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QUARK LIBERATION IN (1 + 1) - DIMENSIONS*

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ABSTRACT

The theory of a massive vector field (gluon) coupled to a massive fermion (quark) is studied in (1 + 1) - dimensions. The model is shown to be equivalent to a theory of two interacting scalar bosons. In the limit of strong coupling the particle spectrum is found to possess a free heavy quark and light quark-antiquark bound state mesons. Perturbative analysis of gluon effects indicates the absence of quark-meson bound states.

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I. INTRODUCTION

Recent experimental investigations have stimulated speculations concerning the existence of a free heavy quark.[1] One possible explanation for a phenomenon of "quark near-confinement" is that the quarks are bound inside the hadrons by gluon fields possessing a small but finite (bare) mass.[2] It has been further speculated that, once free, quarks travelling through matter may attract nucleons and form hadronic clumps with non-integral charge.[3] Since many of the features of quark interactions can be understood in terms of an effective one-dimensional string potential,[4] it is instructive to examine the consequences of the above speculations for a (1+1)-dimensional field theory.

In this paper, a model of a massive vector field (gluon) coupled to a massive fermion (quark) current will be considered. In Sec. II, the model is shown to be equivalent to a theory of two interacting scalar bosons. For various limits in the values of the coupling constants and masses, the Bose form of the theory reproduces known results. In Sec. III, the particle spectrum of the theory in the limit of large coupling compared to the bare gluon mass is shown to contain a heavy quark and light quark-antiquark bound state mesons. In Sec. IV, the theory is shown not to possess a quark-meson bound state in this limit.

II. BOSONIZATION

The model of a massive vector U(1) (gluon) field interacting with a massive fermion (quark) is defined by the Lagrangian density[1]

$$\mathcal{L} = +\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2} F_{\mu\nu} (\partial^\mu A^\nu - \partial^\nu A^\mu) + \frac{\mu_0^2}{2} A_\mu A^\mu + j_\mu A^\mu + \bar{\psi}(i\not{\partial} - m)\psi, \tag{2.1}$$

where

$$j_\mu = -e: \bar{\psi} \gamma_\mu \psi:, \tag{2.2}$$

and where the coupling constant $\underline{e}$, the bare gluon mass $\underline{\mu}_0$, and the fermion mass $\underline{m}$ all have the dimensions of inverse length. Due to the peculiar kinematics of two-dimensional space-time, a massive vector field theory in (1+1)-dimensions may be transformed into a scalar field theory. Setting

$$F_{\mu\nu} = \mu_0 \varepsilon_{\mu\nu} \phi \quad \text{and} \quad A_\mu = \mu_0^{-1} \varepsilon_{\mu\nu} \Phi^\nu, \tag{2.3}$$

Equation (2.1) can be re-expressed as

$$\mathcal{L} = (\partial_\mu \phi) \Phi^\mu - \tfrac{1}{2}\mu_0^2 \phi^2 - \tfrac{1}{2}\Phi_\mu \Phi^\mu + \mu_0^{-1} j^\mu \varepsilon_{\mu\nu} \Phi^\nu + \bar{\psi}(i\not{\partial} - m)\psi. \tag{2.4}$$

The non-dynamical variable, Φ^μ, may be eliminated by using the Euler-Lagrange equations:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi^\nu)} \right) = 0 = \frac{\partial \mathcal{L}}{\partial \Phi^\nu} = \partial_\nu \phi - \Phi_\nu + \mu_0^{-1} j^\mu \varepsilon_{\mu\nu}. \tag{2.5}$$

Solving Eq. (2.5) for Φ^μ and substituting the result into Eq. (2.4), one finds

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}\mu_0^2\,\phi^2 + \mu_0^{-1}\, j^\mu\, \epsilon_{\mu\nu}\,(\partial^\nu\phi) - \tfrac{1}{2}\mu_0^{-2}\, j_\mu\, j^\mu$$
$$+ \bar{\psi}(i\not{\partial} - m)\psi\ . \tag{2.6}$$

The fermion contribution to the Lagrangian density may also be replaced by an equivalent Bose scalar form.[5] A theory of a free massive Dirac field in two dimensions,

