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GRAVITY, THE DECAY OF THE FALSE VACUUM AND THE NEW INFLATIONARY UNIVERSE SCENARIO*

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ABSTRACT

I have calculated, at zero temperature and using the thin-wall approximation, the exponential suppression factor in the rate of decay of the false vacuum per unit volume for a real scalar field. The effects of classical gravity are included. Both the false and the true vacua have arbitrary cosmological constants. I speculate on the effects of gravity in the new inflationary universe scenario.

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To appreciate the importance of gravity in the new inflationary universe scenario, I have calculated the bubble decay rate for a single scale field decaying from the false vacuum to the true vacuum including the effects of gravity. This calculation has a large cosmological constant for both the false and true vacua and is performed at zero temperature. Also, to obtain an analytic result, the potential for the scalar field is assumed to allow the use of the thin-wall approximation.

Recently Linde,[1] Albrecht and Steinhardt[2] have renewed interest in the inflation universe for solving the cosmological problems of flatness, homogeneity, isotropy and under-abundance of magnetic monopoles. Their scenario requires that the Higgs potential for the grand unified theory be at or near the Coleman-Weinberg[3] limit. The special nature of this limit is that at zero temperature the effective potential is approximately constant until the Higgs field gets close to its grand unified vacuum expectation value, σ. At a temperature, T, much less than σ, this plateau has a bump of height $O(T^4)$ at a distance $O(T)$ from the origin. This bump allows the universe to supercool in the symmetric phase, but since the bump disappears as the temperature goes to zero, the phase transition must eventually take place.

With the Coleman-Weinberg potential, the universe cools many orders of magnitude below the critical temperature $T_c \approx \sigma \approx 10^{15}$ GeV before the phase transition takes place. Linde and Albrecht and Steinhardt disagree on how this phase transition takes place. Linde assumes it is by bubble nucleation whereas Albrecht and Steinhardt argue for the transition occurring by thermal fluctuations. This disagreement is not important to my discussion. What is important is that both groups have the transition occurring at a temperature less than 10^8 GeV. Therefore, the length scale associated with the transition is greater than $(10^8\ \mathrm{GeV})^{-1}$.

After the critical fluctuation has occurred, the Higgs field in the center of the fluctuating region is a long way from its equilibrium value and that in the subsequent "slow roll down the hill" the universe continues to inflate. This inflation is then purported to solve all the forementioned cosmological problems.

However, as Hut and Klinkhamer,[4] and Hawking and Moss[5] have pointed out there is another length scale associated with this problem. The Schwarzchild radius for an energy density of $(10^{15}\ \mathrm{GeV})^4$, which is typical of supercooled grand unified theories in the symmetric phase, is $(10^{11}\ \mathrm{GeV})^{-1}$. In this letter, I provide evidence that suggests that the phase transition occurs by bubble nucleation, close to a temperature of 10^{11} GeV, stimulated by classical gravity.

The zero temperature, thin-wall calculation is a direct extension of the work of Coleman and DeLuccia[6] and I will follow their paper except for a few minor changes in notation. For a single scalar field including gravitation, the action is given by

$$S = \int d^4x \sqrt{-g}\left[\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi) - (16\pi G)^{-1}R\right] \qquad (1)$$

where R is the curvature scalar. In this theory, a cosmological term need not be included explicitly as adding a constant to $U(\phi)$ is equivalent to adding such a term. The potential $U(\phi)$ is chosen so as to have both a false vacuum at ϕ_f, $U_f \equiv U(\phi_f)$ and a true vacuum at ϕ_t,

$U_t \equiv U(\phi_t)$. The barrier between the two minima is assumed to be large, so that the thin-wall approximation is applicable, at least in a flat space-time.

The decay of the false vacuum proceeds by the nucleation of bubbles of true vacuum in the false vacua. The bubble nucleation rate per unit volume equals $A\exp[-B/\hbar](1+O(\hbar))$, where B is the action for the bounce. The bounce, ϕ_b, is the solution of the Euclidean equations of motion with minimum action, which interpolates between the true and the false vacua.

I will assume like the authors of Ref. 6 that the solution of minimum action is invariant under four-dimensional rotations. If this is false, then this calculation puts only a lower bound on the bubble nucleation rate. The most general rotationally invariant Euclidean metric is

$$ds^2 = d\xi^2 + \rho^2(\xi)d\Omega^2 \tag{2}$$

where ξ is the radial co-ordinate and ρ is the radius of curvature. Also the ϕ field is now only a function of ξ. With this symmetry the Euclidean equations of motion are just the Coleman-DeLuccia equations

$$\phi'' + \frac{3\rho'}{\rho}\phi' = \frac{dU}{d\phi} \tag{3}$$

$$\rho'^2 = 1 + \frac{\kappa\rho^2}{3}(\tfrac{1}{2}\phi'^2 - U) \tag{4}$$

where $\kappa = 8\pi G$ and prime denotes $d/d\xi$.

