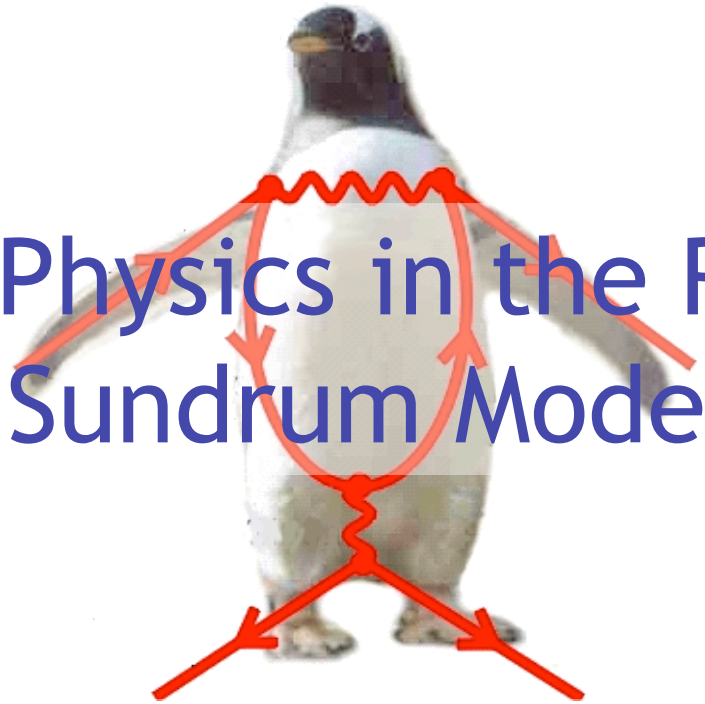

Flavor Physics in the Randall-Sundrum Model

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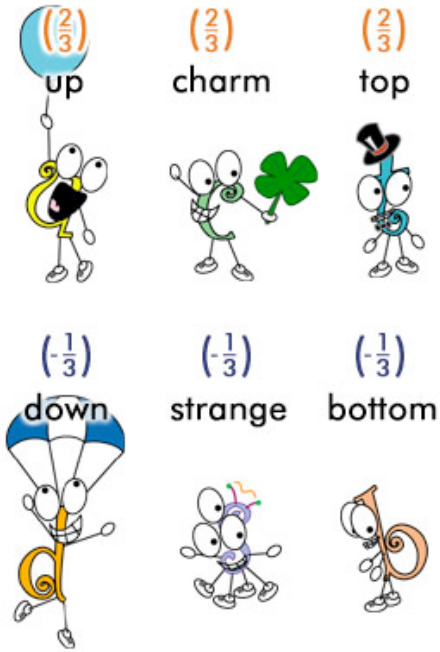
(arXiv:0807.4537 and paper in preparation)

Flavor in RS

- Already many studies in literature, in particular:
  - Huber, Shafi 1999; Huber 2003
  - Burdman 2003
  - Agashe, Perez, Soni 2004
  - Agashe, Papucci, Perez, Pirjol 2005
  - Davidson, Isidori, Uhlig 2007
  - Csaki, Falkowski, Weiler 2008



- Here:
  - first complete study of all tree-level processes
  - use exact solutions to bulk equations (some important $\Delta F=1$ effects neglected so far)
  - include correlations and RG evolution at NLO

Flavor in RS

- Study simplest (original) model based on S^1/Z_2 orbifold, Higgs on TeV brane, minimal particle content, and SM gauge group
  - generalization to $SU(2)_L \times SU(2)_R \times U(1)_X$ (custodial symmetry) straightforward [Agashe, Delgado, May, Sundrum 2003]
  - generalization to Higgs in bulk possible [Agashe, Contino, Pomarol 2004]
- Even in simplest case rich structure of flavor effects arises, some of which are subtle
- Find that complicated extensions not necessarily required by EWPT

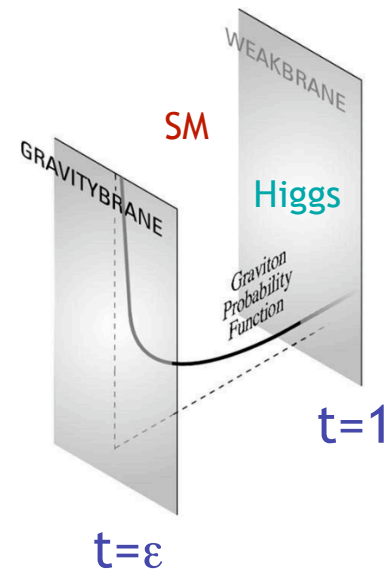
Notations and parameters

- Warp factor:

$$\varepsilon = e^{-kr\pi} \approx 10^{-16} \qquad L = -\ln(\varepsilon) \approx 37$$

- Variable along extra dimension:

$$t = \varepsilon\, e^{-kr|\phi|}$$



- KK scale:

$$M_{KK} = k\varepsilon = O(\text{TeV}) \qquad M_{g1} \approx 2.45\, M_{KK}$$

- Fermion c_i parameters:

$$c_{Qi} = m_{Qi}/k,\ c_{qi} = -m_{qi}/k$$

Bulk fermions

- KK decomposition (up-quark sector):

$$u_L(x,\phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \boldsymbol{C}_n^{(Q)}(\phi)\, a_n^{(U)}\, u_L^{(n)}(x)\,, \qquad u_R(x,\phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \boldsymbol{S}_n^{(Q)}(\phi)\, a_n^{(U)}\, u_R^{(n)}(x)$$

$$u_L^c(x,\phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \boldsymbol{S}_n^{(u)}(\phi)\, a_n^{(u)}\, u_L^{(n)}(x)\,, \qquad u_R^c(x,\phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \boldsymbol{C}_n^{(u)}(\phi)\, a_n^{(u)}\, u_R^{(n)}(x)\,,$$

expansion coefficients (3-comp. vectors for each n)

exact mass eigenstates

  - $C_n(\phi)$ and $S_n(\phi)$ are even/odd solutions to the bulk equations of motion [Grossman, MN 1999; Gherghetta, Pomarol 2000]

Bulk fermions

- Exact field equations (with Yukawa couplings) are incompatible with orthonormality relations for fermion profiles
- Generalized relations:

$$\int_{-\pi}^{\pi} d\phi\, e^{\sigma(\phi)}\, C_m^{(A)}(\phi)\, C_n^{(A)}(\phi) = \delta_{mn} + \Delta C_{mn}^{(A)}$$

$$\int_{-\pi}^{\pi} d\phi\, e^{\sigma(\phi)}\, S_m^{(A)}(\phi)\, S_n^{(A)}(\phi) = \delta_{mn} + \Delta S_{mn}^{(A)}$$

  - extra terms are $O(v/M_{KK})$ for KK modes, but $O(1)$ for light SM particles!

Zero-mode approximation (ZMA)

- Used in all previous work on flavor in RS
- Amounts to keeping leading terms in $x_n = m_n / M_{KK}$ in profiles for light SM fermions:

$$C_n^{(A)}(\phi) \quad \rightarrow \quad \sqrt{\frac{2L\epsilon}{\pi}}\, F(c_A)\, t^{c_A}$$

$$S_n^{(A)}(\phi) \quad \rightarrow \quad 0 \qquad \text{(ZMA)}$$

$t = \varepsilon\, e^{-kr|\phi|}$

with: $\varepsilon = e^{-kr\pi} \approx 10^{-16}$ $\quad L = -\ln(\varepsilon) \approx 37$

$$F(c) = \sqrt{\frac{1+2c}{2(1-\epsilon^{1+2c})}}$$

exponentially small if $c < -1/2$

[Grossman, MN 1999]

- *Important finding*: $S_n(\phi)$ profiles cannot always be neglected, but give rise to some leading effects

Scaling relations

- Effective Yukawa matrices (in ZMA):

$$\boldsymbol{Y}_q^{\text{eff}} = \boldsymbol{F}(c_Q)\,\boldsymbol{Y}_q\,\boldsymbol{F}(c_q) = \boldsymbol{U}_q\,\boldsymbol{\lambda}_q\,\boldsymbol{W}_q^\dagger$$

diagonal

  - natural to assume that Y_q are anarchical matrices
  - flavor hierarchies explained by fact that $F(c_{Q,q})$ profiles are exponentially small and hierarchical (→ Froggatt-Nielsen mechanism)

[Froggatt-Nielsen 1979; Huber, Shafi 2000]

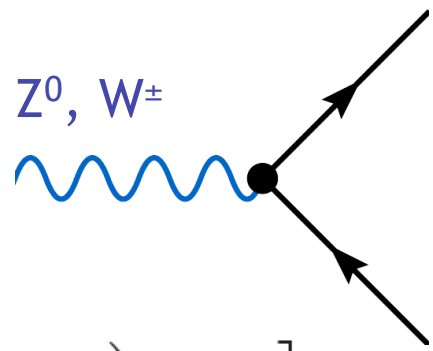
- Relations for quark masses and CKM parameters:

$$m_{q_i} = \mathcal{O}(1) \times \frac{v}{\sqrt{2}}\,F(c_{Q_i})\,F(c_{q_i})$$

$$\lambda = \mathcal{O}(1)\,\frac{F(c_{Q_1})}{F(c_{Q_2})} \qquad A = \mathcal{O}(1)\,\frac{F^3(c_{Q_2})}{F^2(c_{Q_1})\,F(c_{Q_3})} \qquad \bar{\rho},\,\bar{\eta} = \mathcal{O}(1)$$

Gauge interactions with SM fermions

- Couplings of light weak gauge bosons
  - flavor violation from modification of gauge-boson profiles due to EWSB on IR brane



$L = -\ln(\varepsilon) \approx 37$

$$\chi_{W,Z}(\phi) = \frac{1}{\sqrt{2\pi}}\left[1 + \frac{m_{W,Z}^2}{4M_{\rm KK}^2}\left(1 - \frac{1}{L} + t^2\left(1 - 2L - 2\ln t\right)\right) + \ldots\right]$$

[Csaki, Ehrlich, Terning 2002]

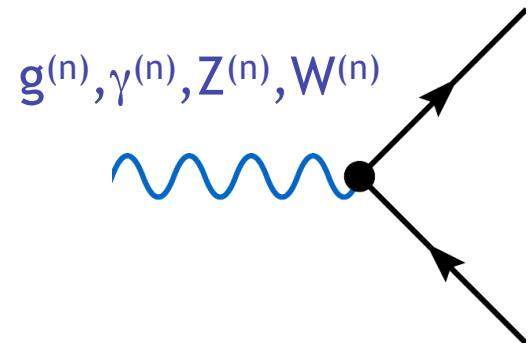
$t = \varepsilon\, e^{-kr|\phi|}$ nontrivial ϕ dependence

  - flavor violation from non-orthonormality of fermion profiles (mixing between $SU(2)_L$ doublets and singlets)

Gauge interactions with SM fermions

- Couplings of KK gauge bosons:
  - flavor violation from nontrivial profiles
  - dominant corrections from KK gluons

  [Burdman 2003; Agashe, Perez, Soni 2004]



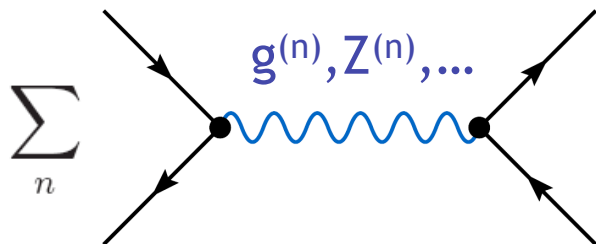
- *Both cases:*
  Fact that flavor-violating effects live near IR brane implies suppression ~ $F(c_i)\ F(c_j)$

RS-GIM mechanism !

