Kinematic Constraints and M_{T2}

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Beyond the Standard Model: from the Tevatron to the LHC
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Outline

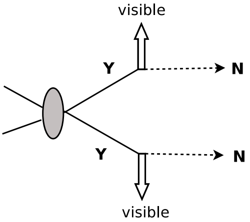
What is M_{T2}?

What is kinematic constraints?

What is their relation?

What is the use?

New physics and missing particles at hadron colliders



What are the masses for $Y, N \ldots$?

- M_{T2}
- Kinematic constraints

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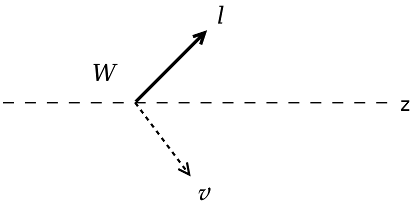
Transverse mass: M_T



α_ℓ, α_ν 2+1 momenta:

$$\alpha_\ell = (E_T^\ell, p_x^\ell, p_y^\ell), \quad \alpha_\nu = (E_T^\nu, p_x^\nu, p_y^\nu)$$

$$E_T^\ell = \sqrt{(p_x^\ell)^2 + (p_y^\ell)^2 + m_\ell^2}, \quad E_T^\nu = \sqrt{(p_x^\nu)^2 + (p_y^\nu)^2 + m_\nu^2}.$$

Transverse mass: the 2+1 dimensional invariant mass

$$M_T^2 = (\alpha_\ell + \alpha_\nu)^2.$$

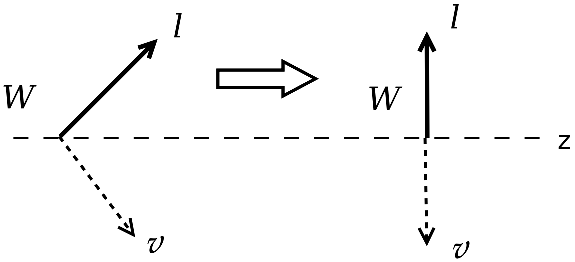
Properties of M_T

$$M_T \leq M_W.$$

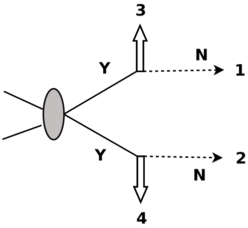
The equality holds for $\eta_\ell = \eta_\nu$.

The 4d invariant mass equals the 2+1d invariant mass when ℓ and ν have the same rapidity.

- Same rapidity $\Longrightarrow$ a common longitudinal boost to transverse plane.
- M_T invariant under longitudinal boosts.



M_{T2} (S-transverse mass) *Lester etal.*



- Trial N mass, μ_N.
- Consider all partitions of $\not{p}_T = \mathbf{p}_T^{(1)} + \mathbf{p}_T^{(2)}$.

$$M_{T2}(\mu_N) \equiv \min_{\mathbf{p}_T^{(1)}+\mathbf{p}_T^{(2)}=\not{p}_T} \left[\max\{M_T(1,3;\mu_N), M_T(2,4;\mu_N)\}\right]$$

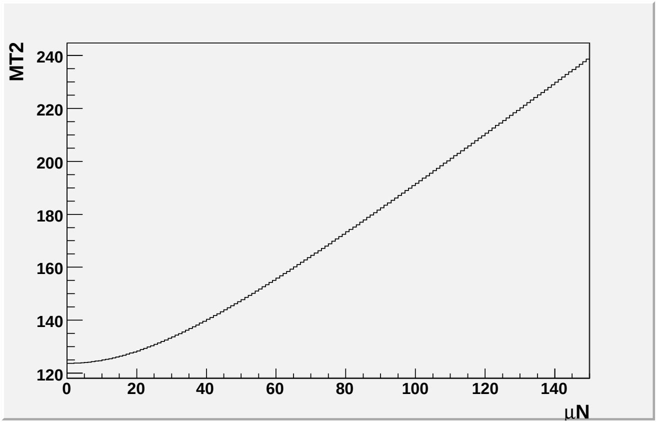
In other words...

$$M_{T2}(\mu_N) \equiv \min_{\mathbf{p}_T^{(1)}+\mathbf{p}_T^{(2)}=\not{\mathbf{p}}_T} \left[\max\{M_T(1,3;\mu_N), M_T(2,4;\mu_N)\}\right]$$

