

Trigger Level Cuts

- Additional selection criteria
  - At least 6 jets
  - $|\eta_j| < 3 \quad j = 1, \ldots, 6$
  - $\Delta R_{ij} > 0.5$ (jets with cone size 0.5)
  - $p_{T,6} > 30\ \mathrm{GeV}$

Additional Event Selection

pp → QQ → 6 jets: After Basic Trigger-Level Cuts

	QQ	QQ	QQ	QQ	QCD	$\bar{t}t$
m_Q (GeV)	290	420	660	880	–	172
σ (pb) Trigger + Event Selection	98	30	2.8	0.42	10650	17

S / B ~ 1 / 110

S / B ~ 1 / 25400

Very bad !!

- Next Aim: Increase Signal / Background
- Jets from Signal are, on average, harder than jets from background, so use hard kinematic cuts

pp → QQ → 6 jets: Kinematic Cuts

- Aim: Optimize Signal / Background

1) Cut on Sum p_T of the first 6 jets:

	Cut on $\sum_{i=1}^{6} p_{T,i}$
• m_Q = 290 GeV	600 GeV
• m_Q = 420 GeV	700 GeV
• m_Q = 660 GeV	1100 GeV
• m_Q = 880 GeV	1500 GeV

2) Cut on p_T of the 6^{th} jet (optimize):

Consider $p_{T,6}$ > 30 GeV, or 60 GeV, or 90 GeV, or 120 GeV

S / B After Kinematic Cuts: m_Q = 290 GeV

Cuts $\sum_{j=1}^{6} p_{T,j}$ (GeV)	Cuts $p_{T,6}$ (GeV)	QQ	QCD	$\bar{t}t$
m_Q (GeV)		290	–	172
600	30	83	8350	14
600	60	36	430	4.5
600	90	7.4	28	0.63
600	120	1.5	3.2	0.067

S / B ~ 1 / 2

Not important

S / B After Kinematic Cuts: m_Q = 880 GeV

		QQ	QCD	$\bar{t}t$
m_Q (GeV)		880	–	172
Cuts				
$\sum_{j=1}^{6} p_{T,j}$ (GeV)	$p_{T,6}$ (GeV)			
1500	30	0.28	210	0.79
1500	60	0.24	29	0.32
1500	90	0.18	6.4	0.10
1500	120	0.12	1.6	0.032

S / B ~ 1 / 13

Not important

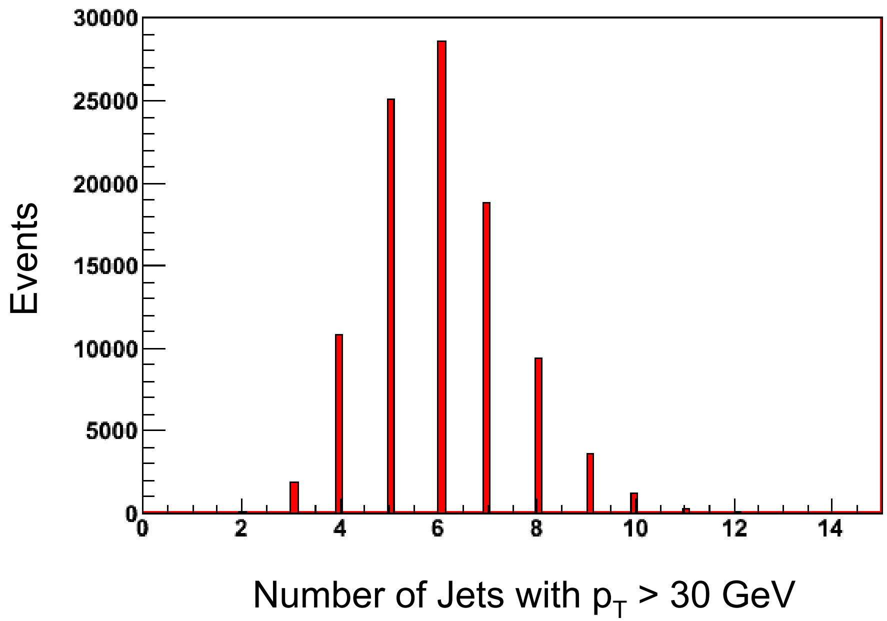
S / B now much more manageable !!!

pp → QQ → 6 jets: Looking for the Resonance

QQ → 3j+3j = 6j (Two Three-Body Resonances)

- How select correct six jets in each event? Not easy!
- Many extra radiated jets

e.g. m_Q = 290 GeV



- Maybe 6 jets from Q are hardest 6 jets in every event?

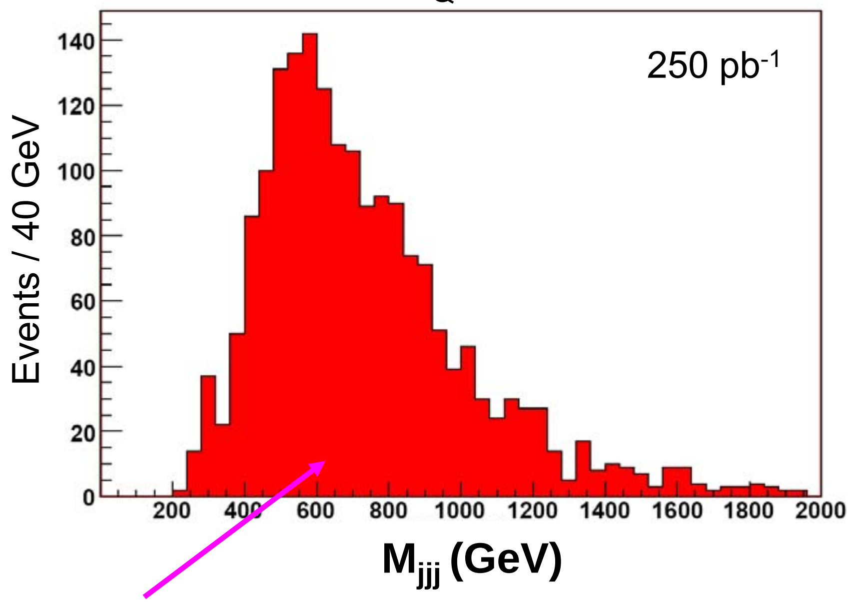
Only true ~60% of the time!

Jets from Initial State Radiation & other radiated jets can be harder than signal jets

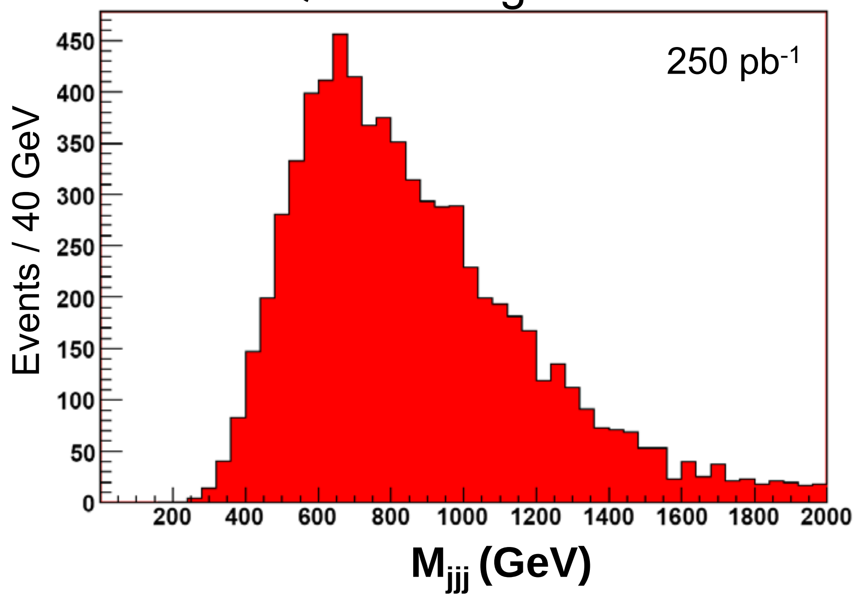
pp → QQ → 6 jets: Looking for the Resonance

- Assume 6 correct jets are 6 hardest jets in an event
- How select correct 3 jets for each Q?
- 10 possible pairs of jet-triplets; choose pair of triplets with smallest $|M_{1,jjj} - M_{2,jjj}|$

