Renormalization and BSM Physics: A Cautionary Tale

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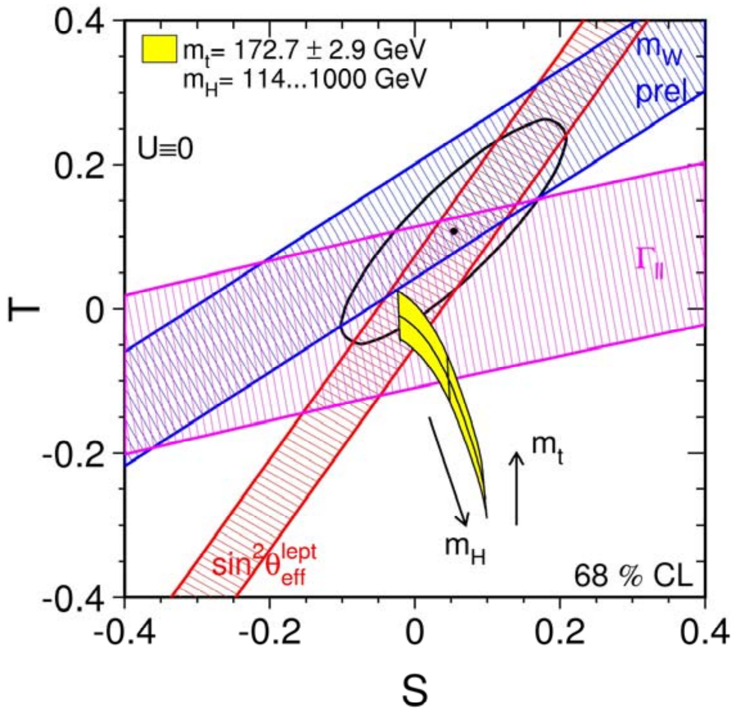
Beyond the SM: From the Tevatron to the LHC

Fermilab, September, 2008

M.-C. Chen, S. Dawson, and C. Jackson, hep-ph/08xxxx

The Usual Approach

- Build the model of the week
- Assume new physics contributes primarily to gauge boson 2-point functions
- Calculate contributions of new particles to S, T, U
- Extract limits on parameters of model
- CLAIM: This approach must be modified when $\rho = M_W^2/(M_Z^2 c_\theta^2) \neq 1$ at tree level
- $\rho \neq 1 = \alpha T$ can be compensated by heavy Higgs



Chivukula, Hoelbling, Evans, hep-ph/0002022; Peskin & Wells, hep-ph/0101342

Standard Model Renormalization

- EW sector of SM is SU(2) x U(1) gauge theory
  - 3 inputs needed: g, g', v, plus fermion/Higgs masses
  - Trade g, g', v for precisely measured G_μ, M_Z, α

$$g^2 = \frac{4\pi\alpha}{s_\theta^2} \qquad g'^2 = \frac{4\pi\alpha}{c_\theta^2} \qquad v^2 = \left(G_\mu \sqrt{2}\right)^{-1/2}$$

  - SM has $\rho = M_W^2/(M_Z^2 c_\theta^2) = 1$ at tree level
    - s_θ is derived quantity
  - Models with $\rho=1$ at tree level include
    - MSSM
    - Models with singlet or doublet Higgs bosons
    - Models with extra fermion families

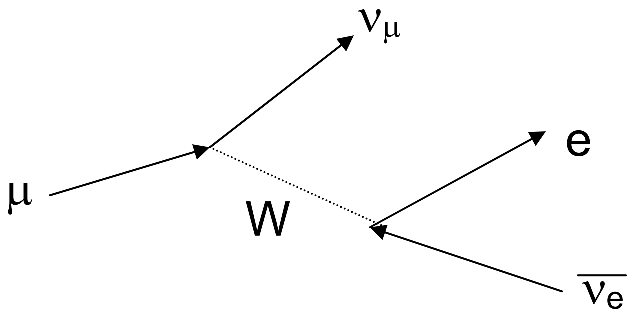
Muon Decay in the SM

- At tree level, muon decay related to input parameters:
- One loop radiative corrections included in parameter Δr_{SM}

$$\boxed{G_\mu = \frac{\pi\alpha}{\sqrt{2} s_\theta^2 M_W^2}(1+\Delta r_{SM})}$$

- Dominant contributions from 2-point functions

$\boxed{\Delta r \text{ is a physical parameter}}$



Various Schemes for s_θ in SM

- On-shell: $s_W^2 = 1 - M_W^2/M_Z^2$

$$\Delta r_H^{os} \approx \frac{11\alpha}{48\pi s_W^2}\log\left(\frac{M_H^2}{M_W^2}\right)+\ldots$$