[Agashe, Perez, Soni 2004; Agashe, Papucci, Perez, Pirjol 2005]

Summing over KK modes

- Tree-level heavy boson exchange generates local 4-fermion couplings at low energy



- Sums over KK tower can be performed using completeness and orthogonality of gauge boson profiles along with integral form of bulk EOM

[Hirn, Sanz 2007]

Summing over KK modes

- Sum over profiles for KK gluons and photons:

$$\sum_{n=1}^{\infty} \frac{\chi_n(\phi)\,\chi_n(\phi')}{m_n^2} = \frac{1}{4\pi M_{\rm KK}^2}\left[L\,t_<^2 - t^2\left(\frac{1}{2} - \ln t\right) - t'^2\left(\frac{1}{2} - \ln t'\right) + \frac{1}{2L}\right]$$

- Sum over profiles for Z, W and KK excitations:

$$\sum_{n=0}^{\infty} \frac{\chi_n(\phi)\,\chi_n(\phi')}{m_n^2} = \frac{1}{2\pi m_0^2} + \frac{1}{4\pi M_{\rm KK}^2}\left[L\,t_<^2 - L\,(t^2 + t'^2) + 1 - \frac{1}{2L} + \mathcal{O}\left(\frac{m_0^2}{M_{\rm KK}^2}\right)\right]$$

$L = -\ln(\varepsilon) \approx 37$

$\Delta F=2$ processes
(dominated by KK gluons)

$\Delta F=1$ processes
(dominated by Z boson)

Overlap integrals

- Flavor-violating effects described by matrices

$$(\Delta'_Q)_{mn} = \frac{2\pi}{L\epsilon}\int_\epsilon^1 dt\, t^2 \left[a_m^{(Q)\dagger}\, \boldsymbol{C}_m^{(Q)}(\phi)\, \boldsymbol{C}_n^{(Q)}(\phi)\, a_n^{(Q)} + a_m^{(q)\dagger}\, \boldsymbol{S}_m^{(q)}(\phi)\, \boldsymbol{S}_n^{(q)}(\phi)\, a_n^{(q)}\right]$$

$$(\Delta_Q)_{mn} = \frac{2\pi}{L\epsilon}\int_\epsilon^1 dt\, t^2 \left(\frac{1}{2} - \ln t\right) \left[a_m^{(Q)\dagger}\, \boldsymbol{C}_m^{(Q)}(\phi)\, \boldsymbol{C}_n^{(Q)}(\phi)\, a_n^{(Q)} + a_m^{(q)\dagger}\, \boldsymbol{S}_m^{(q)}(\phi)\, \boldsymbol{S}_n^{(q)}(\phi)\, a_n^{(q)}\right]$$

$$(\delta_Q)_{mn} = a_m^{(q)\dagger}\left(\delta_{mn} + \Delta \boldsymbol{S}_{mn}^{(q)}\right) a_n^{(q)}$$

and corresponding matrices Δ'_q, Δ_q, δ_q in right-handed sector

- Hermitian matrices in flavor space

CP violation possible with N≥2 generations!

Overlap integrals

- ZMA gives for SM fermions:

$$\Delta'_Q \to \boldsymbol{U}_q^\dagger \,\mathrm{diag}\left[\frac{2F^2(c_{Q_i})}{3+2c_{Q_i}}\right]\boldsymbol{U}_q$$

$$\Delta_Q \to \boldsymbol{U}_q^\dagger \,\mathrm{diag}\left[\frac{5+2c_{Q_i}}{(3+2c_{Q_i})^2}\,F^2(c_{Q_i})\right]\boldsymbol{U}_q$$

m_q/M_{KK}

$$\boxed{\boldsymbol{\delta}_Q \to \boldsymbol{x}_q\,\boldsymbol{W}_q^\dagger \,\mathrm{diag}\left[\frac{1}{1-2c_{q_i}}\left(\frac{1}{2F^2(c_{q_i})}-1+\frac{2F^2(c_{q_i})}{3+2c_{q_i}}\right)\right]\boldsymbol{W}_q\,\boldsymbol{x}_q}$$

- Scaling relations:

$$\left(\Delta_Q^{(\prime)}\right)_{ij} \sim F(c_{Q_i})\,F(c_{Q_j})\,, \qquad \left(\Delta_q^{(\prime)}\right)_{ij} \sim F(c_{q_i})\,F(c_{q_j})$$

$$(\delta_Q)_{ij} \sim \frac{v^2}{M_{\mathrm{KK}}^2}\,F(c_{Q_i})\,F(c_{Q_j})\,, \qquad (\delta_q)_{ij} \sim \frac{v^2}{M_{\mathrm{KK}}^2}\,F(c_{q_i})\,F(c_{q_j})$$

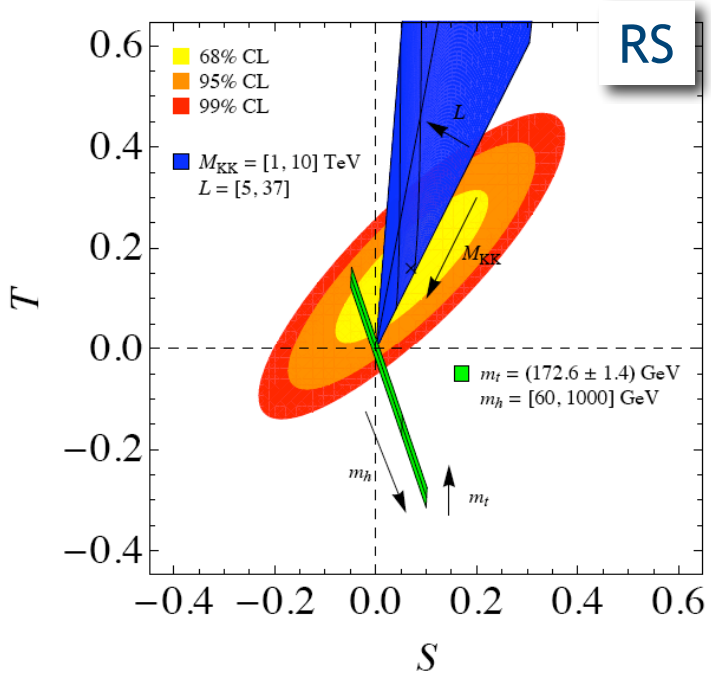
but effects enhanced by M_{KK}^2/m_Z^2 !

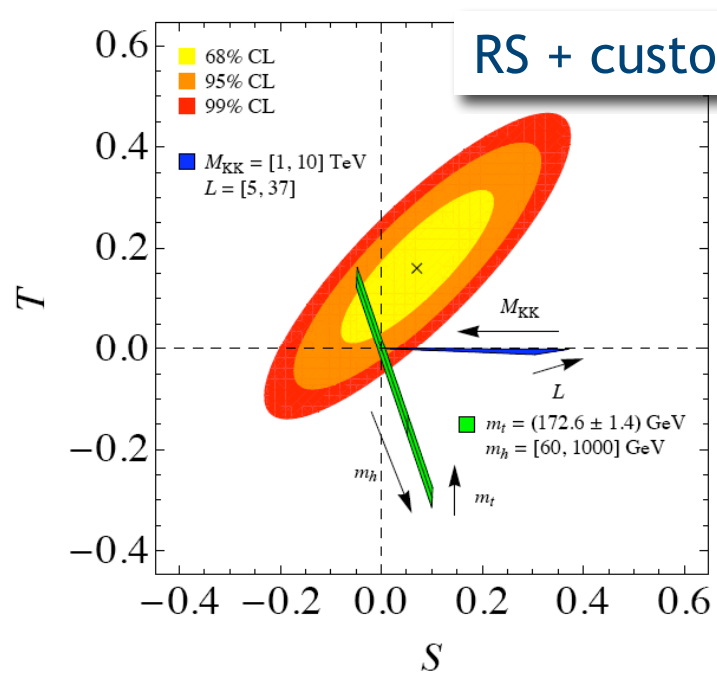
Electroweak Precision Tests

S and T, Zbb couplings, t→cZ

Electroweak precision tests (S,T)

- T requires custodial symmetry or small L (little RS)
  [Agashe, Delgado, May, Sundrum 2003] [Davoudiasl, Perez, Soni 2008]
- Naturally heavy Higgs, m_H=O(TeV), helps:





Electroweak precision tests (S,T)