M_{T2} is the minimum M satisfying

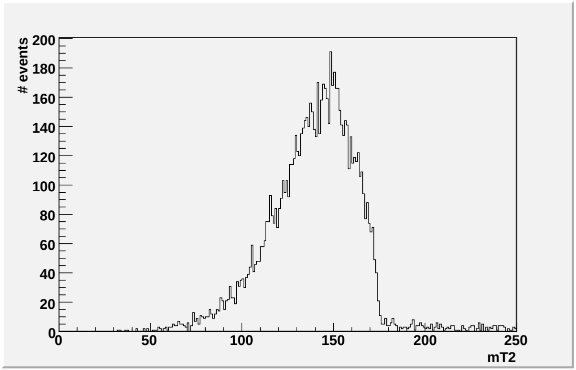
$$\begin{aligned}(\alpha_1+\alpha_3)^2 &= M^2,\\ (\alpha_2+\alpha_4)^2 &\le M^2,\\ \mathbf{p}_T^{(1)}+\mathbf{p}_T^{(2)} &= \not{\mathbf{p}}_T.\end{aligned}$$

Example event



M_{T2} endpoint

M_{T2} endpoint is m_Y if $\mu_N = m_N$.



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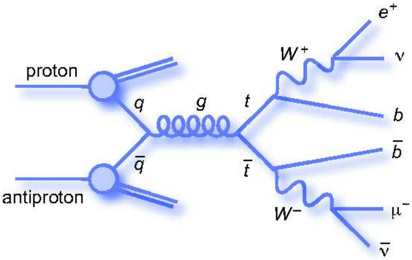
An example: $t\bar{t}$



$$
\begin{aligned}
&p_\nu^2 = p_{\bar\nu}^2 = 0,\\
&(p_\nu + p_{l^+})^2 = (p_{\bar\nu} + p_{l^-})^2 = m_W^2,\\
&(p_\nu + p_{l^+} + p_b)^2 = (p_{\bar\nu} + p_{l^-} + p_{\bar b})^2 = m_t^2,\\
&p_\nu^x + p_{\bar\nu}^x = \not{p}^x, \;\; p_\nu^y + p_{\bar\nu}^y = \not{p}^y.
\end{aligned}
$$

$p_\nu, p_l \ldots$: 4-momenta.

Assuming m_t, m_W, m_ν on shell, we can solve the system for p_ν and $p_{\bar\nu}$.

- Can we determine masses for new physics with the same topology?

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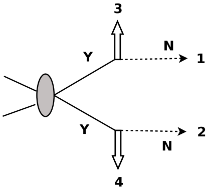
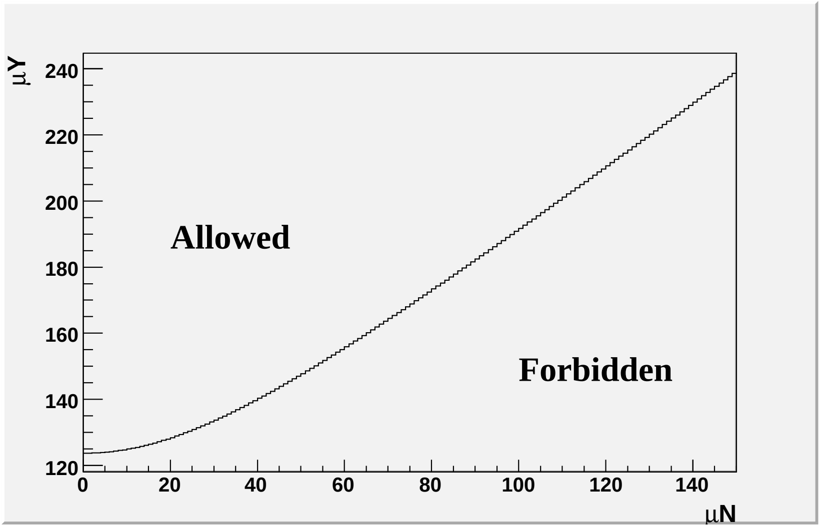
“Minimal” kinematic constraints



$$(p_1 + p_3)^2 = (p_2 + p_4)^2 = \mu_Y^2,$$
$$p_1^2 = p_2^2 = \mu_N^2,$$
$$p_1^x + p_2^x = \not{p}^x, p_1^y + p_2^y = \not{p}^y.$$

What is the mass region consistent with the above equations?

Allowed mass region for one event



Relation to M_{T2}

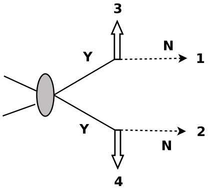
Claim: $m_{T2}(\mu_N)$ is the boundary of the allowed region.

$\mu_Y \geq m_{T2}(\mu_N) \Leftrightarrow (\mu_Y, \mu_N)$ in the allowed region.