Signal m_Q = 290 GeV



QCD Background

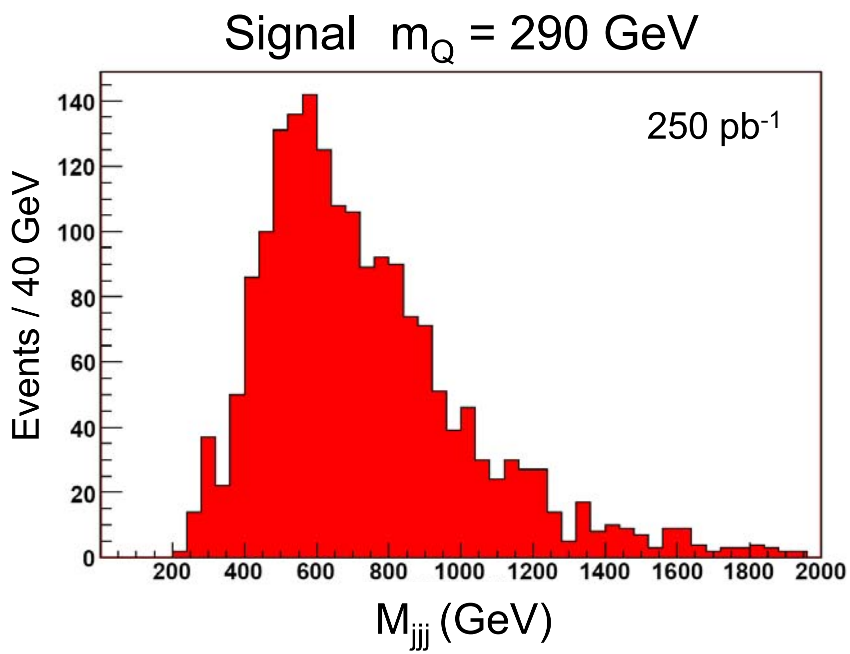


- Tail much larger than jet resolution
- Large combinatoric background within signal

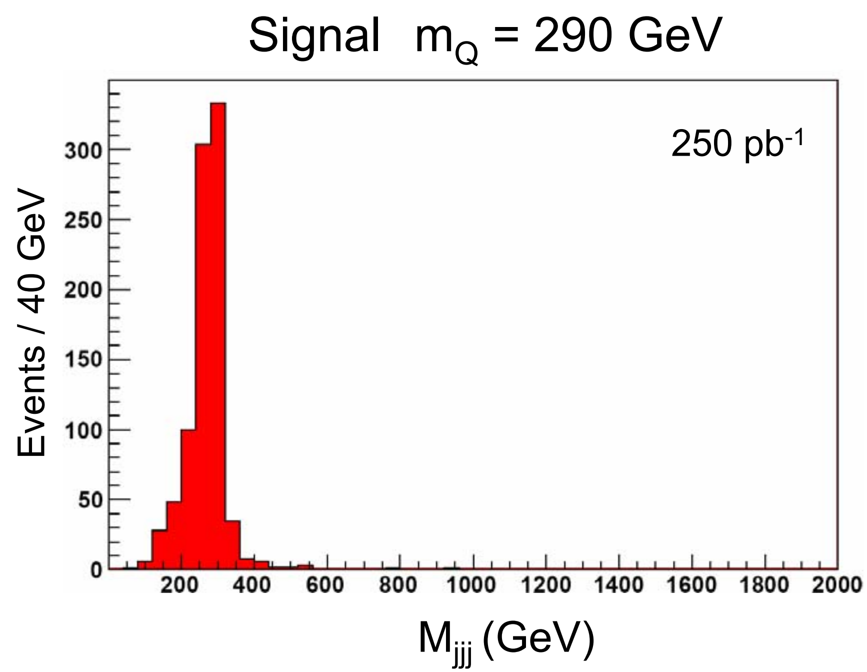
- Mismatching of jet-triplets
- Cuts shape background

pp → QQ → 6 jets: MC Info

- Use Monte-Carlo simulation information to match correct jets to Q's



No MC info



Using MC info

pp → QQ → 6 jets: MC Info - which triplets are correct?

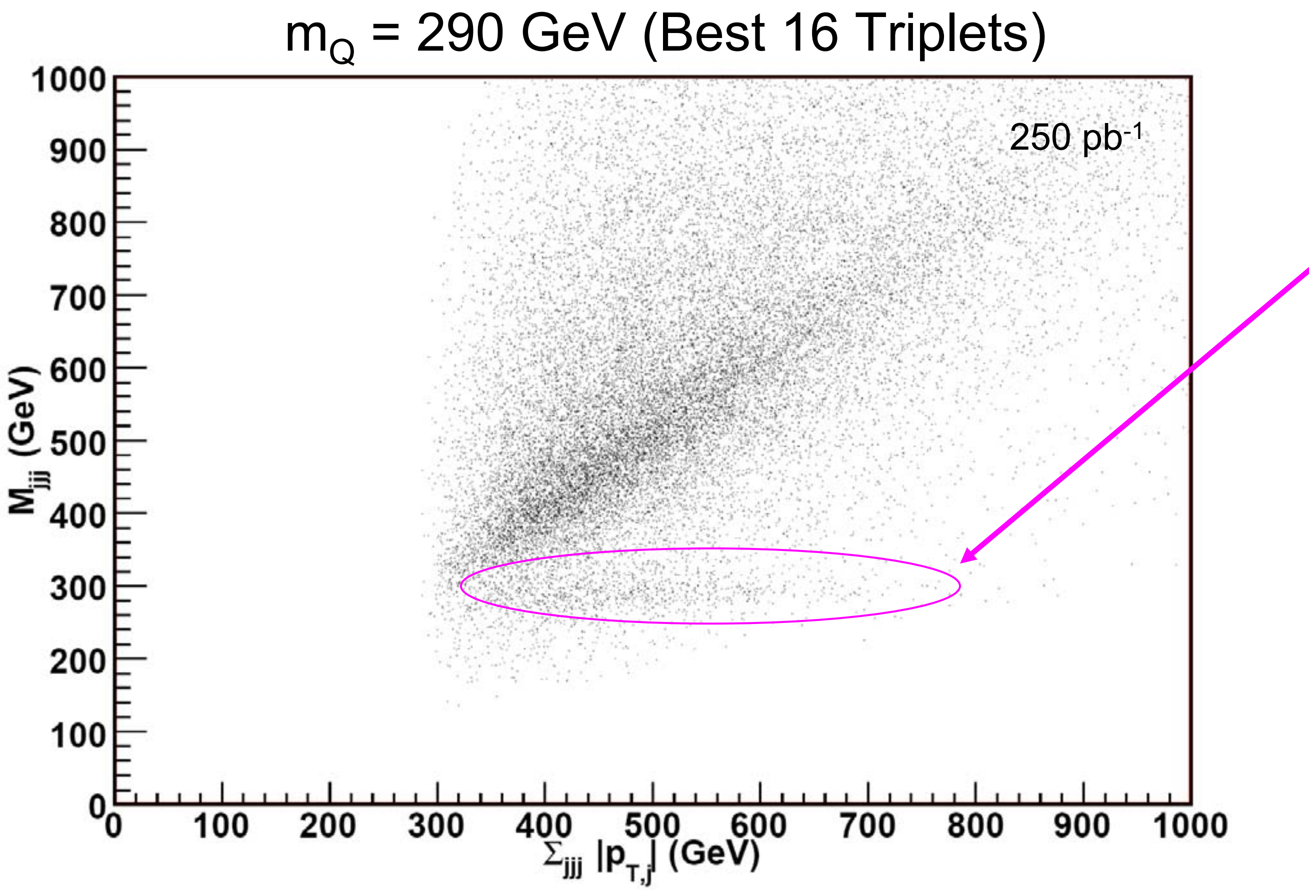
- Use Monte-Carlo simulation information to find which triplets are correct most often (shown are top 16)

p_T Ordered Jet Triplet	Correct Matching Fraction	p_T Ordered Jet Triplet	Correct Matching Fraction
235	0.060	136	0.031
234	0.057	135	0.031
245	0.053	345	0.031
145	0.047	256	0.024
236	0.045	134	0.021
146	0.040	346	0.021
156	0.034	126	0.016
246	0.034	356	0.016

- Jets ordered in p_T
- Triplet 235 is correct 6% of the time
- Triplet 123 not among top 16 combinations
- How make use of this MC info?

