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Fermilab

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Theory prediction for $\Delta\Gamma$ and $\Delta\Gamma/\Delta m$ in the B_s system

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Outline

1. $\mathrm{B}-\overline{\mathrm{B}}$ mixing basics
2. Mass difference Δm
3. Width difference $\Delta\Gamma$
4. CP violation in mixing
5. Summary and Outlook

1. $\mathrm{B}-\overline{\mathrm{B}}$ mixing basics

Schrödinger equation:

$$i\frac{d}{d\,t}\begin{pmatrix} |\,B(t)\,\rangle \\ |\,\overline{B}(t)\,\rangle \end{pmatrix} = \left(M - i\,\frac{\Gamma}{2}\right)\begin{pmatrix} |\,B(t)\,\rangle \\ |\,\overline{B}(t)\,\rangle \end{pmatrix}$$

3 physical quantities in $\mathrm{B}-\overline{\mathrm{B}}$ mixing:

$$|M_{12}|\,,\quad |\Gamma_{12}|\,,\quad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

Two mass eigenstates:

Lighter eigenstate: $|\,B_L\,\rangle = p|\,B^0\,\rangle + q|\,\overline{B}^0\,\rangle.$

Heavier eigenstate: $|\,B_H\,\rangle = p|\,B^0\,\rangle - q|\,\overline{B}^0\,\rangle$ with $|p|^2 + |q|^2 = 1.$

with masses $M_{L,H}$ and widths $\Gamma_{L,H}$.
Here B represents either B_d or B_s.

To determine $|M_{12}|$, $|\Gamma_{12}|$ and ϕ measure

$$\Delta m = M_H - M_L \simeq 2|M_{12}|,$$

$$\Delta\Gamma = \Gamma_L - \Gamma_H \simeq -\Delta m \, \mathrm{Re} \frac{\Gamma_{12}}{M_{12}} = 2|\Gamma_{12}| \cos\phi$$

and

$$a_{\mathrm{fs}} = \mathrm{Im} \frac{\Gamma_{12}}{M_{12}} = \left| \frac{\Gamma_{12}}{M_{12}} \right| \sin\phi.$$

a_{fs} is the CP asymmetry in flavour-specific B decays.
a_{fs} measures CP violation in mixing.

Standard Model expectations:

$$\Delta m \sim 20\,\mathsf{ps}^{-1}$$

$$\Delta\Gamma \sim 0.1\,\mathsf{ps}^{-1}$$

$$\frac{\Delta\Gamma}{\Delta m} \simeq \left|\frac{\Gamma_{12}}{M_{12}}\right| = \mathcal{O}\left(\frac{m_b^2}{M_W^2}\right) \sim 4\cdot 10^{-3}$$

$$a_{\mathrm{fs}} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi \sim 2\cdot 10^{-5}$$

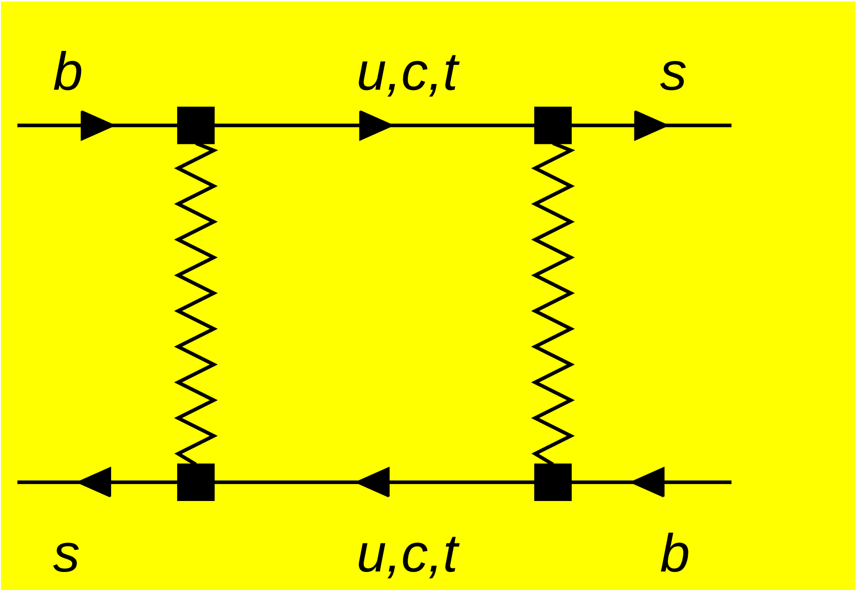
$$\phi = \mathcal{O}\left(|V_{us}|^2\frac{m_c^2}{m_b^2}\right) \sim 5\cdot 10^{-3} = 0.3^\circ$$