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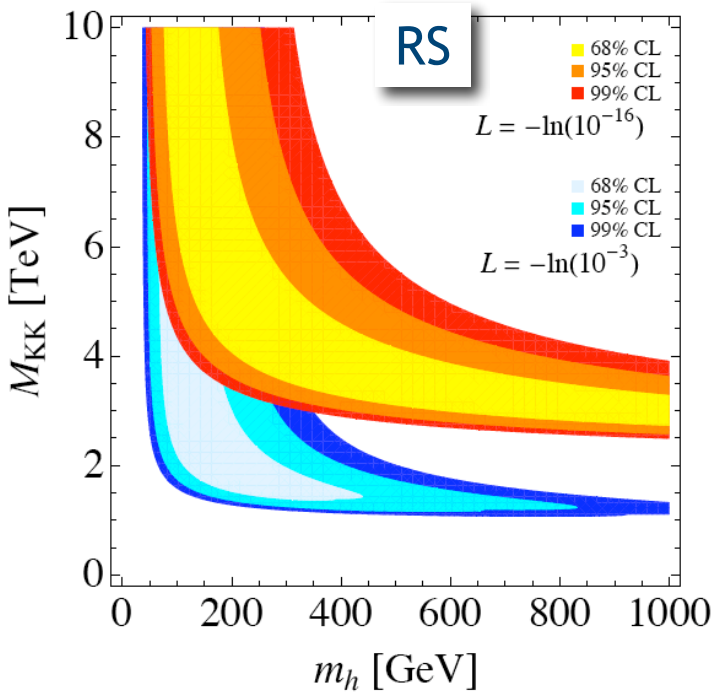
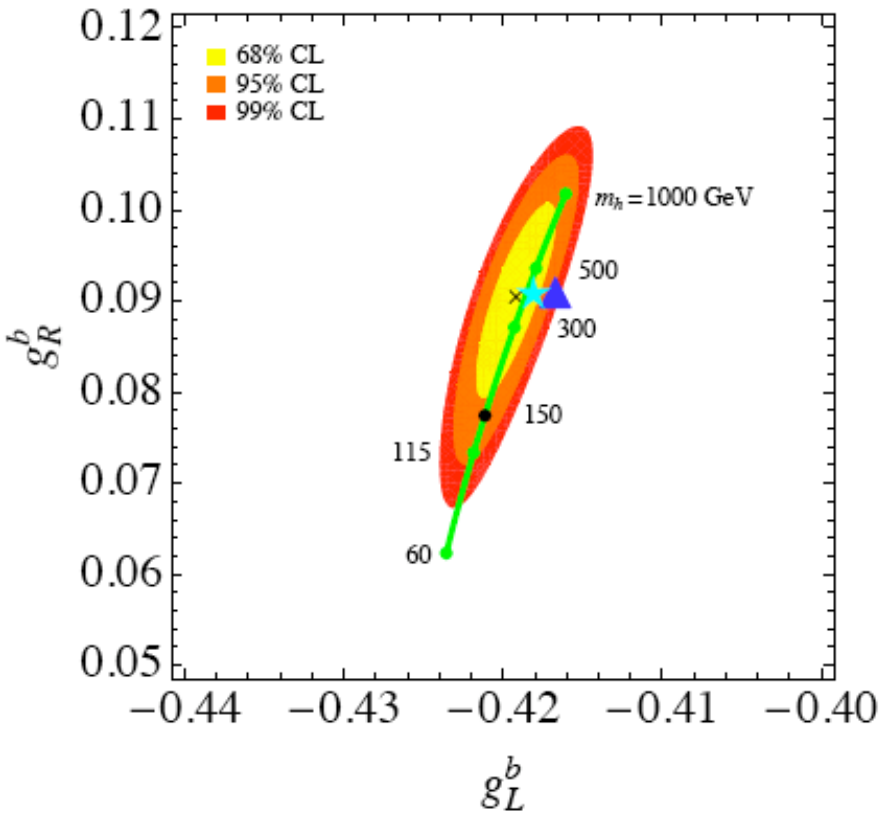


cf. M_{KK}>2.4 TeV in RS model with custodial symmetry

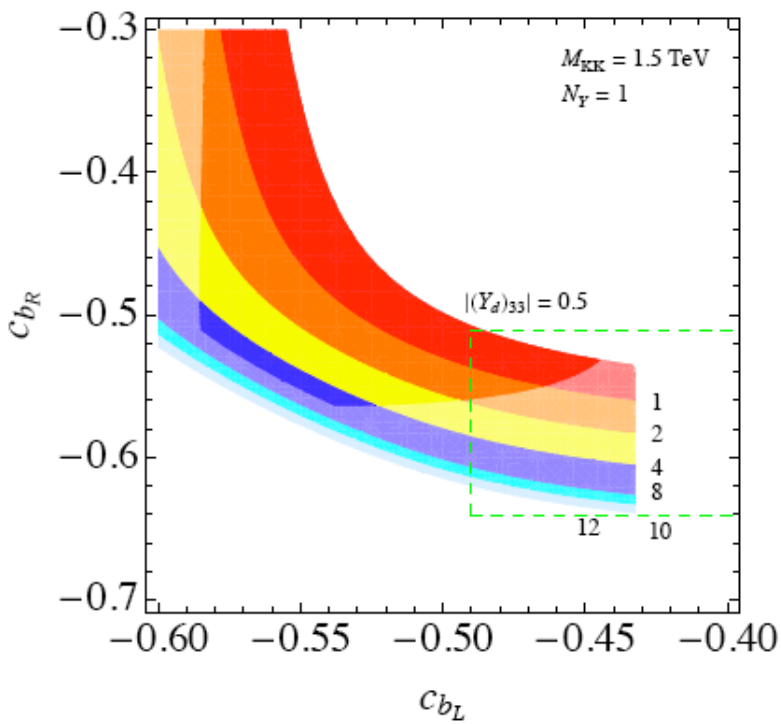
Zbb couplings

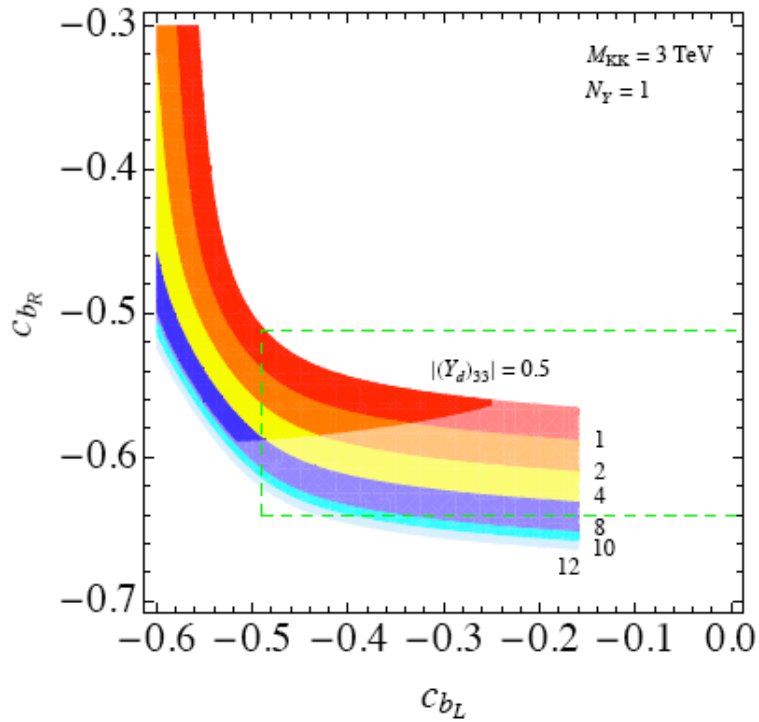
- Improvement compared with SM possible for heavy Higgs (A_{LR}^b)
- Derive bounds requiring that R_b remains less than 3σ away from experiment
- Possibe to satisfy constraints from experimental data for M_{KK} as low as 1 TeV



Zbb couplings

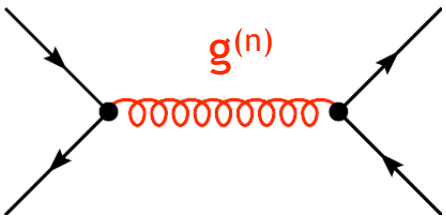
- Allowed range of c_{bi} parameters:



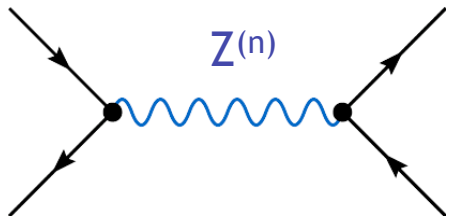


Tree-level FCNCs

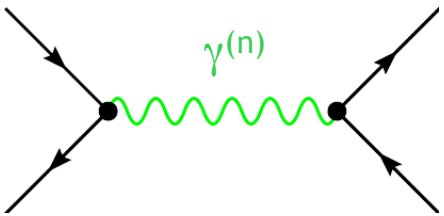
- Generic diagrams:



dominant for $\Delta F=2$ processes



dominant for $\Delta F=1$ processes

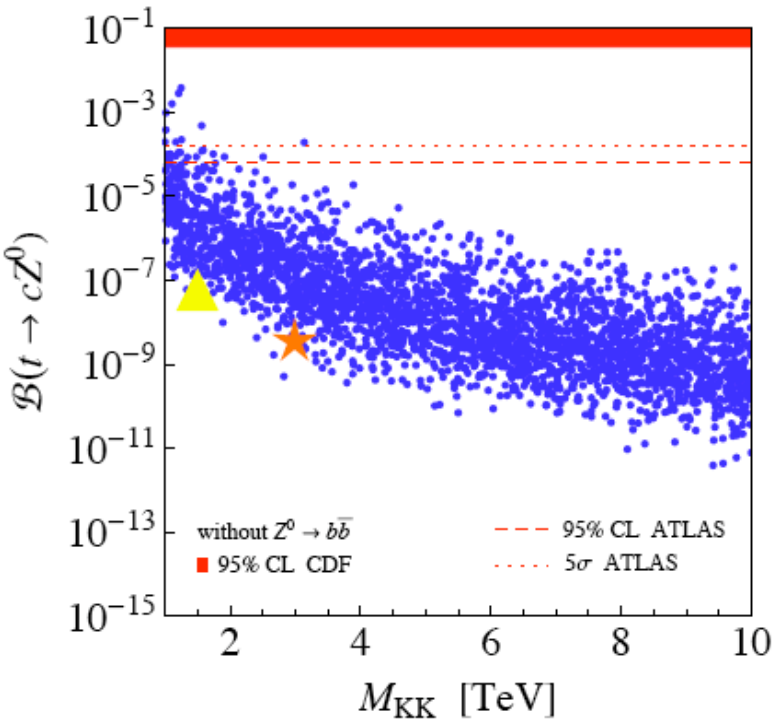


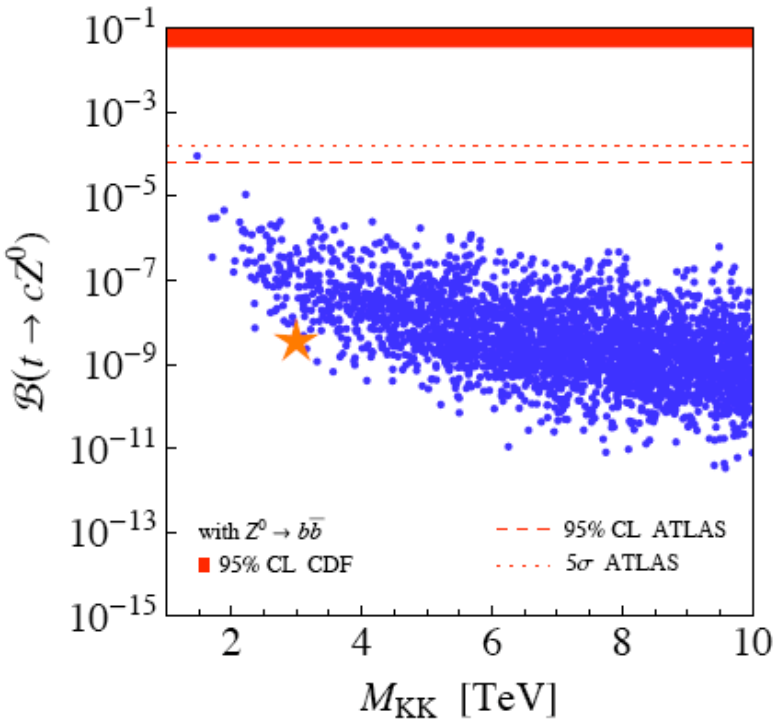
always small

  - likely to give largest flavor-violating effects
- Will consider both $\Delta F=2$ (mixing) and $\Delta F=1$ (rare decays) processes
- No tree-level contribution to $B \to X_s \gamma$