- Effective mixing angle:

$$L = \frac{e}{2c_\theta^{eff} s_\theta^{eff}}\bar{e}\gamma^\mu\left(\frac{1}{2}-2s_\theta^{eff\,2}-\frac{1}{2}\gamma_5\right)eZ_\mu$$

$$\Delta r_H^{eff} \approx \frac{\alpha}{48\pi s_{\theta,eff}^2}\log\left(\frac{M_H^2}{M_W^2}\right)+\ldots$$

- "M_Z" scheme:

$$s_Z^2 c_Z^2 = \frac{\alpha\pi}{M_Z^2\sqrt{2}G_\mu}$$

$$\Delta r_H^Z \approx \frac{11\alpha s_Z^2}{48\pi s_Z^2(c_Z^2 - s_Z^2)}\log\left(\frac{M_H^2}{M_W^2}\right)+\ldots$$

- All schemes identical at tree level
- One-loop results show strong scheme dependence
- (Of course in SM, state of the art way beyond one-loop, but BSM conclusions often drawn from one-loop results)

Models with $\rho \neq 1$ at tree level are different from the SM

$$\boxed{\rho = M_W^2/(M_Z^2 c_\theta^2) \neq 1}$$

- SM with Higgs Triplet
- Left-Right Symmetric Models
- Little Higgs Models
- …..many more
- These models need additional input parameter
- Decoupling is not always obvious beyond tree level

Higgs Triplet Model

Simplest extension of SM with $\rho \neq 1$

- SM: SU(2) x U(1)
  - Parameters, g, g', v, M_h
- Add a real triplet

$$H = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + h^0 + i\chi^0) \end{pmatrix} \qquad \Phi = \begin{pmatrix} \eta^+ \\ v' + \eta^0 \\ \eta^- \end{pmatrix}$$

  - $v_{SM}^2 = (246\text{ GeV})^2 = v^2 + 4v'^2$
  - Real triplet doesn't contribute to M_Z

$$\boxed{M_W^2 = \frac{g^2 v^2}{4}\left(1 + \frac{4v'^2}{v^2}\right)}$$

- At tree level, $\rho = 1 + 4v'^2/v^2 \neq 1$
- PDG: $v' < 12$ GeV

Motivated by Little Higgs models

Neglects effects of scalar loops

Scalar Potential

$$\boxed{V = \mu_1^2 |H|^2 + \mu_2^2 |\Phi|^2 + \lambda_1 |H|^4 + \frac{\lambda_2}{4} |\Phi|^4 + \frac{\lambda_3}{2} |H|^2 |\Phi|^2 + \lambda_4 H^+ \sigma^a H \Phi_a}$$

Forbidden by T-parity

- λ_4 has dimensions of mass → doesn't decouple
- Mass Eigenstates:

$$\begin{pmatrix} H^0 \\ K^0 \end{pmatrix} = \begin{pmatrix} c_\gamma & s_\gamma \\ -s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} h^0 \\ \eta^0 \end{pmatrix} \qquad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} c_\delta & s_\delta \\ -s_\delta & c_\delta \end{pmatrix} \begin{pmatrix} \phi^\pm \\ \eta^\pm \end{pmatrix}$$

- 6 parameters in scalar sector: Take them to be:

M_{H0}, M_{K0}, M_{H+}, v, δ, γ

$\boxed{\tan\delta = 2\,v'/v}$

δ small since it is related to ρ parameter

Decoupling at Tree Level

- Require no mixing between doublet-triplet sectors for decoupling

$$\boxed{\left.\frac{\partial^2 V}{\partial h_0 \partial \eta_0}\right|_{\min} = \lambda_3 v v' - \lambda_4 v = 0} \qquad \boxed{v' = \frac{\lambda_4}{\lambda_3}}$$

- v'→0 requires $\lambda_4 \to 0$ (custodial symmetry), or $\lambda_3 \to \infty$ (invalidating perturbation theory)

$$M^2_{K^0} = \mu_2^2 + 12 v'^2 \lambda_2 + \frac{1}{2} v^2 \lambda_3$$

$$M^2_{H^+} = \mu_2^2 + 4 v'^2 \lambda_2 + \frac{1}{2} v^2 \lambda_3 + 2 v' \lambda_4$$