New physics

Standard Model:

M_{12} from dispersive part of box,
only internal t relevant;

Γ_{12} from absorptive part of box,
only internal u, c contribute.



New physics can barely affect Γ_{12}, which stems from tree-level decays.

M_{12} is very sensitive to virtual effects of new heavy particles.

$\Rightarrow$ $\Delta m \simeq 2|M_{12}|$ can change.

and in $\phi \simeq \arg(-M_{12}/\Gamma_{12})$ the GIM suppression of ϕ can be lifted.

$\Rightarrow$ $|\Delta\Gamma| = \Delta\Gamma_{\rm SM}\,|\cos\phi|$ is depleted

and $|a_{\rm fs}|$ is enhanced, $|a_{\rm fs}|$ by up to a factor of 200.

To identify or constrain new physics one wants to measure both the magnitude and phase of M_{12}.

$$\rightarrow \qquad \Delta m = 2|M_{12}|$$

Information on $\arg M_{12}$ can be gained from mixing-induced CP asymmetries, which require tagging.

Two untagged measurements are sensitive to $\arg M_{12}$:

$|\Delta\Gamma| = \Delta\Gamma_{\rm SM}\,|\cos\phi|$ and $a_{\rm fs} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi$, where $\phi = \arg(-\frac{M_{12}}{\Gamma_{12}})$.
The maximal possible value for $a_{\rm fs}$ in the presence of new physics is $a_{\rm fs} = 5 \cdot 10^{-3}$ (corresponding to $\sin\phi = 1$).

2. Mass difference Δm

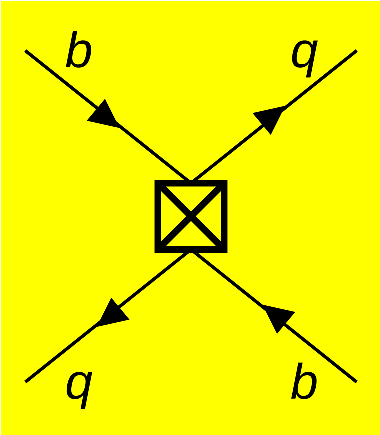
Δm is measured from the B–$\overline{\mathrm{B}}$ oscillations, which are governed by $\cos(\Delta m t)$:

$$\mathcal{A}_0(t) = \frac{\Gamma(B(t) \to f) - \Gamma(B(t) \to \overline{f})}{\Gamma(B(t) \to f) + \Gamma(B(t) \to \overline{f})} = \frac{\cos(\Delta m\, t)}{\cosh(\Delta\Gamma\, t/2)}$$

Eq. (1.81) from *B physics at the Tevatron*

Local four-quark operator:

$$Q = \overline{q}_L \gamma_\nu b_L \, \overline{q}_L \gamma^\nu b_L$$



Theoretical uncertainty dominated by matrix element:

$$\langle \mathrm{B}^0 \, |\mathrm{Q}| \, \overline{\mathrm{B}}^0 \rangle = \frac{2}{3} m_{B_s}^2 \, f_{B_s}^2 \, \widehat{B}$$

$$\Delta m_s = 17.2\,\mathrm{ps}^{-1} \left(\frac{|V_{ts}|}{0.04} \, \frac{f_{B_s}}{230\,\mathrm{MeV}} \right)^2 \frac{\widehat{B}_{B_s}}{1.3}$$

Eq. (8.65) from *B physics at the Tevatron*

Lattice results (Okamoto, Lattice 2005):

$$f_{B_s} = (260 \pm 36)\ \mathrm{MeV} \qquad \text{HPQCD}$$

$$B = 0.84 \pm 0.07, \qquad \Rightarrow \qquad \widehat{B} = 1.52 B = 1.28 \pm 0.11$$