The Euclidean action for a solution of these equations is

$$S_e = 4\pi^2 \int d\xi [\rho^3 U - \frac{3\rho}{\kappa}] + \text{surface terms.} \tag{5}$$

The bounce action,

$$B \equiv S_e(\phi_b) - S_e(\phi_f) \tag{6}$$

can be divided into three regions in the thin wall approximation. Outside the wall, $B_{outside} = 0$. The contribution from the wall is given by,

$$B_{wall} = 2\pi^2 \bar{\rho}^3 S_1 \tag{7}$$

where $\bar{\rho}$ is the radius of curvature of the bubble wall and

$$S_1 = 2\int d\xi [U(\phi_b) - U_f + (U_f - U_t)(\phi_b - \phi_f)/(\phi_t - \phi_f)]. \tag{8}$$

The contribution from inside the bubble is,

$$B_{inside} = \frac{12\pi^2}{\kappa} \left\{ U_t^{-1} [(1 - \frac{\kappa U_t}{3} \bar{\rho}^2)^{3/2} - 1] - (t \to f) \right\} \tag{9}$$

To find the critical bubble size, B has to be extremized with respect to $\bar{\rho}$. At the extremum

$$\bar{\rho}^2 = \frac{\bar{\rho}_0{}^2}{1 + 2(\bar{\rho}_o/2\lambda)^2 + (\bar{\rho}_o/2\Lambda)^4} \tag{10}$$

where $\bar{\rho}_o = 3S_1/(U_f - U_t)$ is the critical size without gravity,

$$\lambda^2 = [\kappa(U_f + U_t)/3]^{-1} \tag{11}$$

and

$$\Lambda^2 = [\kappa(U_f - U_t)/3]^{-1} . \tag{12}$$

The bounce action for the critical size bubble is

$$B = B_o\, r[(\bar{\rho}_o/2\lambda)^2, \lambda^2/\Lambda^2] \tag{13}$$

where

$$B_o = \frac{27\pi^2 S_1^4}{2(U_f - U_t)^3} \tag{14}$$

is the critical action without gravity and the function r is given by

$$r(x,y) = \frac{2\left\{(1+3x+2x^2+x^2y^2+x^3y^2)-(1+2x+x^2y^2)^{3/2}\right\}}{x^2(1-y^2)(1+2x+x^2y^2)^{3/2}}. \tag{15}$$

The Coleman-DeLuccia calculations are the limits λ^2/Λ^2 goes to $\pm$ 1. For these limits Eq.(10) and (15) reduce to

$$\bar{\rho} = \frac{\bar{\rho}_o}{1 \pm (\bar{\rho}_o/2\Lambda)^2} \tag{16}$$

and

$$r[\pm(\bar{\rho}_o/2\Lambda)^2, 1] = \frac{1}{[1 \pm (\bar{\rho}_o/2\Lambda)^2]^2} \tag{17}$$

which are the results they obtained.

For the scenario mentioned at the beginning of this paper, both U_f and U_t are positive so the interest is in B/B_o for $\lambda^2 > 0$. Figure 1 is a plot of B/B_o for various values of $\bar{\rho}_o/2\lambda$ and λ/Λ. For $(\bar{\rho}_o/2\lambda)^2 \gg 1$, the following limits are worth noting:

(i) for $(\bar{\rho}_o/2\Lambda)^2.\lambda^2/\Lambda^2 \ll 1$, then

$$r[(\bar{\rho}_o/2\lambda)^2, \lambda^2/\Lambda^2] = 2^{1/2}(\bar{\rho}_o/2\lambda)^{-3} \tag{18}$$

(ii) for $(\bar{\rho}_o/2\Lambda)^2.\lambda^2/\Lambda^2 \gg 1$, then

$$r[(\bar{\rho}_o/2\lambda)^2,\lambda^2/\Lambda^2] = \frac{2\Lambda^4}{\lambda^2(\lambda^2+\Lambda^2)} (\bar{\rho}_o/2\lambda)^{-4} \ . \tag{19}$$

Therefore, the falloff crosses over from a cubic to a quartic power around

$$(\bar{\rho}_o/2\Lambda).\lambda/\Lambda = 1 \ . \tag{20}$$

For a later discussion of the paper by Hawking and Moss, I would like to emphasize that for positive λ^2,

$$\bar{\rho}^2 \leq \frac{2\lambda^2\Lambda^2}{\lambda^2+\Lambda^2} = [\kappa U_f/3]^{-1} \ . \tag{21}$$

That is , the critical bubble size in the presence of gravity is always smaller or equal to the scale factor for the false vacuum de Sitter space universe regardless of the size of the critical bubble in flat space-time. Also, for positive λ^2, $B \leq B_o$.

The range of validity of these results is that, first, for the semi-classical approach to be reliable $B \gg 1$. Second, the thin-wall approximation is only a good approximation when the thickness of the wall is small compared to $\bar{\rho}_o$, $|\lambda|$ and Λ.