pp → QQ → 6 jets: Correlation No. 1

- Find Correlation in: M_{ijk} vs $\sum_{i=1}^{3} |p_{T,i}|$



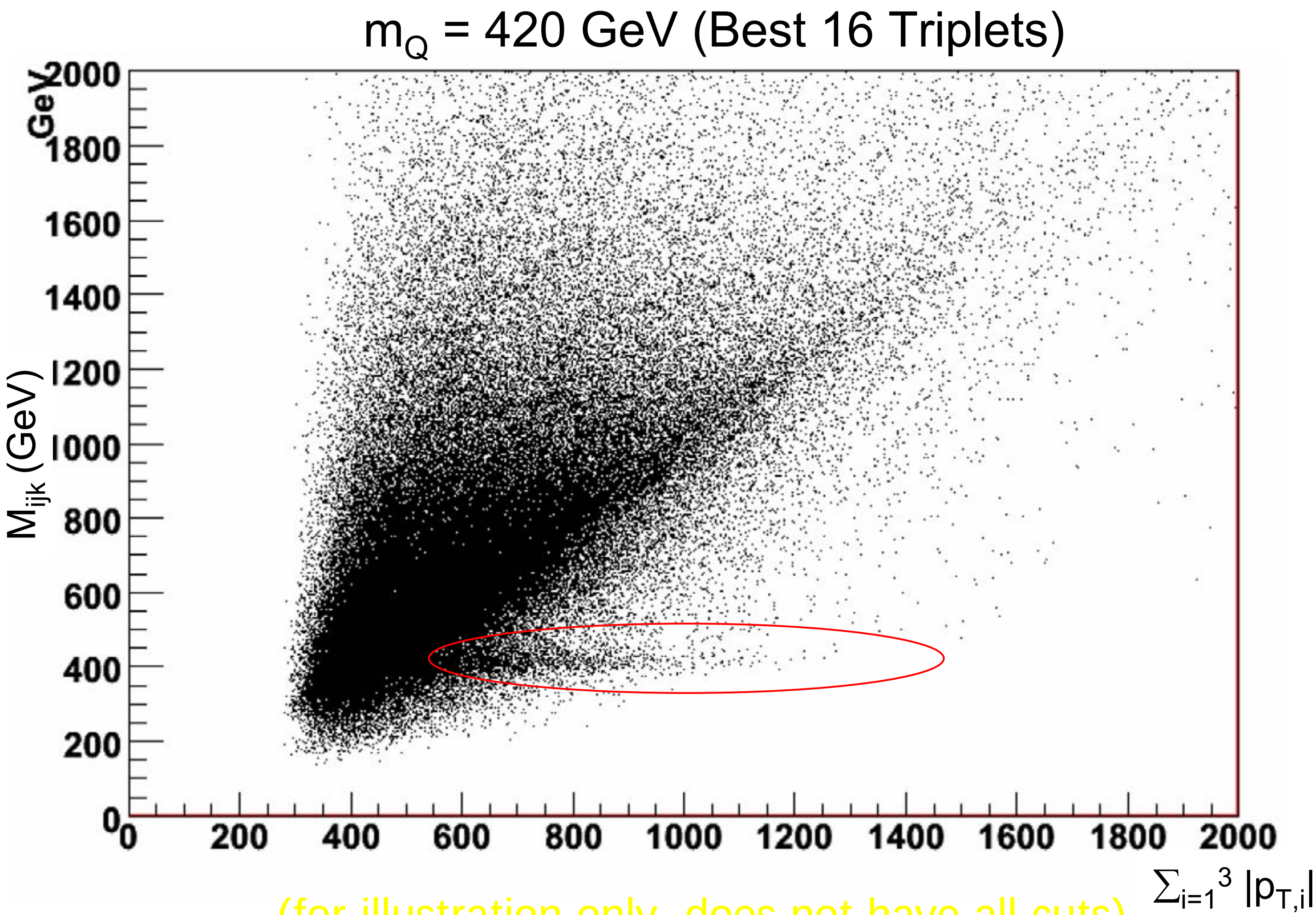
Kinematic Feature: Horizontal Branch

- Contains Correct Jet Triplets !!!
- Region of high Signal to combinatoric Background contrast
- Triplets Highly Boosted

Use 16 Best Triplets to increase signal efficiency (16 entries/event) ← Use Jet Ensemble

pp → QQ → 6 jets: Correlation No. 1

- Find Correlation in: $\mathbf{M_{ijk}}$ **vs** $\sum_{i=1}^{3} |p_{T,i}|$

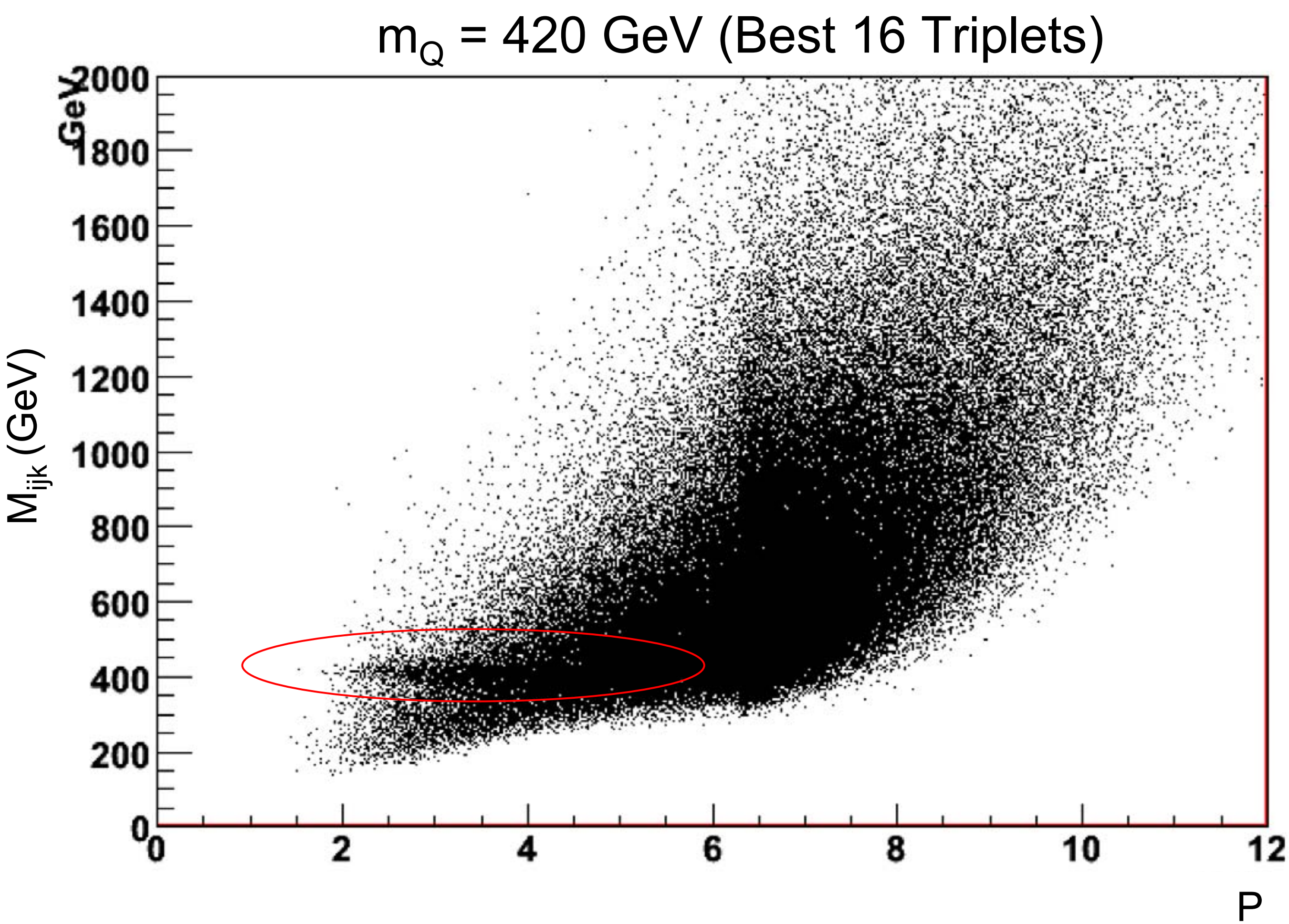


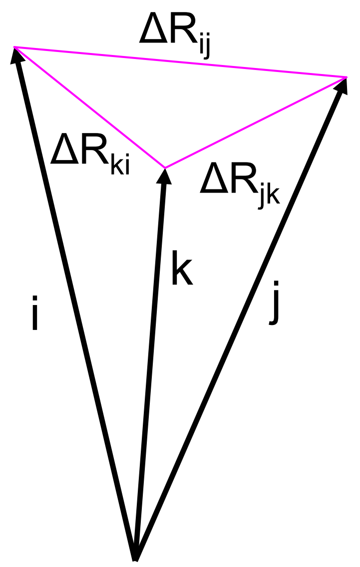
(for illustration only, does not have all cuts)

pp → QQ → 6 jets: Correlation No. 2

- Find Correlation in: **M_{ijk} vs Perimeter (P)**, where

$$P \equiv \Delta R_{ij} + \Delta R_{jk} + \Delta R_{ki}, \quad \Delta R_{ij} = \sqrt{(\phi_i - \phi_j)^2 + (\eta_i - \eta_j)^2}$$