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi, \tag{2.7}$$

is equivalent to the theory of a Bose field defined by

$$\mathcal{H} = N_m[\tfrac{1}{2}\pi_\eta^2 + \tfrac{1}{2}(\partial_1\eta)^2 - cm^2\cos(2\sqrt{\pi}\eta)], \tag{2.8}$$

where N_m denotes normal ordering with respect to the mass $\underline{m}$ and $\underline{c}$ is a numerical constant. Every local charge-zero operator in the Dirac theory corresponds to some local function in the Bose theory. In particular,

$$:\bar{\psi}\,\psi: = -mc\, N_m \cos\,(2\sqrt{\pi}\eta) \tag{2.9}$$

and

$$j^\mu = \frac{-e}{\sqrt{\pi}}\,\epsilon^{\mu\nu}\,\partial_\nu\,\eta\ . \tag{2.10}$$

Substituting these correspondences into Eq. (2.6), one finds

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}\mu_0^2\,\phi^2 - \alpha(\partial_\mu\eta)(\partial_\mu\phi) + \tfrac{1}{2}\alpha^2(\partial_\mu\eta)^2$$
$$+ cm^2\cos\,(2\sqrt{\pi}\eta) + \tfrac{1}{2}(\partial_\mu\eta)^2, \tag{2.11}$$

where

$$\alpha = \frac{e}{\mu_0\sqrt{\pi}}\ . \tag{2.12}$$

Defining $\phi' = \phi - \alpha\eta$, Eq. (2.11) may be rewritten as

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi')^2 + \tfrac{1}{2}(\partial_\mu\eta)^2 - \tfrac{1}{2}\mu_0^2(\phi' + \alpha\eta)^2 + cm^2\cos(2\sqrt{\pi}\eta). \tag{2.13}$$

The quadratic mass term may be diagonalized via

$$\begin{pmatrix}\phi'\\ \eta\end{pmatrix} = \frac{1}{\sqrt{1+\alpha^2}}\begin{pmatrix}\alpha & 1\\ -1 & \alpha\end{pmatrix}\begin{pmatrix}\phi''\\ \eta''\end{pmatrix} \tag{2.14}$$

resulting in the Lagrange density

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)^2 + \tfrac{1}{2}(\partial_\mu\eta)^2 - \tfrac{1}{2}\mu_0^2(1+\alpha^2)\eta^2 \tag{2.15}$$
$$+ cm^2\cos\left[2\sqrt{\pi}\left(\frac{-\phi}{\sqrt{1+\alpha^2}} + \frac{\alpha\eta}{\sqrt{1+\alpha^2}}\right)\right]$$

(double primes on field variables have been dropped). As promised, the original Lagrangian, Eq. (2.1), has been transformed into a theory of two interacting scalar fields.

As a check on the result, one may first consider the limit $\mu_0 \to 0$,

$$\mathcal{L} \to \tfrac{1}{2}(\partial_\mu\phi)^2 + \tfrac{1}{2}(\partial_\mu\eta)^2 - \tfrac{1}{2}\left(\frac{e^2}{\pi}\right)\eta^2 + cm^2\cos(2\sqrt{\pi}\eta) \tag{2.16}$$

Aside from a decoupled massless (Goldstone) field, there remains the Bose form of the massive Schwinger model, as found by Coleman in this limit.[6] In the limit $m \to 0$,

$$\mathcal{L} \to \tfrac{1}{2}(\partial_\mu\phi)^2 + \tfrac{1}{2}(\partial_\mu\eta)^2 - \tfrac{1}{2}\mu_0^2(1+\alpha^2)\eta^2, \tag{2.17}$$

which describes a free massless (fermion) field and a free gluon field of mass $\mu_0(1+\alpha^2)^{\frac{1}{2}}$. This result is in complete agreement with the massless fermion limit studied by Brown[7], Thirring and Wess.[8]

III. STRONG COUPLING LIMIT

For the remainder of the paper, the strong coupling ($e \gg \mu_0$, $e \gg m$) limit will be considered for which the