Hence:

$$\Delta m = 23^{+9}_{-7}\,\mathrm{ps}^{-1}$$

One can also predict $\Delta m = \Delta m_{B_s}$ from the ratio

$$\Delta m \, = \, 20.0 \, \xi^2 \, \frac{\Delta m_{B_d}}{R_t^2}$$

where

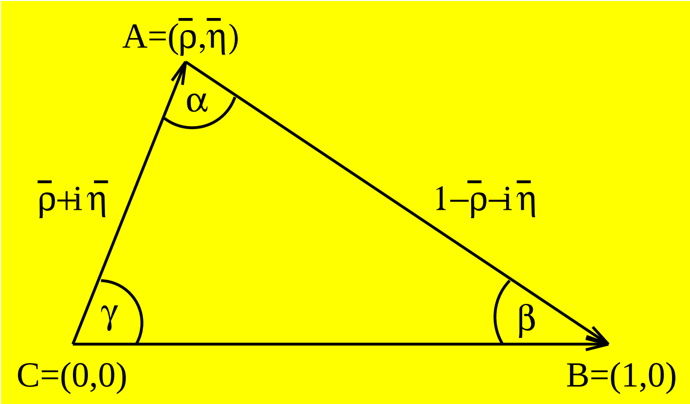
$$\xi = \frac{f_{B_s}\sqrt{\widehat{B}_{B_s}}}{f_{B_d}\sqrt{\widehat{B}_{B_d}}} \, = \, 1.21^{+0.05}_{-0.04}$$

HPQCD,JLQCD

and R_t is one side of the unitarity triangle:

$$R_t \, = \, \sqrt{(1-\overline{\rho})^2 + \overline{\eta}^2} \, \simeq \, \frac{|V_{td}|}{|V_{ts} V_{cb}|}$$

$$= 0.97 \pm 0.19$$



The 2σ CL range of

$$R_t = 0.863^{+0.109}_{-0.107} \qquad \text{CKMFitter}$$

implies

$$\Delta m = 20^{+8}_{-6}\ \text{ps}^{-1}$$

Within the Standard Model a measurement of Δm gives a good constraint on R_t.

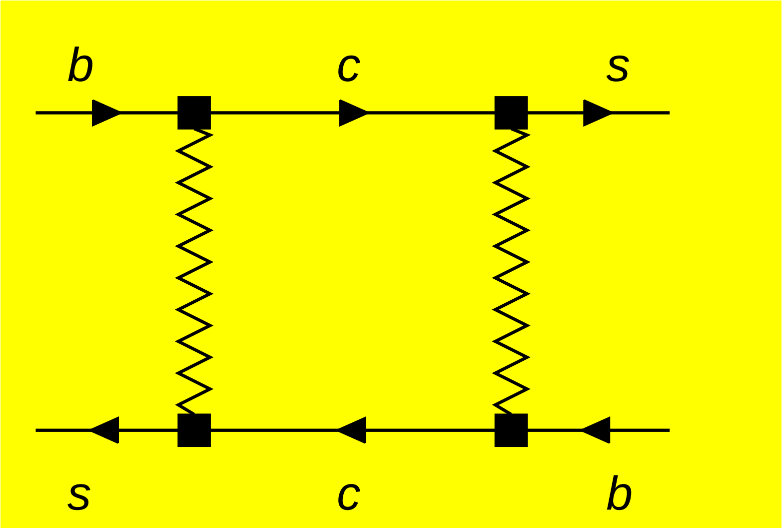
However, even a measurement which is consistent with the Standard Model leaves the possibility of $\mathcal{O}(1)$ effects from new physics in M_{12}.

3. Width difference $\Delta\Gamma$

Consider $\Delta\Gamma_{B_s}$ in the Standard Model:

$B_s \sim \bar{b}s, \qquad \overline{B}_s \sim b\bar{s}$

Effect from internal up quarks negligible.