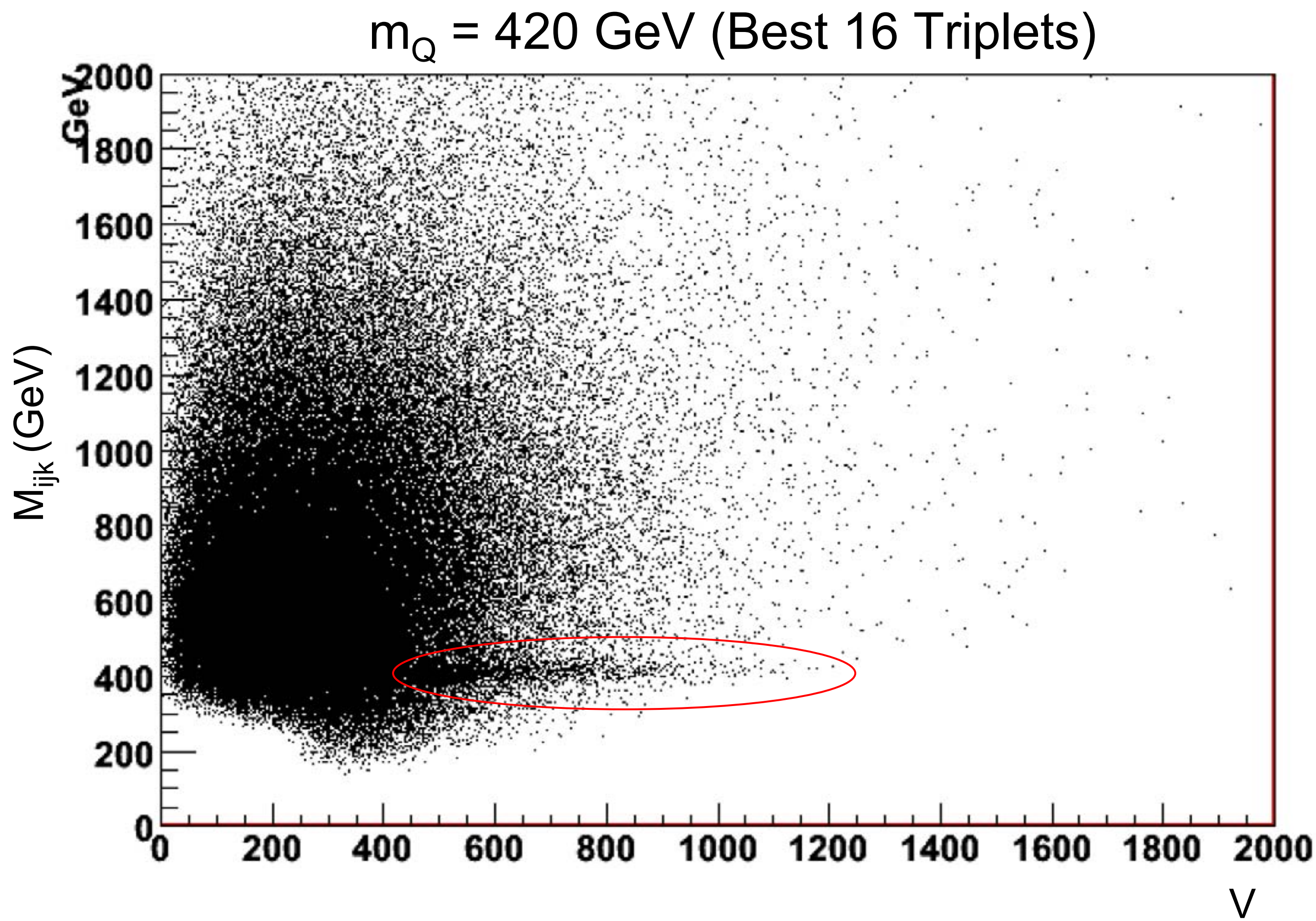


P ~ Physical separation of jet triplet

pp → QQ → 6 jets: Correlation No. 3

- Find Correlation in: **M_{ijk} vs V**, where

$$V \equiv |\vec{p}_{T,i} + \vec{p}_{T,j} + \vec{p}_{T,k}|$$



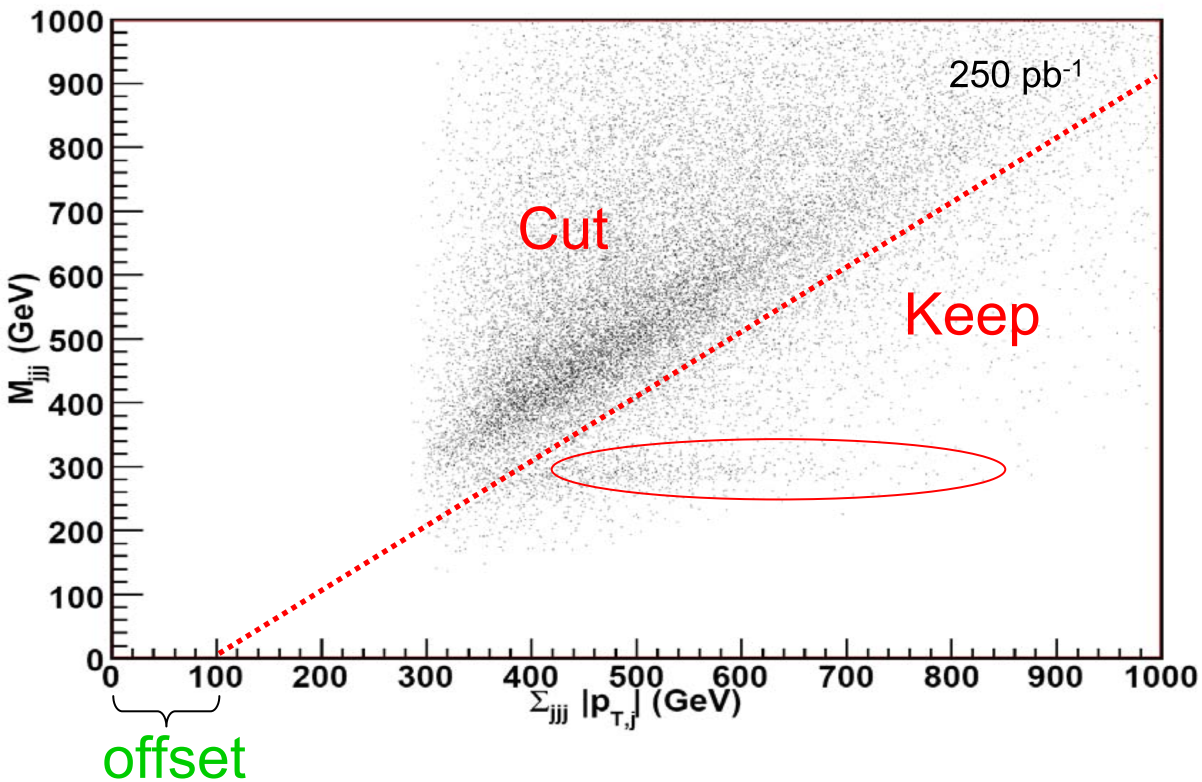
pp → QQ → 6 jets: Cuts

- Use Correlation: M_{ijk} vs $\sum_{i=1}^{3} |p_{T,i}|$

Keep Horizontal Branch: i.e. keep *any* of best 16 Triplets with

$M_{ijk} < \sum_{i=1}^{3} |p_{T,i}|$ - offset,

where offset = 0 GeV, 100 GeV, 200 GeV, or 300 GeV



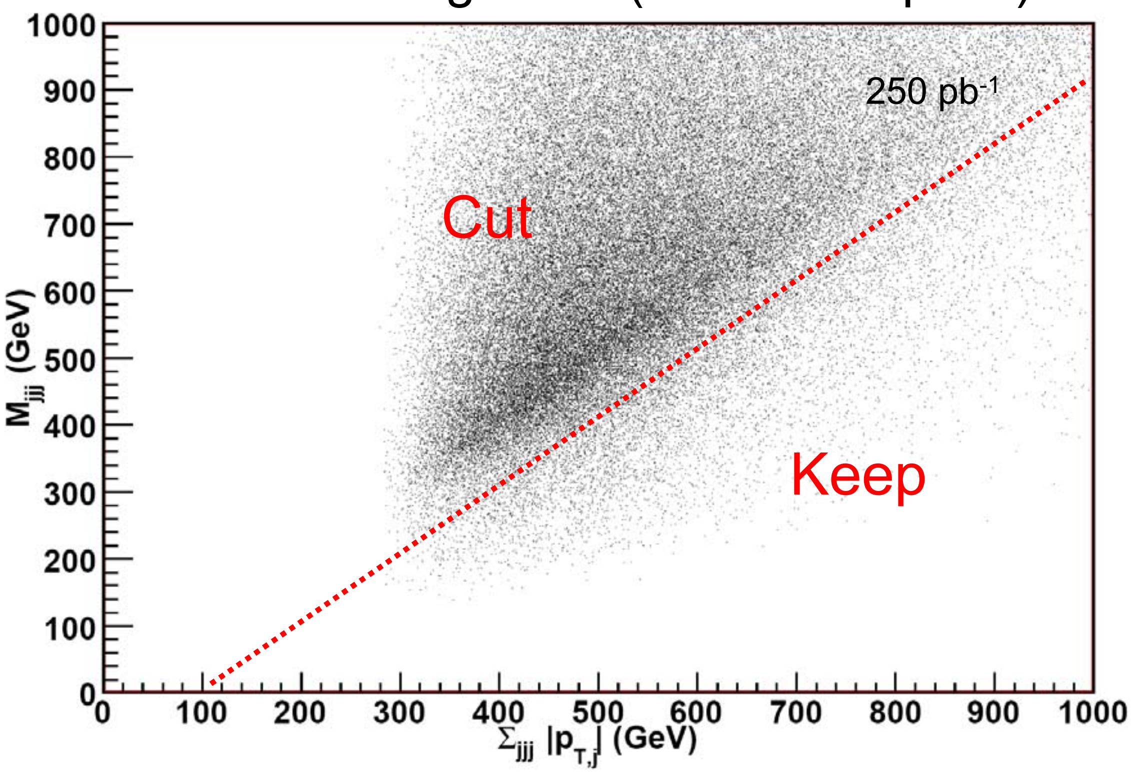
m_Q = 290 GeV
(Best 16 Triplets)

pp → QQ → 6 jets: QCD background

- No Kinematic Feature in QCD background

  e.g. M_{jjj} vs $\sum_{i=1}^{3} |p_{T,i}|$

