Energy Loss of Slowly Moving Magnetic Monopoles in Matter

Sidney D. Drell
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

Norman M. Kroll
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, and Department of Physics, University of California, San Diego, La Jolla, California 92093

and

Mark T. Mueller
Institute for Theoretical Physics and Department of Physics, Stanford University, Stanford, California 94305

and

Stephen J. Parke
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

Malvin A. Ruderman
Department of Physics, Columbia University, New York, New York 10027
(Received 29 November 1982)

The cross section for exciting simple atoms by slowly moving magnetic monopoles is calculated. Including the effects of the monopole magnetic field on the atomic energy levels, an energy loss per unit density is obtained that is much larger than previous studies. For helium $(1/\rho)dE/dx = 15(\beta/10^{-4})[1-(9.29\times10^{-5}/\beta)^2]^{3/2}$ MeV cm^2/g for β in the range 10^{-4} to 10^{-3}. The possibility of using helium as a monopole detector is discussed.

PACS numbers: 14.80.Hv, 29.70.Gn

During the last few years there has been a great deal of theoretical interest in the supermassive ($\sim 10^{16}$ GeV) magnetic monopoles of grand unified theories. Recently Cabrera[1] intensified this interest with his report of a possible monopole event in the superconducting loop experiment at Stanford. The fact that Cabrera's apparatus is sensitive to monopoles of any speed (βc) together with the absence of monopole events in noninduction experiments that are much more sensitive to a flux of fast monopoles ($\beta > 10^{-3}$) indicates that if Cabrera's event was caused by a monopole it was moving slowly. Theoretical models of how supermassive monopoles enter and move in the galactic magnetic fields and in the solar system suggest that their velocities at the Earth's surface would be in the range of the galactic escape velocity ($\beta \sim 10^{-3}$) and Earth's orbital velocity about the sun ($\beta \sim 10^{-4}$), and in any event no less than the escape velocity from the Earth ($\beta \sim 3\times 10^{-5}$). Therefore a quantitative understanding of the mechanisms by which slowly moving monopoles lose energy when passing through matter is important both for a description of monopole interactions in the solar system and for the interpretation and design of noninduction experiments.

In this Letter we calculate this energy loss for simple atoms. We find that when one includes Zeeman splittings, diamagnetic shifts, and crossings of the energy levels caused by the interaction of the atomic electrons with the monopole magnetic field, the calculated energy loss for atomic hydrogen and helium is larger by an order of magnitude or more than that found in previous studies[2] for other materials.

When a monopole passes through matter the time-varying pulse of its field can excite electrons in (or ionize) nearby atoms and molecules. The monopole's energy loss can be observed in the form of subsequent electromagnetic radiation when the excited electrons cascade down to their ground states. For the problem of interest we treat the very heavy monopoles classically as moving on straight line trajectories with velocity β. Atoms of size a will "see" the time-varying field of the monopole as it traverses as a pulse with frequencies $\omega_m \sim \beta/a$. Thus excitations of frequencies $\omega_x \leq \omega_m$ will be induced in the atom. In the case of very slow passage (β much smaller than the velocity of atomic electrons), ω_m might be too small to excite the atoms; e.g., for $\beta \sim 10^{-4}$ and $a \sim 0.5$ Å, $\omega_m \sim 0.4$ eV. We would then expect the adiabatic approximation to be valid and the

 © 1983 The American Physical Society

resulting excitation probability for atoms with ω_x greater than a few electronvolts to be exponentially small.

This picture is, however, inadequate for very slow monopoles because the strength of a Dirac magnetic charge[3] $g = 1/2e$ is so great that its magnetic field will cause very large changes in the energy levels when it passes through an atom. For example, the characteristic energy shift for a monopole at distance a from an atomic electron of mass m is $eg/2ma^2 \sim 7$ eV. Hence, for a monopole passing within the atom substantial level mixings and some level crossings will occur, and the adiabatic approximation can fail badly. In particular, if the ground state and an initial excited state are shifted close together for a monopole near the center of the atom, the two levels will be mixed. There will then be a good chance of finding the atom in an excited state after the monopole has passed. (The transit time of a monopole through an atom $a/\beta \sim 1\times 10^{-15}$ sec for $\beta \sim 10^{-4}$ is very much shorter than the radiative lifetime of an excited atom, $\tau \approx 10^{-8}-10^{-9}$ sec.)