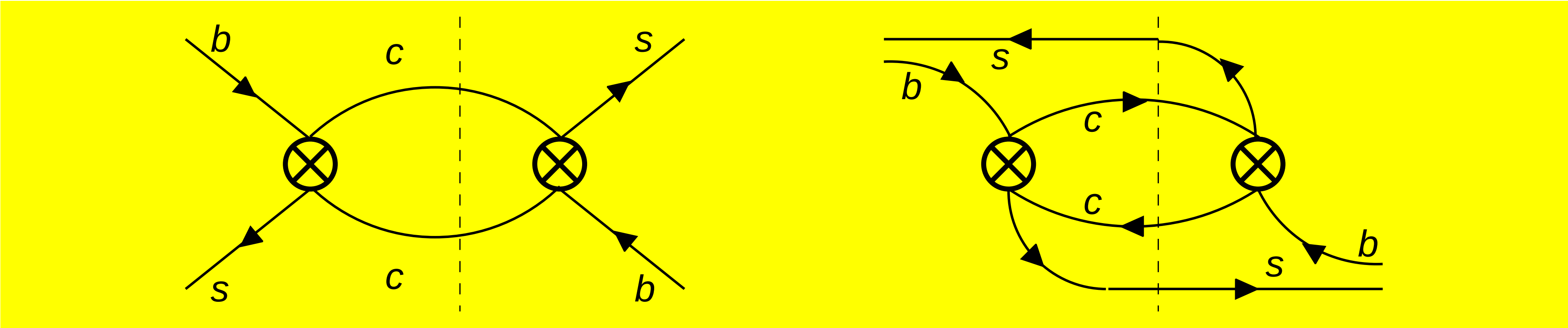
Standard Model: Identify mass eigenstates with CP eigenstates:

$$|\,B_{L,H}\,\rangle = \frac{1}{\sqrt{2}}\left[|\,B_s\,\rangle \mp |\,\overline{B}_s\,\rangle\right], \qquad B_L \text{ is CP-even and } B_H \text{ is CP-odd}$$

$$\Delta\Gamma_{B_s} \equiv \Gamma_L - \Gamma_H \simeq 2|\Gamma_{12}|$$

$\Delta\Gamma_{B_s}$ stems from decays into final states which are common to B_s and $\overline{B}_s$ and contain a $(c,\bar{c})$ pair. CP-even final states like $D_s^+ D_s^-$ contribute positively to $\Delta\Gamma_{B_s}$, while decays into CP-odd states diminish $\Delta\Gamma_{B_s}$.

First step: Express W-mediated $b \to c\bar{c}s$ decay through an effective hamiltonian $\mathcal{H}_{eff}$ with four-quark operators.



Optical theorem:

$$\Gamma_{12} = -\frac{1}{2M_{B_s}} \,\mathsf{Abs}\, \langle B_s | \, i \int d^4x \; T \, \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) | \bar{B}_s \rangle$$

Second step: Expand in Λ_{QCD}/m_b using an operator product expansion:

$$\Gamma_{12} = -\frac{1}{2M_{B_s}} \text{Abs} \langle B_s | \, i \int d^4x \; T \, \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) | \bar{B}_s \rangle$$

$$\propto G_F^2 \sum_j m_b^{8-d_j} \, c_j \; (\mu/m_b) \; \underbrace{\langle B_s | \mathcal{O}_j \, (\mu) \, | \overline{B}_s \rangle}_{\mathcal{O}\left(\Lambda_{QCD}^{d_j-3}\right)}$$

c_j: Wilson coefficients containing physics from scales $\geq \mu = \mathcal{O}(m_b)$

$\mathcal{O}_j$: local $\Delta B = 2$ operators with dimension $d_j \geq 6$.

Effect: Expansion of Γ_{12} in Λ_{QCD}/m_b and $\alpha_s(m_b)$.

A new operator emerges:

$$Q_S = \overline{b}_R s_L \, \overline{b}_R s_L$$
$$\langle B_s | Q_S | \overline{B}_s \rangle = -\frac{5}{12} M_{B_s}^2 \frac{M_{B_s}^2}{(\bar{m}_b + \bar{m}_s)^2} f_{B_s}^2 \, B_S$$

Our 1998 prediction including corrections of order α_s and Λ_{QCD}/m_b:

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{210\,\mathrm{MeV}}\right)^2 [0.006\, B \, + \, 0.172\, B_S \, - \, 0.063]$$
$$= 0.14 \pm 0.05.$$

Pathological situation: Both the $1/m_b$ and α_s corrections are large and decrease $\Delta\Gamma$, leading to large uncertainties. Moreover B_S dominates over B, so that $\Delta\Gamma/\Delta m$ depends on B_S/B.