Hamiltonian density for the theory has the form:

$$\mathcal{H} = N_m\left(\tfrac{1}{2}\pi_\phi^{\,2} + \tfrac{1}{2}(\partial_1\phi)^2 + \tfrac{1}{2}\pi_\eta^{\,2} + \tfrac{1}{2}(\partial_1\eta)^2 + \tfrac{1}{2}\mu^2\eta^2 - cm^2\cos(2\sqrt{\pi}\eta - \beta\phi)\right) \quad (3.1)$$

where $\mu \approx \mu_0\alpha$ (3.2)

and $\beta = 2\sqrt{\pi}/\alpha$, (3.3)

In this limit, there is very strong coupling and the (bare) gluon may be represented by a long (length ~ μ_0^{-1}) string so that one might expect "quark near-confinement" and quark attraction of mesons.

In order to be able to determine the scale of the physical parameters, the theory must be renormal-ordered so as to eliminate graphs with virtual η's and ϕ's emitted and absorbed at the same vertex. The renormal-ordering prescription derived by Coleman [9] is

$$N_m \cos(2\sqrt{\pi}\eta + \theta) = \left(\frac{\mu}{m}\right) N_\mu \cos(2\sqrt{\pi}\eta + \theta) \quad (3.4)$$

and

$$N_m \cos(\beta\phi + \theta) = \left(\frac{m_1}{m}\right)^{\beta^2/4\pi} N_{m_1} \cos(\beta\phi + \theta) \quad (3.5)$$

where $\mu(m_1)$ is the mass appropriate for $\eta(\phi)$ field and θ is some angle. Also note because β is very small, renormal-ordering the ϕ field brings a factor into the interaction term which is nearly unity. Ignoring this factor, the Hamiltonian density becomes

$$\begin{aligned}\mathcal{H} &= N_{m_1}\left(\tfrac{1}{2}\pi_\phi^{\,2} + \tfrac{1}{2}(\partial_1\phi)^2\right) \\ &\quad + N_\mu\left(\tfrac{1}{2}\pi_\eta^{\,2} + \tfrac{1}{2}(\partial_1\eta)^2 + \tfrac{1}{2}\mu^2\eta^2\right) \\ &\quad - cm\mu\left[(N_{m_1}\cos\beta\phi)(N_\mu\cos 2\sqrt{\pi}\eta) + (N_{m_1}\sin\beta\phi)(N_\mu\sin 2\sqrt{\pi}\eta)\right].\end{aligned} \quad (3.6)$$

The mass appropriate for the ϕ field can now be found be looking at the coefficient of ϕ^2.

$$m_1^{\,2} \equiv cm\mu\beta^2 \propto \frac{cm\mu_0}{\alpha} \quad . \quad (3.7)$$

Since the η-field is much more massive than the ϕ, the contribution of graphs containing internal η-lines are small and can be neglected. Thus the effective Hamiltonian density for the ϕ-field is

$$\mathcal{H} = N_{m_1}\left(\tfrac{1}{2}\pi_\phi^{\,2} + \tfrac{1}{2}(\partial_1\phi)^2 - \frac{m_1^2}{\beta^2}\cos\beta\phi\right). \quad (3.8)$$

Equation (3.8) is just the familiar Sine-Gordon Hamiltonian density found by Coleman [5] to be equivalent to the massive Thirring model. In fermion language, the effective Hamiltonian interaction in the strong coupling limit represents a point-like, current-current interaction.

The mass spectrum of the Sine-Gordon equation is well-known.[10] The theory's spectrum contains two particles of equal mass, M,

$$M \approx 8\ m_1/\beta'^2 \propto m_1\alpha^2 \text{ where } \beta'^2 = \frac{\beta^2}{1-\beta^2/8\pi}, \quad (3.9)$$

called the soliton and antisoliton which represent the free quark and antiquark. (To guarantee the validity of the approximation, em must be smaller than μ_0^2 in order that the fermion mass be smaller than that of the gluon.) In addition, the spectrum possesses soliton-antisoliton bound states with masses given by

$$M_n = 2M \sin(\beta'^2 n/16) \tag{3.10}$$

$$\propto n\, m_1 \text{ for small } n.$$

These quark-antiquark bound states, it should be noted, are distinct from the mesons found by Coleman for the massive Schwinger model and are much less massive than either the quark or gluon. Thus, for large α, the thoery contains a heavy quark, a heavy gluon, and a number of very light mesons.

