

Lepton-Photon 2005 conference
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Quark mixing and CP violation – the CKM matrix

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Outline

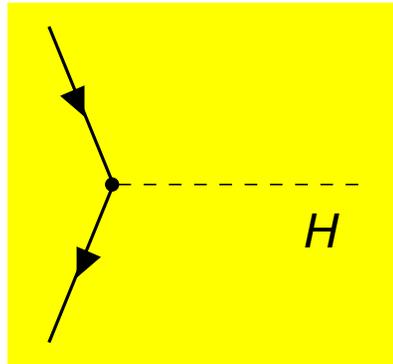
1. Flavour in the Standard Model
2. CKM elements from tree-level decays
3. CKM elements from FCNC processes
4. CP violation in $b \rightarrow s$ penguin decays
5. Summary

1. Flavour in the Standard Model

Origin of the Cabibbo-Kobayashi-Maskawa matrix:

Yukawa couplings of the Higgs field:

$$y_{ij} \bar{f}_i f_j (v + H)$$



\Rightarrow quark mass matrix: $m_{ij} = y_{ij}v$

diagonalisation \Rightarrow fermion masses and CKM matrix V .

$V \neq 1$

\Rightarrow couplings of the W -Bosons to quarks of different generations,
flavour physics

y_{ij}, V complex \Rightarrow one physical CP violating phase

The CKM matrix...

... expanded in $\lambda \simeq 0.22$:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3 \left(1 + \frac{\lambda^2}{2}\right) (\bar{\rho} - i\bar{\eta}) \\ -\lambda - iA^2\lambda^5\bar{\eta} & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 - iA\lambda^4\bar{\eta} & 1 \end{pmatrix}$$

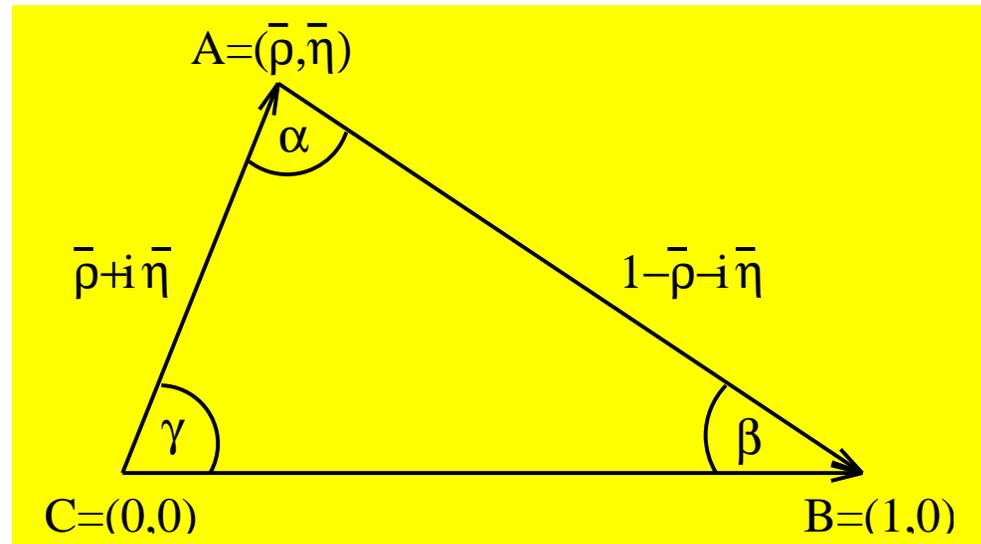
with the Wolfenstein parameters $\lambda, A, \bar{\rho}, \bar{\eta}$

CP violation $\Leftrightarrow \bar{\eta} \neq 0$

Unitarity triangle:

Exact definition:

$$\begin{aligned} \bar{\rho} + i\bar{\eta} &= -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \\ &= \left| \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right| e^{i\gamma} \end{aligned}$$

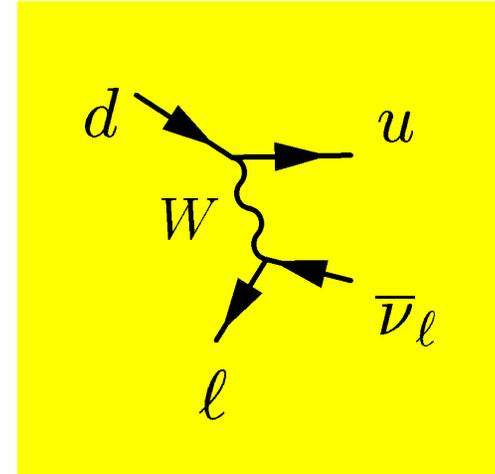


2. CKM elements from tree-level decays

$$V_{ud}$$

V_{ud} is determined from semileptonic $d \rightarrow u \ell^- \bar{\nu}_\ell$ decays through lifetime measurements:

- superallowed ($0^+ \rightarrow 0^+$) nuclear β decay,
- $n \rightarrow p \ell \bar{\nu}_\ell(\gamma)$ or
- $\pi^- \rightarrow \pi^0 \ell \bar{\nu}_\ell(\gamma)$.



All methods involve the hadronic form factor of the vector current:

$$\langle f | \bar{u} \gamma_\mu d | i \rangle$$

Here $(i, f) = (0^+, 0^+)$, (n, p) or (π^\pm, π^0) .

$n \rightarrow p \ell \bar{\nu}_\ell(\gamma)$ further involves the form factor of the axial vector current:

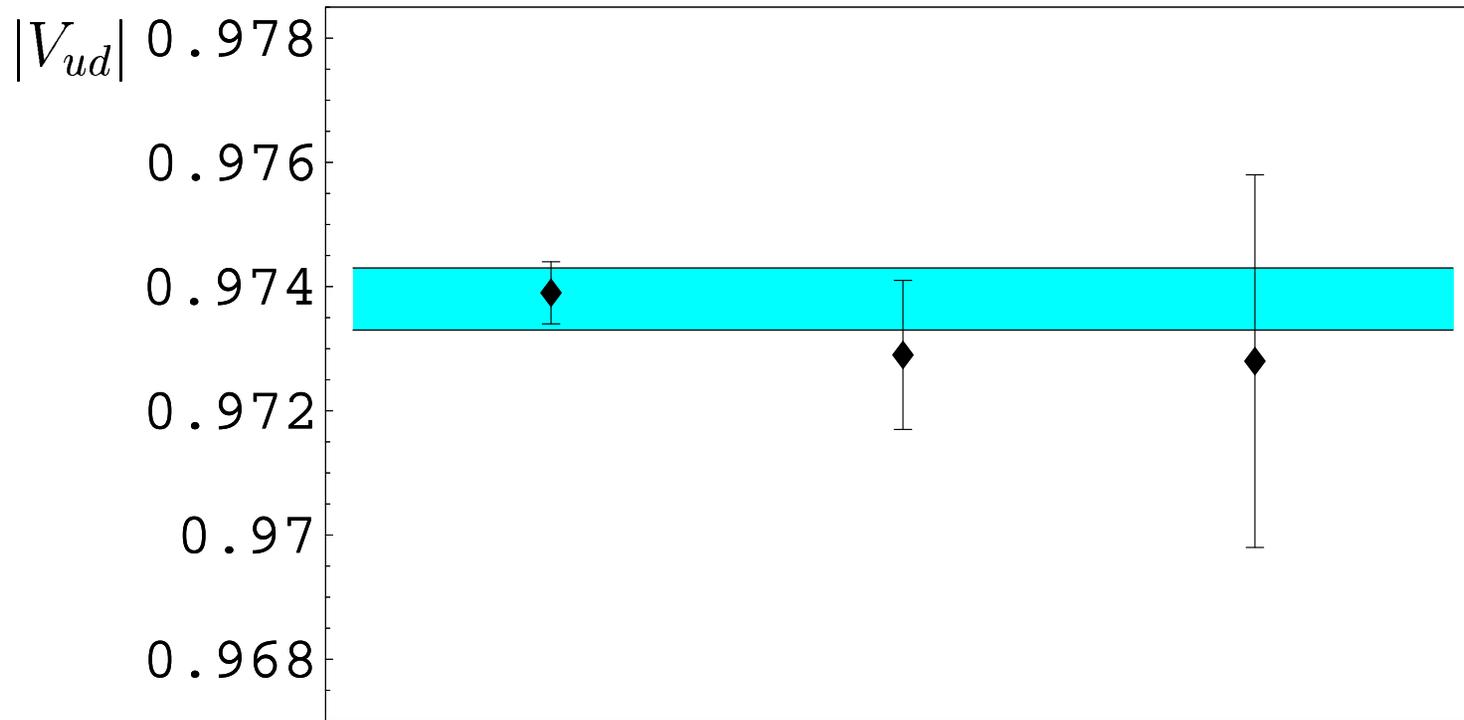
$$\langle f | \bar{u} \gamma_\mu \gamma_5 d | i \rangle$$

Isospin symmetry \Rightarrow normalization of the vector form factor is fixed at $p_i = p_f$.

Ademollo-Gatto theorem: Corrections are $\mathcal{O}\left(\frac{(m_d - m_u)^2}{\Lambda_{\text{had}}^2}\right)$, i.e. second order in the symmetry breaking parameter.

The normalization of the axial form factor is not fixed, but the corresponding parameter G_A can be fixed from asymmetries in the Dalitz plot.

method	theory drawback	theoretical cleanliness	experimental precision in V_{ud}
$0^+ \rightarrow 0^+$	nuclear effects	*	***
$n \rightarrow p \ell \bar{\nu}_\ell(\gamma)$	depends on G_A	**	** (but contradictory)
$\pi^- \rightarrow \pi^0 \ell \bar{\nu}_\ell(\gamma)$		***	*



$0^+ \rightarrow 0^+$

$n \rightarrow p\ell\bar{\nu}_\ell$

$\pi^+ \rightarrow \pi^0\ell\bar{\nu}_\ell$

courtesy of Vincenzo Cirigliano

Average:

$$V_{ud} = 0.9738 \pm 0.0005$$

Theoretical progress: calculation of the leading (two-loop) electroweak $\mathcal{O}(\alpha^2)$ corrections to $n \rightarrow p\ell\bar{\nu}_\ell(\gamma)$ by Czarnecki, Marciano, Sirlin 2004.

$$V_{us}$$

V_{us} from Kaon decays:

Chiral Perturbation Theory (χ PT): The pseudoscalar mesons π, K, \dots are Goldstone bosons of a dynamically broken chiral symmetry of the QCD lagrangian.