- v'→0 implies $M_{K0} \sim M_{H+}$

Require parameters be perturbative

- Generically, want tree level > 1-loop contribution
  - Require $\lambda < (4\pi)^2$
  - Large effects for large mass splittings

$$\lambda_1 = \frac{1}{2v^2}\left(M_{H^0}^2 + s_\gamma^2\left(M_{K^0}^2 - M_{H^0}^2\right)\right)$$

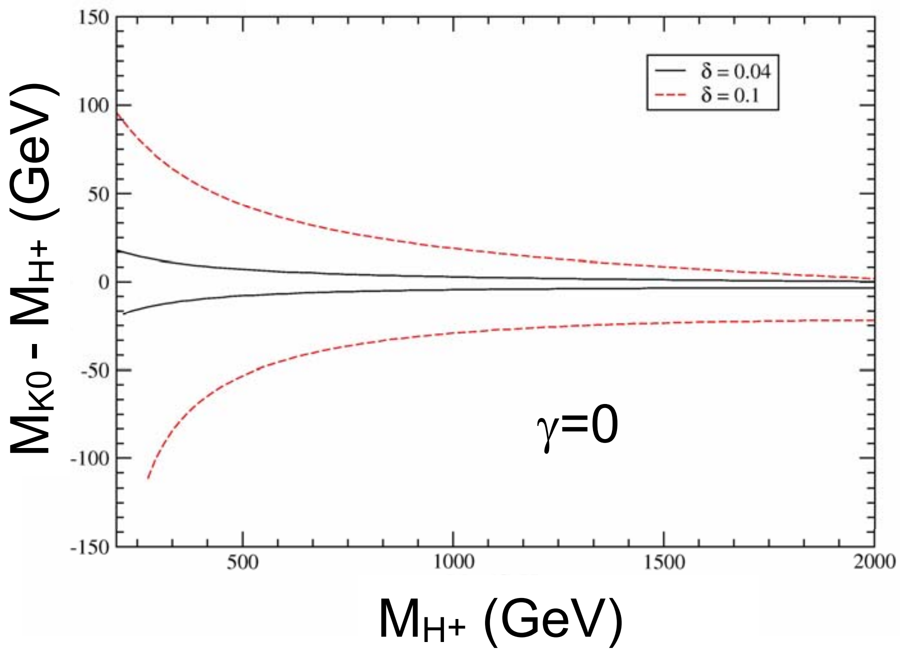
$$\lambda_2 = \frac{2}{v^2}\frac{1}{\tan^2\delta}\left(M_{K^0}^2 - M_{H^+}^2 + s_\gamma^2\left(M_{K^0}^2 - M_{H^0}^2\right) + s_\delta^2 M_{H^+}^2\right)$$

- $\rho \sim 1 + \tan^2\delta$, so δ must be small

$\rightarrow$ Scalar couplings large unless $M_{K0} \sim M_{H^+}$

- Large γ will require $M_{K0} \sim M_{H0}$
- $M_{K0} \rightarrow \infty$ will force small γ