We first consider a slow monopole passing through a hydrogen atom including the effects due to the electron spin. If the monopole impinges with zero impact parameter along the z axis the z component of angular momentum,

$$J_z \equiv [\vec{r}\times(\vec{p}-e\vec{A})+\vec{\sigma}/2-\hat{n}/2]_z, \qquad (1)$$

is conserved; $\vec{r}$ is the electron coordinate relative to the proton *fixed* at the origin and $\hat{n}$ is the unit vector from the monopole to the electron. Since $\hat{n}_z$ changes sign as the monopole moves from the far left ($\hat{n}_z = -1$) to the far right ($\hat{n}_z = +1$), the z component of the electron's angular momentum must change by a compensating amount. Consider, for example, the doubly degenerate ground state of the H atom with principal quantum number $n = 1$ and $m_j = \pm\frac{1}{2}$ when the monopole is far away. An electron initially with $m_j = -\frac{1}{2}$ will spin flip to $m_j = +\frac{1}{2}$ as the monopole traverses left to right, while one with $m_j = +\frac{1}{2}$ will be raised to an excited state with $n > 1$ and $m_j = +\frac{3}{2}$. On the way up, this level will necessarily cross one moving down from $m_j = -\frac{3}{2}$ to the ground state with $m_j = -\frac{1}{2}$.

In order to map out this level crossing we consider the two extremes of the monopole at the origin and at large distance from the atom and interpolate by a perturbative calculation. As the monopole approaches the atom from a large distance the energy levels split in the characteristic Zeeman pattern for a uniform magnetic field. In particular, three of the excited $n = 2$ octet of levels start to move down in energy and the $n = 1$ level with $m_j = +\frac{1}{2}$ starts up toward them. When the monopole is at the origin of coordinates the exact nonrelativistic eigenvalues and eigenvectors for the electron are known.[4] The eigenstates of this system are classified in terms of a principal quantum number $n = 0, 1, 2, \ldots$ and a spin $j = 0, 1, 2, \ldots$. The singlet states with $j = 0$ have the same energies as the s states of the H atom: $E_n = -m\alpha^2/2(n+1)^2$. For $\mu \equiv [j(j+1)]^{1/2} > 0$, the states form two sequences with eigenvalues $E_n = -m\alpha^2/2(n+\mu)^2$ and $E_n = -m\alpha^2/2(n+\mu+1)^2$. Thus the energy eigenstates are in the following sequence of multiplets starting from the ground state: $1, 3, 1, 3^2, 5, 1, \ldots$, where the superscript gives the number of such multiplets. The lowest triplet state with $n = 0$, $j = 1$ is the state of most interest to us and has energy $-m\alpha^2/4$.

When the monopole is near the origin, its effect can be found by use of the multipole expansion about the origin. If the monopole is at a distance small compared to the atomic size, the terms higher than dipole can be neglected and we can treat the dipole with first-order perturbation theory. The results of this calculation can be joined to the Zeeman-shifted levels for large separations of the monopole. The solid lines in Fig. 1 give the energy levels for the lowest relevant states for arbitrary separation of a stationary monopole along the z axis. The energy levels actually *cross* as the monopole "passes" through the origin because they have different eigenvalues of J_z.

The energy-level diagram for a stationary monopole along a path of nonzero impact parameter b can be obtained in a similar manner. This is also displayed in Fig. 1 and follows the solid lines except near the point of closest approach for which the dotted lines are applicable. In this case the symmetry axis rotates as the monopole approaches the atom. Hence J_z is not conserved, and the levels mix and do not cross. The minimum interval between them, $\omega_{\min}$, increases with increasing values of b. For sufficiently large impact parameters and slow enough monopole velocities, such that $(\beta/b) < \omega_{\min}(b)$, the adiabatic approximation is applicable and the electron would just follow these levels. Then an electron in either of the two degenerate ground states would remain in the ground state. For smaller values of b there will be level mixing, the adiabatic approximation will break down, and the electron will be excited. The region of the

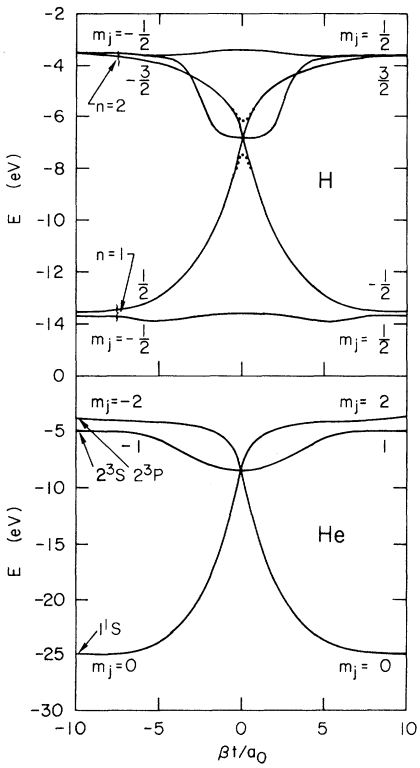

FIG. 1. The energy levels for atomic hydrogen and helium.