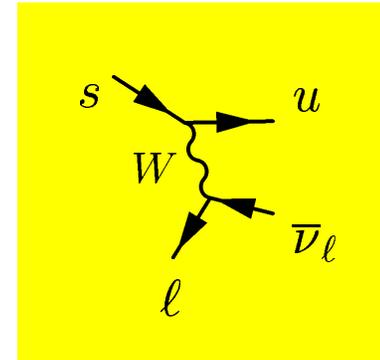
\Rightarrow systematic expansion in $\frac{p}{\Lambda_{\text{had}}}$, $\frac{M}{\Lambda_{\text{had}}}$, $\frac{m_\ell}{\Lambda_{\text{had}}}$ and the electroweak coupling e .

p and M denote meson momenta and masses.

$K\ell 3$ decays:

$$K^0 \rightarrow \pi^- \ell^+ \nu_\ell, \quad K^0 \rightarrow \pi^- \mu^+ \nu_\ell$$

$$K^+ \rightarrow \pi^0 \ell^+ \nu_\ell, \quad K^+ \rightarrow \pi^0 \mu^+ \nu_\ell$$



Dependence on key quantities:

$$\Gamma(K \rightarrow \pi \ell^+ \nu_\ell) \propto V_{us}^2 \left| f_+^{K^0 \pi^-}(0) \right|^2 \left[1 + 2\Delta_{SU(2)}^K + 2\Delta_{\text{em}}^{K\ell} \right]$$

with $\langle \pi^-(p_\pi) | \bar{s} \gamma^\mu u | K^0(p_K) \rangle = f_+^{K^0 \pi^-}(0) (p_K^\mu + p_\pi^\mu) + \mathcal{O}(p_K - p_\pi)$

$$\Delta_{SU(2)}^{K^+} = \frac{f_+^{K^+ \pi^0}(0)}{f_+^{K^0 \pi^-}(0)} - 1, \quad \Delta_{SU(2)}^{K^0} = 0$$

$\Delta_{\text{em}}^{K\ell}$: QED corrections

and $f_+^{K^0 \pi^-}(0) = 1 + \mathcal{O}\left(\frac{(m_s - m_d)^2}{\Lambda_{\text{had}}^2}\right)$

Substantial theory (χ PT) improvements since Lepton-Photon 2003:

Key references:

$\Delta_{\text{em}}^{K\ell}$: Cirigliano, Knecht, Neufeld, Rupertsberger, Talavera 2002

Cirigliano, Neufeld, Pichl 2004

Andre 2004

Moussallam, Descotes 2005

significant effect of $\mathcal{O}(e^2 p^2)$ corrections on differential distributions, must be included in Monte Carlo simulations.

$f_+^{K^0\pi^-}(0) - 1$: Leutwyler, Roos 1984

Post, Schilcher 2002

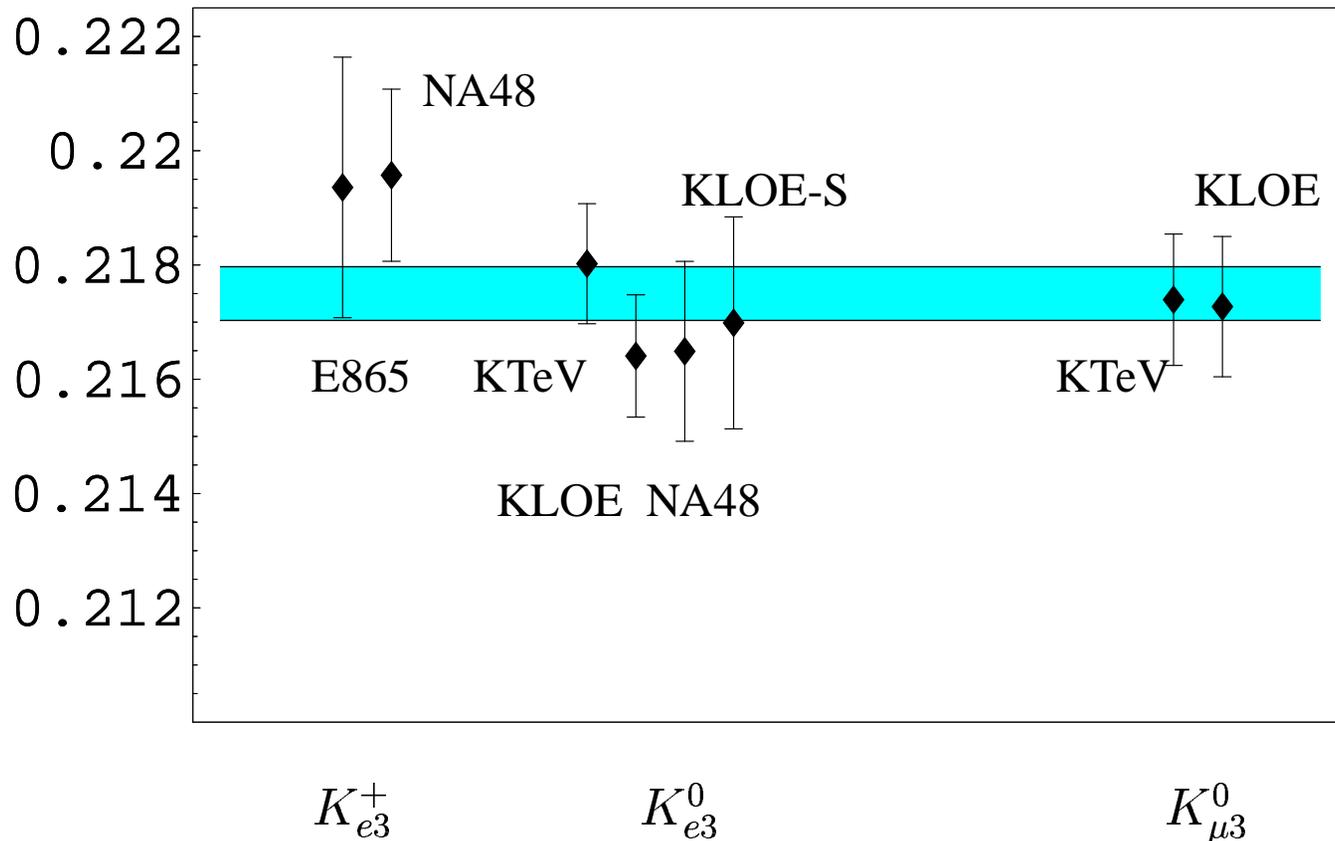
Bijnens, Talavera 2003

Jamin, Oller, Pich 2004

Becirevic et al. (SPQcdR)2004 (lattice!)

Cirigliano et al. 2005

$$|V_{us}|f_+^{K^0\pi^-}(0)$$



courtesy of Vincenzo Cirigliano

$$f_+^{K^0\pi^-} V_{us} = 0.2175 \pm 0.0008,$$

$$\text{theory: } f_+^{K^0\pi^-} = 0.972 \pm 0.012$$

$$\Rightarrow V_{us} = 0.2238 \pm 0.0029 \quad \text{from } K\ell 3$$

$K\mu 2$ decay: $K^+ \rightarrow \mu^+ \nu_\mu (\gamma)$

$$\frac{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu (\gamma))}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu (\gamma))} = \frac{V_{us}^2}{V_{ud}^2} \frac{F_K^2}{F_\pi^2} \frac{M_K^2 - m_\mu^2}{M_\pi^2 - m_\mu^2} \left[1 - \frac{\alpha}{\pi} (C_\pi - C_K) \right]$$

QED corrections: $C_\pi - C_K = 3.0 \pm 1.5$

Marciano 2004

Lattice: $\frac{F_K}{F_\pi} = 1.210 \pm 0.004 \pm 0.013$

MILC 2004

$\Rightarrow V_{us} = 0.2223 \pm 0.0026$ from $K\mu 2$

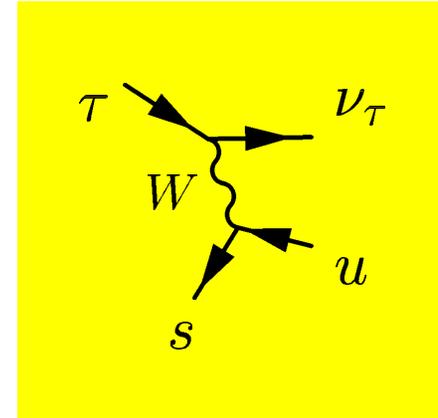
... astonishingly precise, it starts to constrain new physics (charged Higgs).

Marciano 2004

V_{us} from τ decay:

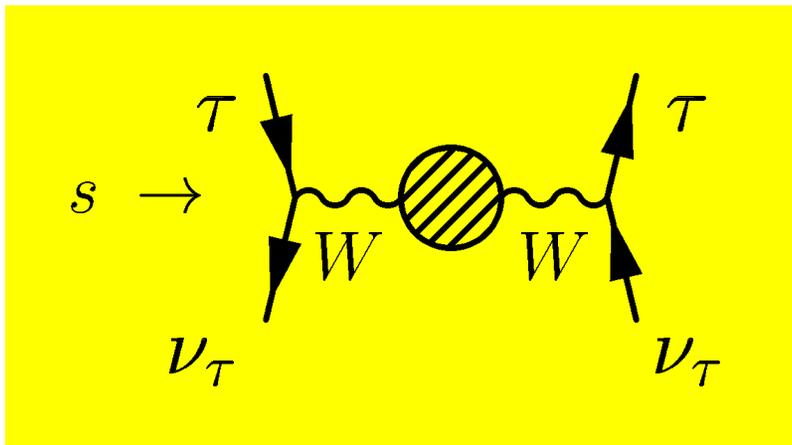
$$R_{\tau s} = \frac{\Gamma^{\Delta S=1}(\tau \rightarrow \text{hadrons } \nu_\tau(\gamma))}{\Gamma(\tau \rightarrow e\bar{\nu}_e\nu_\tau(\gamma))} \propto V_{us}^2$$

$$R_{\tau d} = \frac{\Gamma^{\Delta S=0}(\tau \rightarrow \text{hadrons } \nu_\tau(\gamma))}{\Gamma(\tau \rightarrow e\bar{\nu}_e\nu_\tau(\gamma))} \propto V_{ud}^2$$



Here S is the strangeness. $R_{\tau s,d}$ can be related to the QCD current-current correlators Π_S^T and Π_S^L (with $z = s/M_\tau^2$):

$$R_{\tau s,d} = 12\pi \int_0^1 dz (1-z)^2 [(1+2z) \text{Im} \Pi_{s,d}^T(z) + \text{Im} \Pi_{s,d}^L(z)]$$



$\Pi_{s,d}^{T,L}$ can be computed through the operator product expansion (OPE). The leading term is massless perturbative QCD, subleading operators are m_s^2 and $m_s \langle \bar{q}q \rangle$.