Heavy Scalars → Small Mass Splittings

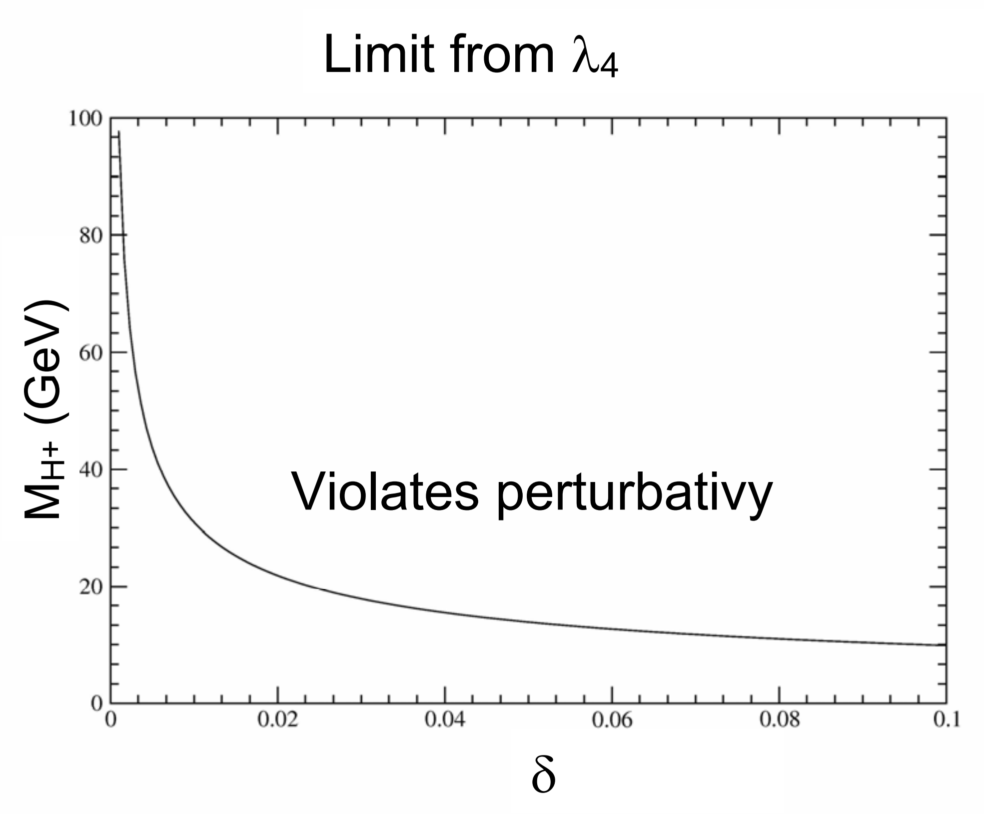


Allowed region is between curves

- Plots are restriction $\lambda_2 < (4\ \pi)^2$

Forshaw, Vera, & White, hep-ph.0302256

Upper Limit on M_{H+} from Perturbativity



$M_{H+} \to \infty$ requires $\delta \to 0$

Renormalization of Triplet Model

- At tree level W mass, related to input parameters:

$$G_\mu = \frac{\pi\alpha}{\sqrt{2} s_\theta^2 c_\theta^2 M_Z^2 \rho} = \frac{\pi\alpha}{\sqrt{2} s_\theta^2 M_W^2} \qquad \rho = \frac{M_W^2}{c_\theta^2 M_Z^2} = 1$$

For $\rho \neq 1$, 4 input parameters

- One loop radiative corrections included in parameter Δr

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2} s_\theta^2 G_\mu}(1+\Delta r)$$

- Study scheme dependence in triplet model:

$$\boxed{\Delta r = -\frac{\delta G_\mu}{G_\mu} - \frac{\delta M_W^2}{M_W^2} + \frac{\delta\alpha}{\alpha} - \frac{\delta s_\theta^2}{s_\theta^2}}$$

Scheme 1: Input 4 Measured Quantities

(M_Z, α, G_μ, $\sin\theta^{eff}$)

- Use effective leptonic mixing angle at Z resonance as 4th parameter

$$L = -i\bar{e}\gamma_\mu(v_e + a_e\gamma_5)eZ^\mu \qquad v_e = \frac{1}{2} - 2s_\theta^{eff\,2}, \qquad a_e = \frac{1}{2}$$

- Variation of s_θ^{eff}:

$$\frac{\delta s_\theta^{eff\,2}}{s_\theta^{eff\,2}} = \left(\frac{c_\theta^{\;eff}}{s_\theta^{\;eff}}\right)\frac{\Pi_{\gamma Z}(M_Z^2)}{M_Z^2} + O(m_e^2)$$

- Could equally well have used ρ or M_W as 4th parameter

- At tree level, SM and triplet model are identical in s_θ^{eff} scheme

$$\boxed{M_W^2 = \frac{\alpha\pi}{\sqrt{2}s_\theta^{eff\,2}G_\mu}(1+\Delta r)}$$

This scheme discussed by: Chen, Dawson, Krupovnickas, hep-ph/0604102; Blank and Hollik hep-ph/9703392

Scheme 1 results

- Compare with SM in effective mixing angle scheme
  - In both SM and triplet model M_W(tree)=79.838 GeV
  - M_W(experiment)=80.399±.025 GeV
- Input parameters: M_Z, $\sin\theta^{eff}$, α, G_μ, M_{H0}, M_{K0}, M_{H+},γ
  - $\cos\delta = M_Z \cos\theta^{eff} / M_W$ predicts $\sin\delta$ =.07 (v'=9 GeV)
  - System is overconstrained (can't let v' run)
- Triplet model has extra contributions to Δr from K^0, H^+
- SM couplings are modified by factors of $\cos\delta$, $\cos\gamma$