IV. QUARK-MESON BOUND STATES ?

In addition to free quarks and mesons, quark-meson bound states may also exist. Such states may be found by examining periodic perturbations about the stationary soliton solution, which, for the Sine-Gordon theory, are

$$\phi \to \phi_{sol} + q(x)\, e^{i\omega t}, \text{ where } \phi_{sol} = \frac{4}{\beta}\tan^{-1} e^{x m_1}. \tag{4.1}$$

Substituting Eq. (4.1) into the Euler-Lagrange equations

$$\partial^2\phi + \frac{m_1^2}{\beta}\sin\beta\phi = 0, \tag{4.2}$$

one obtains

$$-\omega^2 q - \frac{d^2 q}{dx^2} + U_0''(\phi_{sol})q = 0 \tag{4.3}$$

where

$$U_0(\phi) = -\frac{m_1^2}{\beta^2}\cos\beta\phi. \tag{4.4}$$

Eq. (4.3) is in the form of a one-dimensional Schrödinger equation which is exactly solvable. For the given potential, there is known to exist a single bound state of even parity for $\omega = 0$, corresponding to a spurious solution for the problem being considered. [See Eq. (4.1)]. In addition, if the strength of the potential were made any greater than it

is, there would also exist a non-spurious odd parity bound state with mass just below the quark-meson continuum two-particle threshold, $\omega = m_1$.

The non-existence of quark-meson bound states in the Sine-Gordon theory is also known from the exact calculation of the quantum S-matrix for the massive Thirring model [11,12]. Thus the only hope of finding quark-meson bound states in the model, whose Lagrangian is given by (2.1), is to include the gluon interactions not included in the Sine-Gordon theory. In the remainder of this section it will be shown that gluon interactions to lowest order in perturbation theory have the effect of weakening the potential, thereby indicating that there is no odd parity quark-meson bound state.

The first step is to compute the correction to the potential due to the first order gluon interactions with the meson. The interaction term is

$$\mathscr{H}_{int} = -\frac{m_1^2}{\beta^2}\left[(N_{m_1}\cos\beta\phi)(N_\mu \cos 2\sqrt{\pi}\eta) + (N_{m_1}\sin\beta\phi)(N_\mu \sin 2\sqrt{\pi}\eta)\right].$$

Thus, the first order effects of the η-field involve graphs with one internal η-line to which only the second term in Eq. (4.5) can contribute. The graphs produce an effective interaction of the form

$$iS_{eff} \propto \frac{m_1^4}{\beta^4}\int d^2x\, d^2y\, [N_{m_1}\sin(\beta\phi(x))]\,\Delta_E(x-y;\mu)\,[N_{m_1}\sin(\beta\phi(y))]. \tag{4.6}$$

where Δ_E is the Euclidean propagator for a free field of mass $\mu = \frac{e}{\sqrt{\pi}}$, and the effective action has been rotated into Euclidean

space ($x^2 = -i\,x^0$). The proportionality sign indicates a positive constant. Using Wick's Theorem [6], one may show that

$$[N_{m_1}\sin(\beta\phi(x))]\,[N_{m_1}\sin(\beta\phi(0))] \propto |m_1x|^{2/\alpha^2}[1+0(m_1^2x^2)]N_{m_1}[-2\cos(2\beta\phi)+0(m_1^2x^2)]$$
$$+\ |m_1x|^{-2/\alpha^2}[1+0(m_1^2x^2)]N_{m_1}[+2-\tfrac{4\pi}{\alpha^2}(x_\mu\partial^\mu\phi)^2]. \qquad (4.7)$$

Using

$$\int d^2x\ \Delta_E(x-y;\ \mu)\ |x|^n \propto \mu^{-(n+2)} \qquad (4.8)$$

and that $\alpha >> 1$, the effective action becomes:

$$iS_{eff} \propto -\frac{m_1^4}{\beta^4}N_{m_1}\int d^2x[\frac{2}{\mu^2}\cos 2\beta\phi + \frac{2\pi}{\alpha^2}\frac{1}{\mu^4}(\partial^\mu\phi)^2]. \qquad (4.9)$$