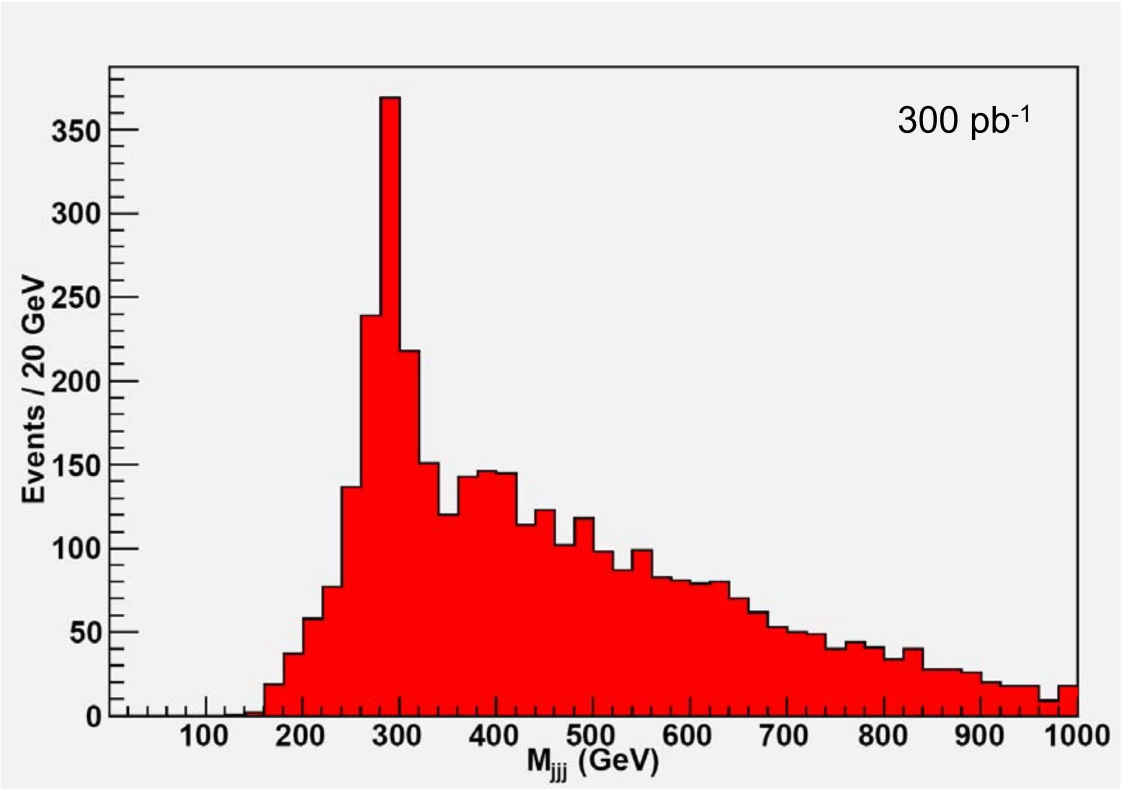
QCD background (Best 16 Triplets)



Cut removes combinatoric background within Signal & QCD background

pp → QQ → 6 jets: Results for Signal

m_Q = 290 GeV, $p_{T,6}$ > 90 GeV, $\sum_{i=1}^{6} p_{T,i}$ > 600 GeV, offset = 100



Much better!

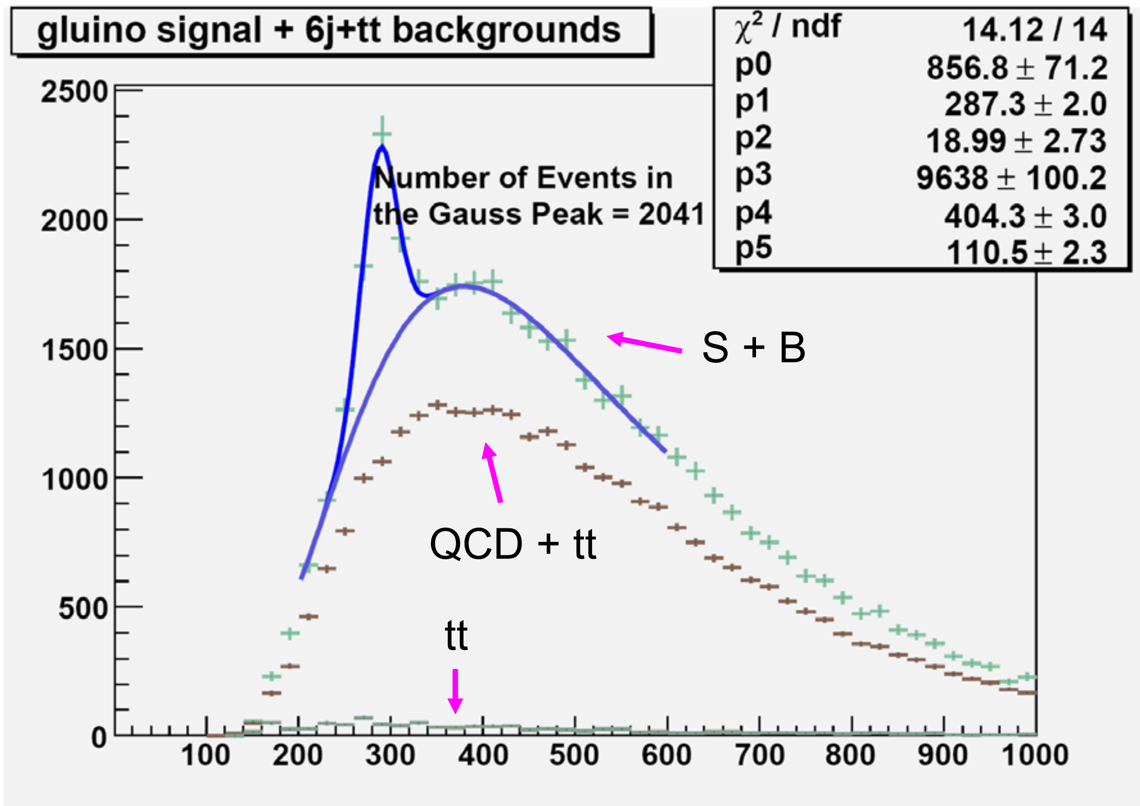
Note:
- Number of entries per event is between 0 and 16
- Average Diagonal Cut Yield for:
  - Signal: 1.66 triplets/event
  - Background: 1.03 triplets/event

<u>Fit</u>
Signal: Gaussian + Landau
Background: Landau

+ do pseudo-experiments

pp → QQ → 6 jets: Results for Signal + Background

m_Q = 290 GeV, $p_{T,6}$ > 90 GeV, $\sum_{i=1}^{6} p_{T,i}$ > 600 GeV, offset = 100