monopole's trajectory where the probability of level transition is greatest occurs when the monopole is closest to the origin where the interval between the energy levels is the smallest. In this region we can model the effects of a monopole by using the dipole approximation and first-order degenerate perturbation theory among the triplet of levels obtained with the monopole at the origin. The time-dependent Schrödinger equation for this system is given by

$$i\frac{d}{dt}\begin{pmatrix} c_1 \\ c_0 \\ c_{-1} \end{pmatrix} = \gamma \begin{pmatrix} \beta t & b/\sqrt{2} & 0 \\ b/\sqrt{2} & 0 & b/\sqrt{2} \\ 0 & b/\sqrt{2} & -\beta t \end{pmatrix} \begin{pmatrix} c_1 \\ c_0 \\ c_{-1} \end{pmatrix}, \quad (2)$$

where $c_i(t)$ is the amplitude for the electron to be in eigenstate i in a J_z-diagonal basis and

$$\gamma = (2-\sqrt{2})\langle r^{-3}\rangle_{j=1}/4m = m\alpha^2/4(4-\sqrt{2})a_0$$

where a_0 is the Bohr radius. These coupled differential equations can be solved[5] in terms of Weber functions.

Starting with the initial condition $|c_1(t\to-\infty)|^2 = 1$, we find for large positive times $|c_1(\infty)|^2 = x^2$, $|c_0(\infty)|^2 = 2x(1-x)$, and $|c_{-1}(\infty)|^2 = (1-x)^2$, where $x \equiv \exp[-3b^2/2\bar{b}_\beta{}^2]$ and $\bar{b}_\beta{}^2 = 3\beta/\pi\gamma$. The large-$b$ limit of $|c_{-1}|^2 \to 1$ corresponds to the adiabatic approximation of no energy loss. The $b\to 0$ limit coincides with the sudden approximation for the nonexcitation probability: $|c_{-1}|^2 \propto b^4$. Integrating over all impact parameters we obtain for the cross section for exciting the electron to an $n=2$ level

$$\sigma_{n=2} = \tfrac{1}{2}\int db\, 2\pi b(|c_1|^2 + |c_0|^2) = \pi\bar{b}_\beta{}^2/2, \quad (3)$$

where the factor $\frac{1}{2}$ recognizes that 50% of the time the electron is initially in the state with $m_j = -\frac{1}{2}$ that is not excited by the monopole as shown in Fig. 1. The corresponding energy loss per atom is $\Delta E = \frac{3}{8}m\alpha^2\sigma_{n=2}$.

The peak contribution to the cross section occurs when

$$b = 0.70\bar{b}_\beta \simeq 0.095(\beta/10^{-4})^{1/2}\langle r\rangle_{j=1},$$

where $\langle r\rangle_{j=1}$ ($\simeq 2.7a_0$) is the radius of the triplet state of the H atom with the monopole at the center. We therefore expect the dipole expansion to be a reasonable approximation up to $\beta \sim 1\times10^{-3}$. The process has a threshold at $\beta = 1.47\times10^{-4}$ due to recoil of the H atom. On account of recoil, the values of β and b when the energy level transfers are taking place are different from the asymptotic values. Taking this into account, we find that Eq. (3) should be corrected by a recoil factor $(\beta_0/\beta)^3$ where the subscript 0 refers to quantities when the pole is at the atomic center. This correction factor is obtained by introducing β_0 and b_0 for β and b in Eq. (2) and using angular momentum conservation, $\beta_0 b_0 = \beta b$. Expressing β_0 in terms of the energy when the levels cross, $\frac{1}{2}M\beta_0{}^2 + \Delta E_0 = \frac{1}{2}M\beta^2$, we can write a threshold-corrected formula

$$\sigma_{n=2} = (3\beta/2\gamma)(1-\beta_c{}^2/\beta^2)^{3/2}, \quad (4)$$

with $\beta_c = (2M\Delta E_0)^{1/2} = 1.20\times10^{-4}$. The energy loss per unit density is

$$37(\beta/10^{-4})(1-\beta_c{}^2/\beta^2)^{3/2}\ \text{MeV cm}^2/\text{g}.$$

Comparison is made with previous calculations[2] in Fig. 2.