Scheme 1, Continued

$$\boxed{\Delta r^{triplet} \approx \Delta\tilde{r}^{SM} + \Delta r^{NP}}$$

- $\Delta\tilde{r}^{SM}$ contains SM particles (including the Higgs), but differs from SM Δr^{SM} because M_Z is input for triplet model and calculated for SM
- Δr^{NP} contains contributions from SM particles multiplied by factors of s_δ and s_γ and contributions from K^0 and H^+ (which need not vanish for s_δ or $s_\gamma = 0$)

Can write $\Delta r^{triplet}$ in this way only because we've chosen a scheme where M_W is the same in both the SM and triplet model

Quadratic dependence on Higgs mass

- Triplet model with $M_{H0} << M_{K0} \approx M_{H\pm}$ and small mixing (Scheme 1)

$$\boxed{\Delta r^{triplet} \approx \Delta\tilde{r}^{SM} + \frac{\alpha}{24\pi s_\theta^2}\frac{M_{K^0}^2 - M_{H^+}^2}{M_{H^+}^2} + \sin\delta(...) + \sin\gamma(...)}$$

Inputs different in triplet model and SM

Triplet model: M_Z=91.1876 GeV is input

SM (in this scheme): M_Z is calculated = 91.453 GeV

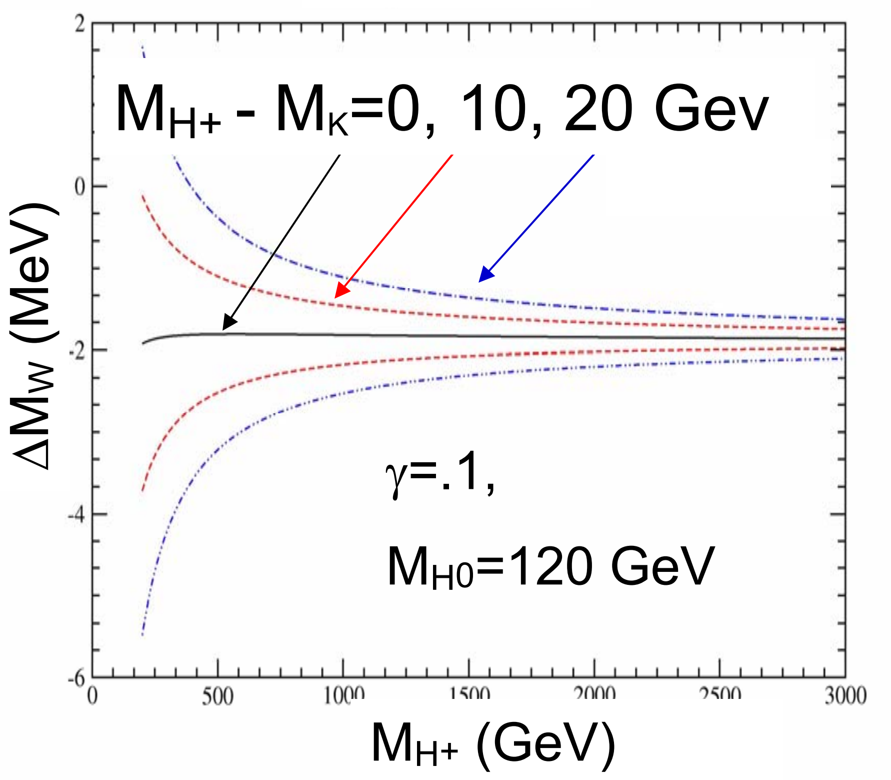
Perturbativity requires M_{K0}~M_{H+} for large M_{H+}

Toussaint, PRD18 (1978) 1626

Scheme 1: M_W(SM)-M_W(Triplet)

- For heavy H^+, perturbativity requires $M_{H+} \sim M_{K^0}$, and predictions of triplet model approach SM