In the leading order of the $1/m_b$ expansion one first encounters a third operator:

$$\widetilde{Q}_S = \overline{b}_R^i s_L^j \, \overline{b}_R^j s_L^i,$$

where i, j are color indices.
Then $\widetilde{Q}_S$ is eliminated in favor of

$$R_0 = \widetilde{Q}_S + Q_S + \frac{Q}{2} = \mathcal{O}(1/m_b),$$

which is one out of five operators appearing at $\mathcal{O}(1/m_b)$. The matrix element of $\widetilde{Q}_S$ is small:

$$\langle B_s | \widetilde{Q}_S | \overline{B}_s \rangle = \frac{1}{12} \, M_{B_s}^2 \, \frac{M_{B_s}^2}{(\bar{m}_b + \bar{m}_s)^2} \, f_{B_s}^2 \, \widetilde{B}_S$$

Becirevic et al. find $\widetilde{B}_S = 0.91 \pm 0.09$.

Eliminate Q_S in favor of $\widetilde{Q}_S$ to find:

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{210\,\mathrm{MeV}}\right)^2 \left[0.116\,B \;+\; 0.042\,\widetilde{B}_S \;-\; 0.035\right]$$
$$= 0.15 \pm 0.05.$$

$\Rightarrow$ $\Delta\Gamma$ is now dominated by the term proportional to B and the $1/m_b$ corrections are smaller.

The size of the $1/m_b$ corrections has decreased from 33% to 19% of the leading order (in both $1/m_b$ and α_s) result.

Remark: $\Delta\Gamma/\Gamma$ above is calculated using $1/\Gamma = \tau_{B_s} \simeq \tau_{B_d}$. In 1998 we used $1/\Gamma \simeq B_{SL}/\Gamma_{SL}$. Better quote:

$$\Delta\Gamma = \left(\frac{f_{B_s}}{260\,\text{MeV}}\right)^2 \left[0.116\,B \;+\; 0.042\,\widetilde{B}_S \;-\; 0.035\right]$$
$$= 0.10 \pm 0.03\,\text{ps}^{-1}.$$

$$\frac{\Delta\Gamma}{\Delta m} = \left[30 \pm 7 + (15 \pm 1)\frac{\widetilde{B}_S}{B}\right] \cdot 10^{-4}$$
$$= (47 \pm 8) \cdot 10^{-4}$$

All numbers are preliminary.

Measurement

Time evolution of any decay of an untagged $\overset{(-)}{B_s} \to f$ decay:

$$\Gamma[f,t] \propto |\langle f | B_L \rangle|^2 e^{-\Gamma_L t} + |\langle f | B_H \rangle|^2 e^{-\Gamma_H t}$$

Consider $f = (J/\psi\phi)_{L=0}$ (i.e. S-wave), which is CP-even:

$$\langle (J/\psi\phi)_{L=0} | B_H \rangle = \langle (J/\psi\phi)_{L=0} | B_s^{\mathrm{CP-odd}} \rangle = 0$$

$\Rightarrow$ Lifetime measured in $\overset{(-)}{B_s} \to (J/\psi\phi)_{L=0}$ determines Γ_L.

Then use $(\Gamma_L + \Gamma_H)/2 \simeq \Gamma_d$ from theory or determine Γ_H from a lifetime measurement from decays to the CP-odd final state $(J/\psi\phi)_{L=1}$ (i.e. P-wave) to find:

$$\Gamma_L - \Gamma_H = \Delta\Gamma$$

$\Delta\Gamma$ beyond the Standard Model

Γ_{12} is a tree-level quantity and is difficult to change significantly in models of new physics. It's safe to assume $\Gamma_{12} = \Gamma_{12,\mathrm{SM}}$.
Then new physics can only enter $\Delta\Gamma$ via $\cos\phi$. Two effects:

- $\Delta\Gamma = \Delta\Gamma_{\mathrm{SM}} \cos\phi$.
- $|\, B_L \,\rangle$ and $|\, B_H \,\rangle$ are no more CP eigenstates.
  $\Rightarrow$ both $|\, B_L \,\rangle$ and $|\, B_H \,\rangle$ can decay into $(J/\psi\phi)_{L=0}$
  $\Rightarrow$ the lifetime measured in $\overset{(-)}{B}_s \to (J/\psi\phi)_{L=0}$ is no more $1/\Gamma_L$.