Compute the flavour SU(3) breaking difference:

$$\delta R_\tau \equiv \frac{R_{\tau d}}{V_{ud}^2} - \frac{R_{\tau s}}{V_{us}^2} \quad \text{Pich, Prades; ALEPH (1998)}$$

With $R_{\tau d} = 3.469 \pm 0.014$ and $R_{\tau s} = 0.1694 \pm 0.0049$ from experiment find:

$$\begin{aligned} V_{us} &= \sqrt{\frac{R_{\tau s}}{R_{\tau d}/|V_{ud}|^2 - \delta R_\tau}} && \text{theory: } \delta R_\tau = 0.218 \pm 0.026 \\ &= 0.2219 \pm 0.0033_{\text{exp}} \pm 0.0009_{\text{th}} = 0.2219 \pm 0.0034, \end{aligned}$$

Gámiz, Jamin, Pich, Prades, Schwab 2003/04

Dominant source of uncertainty in δR_τ is from m_s . δR_τ is small!

Near future: reduce the uncertainty with τ -data from BABAR and BELLE.

V_{ud} and V_{us} summary

$$V_{us} = 0.2238 \pm 0.0029$$

from $K\ell 3$

$$V_{us} = 0.2223 \pm 0.0026$$

from $K\mu 2$

$$V_{us} = 0.2219 \pm 0.0034$$

from $\tau \rightarrow \text{hadrons } \nu_\tau$

Average:

$$V_{us} = 0.2227 \pm 0.0017$$

$$V_{ud} = 0.9738 \pm 0.0005$$

Unitarity check:

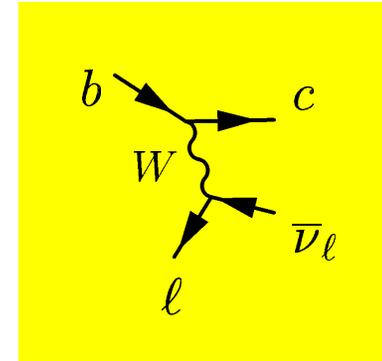
$$V_{us}^2 + V_{ud}^2 + |V_{ub}|^2 - 1 \simeq V_{us}^2 + V_{ud}^2 - 1 = -0.0021 \pm 0.0012$$

The Cabibbo matrix is unitary at the 1.8σ level,
just as at Lepton-Photon 2003:

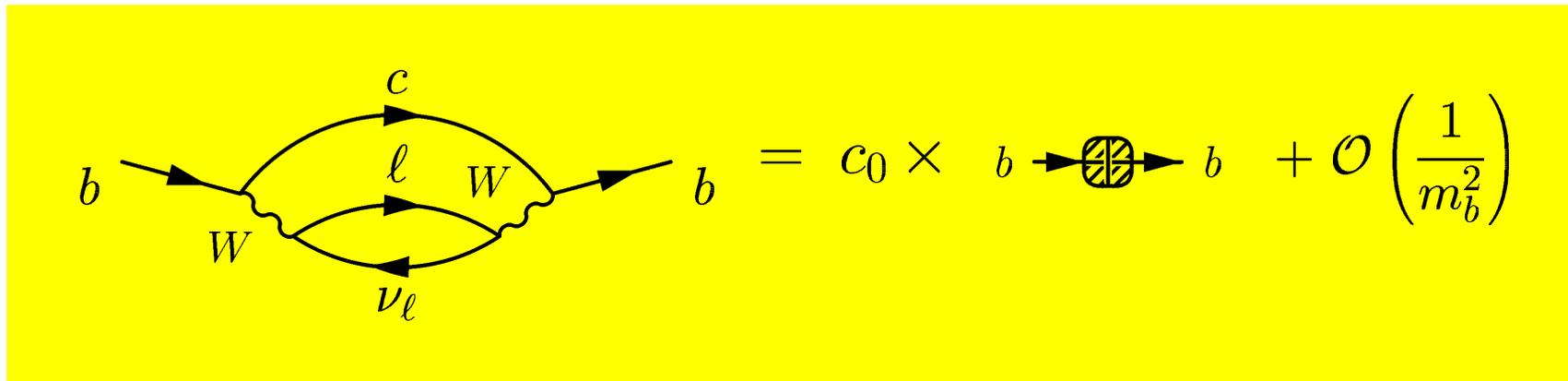
$$V_{us}^2 + V_{ud}^2 - 1 = -0.0031 \pm 0.0017$$

$$V_{cb}$$

Here: Determinations of V_{cb} from
inclusive semileptonic B decays.



In B decay exploit that $m_b \gg \Lambda_{QCD}$. Apply the operator product expansion (OPE) to inclusive decay rates, schematically:



c_0 multiplies the leading dimension-3 operator $\bar{b}b$. It can be calculated in perturbative QCD.

OPE: expansion in Λ_{QCD}/m_b and $\alpha_s(m_b)$

First non-perturbative corrections appear at order Λ_{QCD}^2/m_b^2 and involve

$$-\langle B | \bar{b} D_{\perp}^2 b | B \rangle \propto \mu_{\pi}^2, \quad \langle B | \bar{b} i \sigma_{\mu\nu} G^{\mu\nu} b | B \rangle \propto \mu_G^2.$$

↑

kinetic operator

↑

chromomagnetic operator

μ_G^2 can be determined from spectroscopy.

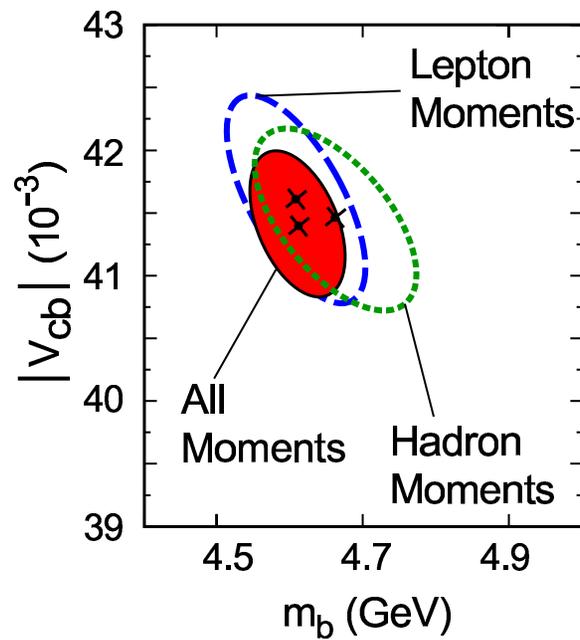
⇒ To order Λ_{QCD}^2/m_b^2 only involve m_b , m_c and μ_{π}^2 .

The OPE can further be applied to certain spectral moments of the $B \rightarrow X \ell \bar{\nu}_{\ell}$ decay, the distributions of the hadron invariant mass M_X and of the lepton energy and the same parameters govern different inclusive decays.

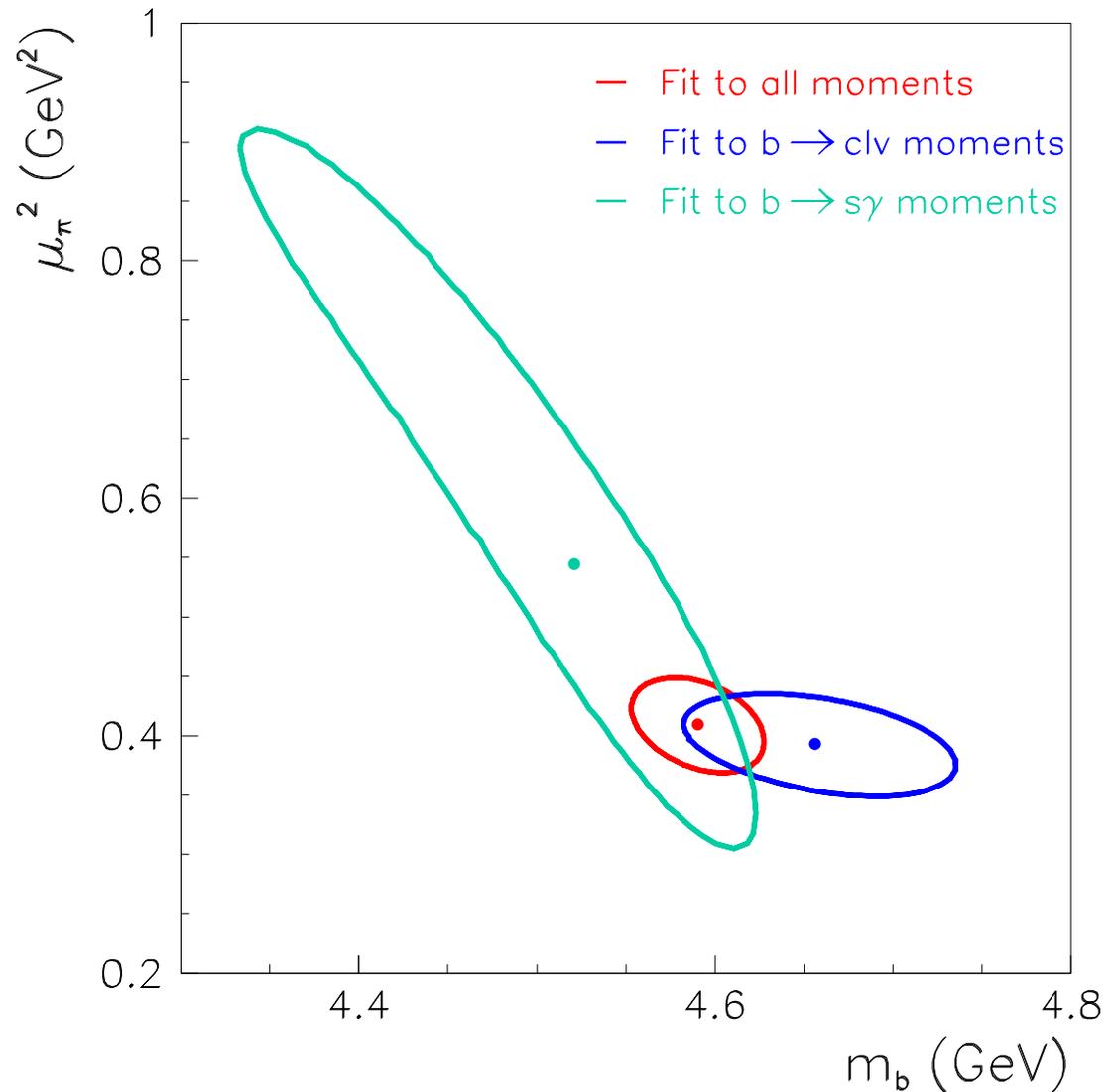
State of the art: Fits to order Λ_{QCD}^3/m_b^3 (involving 7 parameters)

Gambino, Uraltsev; Bauer, Ligeti, Luke, Manohar, Trott

Here $m_b = m_b(1 \text{ GeV})$ is defined in the kinetic scheme.