Atomic helium is of interest as a practical substance for a noninduction experiment that is also amenable to analysis in terms of simple calculations. The energy level diagram for He is also shown in Fig. 1 including the shielding effects between the two electrons. The level mixing for a monopole trajectory with nonzero impact parameter can be calculated in the same way as for H except that, in Eq. (2), $\gamma \propto \langle r^{-3}\rangle_{j=1}$ is changed as a result of shielding due to the presence of a second electron. This change can be calculated by standard variational techniques leading to $\gamma \to Z_{\text{eff}}{}^3\gamma$ where $Z_{\text{eff}} \simeq 1.33$ for this state. Further-

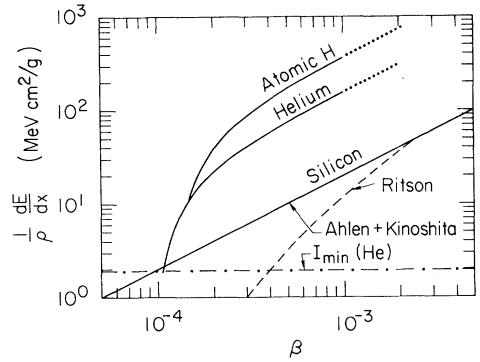

FIG. 2. The energy loss vs velocity for a Dirac monopole. The curves for atomic hydrogen and helium are the results of this calculation including recoil effects.

more, the factor $\frac{1}{2}$ appearing in Eqs. (3) and (4) should be omitted. This yields the value

$$5\times10^{-18}(\beta/10^{-4})(1-\beta_c{}^2/\beta^2)^{3/2}\ \mathrm{cm}^2$$

for the excitation cross section. Here $\beta_c = 9.29 \times10^{-5}$ and the threshold values of $\beta\times10^4$ are 1.06 and 1.03 for $n=2$ ^3P and ^3S excitations, respectively. The energy loss per unit density is

$$15(\beta/10^{-4})(1-\beta_c{}^2/\beta^2)^{3/2}\ \mathrm{MeV\ cm^2/g}$$

and the relative populations of ^3P to ^3S is approximately 1:2.

The fact that the excitation cross section in He is large and exclusively to the triplet $n=2$ levels should provide a unique signature for the passage of the monopole. The ^3P state will decay to ^3S with emission of a 1.15-eV photon, which is not self-absorbed, but which may be difficult to detect efficiently. For pure He the metastable ^3S states are likely to remain excited until the atoms reach the walls of the He container, where electron ejection is likely to occur. The ^3S states may be rendered optically active by the addition of Ne, which is readily excited to the nearby resonant 4s levels by collision with the metastable He atoms. There are also a large variety of additives which will be collionally ionized, producing electrons and ions which could be collected. Some of the uniqueness of signature, which may be lost as the result of the additives, may be regained by the use of timing measurements.

Analogous effects are likely to occur in other atomic and molecular systems. Of particular interest are spherically symmetric systems such as higher-Z noble gas atoms. When a monopole passes through the precise center of such a system it must leave the atom with Z units of angular momentum, a circumstance which implies multiple-electron excitations likely to decay by autoionizing Auger processes and to involve large excitation energies. However, there are also likely to be excitation-inducing level crossings for nonzero impact parameter, which can lead to smaller angular momentum transfer, smaller excitation energies, and single-electron excitations. The situation is complex and our analysis is in the most preliminary stage, but the likelihood of substantial energy loss and observable excitation appears to us to be high. Provided that the probability of single-electron excitation is substantial the kinematic constraint is less severe than for He with thresholds below $\beta = 5\times10^{-5}$.

We have also considered some molecular systems in a preliminary way. The H_2 molecule is clearly the simplest case. There are two positions on the molecular axis, outside the pair of protons, at which the monopole causes a crossing of the singlet (bonding) and triplet (antibonding) states. Thus passage of the monopole near these points will induce a substantial fraction of transitions which cause the dissociation of H_2 into ground-state atoms. The cross section for this process is likely to be comparable to those estimated above. There is also the probability of a dissociation which produces one excited atom, a process which is caused when a monopole passes through the molecule nearly parallel to the molecular axis, but we have not estimated the size of this effect. Organic molecules of the sort used in scintillators as well as scintillating inorganic crystals also have intriguing possibilities, which are being investigated. The π electrons on benzene rings, whose excitation is crucial for the functioning of organic scintillators such as polystyrene plastic, are similar in many ways to electrons on simple spatially fixed loops except that angular momentum can be transferred to the benzene ring in integral multiplets of $6\hbar$. Unknown matrix elements for multielectron transitions of this kind have so far eluded a quantitative calculation.