Rare decay t→cZ

- Unlikely to find effects at LHC except for very light KK scales:





$\Delta F=2$ Processes

K, B_d, B_s, D mixing

Neutral meson mixing

- Effective weak Hamiltonian:

$$H_{\rm eff}^{\Delta S=2} = \sum_{i=1}^{5} C_i\, Q_i^{sd} + \sum_{i=1}^{3} \tilde{C}_i\, \tilde{Q}_i^{sd}$$

- Coefficients in RS (examples):

$$C_1^{\rm RS} = \frac{4\pi L}{M_{\rm KK}^2} \left(\widetilde{\Delta}_D\right)_{12} \otimes \left(\widetilde{\Delta}_D\right)_{12} \left[\frac{\alpha_s}{2}\left(1-\frac{1}{N_c}\right) + Q_d^2\,\alpha + \frac{(T_3^d - \sin^2\theta_W\, Q_d)^2\,\alpha}{\sin^2\theta_W\,\cos^2\theta_W}\right]$$

$$C_4^{\rm RS} = \frac{4\pi L}{M_{\rm KK}^2} \left(\widetilde{\Delta}_D\right)_{12} \otimes \left(\widetilde{\Delta}_d\right)_{12} \left[-2\alpha_s\right]$$

gluon contribution agrees with:
[Csaki, Falkowski, Weiler 2008]

  - for CP violation in K mixing (ε_K), KK gluon contribution to C_4 is by far dominant

Neutral meson mixing

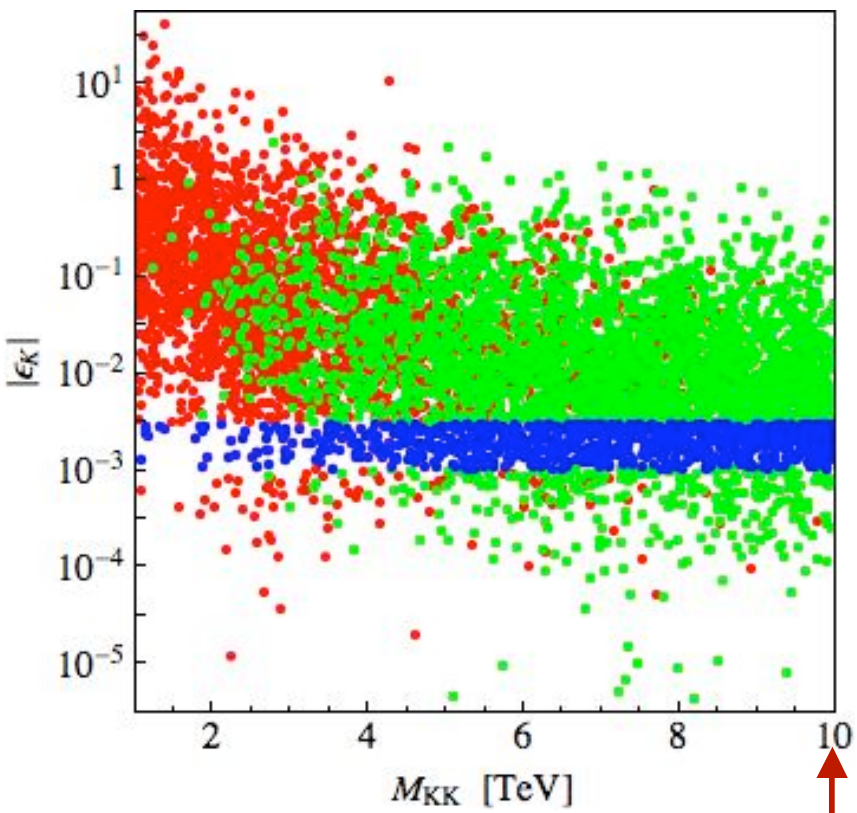
- Presence of tree-level FCNCs mediated by vector bosons potentially disastrous [Bona et al. (UT*fit*) 2007]
- Recent analysis finds typical bound M_{KK}>10 TeV (KK gluon mass >21 TeV) [Csaki, Falkowski, Weiler 2008]
- Reason is enhancement of $\langle Q_4 \rangle$ matrix element from RG evolution and chiral factor $\sim(m_K/m_s)^2$
- In RS model: [Burdman 2003; Agashe, Perez, Soni 2004]

$$[\Delta\epsilon_K]_{\rm RS} \simeq \boxed{-3.8\cdot 10^{-3}} \frac{{\rm Im}\,[(\Delta_D)_{12}(\Delta_d)_{12}]}{10^{-12}\,M_{\rm KK}^2\,[{\rm TeV}^2]} + 5.2\cdot 10^{-6}\,\frac{{\rm Im}\,[(\Delta_D)_{12}^2+(\Delta_d)_{12}^2]}{10^{-12}\,M_{\rm KK}^2\,[{\rm TeV}^2]}$$

  - only second term included in early papers

Neutral meson mixing

- Parameter scan assuming $|Y_{ij}|<3$, $|F(c_i)|<\sqrt{2}$:



red: all points

green: after Zbb constraint

blue: after $|\varepsilon_K|$ constraint
(requires some fine-tuning of 5D Yukawas)

Csaki, Falkowski, Weiler (2008):

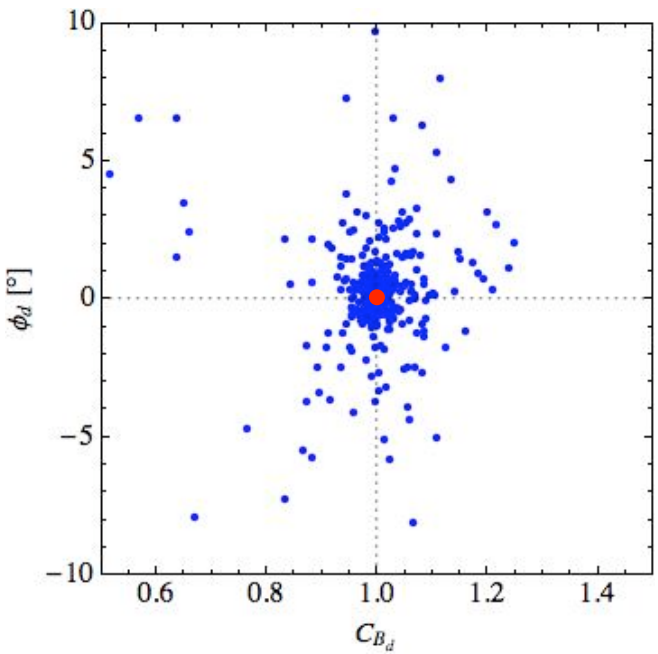
"lower bound"

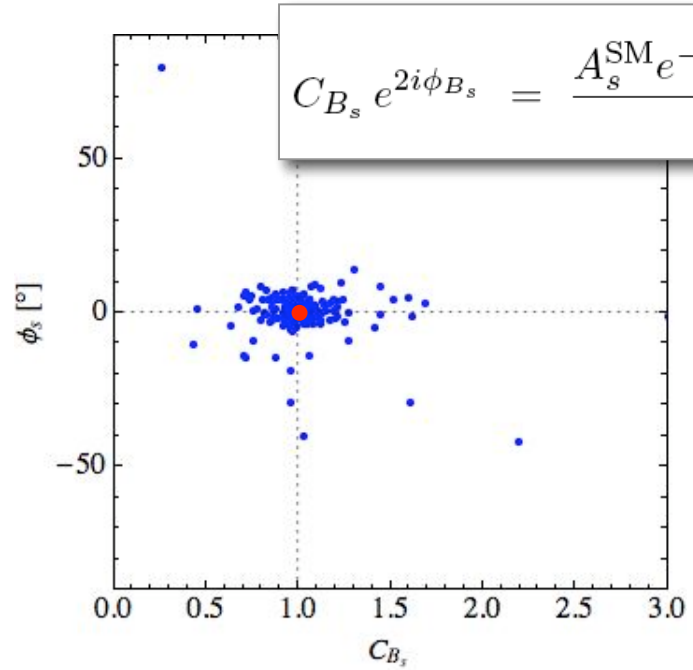
Neutral meson mixing

- Possible ways to reduce fine-tuning:
  - lower L significantly ("little RS"): [Davoudiasl, Perez, Soni 2008]
    e.g., UV scale at 1000 TeV lowers "bound" to M_{KK}>4 TeV
  - impose bulk flavor symmetry to get hierarchical Yukawas (Y_d~1) [Cacciapaglia et al. 2007; Fitzpatrick, Perez, Randall 2007]
  - impose equality of c_i parameters in right-handed down sector ("minimal flavor protection"): $c_{d1}=c_{d2}=c_{d3}$ [Santiago 2008]
  - increase 5D Yukawa couplings to O(10), lowering KK scale to O(1 TeV)
- Assume fine-tuning for now (more in paper …)

Neutral meson mixing

- Either solution leaves large effects in $\Delta F=1$ rare decays as well as $B_{d,s}$ and D mixing
- $B_{d,s}$ mixing amplitudes after $|\varepsilon_K|$ constraint:



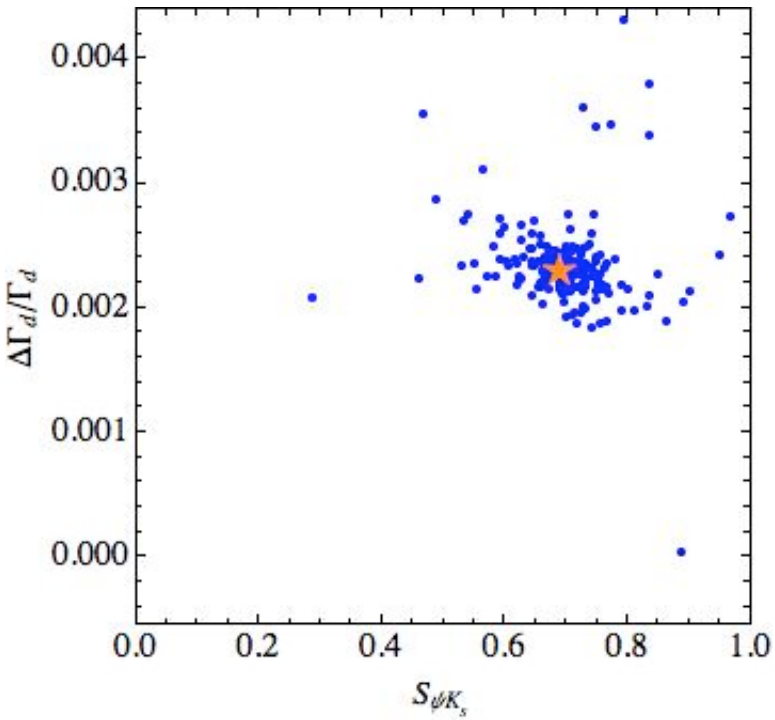


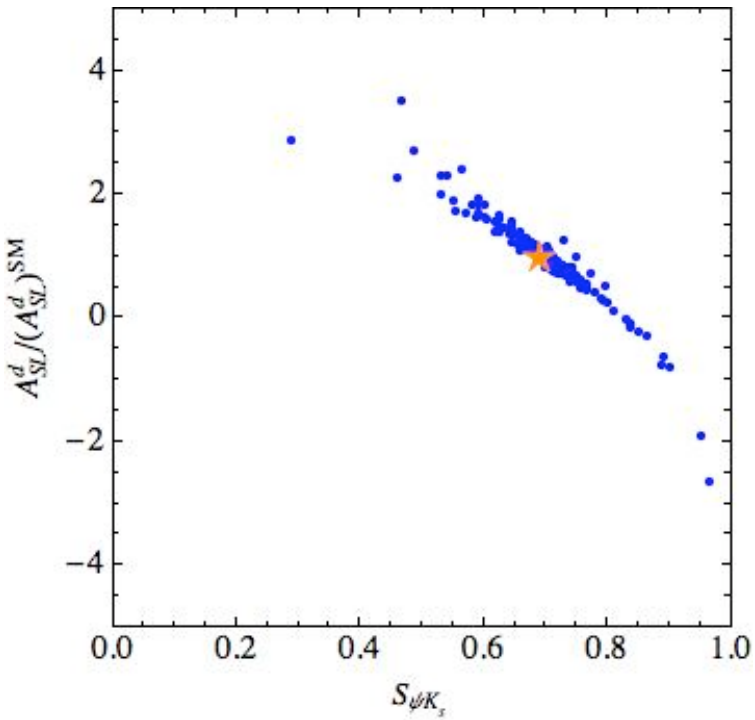
$$C_{B_s}\, e^{2i\phi_{B_s}} = \frac{A_s^{\rm SM} e^{-2i\beta_s} + A_s^{\rm NP} e^{2i(\phi_s^{\rm NP}-\beta_s)}}{A_s^{\rm SM} e^{-2i\beta_s}}$$

SM: $C_{Bi}=1$, $\phi_i=0$

Neutral meson mixing

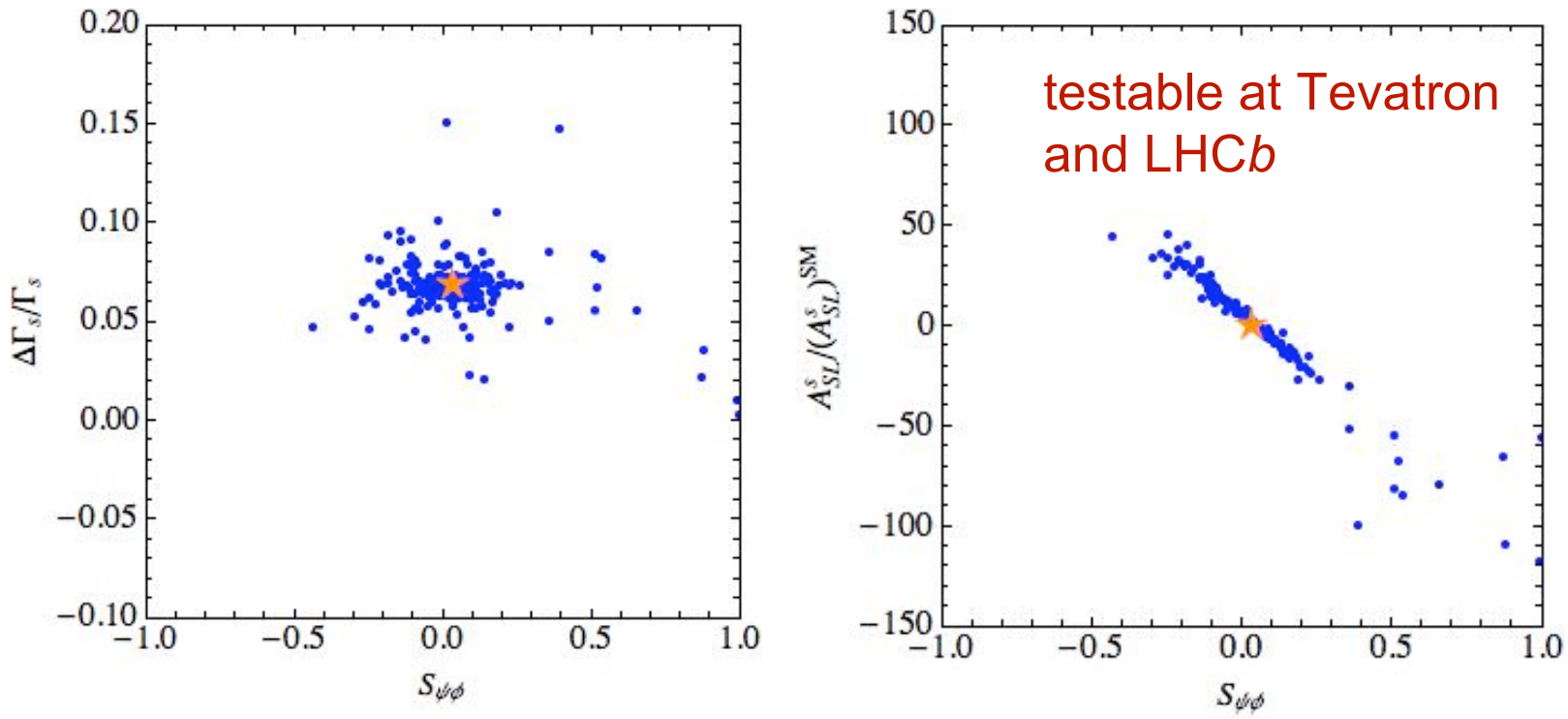
- Corrections to $S(\psi K_S)$ in B_d mixing:





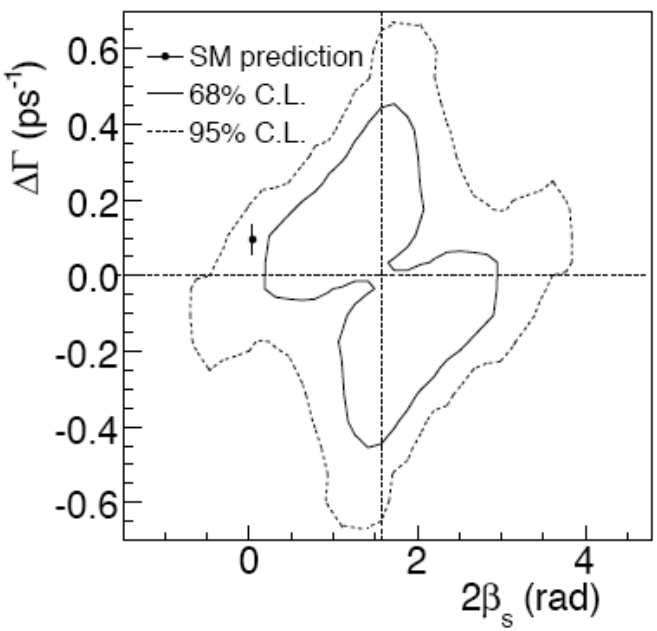
Neutral meson mixing

- Corrections to $S(\psi\phi)$ in B_s mixing:

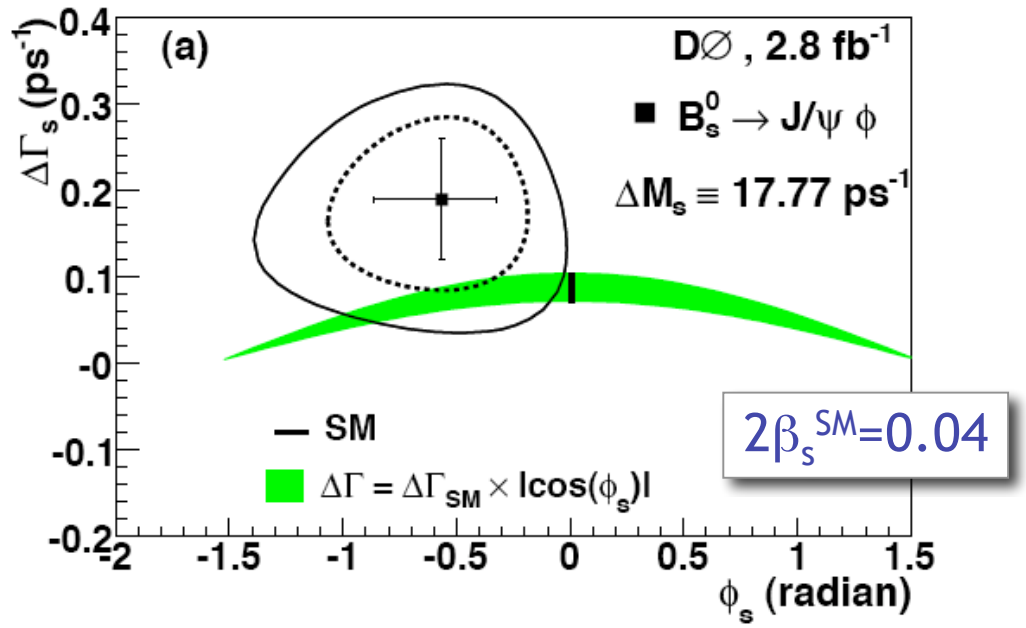


New physics in B_s mixing ?