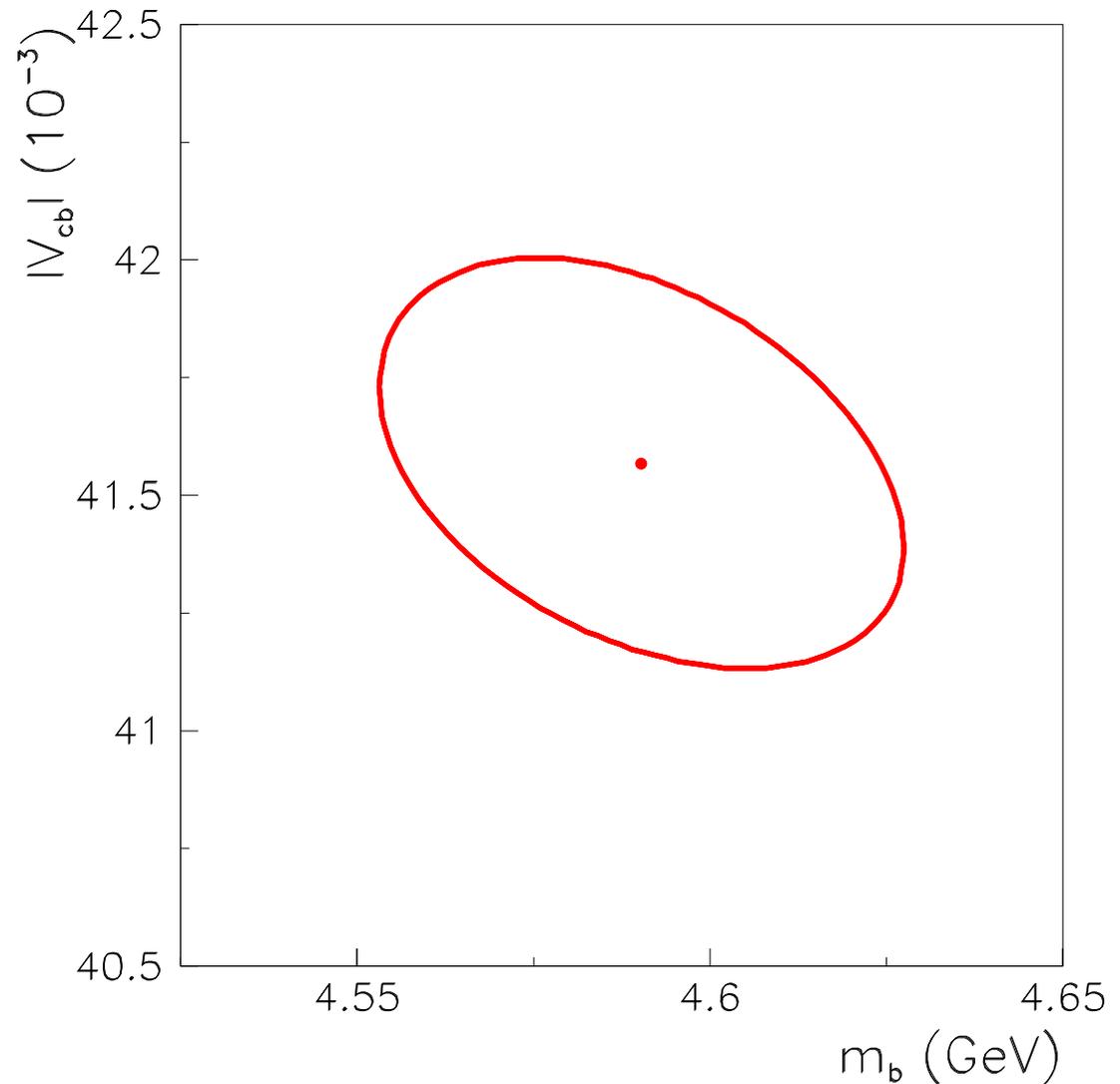


courtesy of Mike Luke



courtesy of Oliver Buchmüller and Henning Flächer

Global fit to hadron
and lepton moments in
 $B \rightarrow X \ell \nu_\ell$ and pho-
ton energy moments in
 $B \rightarrow X_s \gamma$ from BaBar,
BELLE, CDF, CLEO,
DELPHI:



fit and plot courtesy of Oliver Buchmüller and Henning Flächer

V_{cb}^{incl} summary

Result:

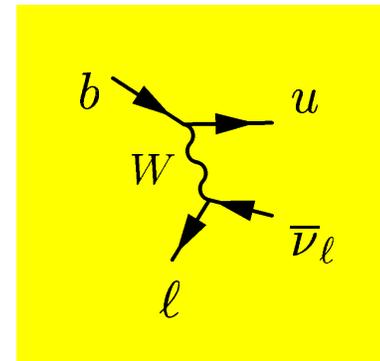
$$\begin{aligned} V_{cb} &= 41.6 \pm 0.3_{\text{exp}} \pm 0.3_{\text{OPE moments}} \pm 0.3_{\text{OPE } \Gamma_{sl}} \\ &= (41.6 \pm 0.5) \cdot 10^{-3} \end{aligned}$$

from inclusive $B \rightarrow X l \nu_l$

compared to CKM 2005: new BaBar photon energy moment included

$$|V_{ub}|$$

Here: Determinations of $|V_{ub}|$ from
inclusive $B \rightarrow X_u \ell \bar{\nu}_\ell$ decays
Exclusive decays: Iain Stewart's talk



Problem: Huge background from $b \rightarrow c \ell \bar{\nu}_\ell$.

\Rightarrow employ cuts on judiciously chosen combinations of
 M_X , E_X and E_ℓ .

$\Rightarrow M_X$ is typically too low for OPE

Still some components of the hadron momentum \vec{P}_X are large. The description of inclusive B decays in this region involves the non-perturbative shape function S , which is a parton distribution function of the B meson.

At leading order in $1/m_b$: The same S governs the photon spectrum in $B \rightarrow X_s \gamma$ and differential decay rates in $B \rightarrow X_u \ell \bar{\nu}_\ell$.

\Rightarrow Extract S from $B \rightarrow X_s \gamma$ for use in $B \rightarrow X_u \ell \bar{\nu}_\ell$.

Goal: Push total uncertainty in $|V_{ub}|$ below 10%:

Theoretical progress:

1. Factorization formula at $\mathcal{O}(\alpha_s)$:

$$d\Gamma \propto H \int_0^{P_+} d\omega J(m_b(P_+ - \omega)) S(\omega)$$



hard QCD

$$\mu \sim m_b$$

jet function

$$\mu \sim M_X \sim m_b \Lambda_{\text{QCD}}$$

shape function

$$\mu \sim \Lambda_{\text{QCD}}$$

where $M_X \sim P_+ \equiv E_X + |\vec{P}_X| \ll P_- \equiv E_X - |\vec{P}_X| \leq m_b$.

Bauer, Manohar; Bosch, Lange, Neubert, Paz

\Rightarrow radiative QCD corrections properly included

2. Factorization of subleading ($\mathcal{O}(1/m_b)$) shape functions $s_i(\omega)$:

Unrelated in $B \rightarrow X_s \gamma$ and $B \rightarrow X_u \ell \bar{\nu}_\ell$, but their moments are known in terms of OPE parameters like μ_π^2 .

Lee, Stewart; Bosch, Neubert, Paz; Beneke, Campanario, Mannel, Pecjak 2004

3. Event generator for $B \rightarrow X_u \ell \bar{\nu}_\ell$:

uses full theoretical information on $B \rightarrow X_s \gamma$ and $B \rightarrow X_u \ell \bar{\nu}_\ell$ shape functions, interpolates to the OPE region

Lange, Neubert, Paz 2005

LNP promotes to cut on the variable P_+ , which is related to the photon energy in $B \rightarrow X_s \gamma$ as $P_+ = M_B - 2E_\gamma$ and allows for the most efficient use of the extracted $S(\omega)$.

Mannel, Recksiegel 1999

V_{ub}^{incl} summary

Improved Wolfenstein parameters:

$$V_{cb} = A\lambda^2$$

$$|V_{ub}| = A\lambda^3 (1 + \lambda^2) \sqrt{\bar{\rho}^2 + \bar{\eta}^2} + \mathcal{O}(\lambda^7)$$

$$\Rightarrow \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \frac{|V_{ub}|}{V_{cb}} \frac{1 - \lambda^2 + \mathcal{O}(\lambda^4)}{\lambda},$$

which is a circle in the $(\bar{\rho}, \bar{\eta})$ plane. With $V_{cb} = 0.0416 \pm 0.0005$ and

$$|V_{ub}| = (4.39 \pm 0.20_{\text{exp}} \pm 0.27_{m_b, \text{th}}) \cdot 10^{-3} = (4.39 \pm 0.34) \cdot 10^{-3}$$

from inclusive semileptonic decays one finds

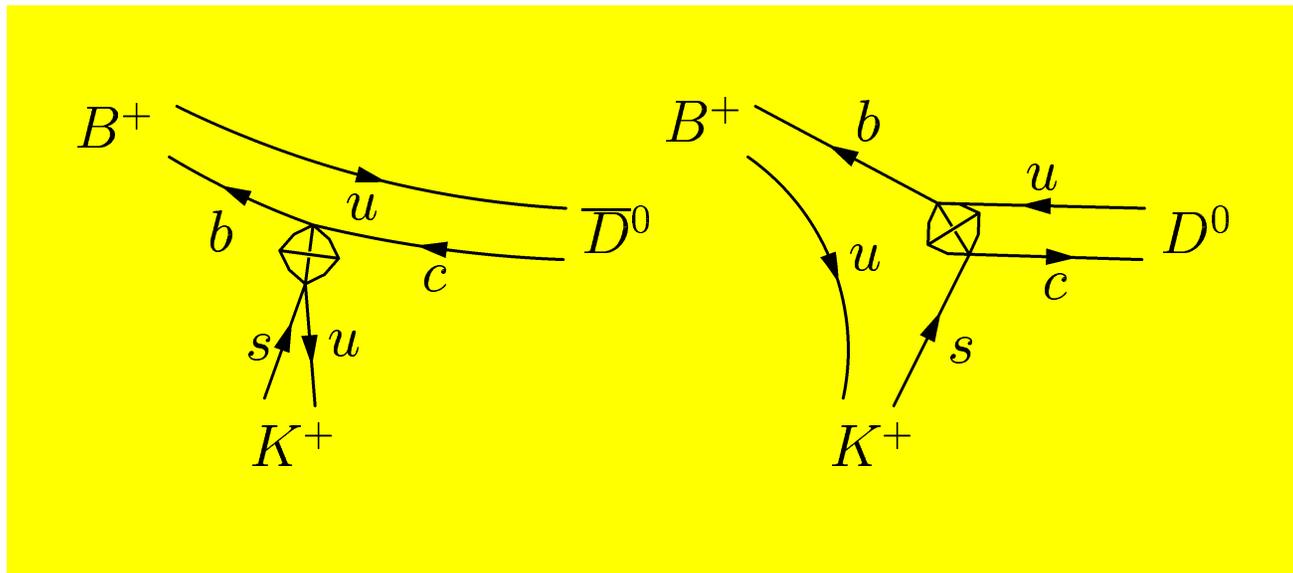
$$\sqrt{\bar{\rho}^2 + \bar{\eta}^2} = 0.45 \pm 0.04$$

$$\arg V_{ub}$$

$\gamma = \arg V_{ub}^*$ from $B \rightarrow D^0 X$:

Basic idea: Use interference of the two tree amplitudes $b \rightarrow c\bar{u}q$ and $b \rightarrow u\bar{c}q$ (with $q = d$ or $q = s$) to get γ from $B \rightarrow (\overline{D^0})X$ branching fractions.