As a result the comparison of the width measured in this decay and Γ_{B_s} yields

$$\Delta\Gamma_{\mathrm{SM}} \cos^2\phi.$$

Grossman 1996, Dunietz, Fleischer, U.N. 2000

$\Rightarrow$ New physics contributions to M_{12} can only diminish $\Delta\Gamma$. Further the described measurements yield no information on the sign of $\Delta\Gamma$.

4. CP violation in mixing

$$a_{\rm fs} = {\rm Im}\,\frac{\Gamma_{12}}{M_{12}} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi.$$

$a_{\rm fs}$ is the CP asymmetry in decays $B \to f$ which are flavour-specific, i.e.

$$\overline{B} \not\to f \text{ and } B \not\to \overline{f}.$$

Examples: $B_{d,s} \to X\ell^+\nu_\ell$ or $B_s \to D_s^-\pi^+$.

$$a_{\rm fs} = \frac{\Gamma(\overline{B}(t) \to f) - \Gamma(B(t) \to \overline{f})}{\Gamma(\overline{B}(t) \to f) + \Gamma(B(t) \to \overline{f})}$$

The time dependence of the decay rates $\Gamma(\overline{B}(t) \to f)$ and $\Gamma(B(t) \to \overline{f})$ drops out.

$a_{\rm fs}$ measures CP violation in mixing.

No tagging is necessary: With $\Gamma[f,t] = \Gamma(B(t) \to f) + \Gamma(\overline{B}(t) \to f)$,

$$a_{\mathrm{fs}}^{\mathrm{untagged}} = \frac{\Gamma[f,t] - \Gamma[\overline{f},t]}{\Gamma[f,t] + \Gamma[\overline{f},t]} = \frac{a_{\mathrm{fs}}}{2}\left[1 - \frac{\cos(\Delta m\, t)}{\cosh(\Delta\Gamma\, t/2)}\right]$$

Even time-integrated rates are useful:

$$A_{\mathrm{fs,unt}} \equiv \frac{\int_0^\infty dt[\Gamma[f,t] - \Gamma[\overline{f},t]]}{\int_0^\infty dt[\Gamma[f,t] + \Gamma[\overline{f},t]]} = \frac{a_{\mathrm{fs}}}{2}\,\frac{x^2 - y^2}{x^2 - 1} \simeq \frac{a_{\mathrm{fs}}}{2},$$

where $x = \Delta m/\Gamma$, $y = \Delta\Gamma/(2\Gamma)$.

$\Rightarrow$ Only need to count e.g. positive vs. negative leptons from untagged B^0 decays.

a_{fs} in the Standard Model

$$
\begin{aligned}
a_{\mathrm{fs}} &= \mathrm{Im}\,\frac{\Gamma_{12}}{M_{12}} \\
&= (2.1 \pm 0.4) \cdot 10^{-5}
\end{aligned}
$$

Negligible. $\Rightarrow$ If you see it, it's new physics.

New physics can lift $a_{\mathrm{fs}}(B_s)$ to $\sim 5 \cdot 10^{-3}$.

5. Summary and Outlook

- Large effects from new physics are still possible in Δm.
- The large size of the $1/m_b$ corrections in $\Delta\Gamma$ seems to be an artifact of a poor choice of operators in the leading order of the $1/m_b$ expansion. The calculation of $1/m_b^2$ effects is underway.
- The choice of $\widetilde{Q}_S$ instead of Q_S further diminishes the dependence of $\Delta\Gamma/\Delta m$ on hadronic parameters.
- A more precise measurement of $\Delta\Gamma$ will constrain new physics by excluding a region in the complex M_{12} plane through $\Delta\Gamma/\Delta m$.
- $a_{\rm fs}$ will contribute to constrain M_{12}, once experimental upper bounds $|a_{\rm fs}| \leq 5 \cdot 10^{-3}$ become available.