- No large effects in perturbative regime

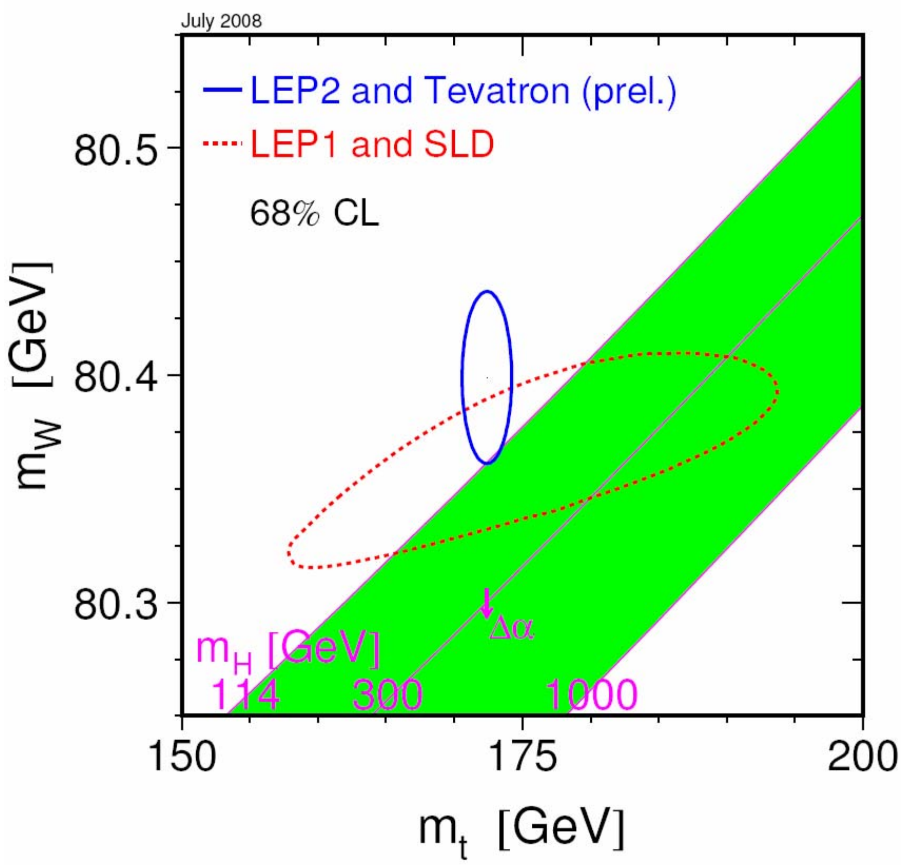


- SM not exactly recovered at large M_H^+ due to different M_Z inputs

Similar conclusions from Chivukula, Christensen, Simmons: arXiv:0712.0546

Reminder of Experimental Status

$M_W = 80.399 \pm 0.025$ GeV



Scheme 2: v' as 4th Input

- Alternative approach: Input M_Z, G_μ, α, v'

$$G_\mu = \frac{1}{\sqrt{2}(v^2 + 4v'^2)}$$

- NaIvely, more natural approach to SM limit
- Naturally connects with SM M_Z scheme in $v' \rightarrow 0$ limit

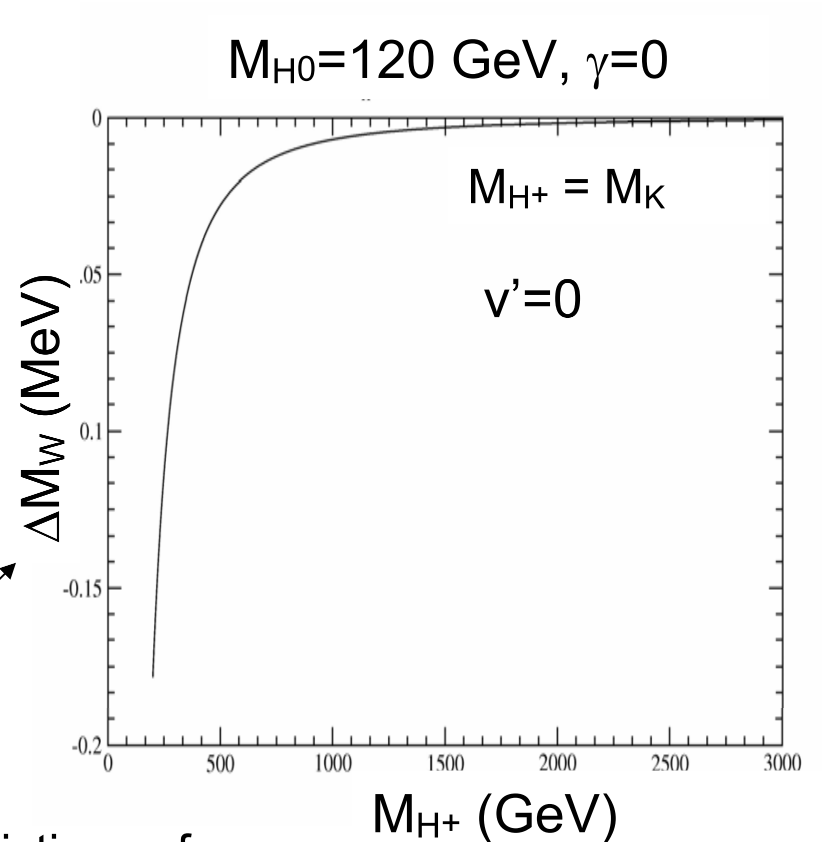
$$s_Z^2 c_Z^2 = \frac{\pi\alpha}{M_Z^2}\left(\frac{1}{\sqrt{2}G_\mu} - 4v'^2\right)$$