1 fb^{-1}

$S_{16}^{res}/B_{16}^{res}$ = 0.36

$S_{16}^{res}/\sqrt{B_{16}^{res}}$

$0.1\ fb^{-1}$	$1\ fb^{-1}$	$10\ fb^{-1}$
7.9	24	76

pp → QQ → 6 jets: All Results

m_Q (GeV)	$\sum_{j=1}^{6} p_{T,j}$ Cut (GeV)	$p_{T,6}$ Cut (GeV)	Diagonal Cut (GeV)	Diagonal Cut Yield on 16-Ensemble QQ (QCD)	$S_{16}^{res}/B_{16}^{res}$ All Cuts	$S_{16}^{res}/\sqrt{B_{16}^{res}}$ 0.1 fb^{-1}	1 fb^{-1}	10 fb^{-1}
290	600	30	-100	1.53 (0.74)	0.01	3.4	11	34
290	600	60	-100	1.40 (0.76)	0.09	7.1	23	72
290	600	90	-100	1.66 (1.03)	0.36	7.9	24	76
290	600	30	-200	0.43 (0.15)	0.02	2.8	8.9	28
290	600	60	-200	0.42 (0.19)	0.16	5.9	19	60
290	600	90	-200	0.63 (0.36)	0.50	6.6	20	63
290	600	60	-300	0.14 (0.05)	0.25	4.5	14	42
290	600	90	-300	0.23 (0.12)	0.72	5.0	15	45
420	700	30	-200	0.47 (0.21)	0.01	1.1	3.4	11
420	700	60	-200	0.45 (0.23)	0.06	2.4	7.5	24
420	700	90	-200	0.60 (0.36)	0.22	3.1	9.7	31
420	700	120	-200	0.89 (0.55)	0.51	3.1	9.9	32
420	700	60	-300	0.15 (0.06)	0.09	1.8	5.9	18
420	700	90	-300	0.23 (0.12)	0.32	2.6	8.5	27
420	700	120	-300	0.39 (0.23)	0.65	2.6	8.7	28
420	700	60	-400	0.06 (0.02)	0.12	1.4	4.5	14
420	700	90	-400	0.09 (0.04)	0.43	2.0	6.7	21
420	700	120	-400	0.17 (0.09)	0.77	1.9	6.5	21
660	1100	60	-200	0.94 (0.61)	0.02	0.5	1.5	4.8
660	1100	90	-200	0.88 (0.55)	0.06	0.7	2.3	7.5
660	1100	120	-200	0.89 (0.60)	0.15	0.8	2.7	8.8
660	1100	60	-300	0.34 (0.20)	0.02	0.4	1.3	4.1
660	1100	90	-300	0.33 (0.20)	0.09	0.6	2.0	6.3
660	1100	120	-300	0.37 (0.25)	0.21	0.8	2.3	7.5
660	1100	60	-400	0.13 (0.07)	0.04	0.4	1.0	3.3
660	1100	90	-400	0.13 (0.07)	0.12	0.5	1.6	5.2
660	1100	120	-400	0.16 (0.10)	0.31	0.7	2.0	6.5

Summary

↓

m_Q ~ 300 GeV can be detected with only ~ 100 pb^{-1}

m_Q ~ 400-450 GeV can be detected with ~ 1 fb^{-1}

m_Q ~ 650-700 GeV can be detected with ~ 10 fb^{-1}

(Background uncertain)
(Problem at very low m_Q)

Comments about Tevatron

- Can do this analysis also at Tevatron
- In fact: for low m_Q, trigger level cuts at LHC wipe out signal

  → *need* Tevatron to search for low m_Q
- Rutgers CDF group looking for six-jet signal in Tevatron data
- Currently first trying to find all-hadronic top signal using this analysis method - difficult, since at Tevatron don't have many boosted top events

  (This is easier at LHC, perhaps possible even without b-tagging and W mass resonance…)
- Stay tuned!

Conclusions

- New Physics searches with Multi-Jet Signals are possible
- Kinematic Cuts and Features/Correlations allow extraction of signal
  pp → QQ → 6 jets from background
- Use an Ensemble of reconstructed objects (here: jets) to
  increase signal efficiency
- Reach (LHC): m_Q ~ 300 GeV with 100 pb^{-1}
  ~ 400-450 GeV with 1 fb^{-1}
  ~ 650-700 GeV with 10 fb^{-1}
- PGS analysis robust when compared with full CMSSW analysis
- Need Tevatron for low m_Q
- Rutgers HEX group is doing analysis on Tevatron data and will soon
  do analysis on real LHC data
- Techniques more widely applicable (multi-jets w/ leptons + MET, t-$\bar{t}$, etc.)

Backup Slides

Several possibilities for Multi-Jet Production:

Lorentz and gauge invariant color flow restrictions on production of new heavy particle Q

	$SU(3)_C$	Lowest Multiplicity Production Modes	Low Multiplicity Decay Modes	Hadron Collider Signatures
Fermion	**8**	gg , $q\overline{q} \to QQ$	$Q \to qqq$, $\overline{qqq}$	$(jjj)(jjj)$
	6 , $\overline{\mathbf{6}}$	$g\overline{q} \to Q$, $gq \to \overline{Q}$ gg , $q\overline{q} \to Q\overline{Q}$	$Q \to g\overline{q}$, $\overline{Q} \to gq$ $Q \to q\overline{q}\overline{q}$, $\overline{Q} \to \overline{q}qq$	(jj) (jjj) $(jj)(jj)$ $(jjj)(jj)$ $(jjj)(jjj)$
	3 , $\overline{\mathbf{3}}$	$g\overline{q} \to \overline{Q}$, $gq \to Q$ gg , $q\overline{q} \to Q\overline{Q}$	$Q \to gq$, $\overline{Q} \to g\overline{q}$ $Q \to qq\overline{q}$, $\overline{Q} \to \overline{q}\overline{q}q$	(jj) (jjj) $(jj)(jj)$ $(jjj)(jj)$ $(jjj)(jjj)$
Scalar	**8**	gg , $q\overline{q} \to Q$ gg , $q\overline{q} \to QQ$	$Q \to gg$, $q\overline{q}$ $Q \to ggg$, $gq\overline{q}$	(jj) (jjj) $(jj)(jj)$ $(jjj)(jj)$ $(jjj)(jjj)$
	6 , $\overline{\mathbf{6}}$	$qq \to Q$, $\overline{q}\overline{q} \to \overline{Q}$ gg , $q\overline{q} \to Q\overline{Q}$	$Q \to qq$, $\overline{Q} \to \overline{q}\overline{q}$ $Q \to gqq$, $\overline{Q} \to g\overline{q}\overline{q}$	(jj) (jjj) $(jj)(jj)$ $(jjj)(jj)$ $(jjj)(jjj)$
	3 , $\overline{\mathbf{3}}$	$\overline{q}\overline{q} \to Q$, $qq \to \overline{Q}$ gg , $q\overline{q} \to Q\overline{Q}$	$Q \to \overline{q}\overline{q}$, $\overline{Q} \to qq$ $Q \to g\overline{q}\overline{q}$, $\overline{Q} \to gqq$	(jj) (jjj) $(jj)(jj)$ $(jjj)(jj)$ $(jjj)(jjj)$

Many models:
e.g.
Farhi & Susskind (1979),
Marciano (1980),
Konishi & Tripiccione (1983),
Zoupanos (1983),
Lust et. al. (1986),
Frampton & Glashow (1987),
Kang & White (1987),
Fukazawa et. al. (1991),
Martin (1992)…

more recently:
Dobrescu, Kong, Mahbubani (2007)
Kilic, Okui, Sundrum (2008)

Several possibilities for Multi-Jet Production:

- Focus on this

$pp \rightarrow QQ \rightarrow 3j+3j = 6j$

$Q = SU(3)_C$ octet fermion

(e.g. gluino in MSSM)