We would like to thank our theoretical and experimental colleagues for many enlightening discussions. N.K., S.P., and M.R. also thank the Aspen Center for Physics. This work was supported in part by the Department of Energy under Contracts No. DE-ARC03-76SF-00515 and No. DE-AT03-S1ER40029, and by the National Science Foundation under Contracts No. PHY78-26847 and No. PHY80-23721.

[1]B. Cabréra, Phys. Rev. Lett. 48, 1378 (1982).

[2]S. Ahlen and K. Kinoshita, Phys. Rev. D 26, 2347 (1982); D. Ritson, SLAC Report No. SLAC-PUB-2950, 1982 (unpublished).

[3]We will use the Dirac value but all our results are easily generalized to integer multiples.

[4]W. V. R. Malkus, Phys. Rev. 83, 899 (1951); Y. Kazama, C. N. Yang, and A. S. Goldhaber, Phys. Rev. D 15, 2287 (1977).

[5]C. Zener, Proc. Roy. Soc. London, Ser. A 137, 696 (1932).

Binding Energies of Two-Delta Bound States

Hiroshi Sato
Institute for Nuclear Study, University of Tokyo, Midoricho, Tanashi, Tokyo, 188 Japan

and

Koichi Saito
Department of Physics, Faculty of Science, Tohoku University, Sendai, Miyagi, Japan

(Received 28 June 1982)

Bound states of the two-delta system are investigated by employment of the realistic one-boson exchange potential. It is found that there exist many bound states in each isospin channel and also that the tensor interaction plays an important role in producing these bound states. The relationship between these bound states and dibaryon resonances is discussed.

PACS numbers: 14.20.Pt, 21.30.+y, 21.40.+d

Experimental studies on the proton-proton, pion-deuteron, and γ-deuteron scatterings suggest the existence of several dibaryon resonances.[1] These dibaryon resonances are believed to be deuteron-type bound states (d^* states) of baryons and their threshold effects[2] or six-quark states. The d^* states of baryons have been studied in terms of bound states of the N-Δ (Ref. 3) and the Δ-Δ (Ref. 4) systems or the πNN and $\pi\pi NN$ states.[5] The existence of d^* states in the N-Δ and Δ-Δ systems has been predicted by Dyson and Xuong in the early sixties from a symmetry argument on two-baryon states.[6] Although the d^* model is a reasonable and interesting extension of ordinary nuclear physics compared to the introduction of the exotic six-quark states, there exists a serious objection to this model, especially to the $T=0$ bound states in the Δ-Δ system.[7,8] The lowest $T=0$ bound state in the Δ-Δ system, which has been studied by Kamae and Fujita (KF) with nonrelativistic S-state one-boson exchange potentials (OBEP) having a reasonable hard core,[4] is the 3^+ state and has a binding energy of about 100 MeV.[9] On the other hand, phase-shift analysis indicates the existence of a negative-parity state ($^{2S+1}L_J = {}^1F_3$) at 2.2 GeV (the corresponding binding energy is 260 MeV).[10] The objection to the d^* model is how to explain this negative-parity state within the model. Here, although this objection looks like a real puzzle, we have two questions about it. The first question is on the reliability of the KF calculations without the tensor part of the OBEP, because it is well known that the tensor interaction plays an essential role in producing the deuteron state. The second is on the experimental level assignment, i.e., whether the state found is really the $T=0$, 3^+ state or not,[9] and also whether the dibaryon resonance of the $T=0$ negative-parity state at 2.2 GeV is the $^1F_3$ state or not.[10] In this Letter we study the bound states of the Δ-Δ system with the OBEP having a tensor part, and show that the second question is partially resolved. We employ the following nonrelativistic OBEP as the Δ-Δ potential:

$$V_{\Delta\Delta}(r) = \begin{cases} \infty, & r \leq r_c \\ [V_\eta(r) + V_\sigma(r) + V_\omega(r)] + [V_\pi(r) + V_\rho(r) + V_\delta(r)](\vec{\tau}_1 \cdot \vec{\tau}_2), & r > r_c \end{cases}$$

© 1983 The American Physical Society