The derivative term provides a small correction to the original Hamiltonian density and can be brought into standard form by wave function renormalization. This results in a small and irrelevant corrections to β. The gluon interaction term

$$-\lambda U_1(\phi) \equiv -\lambda\frac{m_1^2}{\beta^2}\cos(2\beta\phi) \qquad (4.10)$$

where

$$\lambda = c'\frac{m}{e} \text{ and } c' > 0,$$
$$(\text{note } \lambda << 1),$$

produces a correction, $\lambda\phi_1$, to the Euler-Lagrange equation for ϕ, Eq. (4.2):

$$\phi \to \phi_{sol} + \lambda\phi_1, \qquad (4.11)$$

$$-\frac{d^2\phi_1}{dx^2} + U_0''(\phi_{sol})\phi_1 + U_1'(\phi_{sol}) = 0$$

and a perturbation to the Schrödinger potential in Eq. (4.3).

$$-\omega^2 q - \frac{d^2q}{dx^2} + \left(U_0''(\phi_{sol}) + \lambda U_1''(\phi_{sol}) + \lambda\phi_1 U_0'''(\phi_{sol})\right)q = 0\ . \qquad (4.12)$$

The general solution to Eq. (4.11) is

$$\phi_1 = c_1\ \mathrm{sech}(x\,m_1) + c_2\left(\sinh(x\,m_1) + (x\,m_1)\ \mathrm{sech}(x\,m_1)\right)$$
$$+\ \tfrac{1}{2}\left(\frac{8}{\beta}\right)\left((x\,m_1)\ \mathrm{sech}(xm_1) - \mathrm{sech}(xm_1)\ \tanh(x\,m_1)\right), \qquad (4.13)$$

but in order for $\phi = \phi_{sol} + \lambda\phi_1$ to be finite for all x, c_2 must be set to zero. The value of c_1 just determines a small translation of the soliton, ϕ_{sol}, since sech $(x\,m_1)$ is proportional to $d\phi_{sol}/dx$. For convenience, c_1 will be set to zero. Substituting the result into Eq. (4.12), one finds that the new Schrödinger potential is

$$V(x) \equiv V_0(x) + \lambda V_1(x), \text{ where}$$
$$V_0(x) = U''(\phi_{sol}) = m_1^2(1-2\mathrm{sech}^2 x\ m_1) \qquad (4.14)$$

and

$$V_1(x) = U_1''(\phi_{sol}) + \phi_1 U_0'''(\phi_{sol})$$
$$= 8m_1^2\tanh(xm_1)\mathrm{sech}^2(xm_1)(5\tanh(xm_1) - xm_1) - 4m_1^2\ . \qquad (4.15)$$

The effect of the perturbation on the potential is to slightly narrow the zeroth order potential without changing its depth.[3] This is indicated by the fact that $V_1(0) = V_1(\pm\infty) = 0$ and $V_1(x)$ is small and positive in the region where it and $V_0(x)$ are appreciably different from zero. Hence, the total potential is weakened and there is no indication that gluon interactions can lead to a quark-meson bound state in the theory for the range of

parameter $em \ll \mu_0^2$ and $e \gg m$.

V. CONCLUSIONS

The (1+1)-dimensional theory of a massive vector field interacting with a massive fermion field appears to display the features of quark near-confinement in the strong coupling limit. The theory contains a heavy free quark, a massive gluon, and light bound state mesons. The theory, however, does not contain quark-meson bound states in this limit, indicating that quarks travelling through matter(in (1 + 1) - dimensions)do not attract and bind mesons forming massive clumps.

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FOOTNOTES

1. Notation: $g_{00} = -g_{11} = 1$. $\varepsilon^{01} = -\varepsilon^{10} = 1$. $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, x is always the space coordinate.
2. At this point techniques reported by Coleman [6] in analyzing the massive Schwinger model are extensively employed.
3. The perturbation also produces a small constant shift in the potential, $-4\lambda m_1^2$, that should be subtracted when comparing the perturbed and unperturbed potentials.

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