- Tantalizing hints from flavor-tagged analysis of mixing-induced CP violation in $B_s \to J/\psi\ \phi$ decay
- Combined probability regions for $\phi_s = 2\beta_s$ and $\Delta\Gamma_s$



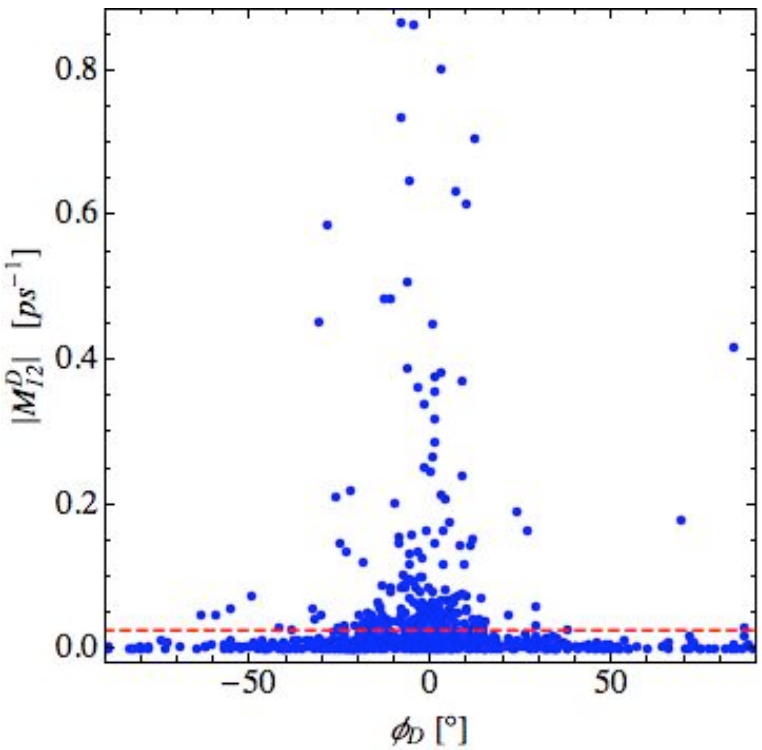
CDF (Dec. 2007)



D0 (Feb. 2008)

Neutral meson mixing

- D mixing amplitude (without uncalculable SM contribution) after $|\varepsilon_K|$ constraint:



→ very large effects possible, including large CP violation (could be testable at LHC*b*)

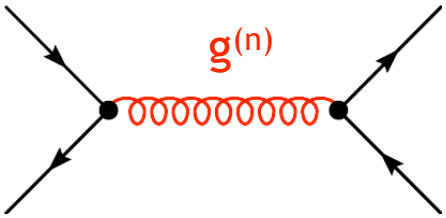
← maximal allowed SM effect (with no significant CP phase)

$\Delta F=1$ Processes

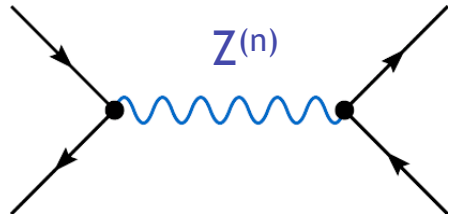
Rare K and B decays relevant to Tevatron and LHC (and Project X)

Tree-level FCNCs

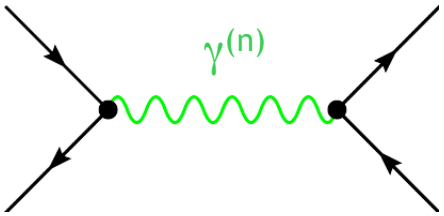
- Generic diagrams:



dominant for $\Delta F=2$ processes



dominant for $\Delta F=1$ processes



always small

  - consider $b \to s+qq$ for example
  - analogous results hold for decays of the type $b \to s+l^+l^-$, $b \to s+\nu\nu$ as well as $b \to d$ and $s \to d$ FCNCs

Rare FCNC decays via b→s+qq

- Effective weak Hamiltonian:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{ps}^* \left(C_1\, Q_1^p + C_2\, Q_2^p + \sum_{i=3,\ldots,10} C_i\, Q_i + C_{7\gamma}\, Q_{7\gamma} + C_{8g}\, Q_{8g} \right)$$

$$+ \sum_{i-3,\ldots,10} \left(C_i^{\rm RS}\, Q_i + \tilde{C}_i^{\rm RS}\, \tilde{Q}_i \right) .$$

  - list RS contributions to Wilson coefficients of QCD and electroweak penguin operators $Q_{3,\ldots,10}$
  - analogous expressions for coefficients of opposite-chirality operators

b→s+qq

- Wilson coefficients:

$$C_3^{\rm RS} = \frac{\pi\alpha_s}{M_{\rm KK}^2}\frac{(\Delta_D)_{23}}{2N_c} - \frac{\pi\alpha}{6\sin^2\theta_W\cos^2\theta_W M_{\rm KK}^2}\left[L\left(\frac{1}{2}-\frac{\sin^2\theta_W}{3}\right)(\Delta'_D)_{23} + \frac{M_{\rm KK}^2}{m_Z^2}(\delta_D)_{23}\right]$$

$$C_4^{\rm RS} = C_6^{\rm RS} = -\frac{\pi\alpha_s}{2M_{\rm KK}^2}(\Delta_D)_{23}$$

sources: g, γ, Z

$$C_5^{\rm RS} = \frac{\pi\alpha_s}{M_{\rm KK}^2}\frac{(\Delta_D)_{23}}{2N_c}$$

$$C_7^{\rm RS} = \frac{2\pi\alpha}{9M_{\rm KK}^2}(\Delta_D)_{23} - \frac{2\pi\alpha}{3\cos^2\theta_W M_{\rm KK}^2}\left[L\left(\frac{1}{2}-\frac{\sin^2\theta_W}{3}\right)(\Delta'_D)_{23} + \frac{M_{\rm KK}^2}{m_Z^2}(\delta_D)_{23}\right]$$

$$C_8^{\rm RS} = C_{10}^{\rm RS} = 0$$

$$C_9^{\rm RS} = \frac{2\pi\alpha}{9M_{\rm KK}^2}(\Delta_D)_{23} + \frac{2\pi\alpha}{3\sin^2\theta_W M_{\rm KK}^2}\left[L\left(\frac{1}{2}-\frac{\sin^2\theta_W}{3}\right)(\Delta'_D)_{23} + \frac{M_{\rm KK}^2}{m_Z^2}(\delta_D)_{23}\right]$$

b→s+qq

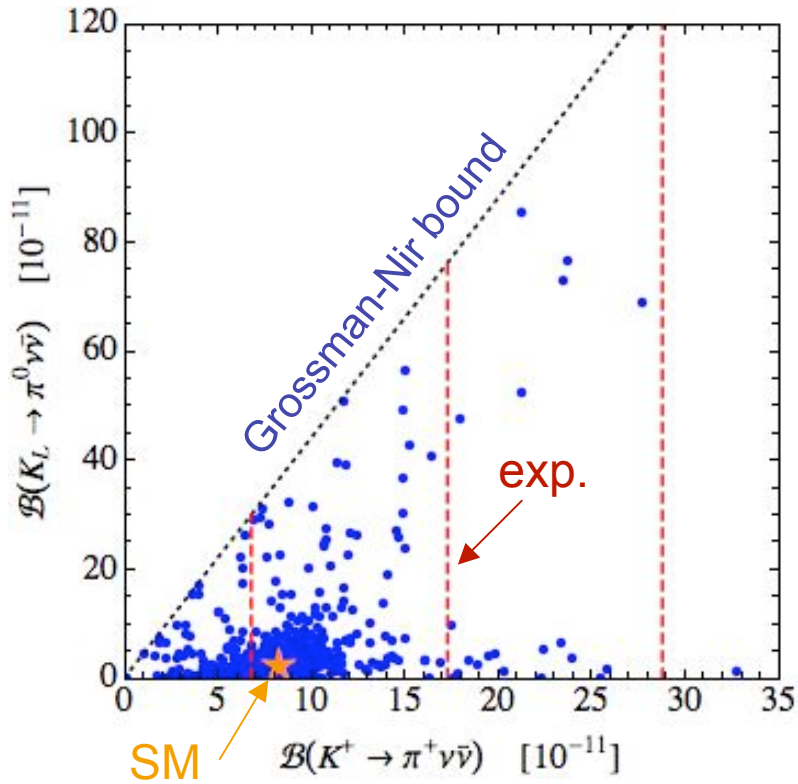
- Only 4 independent couplings:

	LHD	RHD
g, γ	$(\Delta_D)_{23}$	$(\Delta_d)_{23}$
Z	$L\left(\frac{1}{2}-\frac{\sin^2\theta_W}{3}\right)(\Delta'_D)_{23}+\frac{M_{\rm KK}^2}{m_Z^2}(\delta_D)_{23}$	$L\,\frac{\sin^2\theta_W}{3}(\Delta'_d)_{23}+\frac{M_{\rm KK}^2}{m_Z^2}(\delta_d)_{23}$

  - first two can be made small by rescaling 5D Yukawa couplings, but latter two can not!