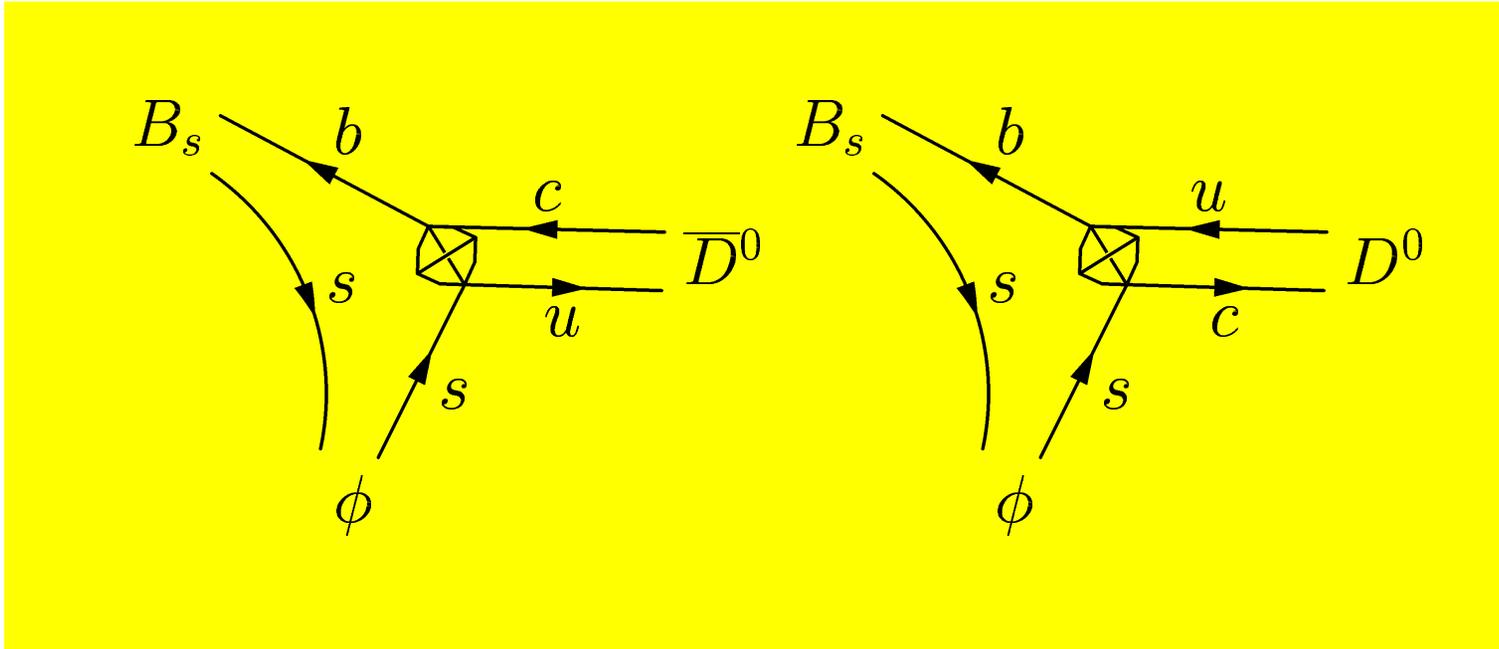
Prototype: Gronau-London-Wyler (GLW) method:



Interference if both $D^0 \rightarrow f$ and $\overline{D^0} \rightarrow f$ are allowed.

Need four measurements to solve for $|A(b \rightarrow c)|$, $|A(b \rightarrow u)|$, the strong phase δ and the weak phase γ . E.g. measure the branching fractions of $B^+ \rightarrow (\overline{D^0})[\rightarrow K^\pm \pi^\mp]K^+$ and $B^\pm \rightarrow (\overline{D^0})[\rightarrow \pi^+ \pi^-]K^\pm$.

This works with non-flavour-specific $B \rightarrow D^0 X$ decays as well:
 E.g. both B_s and \bar{B}_s can decay to $(\bar{D}^0)\phi$:



One can still solve for all hadronic parameters and γ , if at least three pairs of $(\bar{B}_s) \rightarrow (\bar{D}^0)[\rightarrow f_i]\phi$ and $(\bar{B}_s) \rightarrow (\bar{D}^0)[\rightarrow \bar{f}_i]\phi$ branching fractions are measured, where $\bar{f}_i = CP f_i$ (and the f_i 's are not CP eigenstates).

Gronau, Grossman, Shuhmaher, Soffer, Zupan 2004

Can CDF contribute to γ measurements?

γ from α

Tagged analyses of $B^0 \rightarrow \pi\pi$, $B^0 \rightarrow \rho\rho$ or $B^0 \rightarrow \rho\pi$ decays determine α .
(Ignoring discrete ambiguities in extracting α from $\sin(2\alpha)$) one finds:

$$\alpha_{\text{exp}}(\rho\rho) = (96 \pm 13)^\circ, \quad \alpha_{\text{exp}}(\pi\pi) = (99 \pm 9)^\circ$$
$$\alpha_{\text{exp}}(\rho\rho, \pi\pi, \rho\pi \text{ combined}) = (99_{-9}^{+12})^\circ$$

CKM Fitter 2005

The loop-induced penguin decay amplitude is eliminated through an isospin analysis. Combine this information with

$$2\beta_{\text{exp}} = (43.7 \pm 2.4)^\circ$$

from the measured $a_{\text{CP}}^{\text{mix}}$ in $b \rightarrow c\bar{c}s$ decays to eliminate potential new physics in the $B_d - \bar{B}_d$ mixing phase $2\beta_{\text{exp}}$ (again ignoring discrete ambiguities):

$$\gamma = (59_{-12}^{+9})^\circ$$

This is $\gamma = \arg V_{ub}^*$ determined from the tree-level $b \rightarrow u\bar{u}d$ amplitude.

$$\gamma = \arg V_{ub}^* \text{ summary}$$

Combining (almost) all $B^+ \rightarrow D^0 K^{+(*)}$ results gives

$$\gamma = (70_{-14}^{+12})^\circ$$

CKM Fitter 2005, preliminary

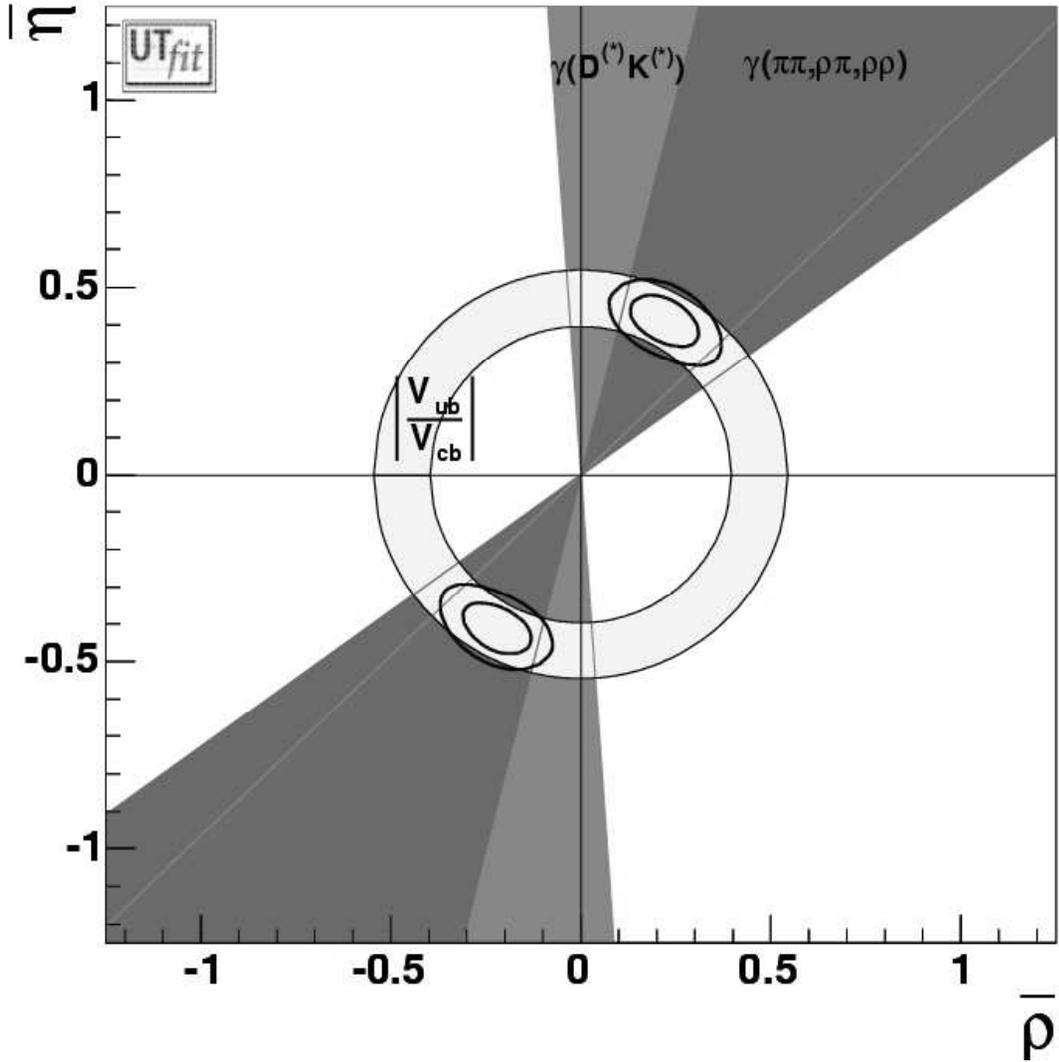
This is $\gamma = \arg V_{ub}^*$ determined from the tree-level $b \rightarrow u\bar{c}d$ amplitude.

Analyses extracting $2\beta + \gamma$ from $B_d \rightarrow D^0\pi$ give only very weak constraints at present, mainly excluding very large values for γ .