Chivukula, Golden, Simmons 1991

- But Techniques useful also for Other Cases

- SM background is pure QCD and all-hadronic decay of $t\text{-}\bar{t}$

	$SU(3)_C$	Lowest Multiplicity Production Modes	Low Multiplicity Decay Modes	Hadron Collider Signatures
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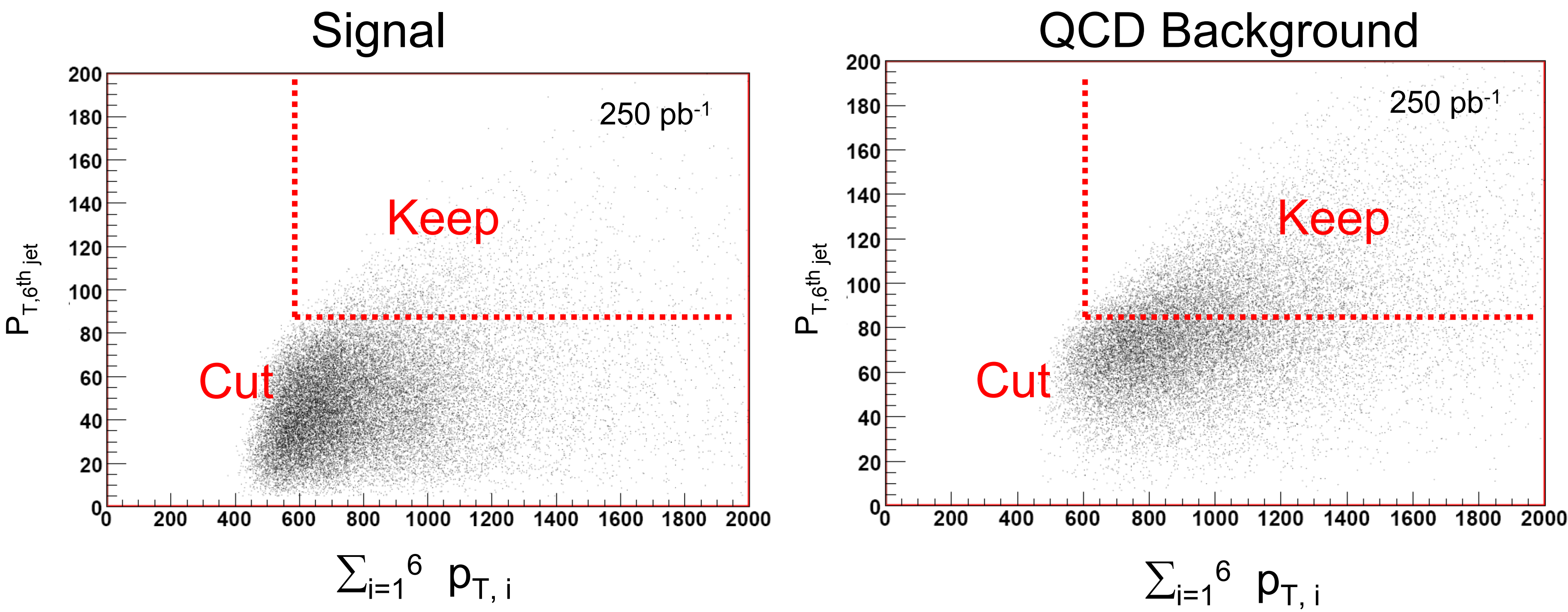
	$\mathbf{6}\ ,\ \overline{\mathbf{6}}$	$g\bar{q} \rightarrow Q\ ,\ gq \rightarrow Q$ $gg\ ,\ q\bar{q} \rightarrow Q\overline{Q}$	$Q \rightarrow g\bar{q}\ ,\ \overline{Q} \rightarrow gq$ $Q \rightarrow q\overline{qq}\ ,\ \overline{Q} \rightarrow \overline{qq}q$	(jj) (jjj) $(jj)(jj)$ $(jjj)(jj)$ $(jjj)(jjj)$
	$\mathbf{3}\ ,\ \overline{\mathbf{3}}$	$g\bar{q} \rightarrow \overline{Q}\ ,\ gq \rightarrow Q$ $gg\ ,\ q\bar{q} \rightarrow Q\overline{Q}$	$Q \rightarrow gq\ ,\ \overline{Q} \rightarrow g\bar{q}$ $Q \rightarrow q\overline{qq}\ ,\ \overline{Q} \rightarrow \overline{qq}q$	(jj) (jjj) $(jj)(jj)$ $(jjj)(jj)$ $(jjj)(jjj)$
Scalar	$\mathbf{8}$	$gg\ ,\ q\bar{q} \rightarrow Q$ $gg\ ,\ q\bar{q} \rightarrow QQ$	$Q \rightarrow gg\ ,\ q\bar{q}$ $Q \rightarrow ggg\ ,\ gq\bar{q}$	(jj) (jjj) $(jj)(jj)$ $(jjj)(jj)$ $(jjj)(jjj)$
	$\mathbf{6}\ ,\ \overline{\mathbf{6}}$	$qq \rightarrow Q\ ,\ \overline{qq} \rightarrow \overline{Q}$ $gg\ ,\ q\bar{q} \rightarrow Q\overline{Q}$	$Q \rightarrow qq\ ,\ \overline{Q} \rightarrow \overline{qq}$ $Q \rightarrow gqq\ ,\ \overline{Q} \rightarrow g\overline{qq}$	(jj) (jjj) $(jj)(jj)$ $(jjj)(jj)$ $(jjj)(jjj)$
	$\mathbf{3}\ ,\ \overline{\mathbf{3}}$	$\overline{qq} \rightarrow Q\ ,\ qq \rightarrow \overline{Q}$ $gg\ ,\ q\bar{q} \rightarrow Q\overline{Q}$	$Q \rightarrow \overline{qq}\ ,\ \overline{Q} \rightarrow qq$ $Q \rightarrow g\overline{qq}\ ,\ \overline{Q} \rightarrow gqq$	(jj) (jjj) $(jj)(jj)$ $(jjj)(jj)$ $(jjj)(jjj)$

pp → QQ → 6 jets: Kinematic Cuts

Example: m_Q = 290 GeV, $p_{T,6} > 90$ GeV, $\sum_{i=1}^{6} p_{T,i} > 600$ GeV



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m_Q=890 GeV	0.5	100000	215.5
6j, $p_T > 20$	407000	315069	0.00077
6j, $p_T > 50$	2400	434011	0.2
6j, $p_T > 80$	120	507428	4.2
6j, $p_T > 110$	14	448520	32.3
$t\bar{t}$	490	490000	1.0

~3-4 events / sec

~1 event every 200s

Background >> Signal !!!

~5 events / sec

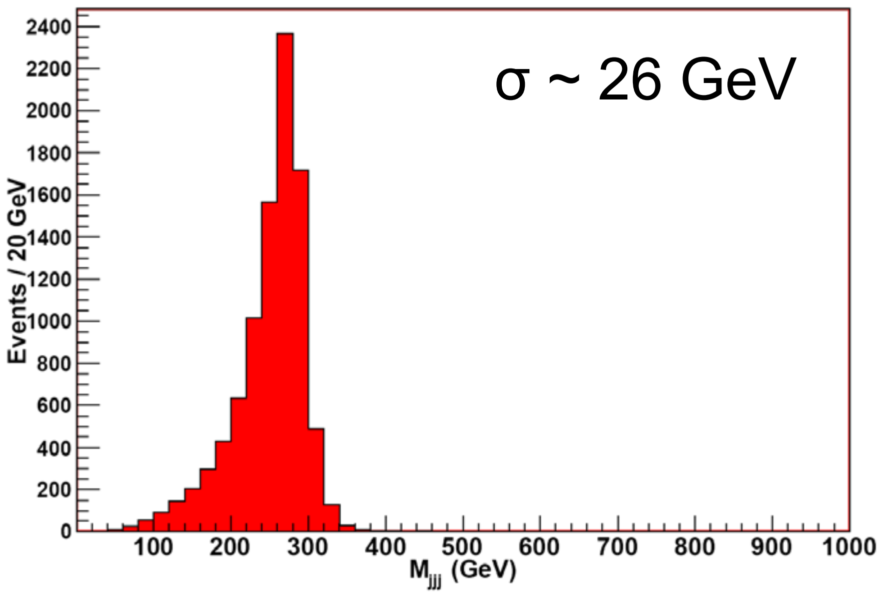
~ 4100 events / sec

Comparing Detector Simulations PGS 4 with CMSSW

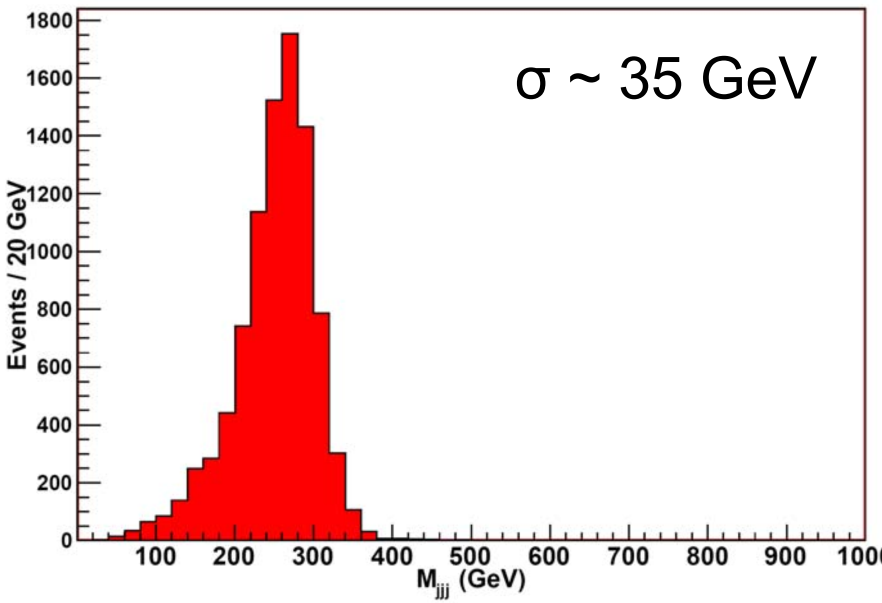
Compare width of resonance of MC matched triplets