- Calculate 1-loop corrections to M_W in usual way

This scheme advocated by Chankowski, Pokorski, & Wagner, hep-ph/0605302

Scheme 2

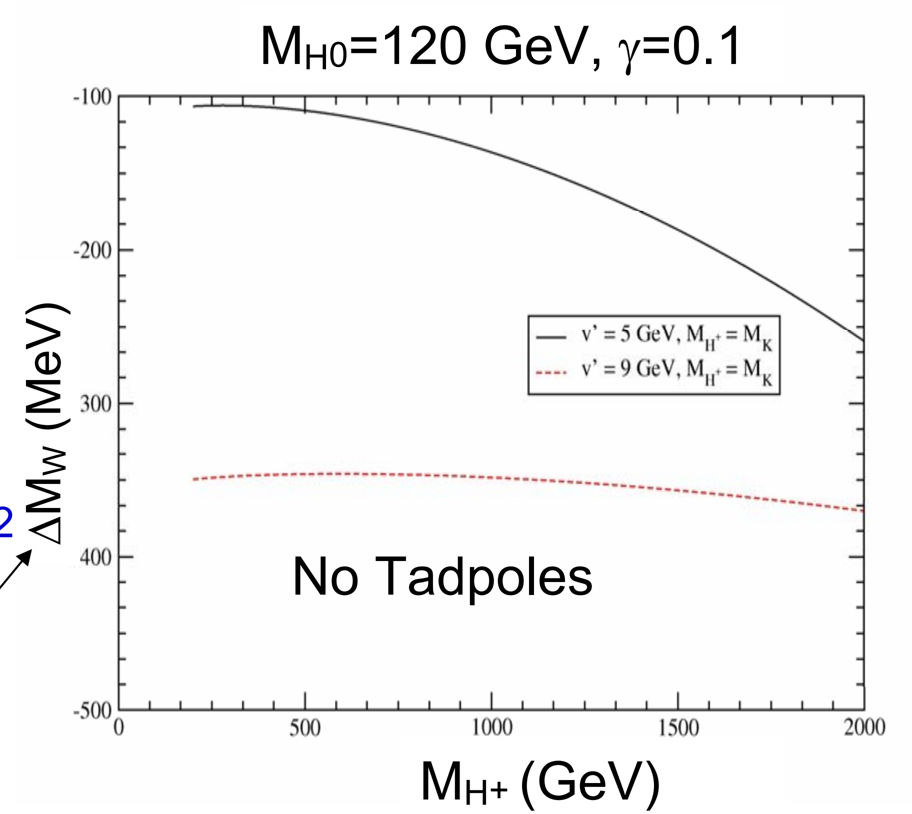
- For v'=0, only consistent solution to minimization of potential is γ=0 (no mixing in neutral sector) and $M_H^+ = M_K$
- No large effects from triplet sector in this case
- Decoupling of heavy Higgs is apparent



Difference between 1-loop predictions of SM in MZ scheme and 1-loop triplet model

Dial up v'

- As soon as $v' \neq 0$, then $\lambda_4 \neq 0$
- Since λ_4 has dimension, decoupling theorem isn't applicable
- Large effects $\sim v'^2 G_\mu (M_{H+}/M_W)^2$