Combining with $\gamma = (59_{-12}^{+9})^\circ$ inferred from α gives (naive average):

$$\gamma = \arg V_{ub}^* = (63_{-9}^{+7})^\circ$$

Unitarity triangle from tree-decays only:



courtesy of Maurizio Pierini

3. CKM elements from FCNC processes

In the Standard Model flavour-changing neutral current (FCNC) processes suffer from several suppression factors:

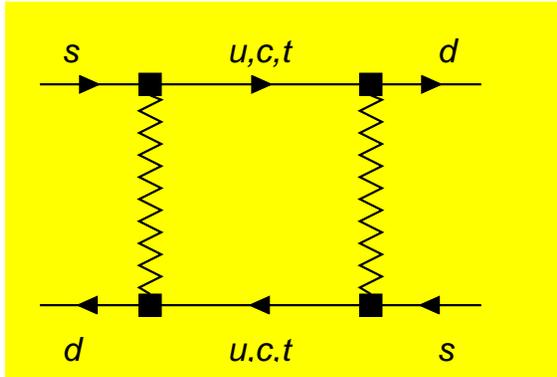
- FCNCs proceed through electroweak loops, no FCNC tree graphs,
- small CKM elements, e.g. $|V_{ts}| = 0.04$, $|V_{td}| = 0.01$,
- GIM suppression, $\propto m_c^2/M_W^2$ in loops with charm,
- helicity suppression in radiative and leptonic decays, e.g. $\propto m_b/M_W$, because FCNCs involve only left-handed fields.

The suppression of FCNC processes is **not** the consequence of any symmetry of the Standard Model. It results from the **particle content** of the Standard Model and the **accidental** smallness of most Yukawa couplings. The suppression is **absent** in generic extensions of the Standard Model.

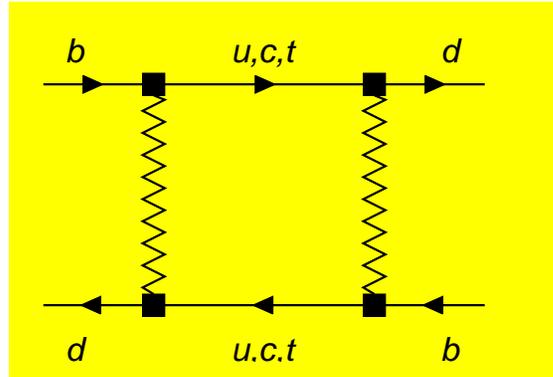
⇒ FCNC processes are **extremely** sensitive to new physics, probing scales in the range **200 GeV–100 TeV**.

Meson-antimeson mixing

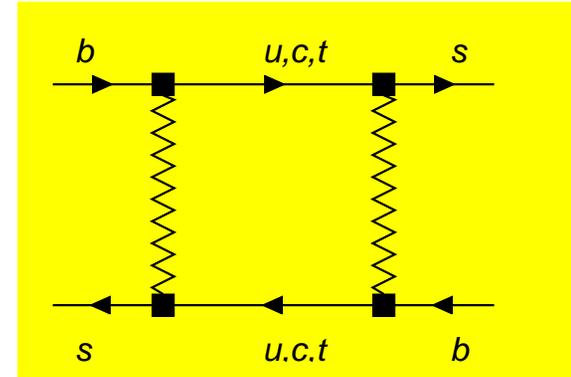
$K - \bar{K}$ mixing



$B_d - \bar{B}_d$ mixing



$B_s - \bar{B}_s$ mixing



Observables:

$$\Delta m_{B_d} \propto |V_{td}|^2 \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2}$$

$$\Delta m_{B_s} \propto |V_{ts}|^2$$

CP:

$$\epsilon_K \text{ gives } \text{Im}(V_{ts} V_{td}^*)^2$$

$$\sin(2\beta), \quad \beta \simeq \arg V_{td}^*$$

$$\sin(2\beta_s), \quad \beta_s \simeq \arg(-V_{ts})$$

$$\bar{\eta}[(1 - \bar{\rho}) + \text{const.}]$$

$$\frac{\bar{\eta}}{1 - \bar{\rho}}$$

$$\bar{\eta}$$

Hadronic matrix elements:

$$\langle \bar{K}^0 | \bar{s}d_{V-A} \bar{s}d_{V-A} | K^0 \rangle \quad \langle \bar{B}^0 | \bar{b}d_{V-A} \bar{b}d_{V-A} | B^0 \rangle \quad \langle \bar{B}_s^0 | \bar{b}s_{V-A} \bar{b}s_{V-A} | B_s^0 \rangle$$

$$\epsilon_K$$

Hadronic matrix element:

$$\langle \bar{K}^0 | \bar{s}d_{V-A} \bar{s}d_{V-A}(\mu) | K^0 \rangle \propto f_K^2 M_K^2 B_K(\mu),$$

where μ is the energy scale.

NEW! Lattice result with 2+1 dynamical staggered fermions:

$$\begin{aligned} B_K^{\overline{\text{MS}}-\text{NDR}}(\mu = 2 \text{ GeV}) &= 0.630 \pm 0.018_{\text{stat}} \pm 0.015_{\text{chiral extrapolation}} \\ &\quad \pm 0.030_{\text{discret.}} \pm 0.130_{\text{perturb. matching}} \\ &= 0.630 \pm 0.135 \quad \text{HPQCD preliminary} \end{aligned}$$

Transform to the usually used \hat{B}_K , which is independent of the renormalisation scale and scheme:

$$\hat{B}_K = 0.85 \pm 0.18$$

Previous results from **quenched lattice QCD** quoted a similar uncertainty, which included an educated guess of the quenching error. In the new **2+1** flavour **unquenched** result the estimate of the uncertainty is on a firm footing now. Dominant error from perturbative matching:

⇒ **Need lattice-continuum matching** at order α_s^2 !

V_{td} from $B_d - \bar{B}_d$ mixing

The hadronic matrix element of $B_d - \bar{B}_d$ mixing drops out from the relation between $a_{\text{CP}}^{\text{mix}}(B_d \rightarrow J/\psi K_S)$ and

$$\begin{aligned} \sin(2\beta) &= 0.69 \pm 0.03, & \cos(2\beta) &> 0 \\ \Rightarrow \arg(\pm V_{td}^*) &= \beta = (21.8 \pm 1.2)^\circ, \end{aligned}$$

but not from the relation between Δm_{B_d} and $|V_{td}|$: With

$$\langle \bar{B}^0 | \bar{b}d_{V-A}\bar{b}d_{V-A} | B^0 \rangle = M_{B_d}^2 f_{B_d}^2 B_{B_d}$$

one has

$$\Delta m_{B_d} \propto |V_{td}|^2 f_{B_d}^2 B_{B_d}.$$

NEW! Lattice result (preliminary) for f_{B_d} with 2+1 dynamical staggered fermions from HPQCD: Iain Stewart's talk

Combining the new HPQCD result for f_{B_d} with the known result for B_{B_d} find

$$f_{B_d} \sqrt{\hat{B}_{B_d}} = (246 \pm 11 \pm 25) \text{ MeV} = (246 \pm 27) \text{ MeV}$$

Effect on $|V_{td}|$ (from $B_d - \bar{B}_d$ mixing alone):

$$|V_{td}|^{\text{old}} = 0.00796 \pm 0.00128 \quad \longrightarrow \quad |V_{td}| = 0.00722 \pm 0.00080$$

for $\Delta m_{B_d} = 0.502 \text{ ps}^{-1}$ and $m_t(m_t) = 168.5 \text{ GeV}$.

Effect on unitarity triangle: Iain Stewart's talk

$|V_{td}|/|V_{ts}|$ from $B-\bar{B}$ mixing

A measurement of the ratio $\Delta m_{B_d}/\Delta m_{B_s}$ will determine $|V_{td}|/|V_{ts}|$ via

$$\left| \frac{V_{td}}{V_{ts}} \right| = \sqrt{\frac{\Delta m_{B_d}}{\Delta m_{B_s}}} \sqrt{\frac{M_{B_s}}{M_{B_d}}} \xi$$

with the hadronic quantity

$$\xi = \frac{f_{B_s} \sqrt{\hat{B}_{B_s}}}{f_{B_d} \sqrt{\hat{B}_{B_d}}}$$

which equals $\xi = 1$ in the limit of exact $SU(3)_F$.

NEW! Lattice result (preliminary) for f_{B_s}/f_{B_d} with 2+1 dynamical staggered fermions from HPQCD: **Iain Stewart's talk**

⇒ Use this to refine the theory prediction for ξ .

The lower bound $\Delta m_{B_s} \geq 14.5 \text{ ps}^{-1}$ implies

$$\left| \frac{V_{td}}{V_{ts}} \right| \leq 0.235$$

which constrains one side of the unitarity triangle

$$R_t \equiv \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = \left| \frac{V_{td}}{V_{ts}\lambda} \right| \leq 1.06$$

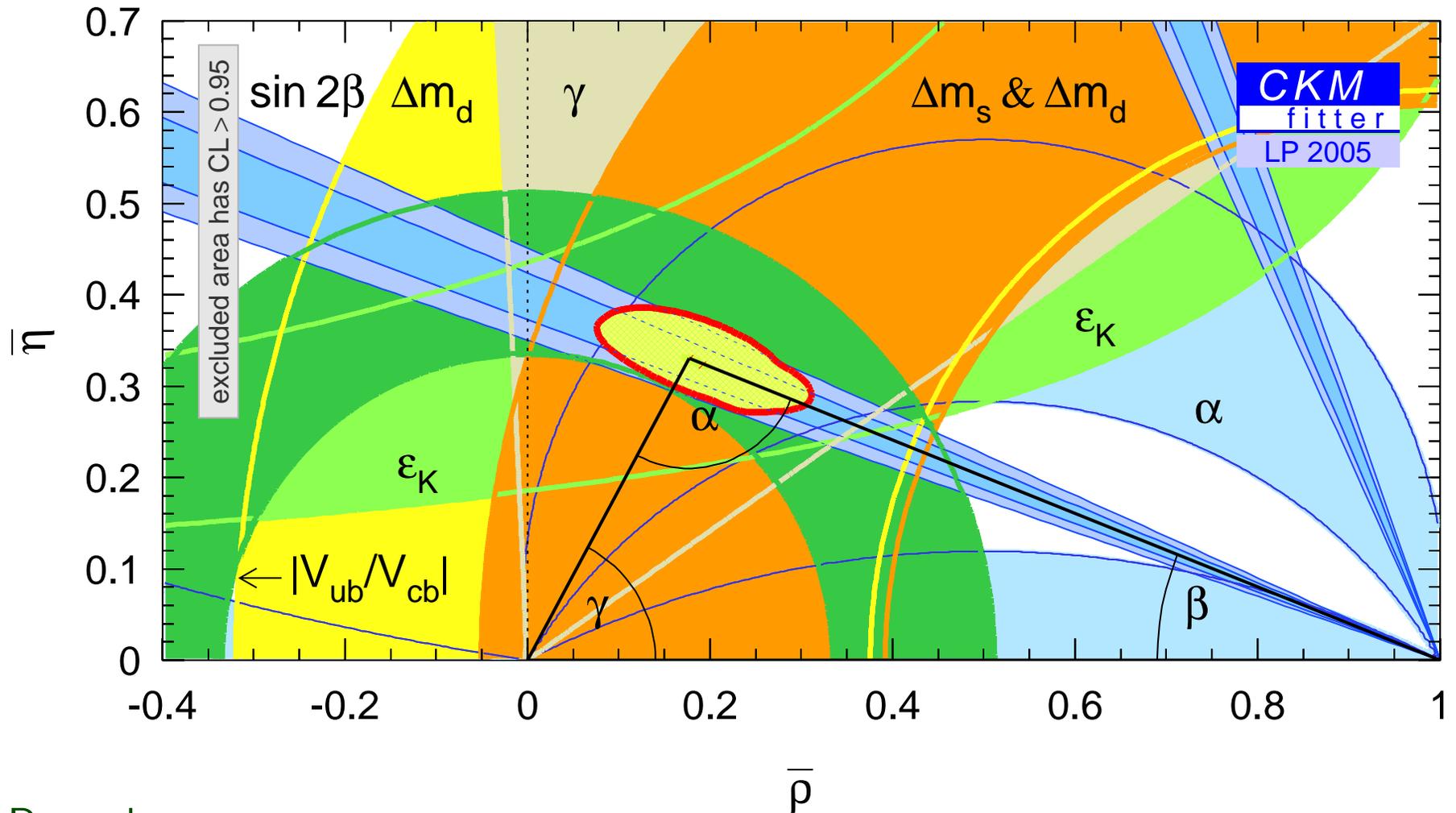
Using $V_{ts} \simeq -V_{cb}$ the combined constraint from Δm_{B_d} and Δm_{B_s} is