Example: m_Q = 290 GeV

PGS 4



PGS 4 smeared to match CMSSW



PGS 4: $\sigma(E_T^{\text{jet}}) = 0.8 \times \sqrt{E_T^{\text{jet}}}$

CMSSW: $\sigma(E_T^{\text{jet}}, |\eta| < 1.4) = (5.8\ \text{GeV}) \oplus (1.25 \times \sqrt{\mathrm{E_T^{jet}}}) \oplus (0.033 \times \mathrm{E_T^{jet}})$

(CMS TDR)

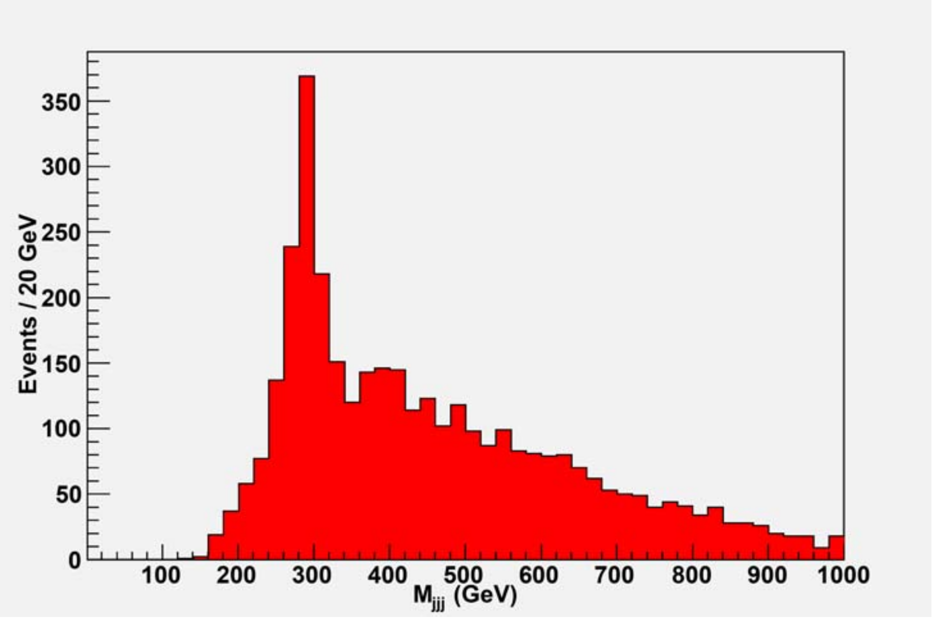
CMSSW wider than PGS 4, but not by too much

Comparing Detector Simulations PGS 4 with CMSSW

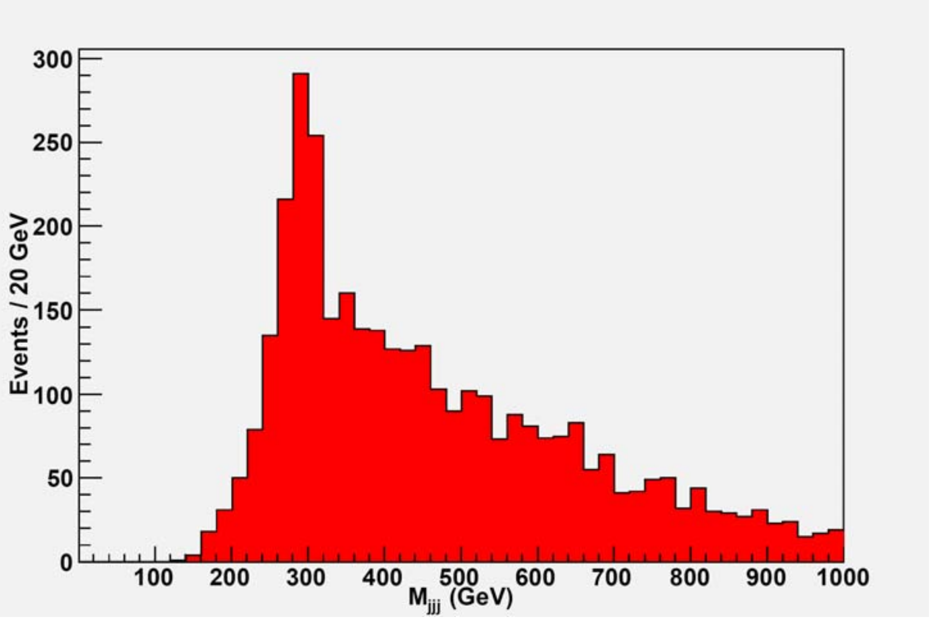
Effect of smearing on analysis cuts

Example: m_Q = 290 GeV

PGS 4



PGS 4 smeared to match CMSSW



Effect on analysis not significant

pp → QQ → 6 jets: All Results (PGS 4 + smearing to match CMSSW)

m_Q (GeV)	$\sum_{j=1}^{6} p_{T,j}$ Cut (GeV)	$p_{T,6}$ Cut (GeV)	Diagonal Cut (GeV)	Diagonal Cut Yield on 16-Ensemble QQ (QCD)	$S_{16}^{res}/B_{16}^{res}$ All Cuts	$S_{16}^{res}/\sqrt{B_{16}^{res}}$ 0.1 fb^{-1}	$S_{16}^{res}/\sqrt{B_{16}^{res}}$ 1 fb^{-1}	$S_{16}^{res}/\sqrt{B_{16}^{res}}$ 10 fb^{-1}
290	600	30	-100	1.53 (0.61)	0.02	4.1	19	41
290	600	60	-100	1.39 (0.77)	0.06	5.4	17	55
290	600	90	-100	1.69 (1.04)	0.26	6.7	20	63
290	600	30	-200	0.43 (0.14)	0.03	3.7	11	35
290	600	60	-200	0.41 (0.20)	0.11	5.2	16	50
290	600	90	-200	0.65 (0.36)	0.38	6.3	18	58
290	600	60	-300	0.13 (0.05)	0.18	4.5	13	39
290	600	90	-300	0.24 (0.12)	0.46	4.8	13	41
420	700	30	-200	0.48 (0.18)	0.01	1.6	4.7	15
420	700	60	-200	0.45 (0.23)	0.05	2.2	6.6	21
420	700	90	-200	0.60 (0.37)	0.17	3.0	8.7	27
420	700	120	-200	0.89 (0.56)	0.42	3.5	9.7	30
420	700	60	-300	0.15 (0.06)	0.06	1.6	4.4	14
420	700	90	-300	0.23 (0.12)	0.22	2.5	6.8	21
420	700	120	-300	0.39 (0.23)	0.53	3.2	7.8	24
420	700	60	-400	0.06 (0.02)	0.08	1.4	3.6	11
420	700	90	-400	0.09 (0.04)	0.29	2.3	5.5	17
420	700	120	-400	0.17 (0.09)	0.66	2.4	6.5	19
660	1100	60	-200	0.94 (0.61)	0.01	0.6	1.4	4.1
660	1100	90	-200	0.88 (0.55)	0.04	1.0	2.2	6.2
660	1100	120	-200	0.90 (0.60)	0.11	1.7	2.7	7.5
660	1100	60	-300	0.35 (0.20)	0.02	0.6	1.2	3.5
660	1100	90	-300	0.34 (0.20)	0.06	1.1	1.9	5.0
660	1100	120	-300	0.38 (0.26)	0.17	1.7	2.5	6.7
660	1100	60	-400	0.13 (0.07)	0.02	0.7	1.0	2.7
660	1100	90	-400	0.14 (0.07)	0.08	1.7	1.6	4.2
660	1100	120	-400	0.17 (0.10)	0.22	2.0	2.3	5.9

19 with PGS 4

0.65 with PGS 4

4.8 with PGS 4

Results qualitatively unchanged

Analysis with PGS 4 robust