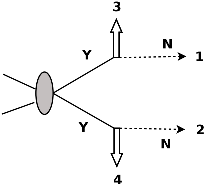
Proof:

- $\Leftarrow$: $\mu_Y \geq \max\{M_T^{(1)}, M_T^{(2)}\} \geq M_{T2}$
- $\Rightarrow$: Given α_1, α_2:

$$\begin{aligned} (\alpha_1+\alpha_3)^2 &= M_{T2}^2, \\ (\alpha_2+\alpha_4)^2 &\leq M_{T2}^2, \\ \mathbf{p}_T^{(1)} + \mathbf{p}_T^{(2)} &= \not{\mathbf{p}}_T. \end{aligned} \quad \overset{?}{\Rightarrow} \quad \begin{aligned} (p_1+p_3)^2 &= M_{T2}^2, \\ (p_2+p_4)^2 &= M_{T2}^2, \\ \mathbf{p}_T^{(1)} + \mathbf{p}_T^{(2)} &= \not{\mathbf{p}}_T. \end{aligned}$$



$$(\alpha_1 + \alpha_3)^2 = M_{T2}^2,$$
$$(\alpha_2 + \alpha_4)^2 \le M_{T2}^2,$$
$$\mathbf{p}_T^{(1)} + \mathbf{p}_T^{(2)} = \not{\mathbf{p}}_T.$$



α_1, α_2 known $\Rightarrow p_{1x,y}, p_{2x,y}$ known, but p_{1z}, p_{2z} are arbitrary, therefore η_1, η_2 arbitrary.

- Set $\eta_1 = \eta_3$:

$$(p_1 + p_3)^2 = (\alpha_1 + \alpha_3)^2 = M_{T2}^2$$

- $\eta_2 = \eta_4 \Rightarrow (p_2 + p_4)^2 = (\alpha_2 + \alpha_4)^2 \le M_{T2}^2,$
  $\eta_2 \to \infty \Rightarrow (p_2 + p_4)^2 \to \infty,$

$$\Rightarrow \exists \ \eta_2, \text{ such that, } (p_2 + p_4)^2 = M_{T2}^2$$

How to calculate M_{T2}?

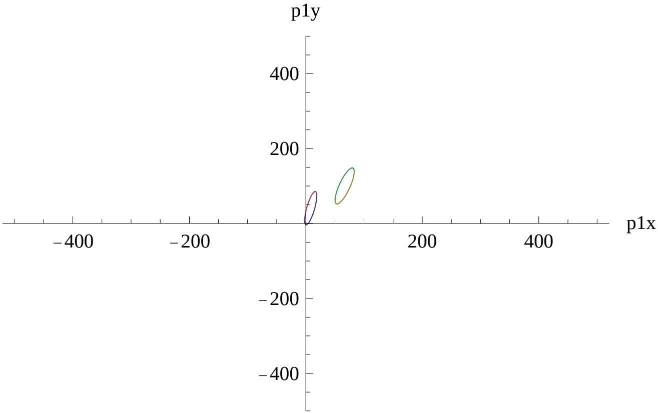
- M_{T2} is the boundary of the allowed region, find the allowed region first.
- Consider one decay chain. Given μ_N,μ_Y,

$$p_1^2 = \mu_N^2,$$
$$(p_1 + p_3)^2 = \mu_Y^2$$
$$\Rightarrow [E_1(p_{1x}, p_{1y}), p_{1x}, p_{1y}, p_{1z}(p_{1x}, p_{1y})].$$

  Physical (real) momentum
  $\Rightarrow$ allowed (p_{1x}, p_{1y}) inside an ellipse (assuming $m_3 > 0$, if $m_3 = 0$ intead, a parabola).
- Another ellipse for the other chain, but
  $p_{2x} = \not{p}_x - p_{1x}, p_{2y} = \not{p}_y - p_{1y}$
  $\Rightarrow$ Two ellipses on the (p1x,p1y) plane. If they overlap, we have physical solutions.