$$|V_{td}|^{\text{old}} = 0.0085_{-0.0019}^{+0.0017} \longrightarrow |V_{td}| = 0.0083_{-0.0020}^{+0.0006} \quad \text{CKM Fitter 2005}$$

showing the impact of the new unquenched lattice results.

More on $B_s - \bar{B}_s$ mixing in Iain Stewart's talk.

Global fit to the unitarity triangle



Remark:

The first UT fit with theory input computed at **NLO** in **QCD** was done in **1995**, from ϵ_K , $|V_{ub}|$ and Δm_{B_d} only.

Numerical results for the CKM elements:

Observable	central \pm CL \equiv 1σ	\pm CL \equiv 2σ	\pm CL \equiv 3σ
$\bar{\rho}$	$0.184^{+0.043}_{-0.042}$	$+0.101$ -0.088	$+0.156$ -0.137
$\bar{\eta}$	$0.325^{+0.023}_{-0.020}$	$+0.049$ -0.045	$+0.075$ -0.064
α (deg)	$97.9^{+6.4}_{-7.3}$	$+14.6$ -14.6	$+20.8$ -22.0
β (deg)	$21.74^{+1.19}_{-0.83}$	$+2.53$ -1.70	$+3.94$ -2.77
$\gamma \simeq \delta$ (deg)	$60.5^{+7.1}_{-6.5}$	$+14.5$ -14.8	$+22.3$ -21.2
$ V_{ub} $ [10^{-3}]	$3.54^{+0.21}_{-0.11}$	$+0.49$ -0.22	$+0.85$ -0.33
$ V_{td} $ [10^{-3}]	$8.12^{+0.50}_{-0.42}$	$+1.03$ -0.89	$+1.37$ -1.25
$ V_{ts} $ [10^{-3}]	$40.47^{+1.35}_{-0.55}$	$+2.41$ -1.10	$+3.06$ -1.65

Table 1: CKMfitter: Preliminary fit results. Lepton-Photon 2005.

$$K \rightarrow \pi \nu \bar{\nu}$$

The rare decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ provide an excellent opportunity to determine the unitarity triangle from $s \rightarrow d$ transitions. With planned dedicated experiments $(\bar{\rho}, \bar{\eta})$ can be determined with a similar precision as today from $b \rightarrow d$ and $b \rightarrow u$ transitions at the B factories.

⇒ Powerful probe of the CKM mechanism of FCNCs.

Key facts:

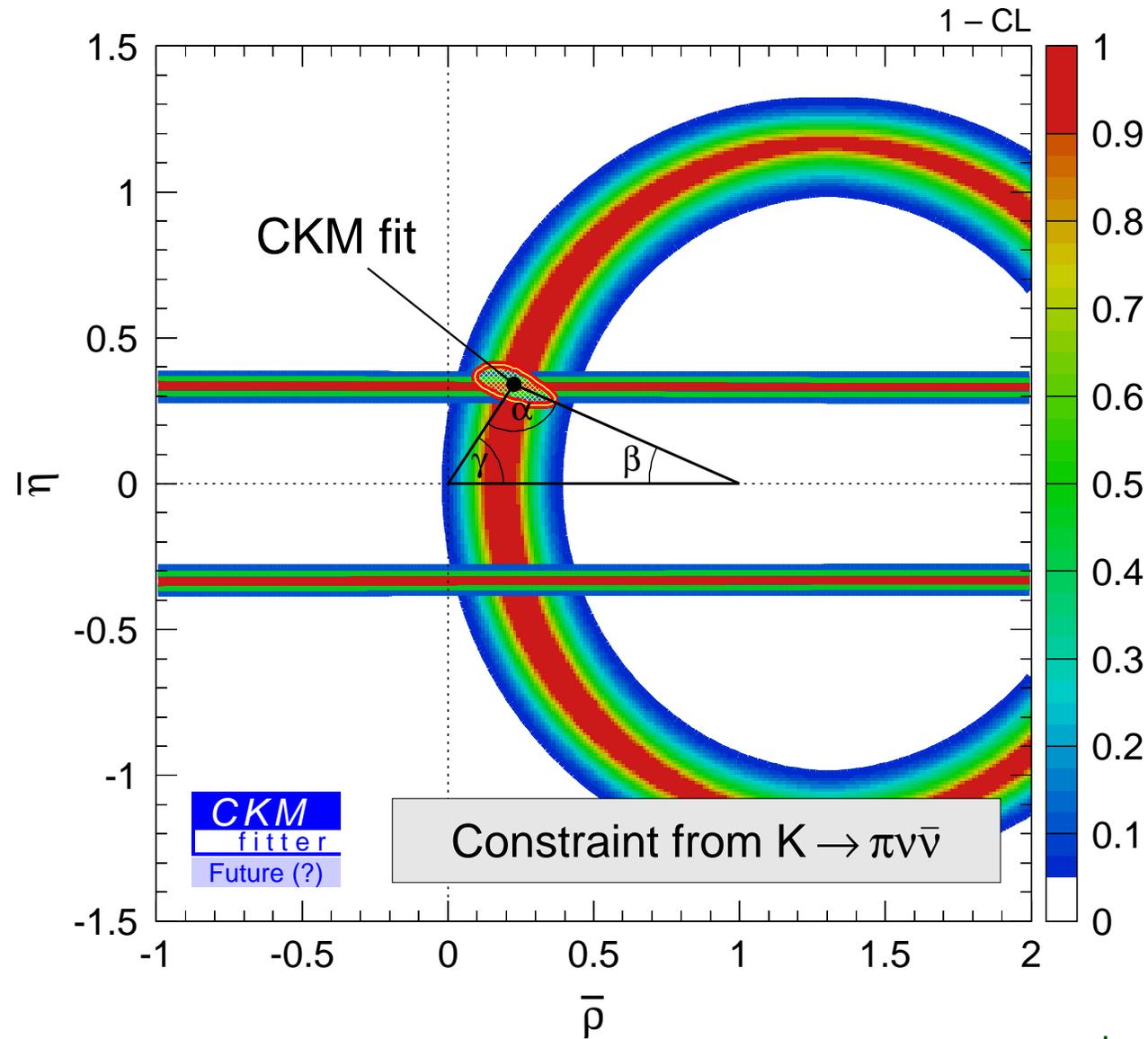
$Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is proportional to $\bar{\eta}^2$, top-dominated, theoretical uncertainty of the next-to-leading order (NLO) prediction below 2%.

$Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ defines an ellipse in the $(\bar{\rho}, \bar{\eta})$ plane, sizeable charm contribution, theoretical uncertainty of the next-to-leading order (NLO) prediction larger, inducing $\mathcal{O}(5 - 10\%)$ uncertainties in extracted CKM parameters.

Parametric uncertainties from V_{cb} and m_t largely drop out, if $\sin(2\beta)$ is calculated from $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$.

Buchalla, Buras

Assume a 10% measurement of $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$:



plot courtesy of Andreas Höcker

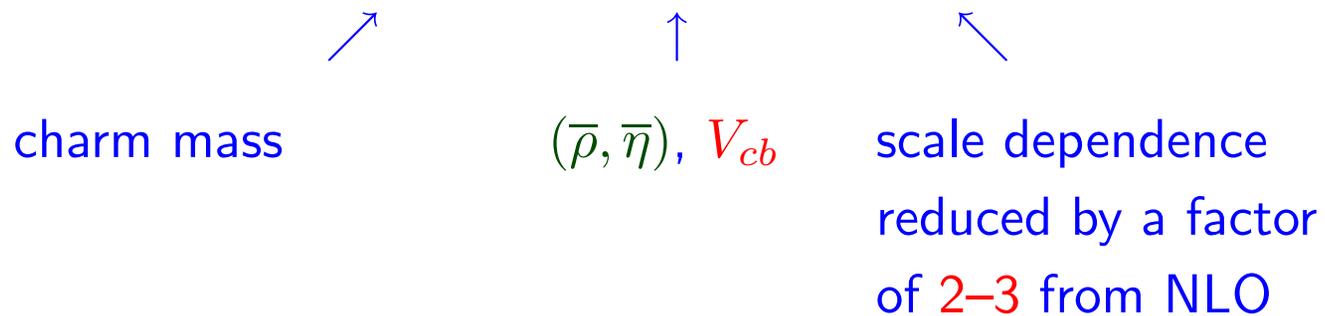
Two expansion parameters in the charm contribution: m_K^2/m_c^2 and $\alpha_s(m_c)$.

$\mathcal{O}(m_K^2/m_c^2)$ corrections: $\mathcal{O}(7\%)$ increase of $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, small uncertainty.

Isidori, Mescia, Smith 2005

NEW! QCD corrections to charm contribution at NNLO (three-loop) (preliminary numerics):

$$Br(K^+ \rightarrow \pi^+ \bar{\nu} \nu) = (8.0 \pm 0.4_{m_c} \pm 0.9_{\text{CKM}} \pm 0.15_{\mu_c}) \cdot 10^{-11}$$



Buras, Gorbahn, Haisch, UN

Perturbation theory at the charm scale is in good shape!