Rare K decays

- Potentially huge effects (even after $|\varepsilon_K|$ constraint) in $K \to \pi\nu\nu$ decays:



Rare K decays

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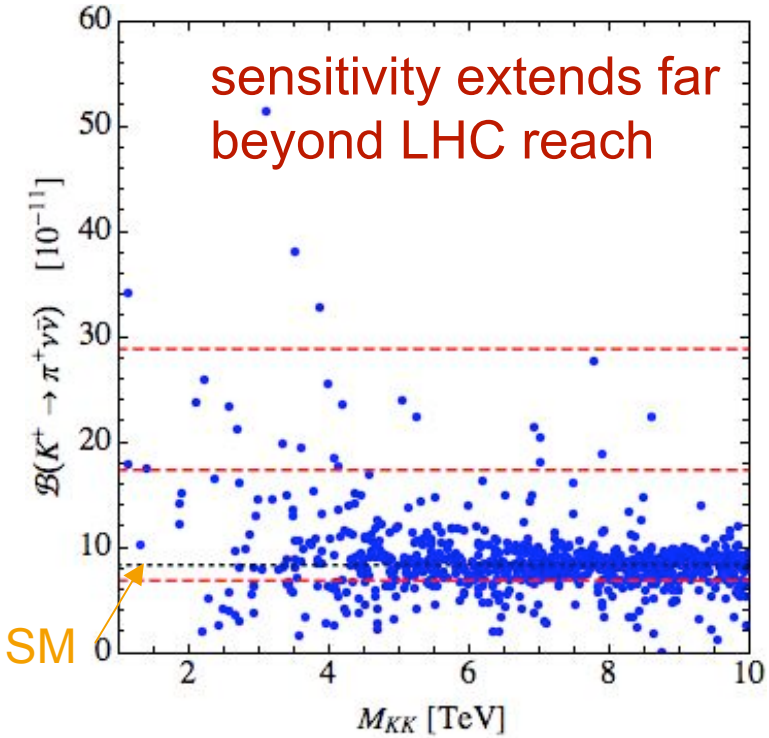
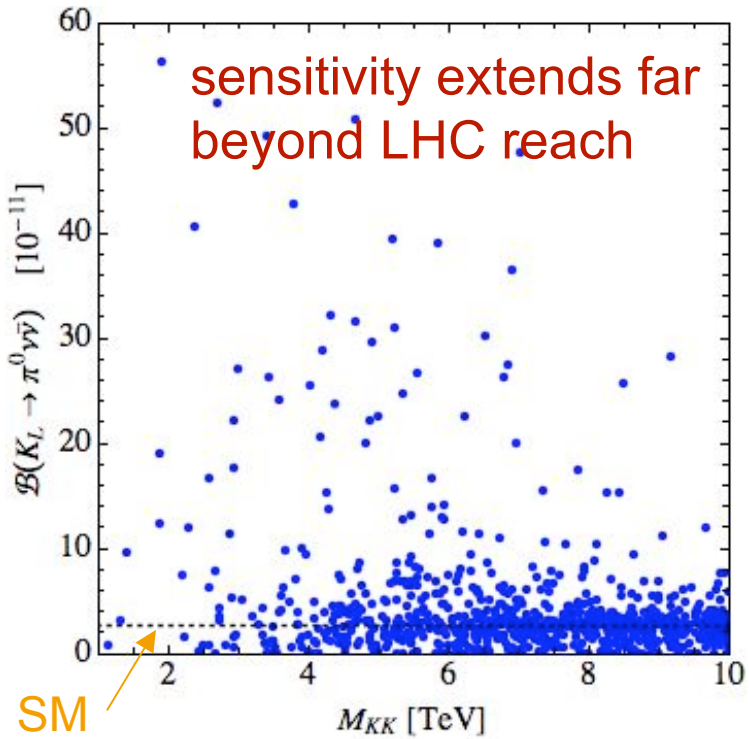
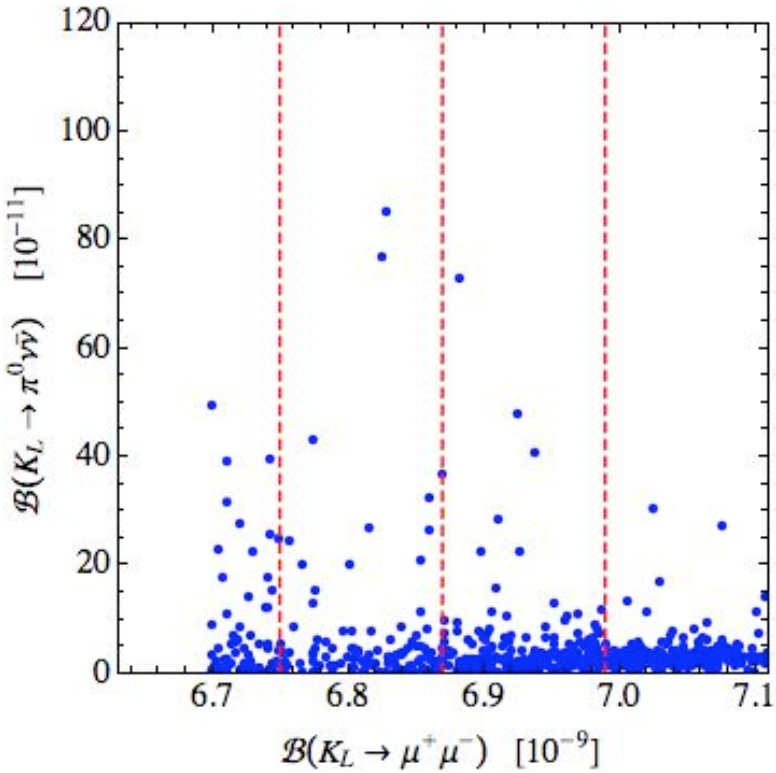
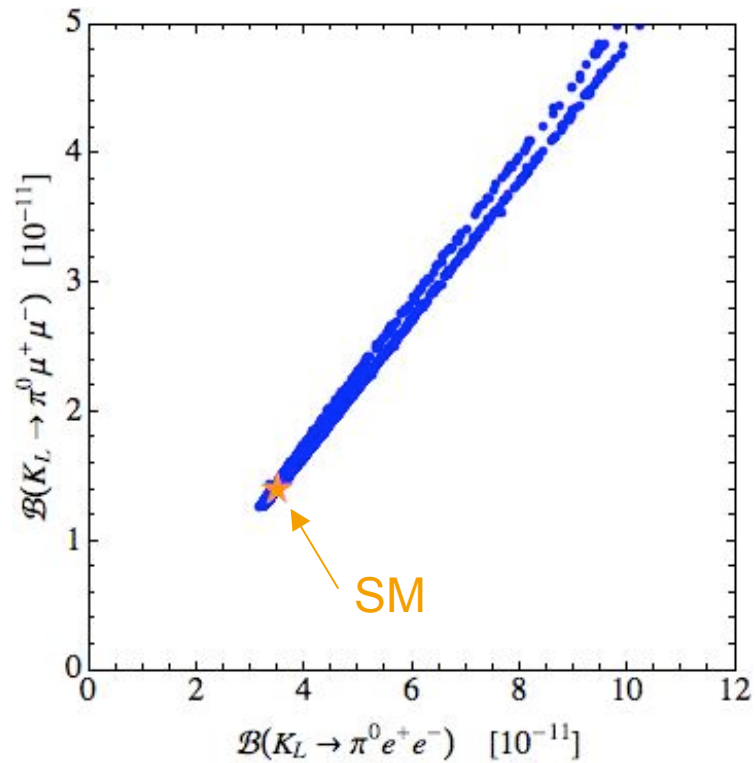




Rare K decays

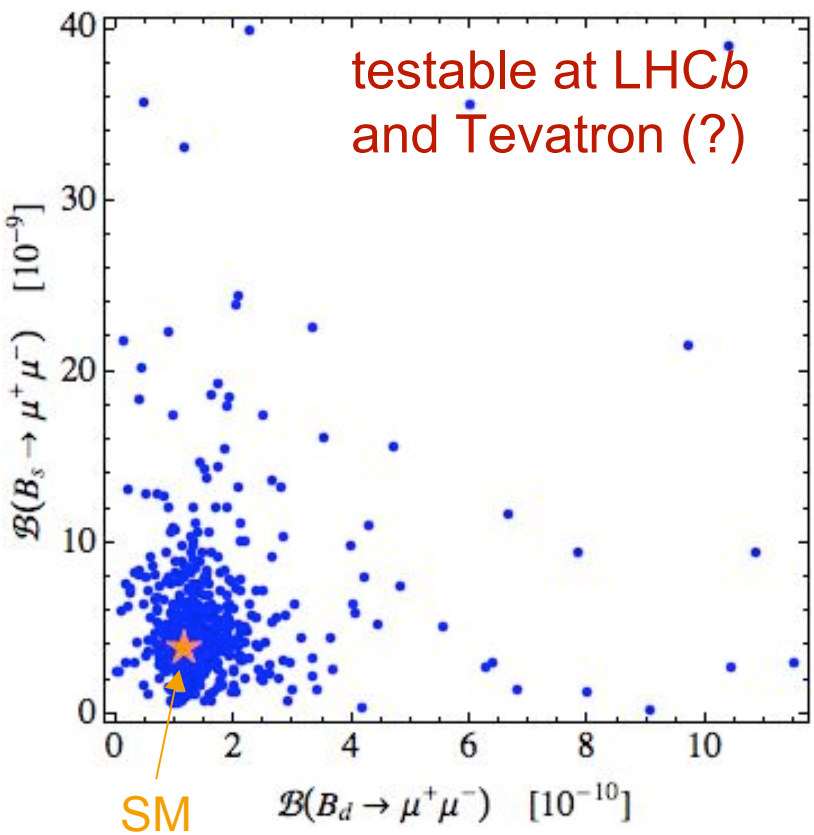
- Other interesting modes:

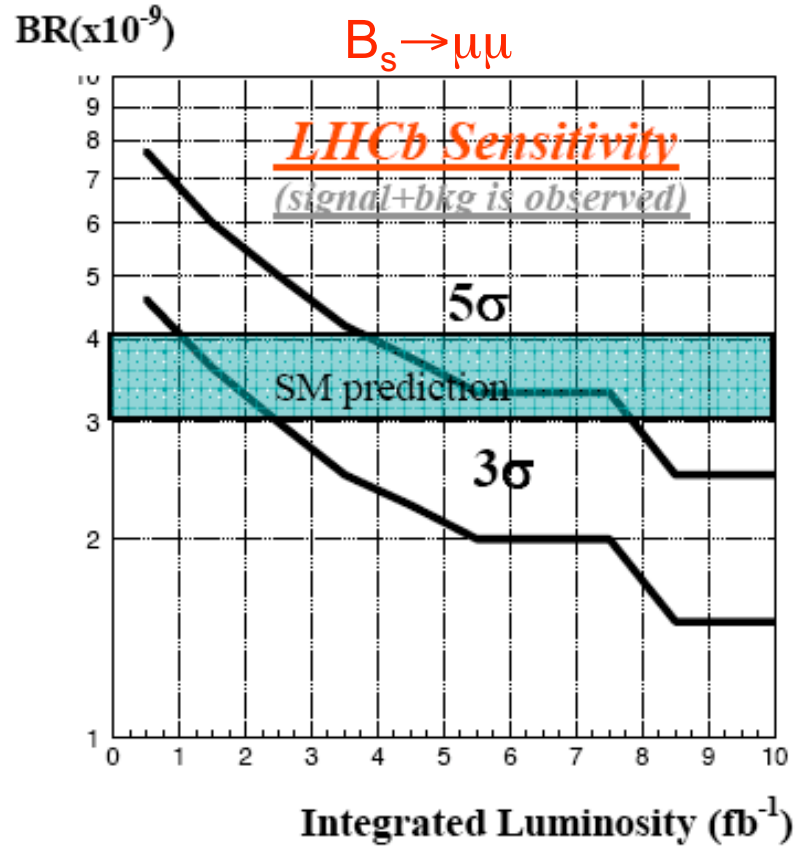




Rare B decays

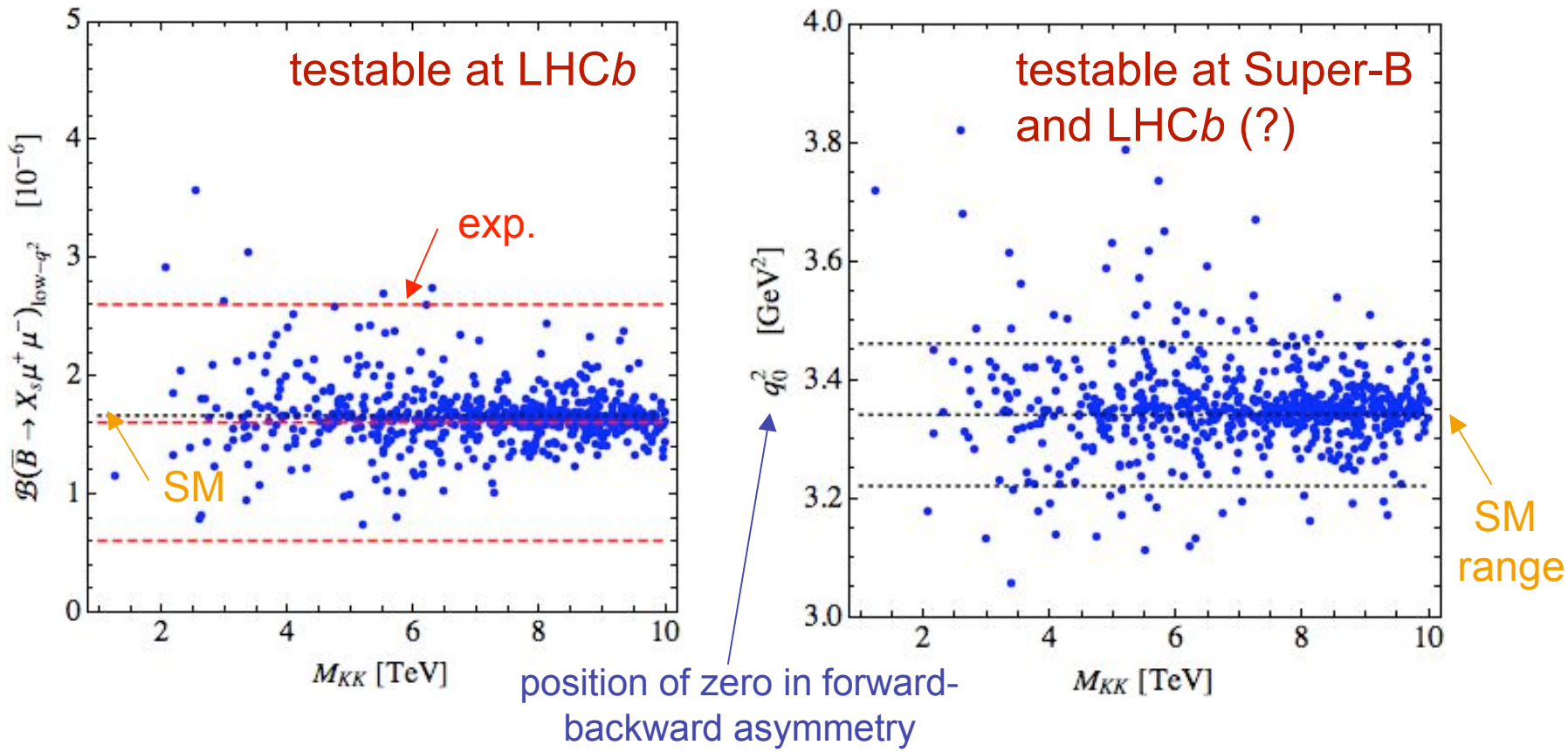
- Large possible effects in $B_{d,s} \to \mu\mu$ decays:





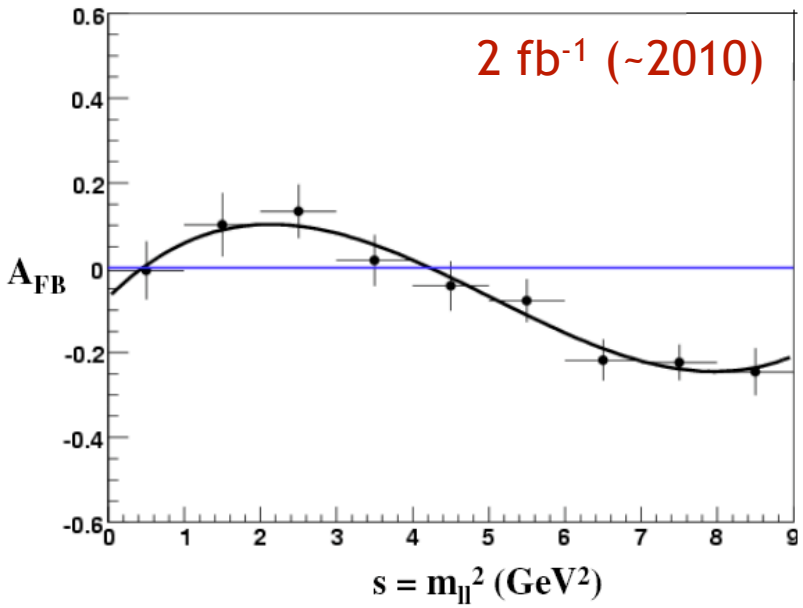
Rare B decays

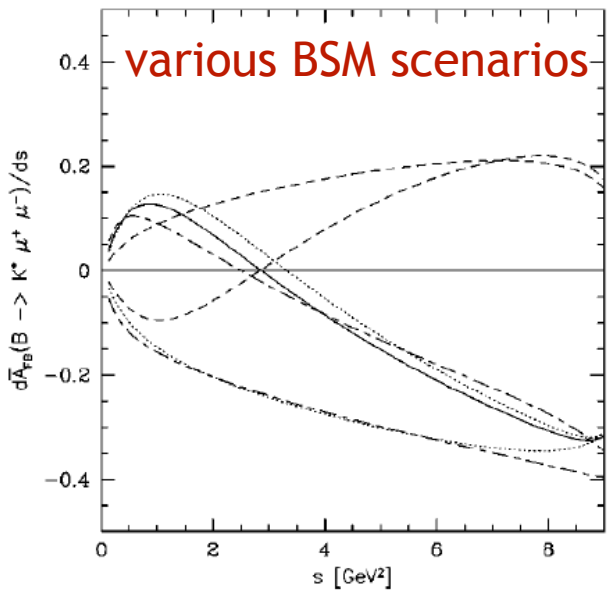
- Large possible effects in $B \to X_s \mu\mu$ and $B \to K^* \mu\mu$:



Rare B decays

- Zero of forward-backward asymmetry sensitive to Wilson coefficients in effective Hamiltonian

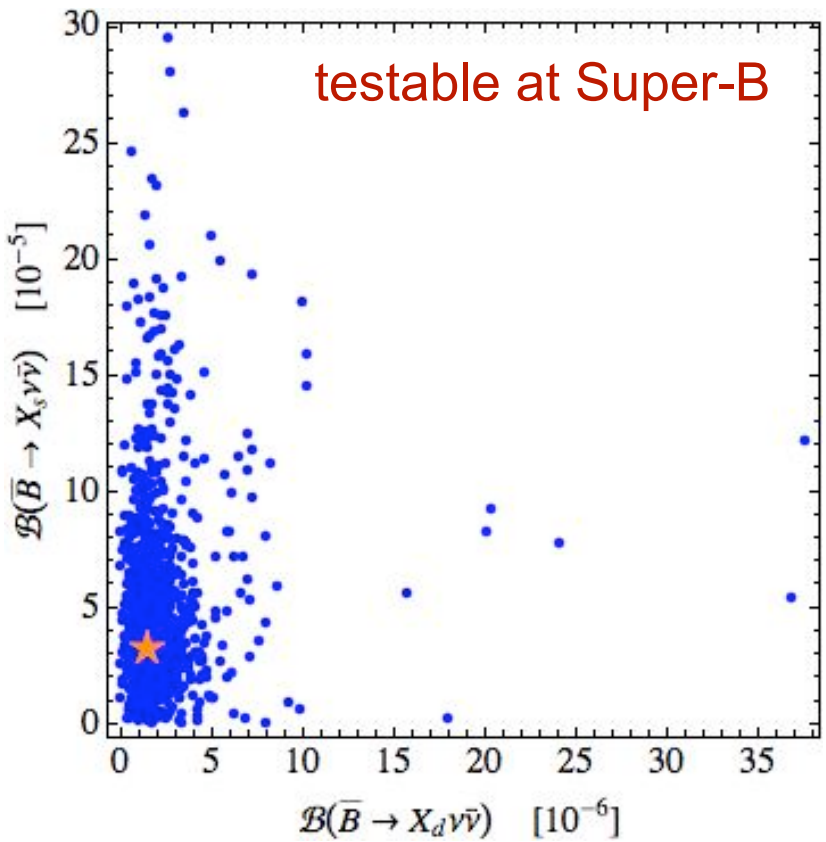




- LHC*b* precision (for 10 fb^{-1}) on zero $\sigma(s_0)$=0.27 GeV^2

Very rare B decays

- Large possible effects in $B \to X\nu\nu$ decays:



Summary

- Randall-Sundrum models offer viable solution to both the gauge and fermion hierarchy problems
- Rich structure of flavor-violating interactions in gauge couplings to fermions (generically not MFV)
  - $\Delta F=1$ FCNC processes dominated by Z exchange
  - $\Delta F=2$ FCNC processes dominated by KK gluon exchange
- Effects naturally of O(1) or larger in modes where deviations from SM are allowed/indicated by the data, while small in other modes (e.g., $B \to X_s \gamma$)
- FCNC processes of K and B_s particularly interesting