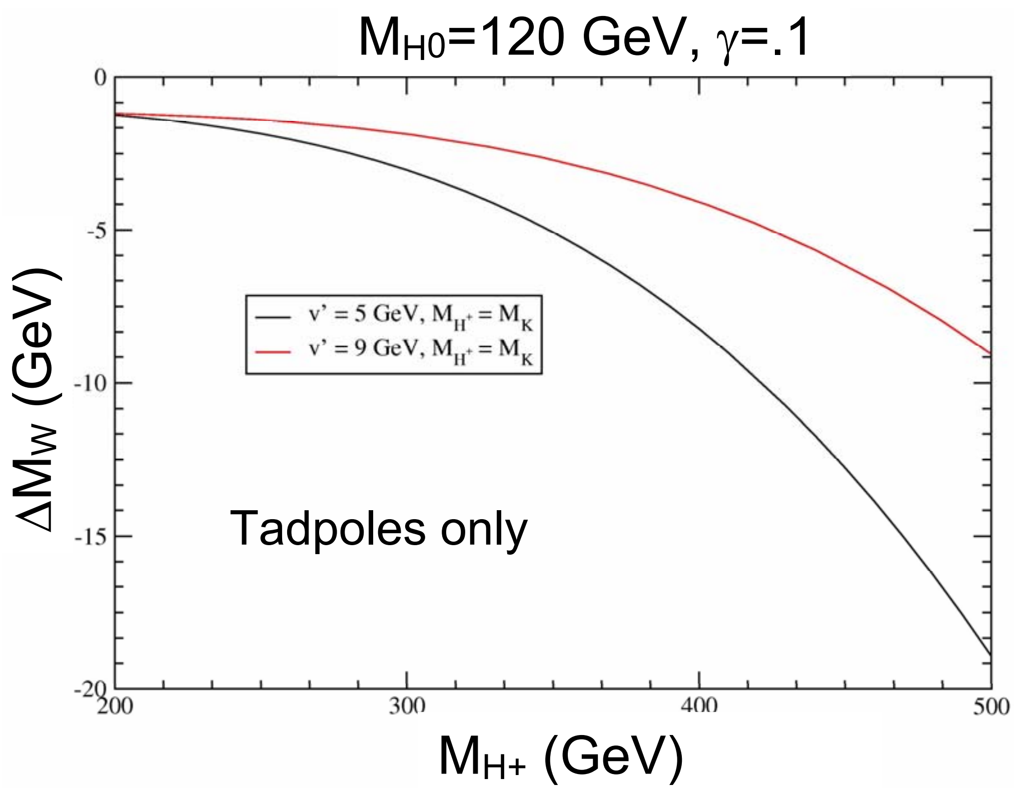


Difference between 1-loop predictions of SM in M_Z scheme and 1-loop triplet model

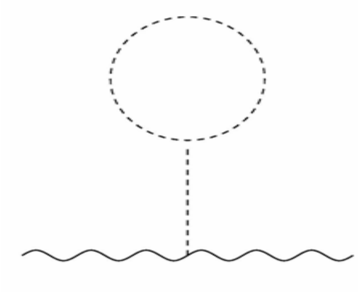
Tadpoles require fine tuning

- In SM, tadpoles cancel
- Not so for non-zero v'
- Tadpole contributions grow with M^2

$$\Delta r_{tadpoles} = \frac{c_Z^2}{c_Z^2 - s_Z^2}\left(-\frac{\Pi_{ZZ}^{tadpole}}{M_Z^2} + \frac{1}{1-4v'^2 G_\mu}\frac{\Pi_{WW}^{tadpole}}{M_W^2} \right)$$



Note units!



Tadpoles generate effective v'

Fine Tuning

- No physical motivation for definition of v' in simplest triplet model
  - Plots use running $\overline{MS}$
  - GUT may have natural way to define v'
- Define v'(renormalized) to cancel tadpoles
  - Numerical effects still large in this scheme
  - Lose predictivity

The Moral of the Story is….

- Models with $\rho \neq 1$ at tree level require 4 input parameters in gauge sector for consistent renormalization
- Non-decoupling effects vanish for $\rho = 1$ limit as expected
- Important to compare NP results with appropriate SM scheme
  - We investigated 2 schemes: One with 4 low energy inputs and one with 3 low energy inputs and a running unknown parameter
- Effects of scalar loops critical
- Same issues arise when using S/T/U formalism if there are tree level contributions
  - *THE CORRECT RENORMALIZATION PROCEDURE IS COMPLICATED AND MATTERS!*