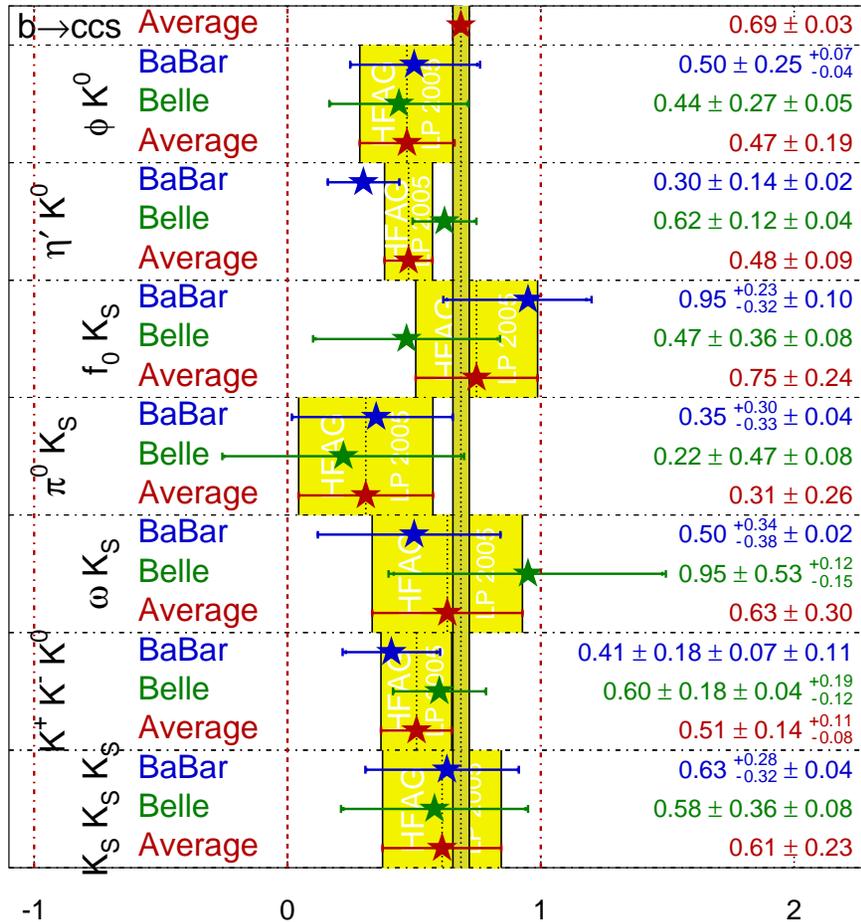
At NNLO we find the following reduced scale uncertainties for parameters extracted from $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$:

$$\frac{\delta|V_{td}|}{|V_{td}|} = 0.011, \quad \delta \sin(2\beta) = 0.006, \quad \delta\gamma = 1.3^\circ$$

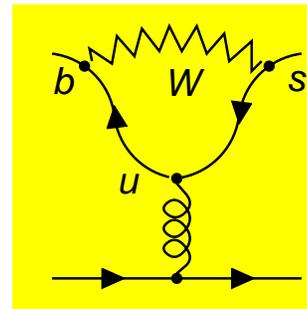
(μ_c -dependence only, numerics preliminary)

4. CP violation in $b \rightarrow s$ penguin decays

$\sin(2\beta^{\text{eff}})/\sin(2\phi_1^{\text{eff}})$ **HFAG**
LP 2005
PRELIMINARY



$b \rightarrow s\bar{q}q$ penguin amplitudes are polluted by a penguin loop with u quark:



$$\propto V_{ub}V_{us}^*$$

$$\propto e^{-i\gamma}$$

$b \rightarrow s\bar{u}u$ transitions further have a colour-suppressed tree contribution. CKM suppression with respect to the leading charm penguin loop:

$$\left| \frac{V_{ub}V_{us}}{V_{cb}V_{cs}} \right| \sim 0.025$$

Naive averages:

Winter 2005:

$$\sin(\beta)^{\text{eff}} = 0.43 \pm 0.07,$$

see right plot:

LP 2005:

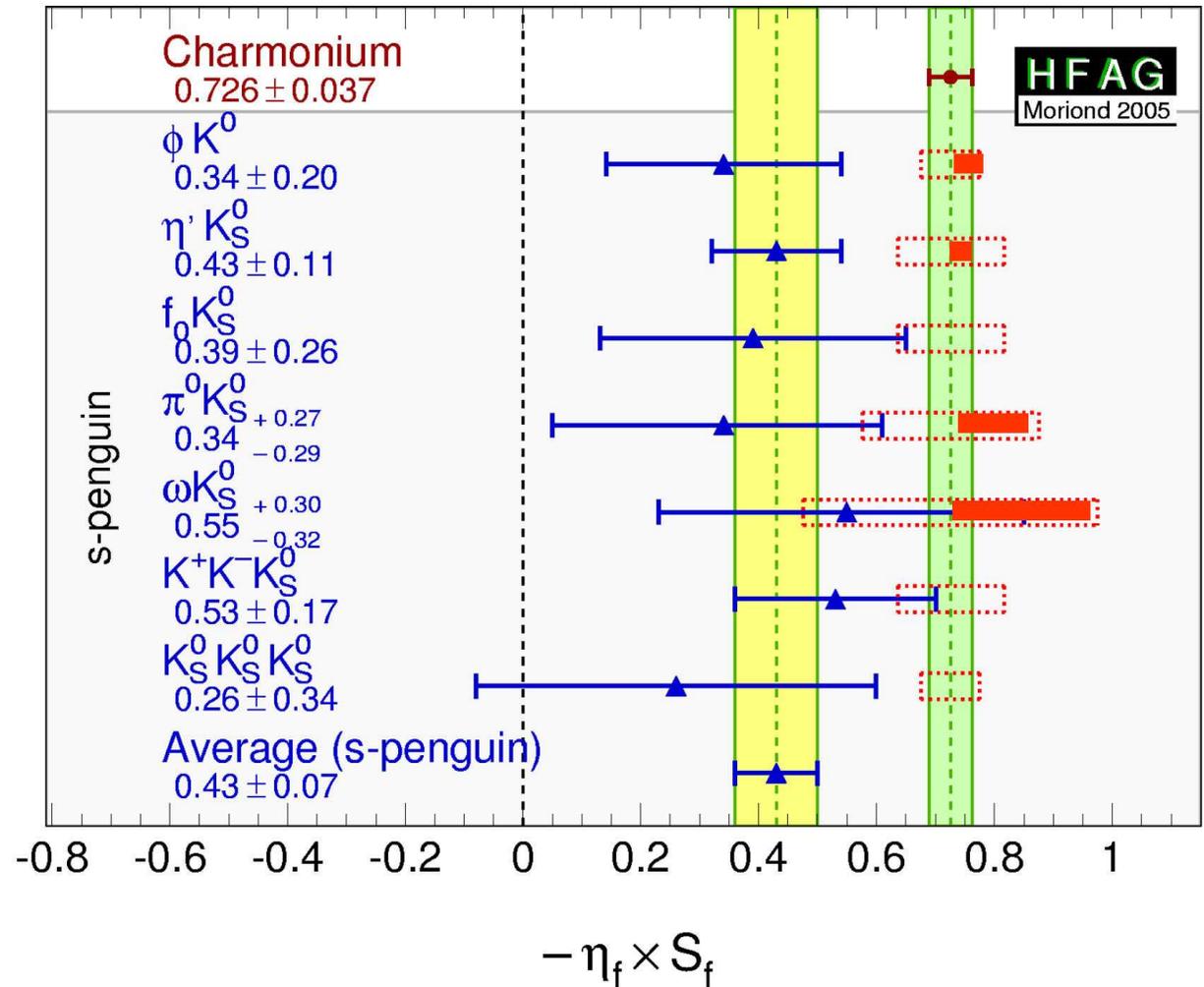
$$\sin(\beta)^{\text{eff}} = 0.51 \pm 0.06,$$

well below

$$\sin(2\beta)^{b \rightarrow c\bar{c}s} = 0.69 \pm 0.03.$$

QCD Factorization finds
 small corrections to
 $\sin(2\beta_{\text{eff}}) - \sin(2\beta)$, which
 are positive, see red bars:

Beneke 2005



Note: To measure a mixing-induced CP asymmetry (S_f term) in a $b \rightarrow s\bar{q}q$ decay of a B_d meson one needs a neutral Kaon in the final state, so that the

$$b(\bar{d}) \rightarrow \bar{q}qs(\bar{d}) \quad \text{and} \quad \bar{b}(d) \rightarrow \bar{q}q\bar{s}(d)$$

decays of B_d and \bar{B}_d can interfere.

In a \bar{B}_s decay, however, one has a flavourless final state:

$$b(\bar{s}) \rightarrow \bar{q}qs(\bar{s}), \quad \bar{b}(s) \rightarrow \bar{q}q\bar{s}(s)$$

and the needed interference occurs in any final state.

$\Rightarrow B_s$ physics is the El Dorado of $b \rightarrow s\bar{q}q$ penguin physics!

5. Summary

- V_{ud} is known with a precision of $5 \cdot 10^{-4}$, $n \rightarrow p \ell \bar{\nu}_\ell$ and $\pi^+ \rightarrow \pi^0 \bar{\ell} \nu_\ell$ are gaining importance.
- V_{us} determinations from $K\ell 3$, $K\mu 2$ and τ decays agree perfectly, V_{us} has a precision of $8 \cdot 10^{-3}$.
- V_{cb} from inclusive $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$ decays is a **success story** of the **operator product expansion**. Many redundant measurements agree, the precision of V_{cb} is 1.2%.
- In inclusive $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$ decays $\mathcal{O}(\alpha_s)$ corrections are understood and $1/m_b$ corrections are attacked. The current precision in $|V_{ub}|$ is 8%.
- Significant progress in finding $\gamma = \arg V_{ub}^*$ from **tree-level** decays, which determine γ to 13% accuracy.
- New **unquenched 2+1** flavor lattice results from HPQCD for B_K , f_{B_d} and f_{B_d}/f_{B_s} improve the extraction of $(\bar{\rho}, \bar{\eta})$ from ϵ_K , Δm_{B_d} and $\Delta m_{B_d}/\Delta m_{B_s}$.

- The **gold-plated** modes $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ will allow to determine $(\bar{\rho}, \bar{\eta})$ from $s \rightarrow d$ transitions and provide a **powerful CKM test**. The precision matches that of the **B factories**. The needed **three-loop** calculation of the charm piece in $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ is available now.
- Standard **QCD** dynamics **cannot** explain sizeable deviations of $\sin(2\beta_{\text{eff}})$ in $b \rightarrow s \bar{q} q$ penguin decays from $\sin(2\beta)$. Further subleading **QCD** effects tend to **increase** $\sin(2\beta_{\text{eff}})$. This remains a hot field to look for **new physics**.

Penguins in $b \rightarrow s\bar{q}q, \dots s \rightarrow d\nu\bar{\nu}$:



Wake-up call for **New Physics?**