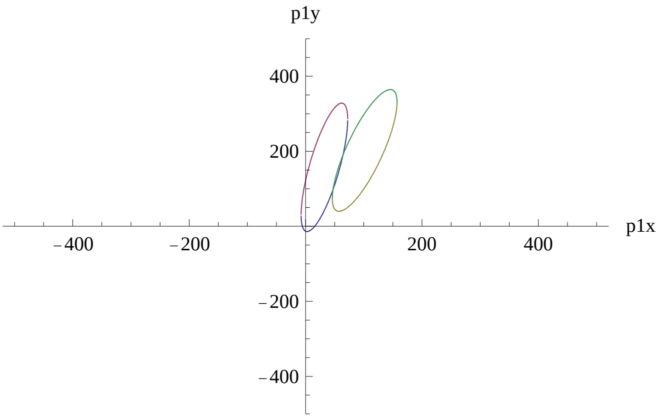
Ellipses

$\mu_N = 1, \mu_Y = 117$



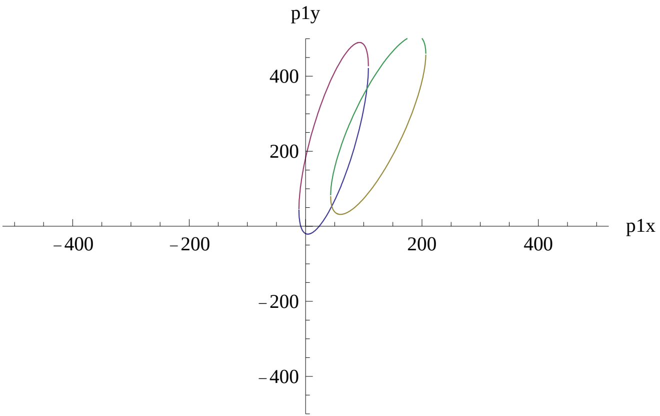
Ellipses

$\mu_N = 1, \mu_Y = 154$



Ellipses

$$\mu_N = 1, \mu_Y = 174$$



- Fix μ_N, sizes of the ellipses increase as μ_Y increases.
- μ_Y=m_{T2} when the two ellipses are tangent to each other.
- Based on this, developed algorism, 2-3 times faster than existing code.

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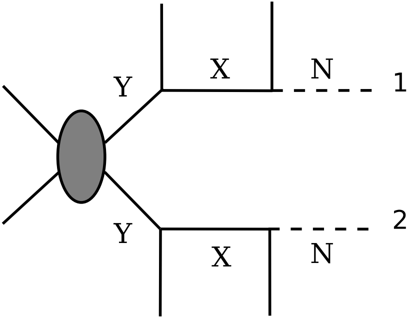
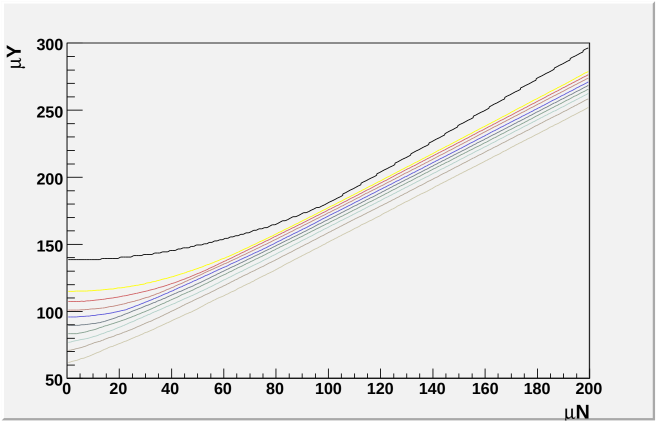
SUSY example

Example $\tilde{\chi}_2^0 \to \tilde{\ell}\ell \to \tilde{\chi}_1^0 \ell\ell$, 3 on-shell particles, masses (180,143,100) GeV.



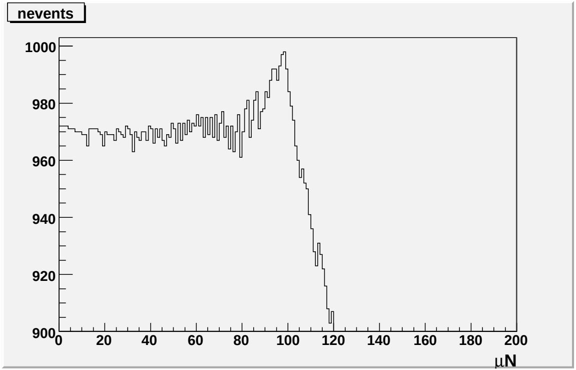
M_{T2} kink, *Cho,Choi,Kim,Park*

- M_{T2} endpoint contour, parton level,1000 events



More constraints

- Go along the M_{T2} endpoint contour, imposing more mass shell constraints from $m_{\tilde{\ell}}$.
- Vary $m_{\tilde{\ell}}$ to maximize number of consistent events.
- Plot number of consistent events as a function of μ_N.



Yes, we can determine the masses of the new particles.

Summary and outlook

- Examples show the power of kinematic constraints.
- M_{T2} is equivalent to “minimal” kinematic constraints.
- Improve/invent methods.
- Find applications at Tevatron/LHC.