In a supercooled universe with a Coleman-Weinberg Higgs potential, the critical bubble size, at temperature T, is $O(T^{-1})$. Thus, at temperatures less than 10^{11} GeV the effects of gravity should be included in calculating the bubble nucleation rate, as the size of the bubble and the scale factor for the universe are comparable. Unfortunately, the results of the zero temperature, thin-wall calculation cannot be taken over directly to this scenario, but they give us an indication of what to expect, that is, a similar modification of the bounce action and the corresponding bubble nucleation rate. I would like to emphasize at this point, that multiplying the bounce action by a number of order unity can change the nucleation rate by many orders of magnitude. Therefore, it is unlikely that such a universe could cool much below 10^{11} GeV, let alone cool to less than 10^{8} GeV required by authors in Refs. 1 and 2.

Hawking and Moss have also come to this conclusion. They argue that the universe cannot cool below the Hawking temperature, $(1/2\pi)[\kappa U_f/3]^{-1/2}$. In discussing the phase transition, these authors discard bubbles which in flat space-time are larger than the scale factor of the universe. This is puzzling, as I have shown in this paper, see Eq. (21), that gravity can drastically modify the size of such bubbles, making them smaller than the scale factor of the universe. These authors argue for a homogeneous transition of the whole universe.

The finite temperature calculation, without the use of the thin-wall approximation is presently being performed.

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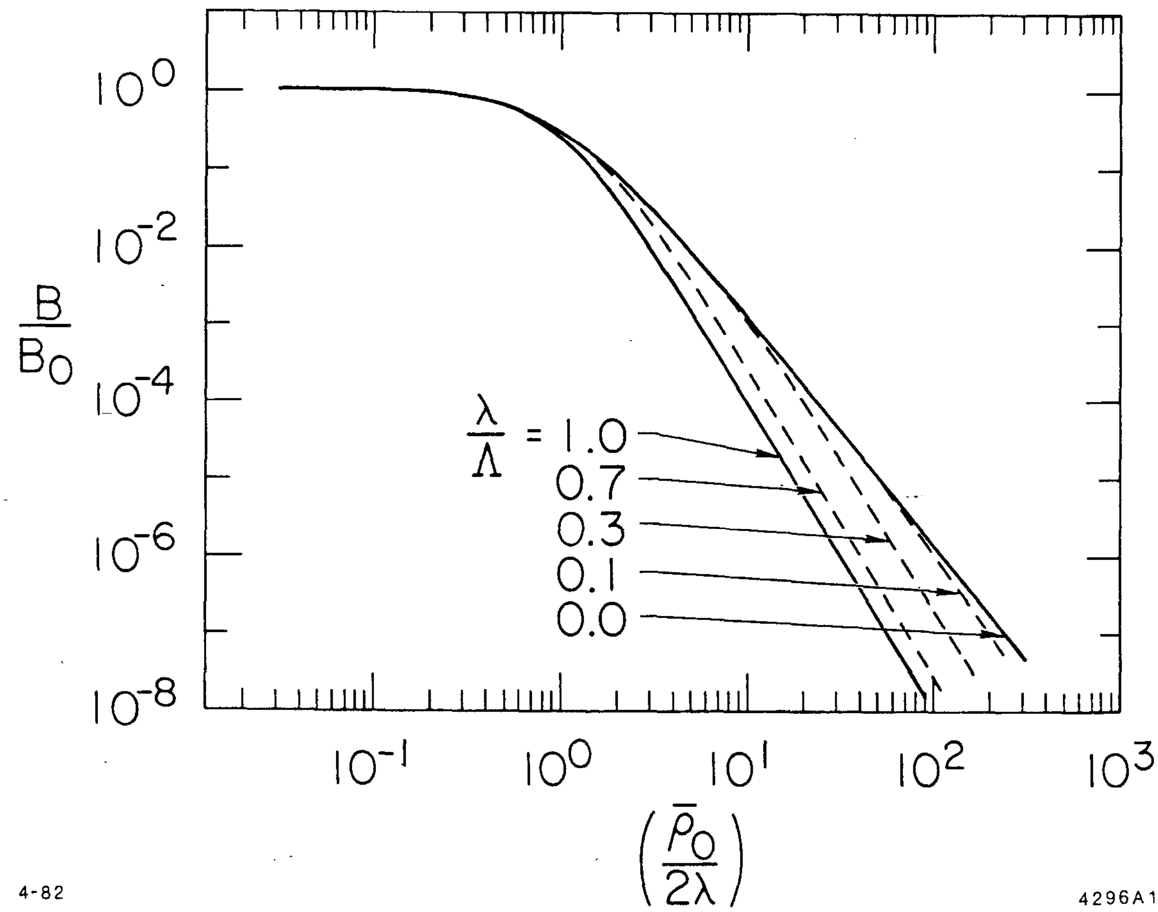
Figure Caption

The ratio B/B_o (=r) as a function of $(\bar{\rho}_o/2\lambda)$ for various values of λ/Λ.



10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
$\frac{B}{B_0}$
$\frac{\lambda}{\Lambda}$ = 1.0
0.7
0.3
0.1
0.0
10^{-1}
10^0
10^1
10^2
10^3
$\left(\frac{\bar{\rho}_0}{2\lambda}\right)$
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Fig. 1