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GLUONIC TWO GOES TO FOUR

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ABSTRACT

The cross section for two gluon to four gluon scattering is given in a form suitable for fast numerical calculations.

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Theoretical predictions for four-jet production at hadron colliders allow detailed tests of QCD. Moreover, at SSC energies, four jets become a serious background to many interesting processes which probe new physics, e.g. pair production of electroweak bosons [1]. Hence a detailed knowledge of four-jet event characteristics is crucial for good background rejection. Although some individual contributions to four-jet production have already been analysed (see e.g. ref. [2]), the two gluon to four gluon scattering, which is the dominant contribution for a wide range of subprocess energies, has remained beyond the scope of previous computational techniques. Here we outline our calculation of the cross section for this process, in the tree approximation of perturbative QCD. The final cross section is presented in a form suitable for fast numerical calculations.

Our calculation makes use of techniques developed in ref.[3], based on the application of extended supersymmetry. We adopt the convention that all particles involved in a scattering process are incoming. An outgoing particle of momentum p and helicity s will be represented as an incoming antiparticle of momentum -p and helicity -s. Let $M(z^1_{s_1},..,z^n_{s_n})$ denote the amplitude for the process with the incoming particles $z^1,..,z^n$ of helicities $s_1,..,s_n$ and momenta $p_1,..,p_n$. The momenta satisfy the conservation equation, $\sum_{i=1}^{i=6} p_i = 0$. We find that all nonvanishing six-gluon helicity amplitudes can be obtained by crossing and/or complex conjugation from two amplitudes, $M(g^1_-,g^2_-,g^3_-,g^4_-,g^5_+,g^6_+)$ and $M(g^1_-,g^2_-,g^3_-,g^4_+,g^5_+,g^6_+)$. These amplitudes can be expressed in terms of the amplitudes for processes involving a smaller number of gluons plus spin one-half massless gluinos λ and spin zero massless scalar gluons ϕ, using supersymmetry relations (on-shell Ward-Takahashi identities). The first of two relations is very simple:

$$\left|M(g^1_-,g^2_-,g^3_-,g^4_-,g^5_+,g^6_+)\right| = \frac{s_{56}}{s_{23}}\left|M(g^1_-,g^4_-,\phi^2_-,\phi^3_-,\phi^5_+,\phi^6_+)\right| \tag{1}$$

where the Lorentz invariants s_{ij} are defined as usual, $s_{ij} = (p_i+p_j)^2$, and the scalar product is given by

$$p_ip_j = p_i^xp_j^x + p_i^yp_j^y + p_i^zp_j^z - E_iE_j \tag{2}$$

($E_i=p_i^t$; all particles are on mass-shell, $p_ip_i=0$). The second supersymmetry relation is more complicated. However, it simplifies considerably in the c.m.s. of particles 1 and 4:

$$\begin{aligned}\left|M(g^1_-,g^2_-,g^3_-,g^4_+,g^5_+,g^6_+)\right| = \frac{1}{s_{23}s_{56}}\Big|&(s_{23}+s_{12}+s_{13})^2M(g^1_-,g^4_+,\phi^2_-,\phi^3_-,\phi^5_+,\phi^6_+)\\ &-2i\sqrt{-s_{14}}\,(s_{23}+s_{12}+s_{13})(p_5^x+p_6^x-ip_5^y-ip_6^y)\,M(\lambda^1_-,\lambda^4_+,\phi^2_-,\phi^3_-,\phi^5_+,\phi^6_+)\\ &-s_{14}\,(p_5^x+p_6^x-ip_5^y-ip_6^y)^2\,M(\phi^1_-,\phi^4_+,\phi^2_-,\phi^3_-,\phi^5_+,\phi^6_+)\Big|\end{aligned} \tag{3}$$

Particles 1 and 4 are chosen to move along the negative and positive z-axis, respectively.

We calculate the two goes to four cross section by first computing the amplitudes for the scattering of two gluons, two fermions and two scalars into four scalars, and subsequently using eqs. (1) and (3) to obtain the appropriate gluonic amplitudes. These amplitudes are calculated in the c.m.s. of particles 1 and 4, and then reexpressed in terms of Lorentz invariants. Before presenting the result, let us adopt some convenient notation.

The squares of the absolute values of all helicity amplitudes will be generated from two generic functions, $A_0(p_1,p_2,p_3,p_4,p_5,p_6)$ and $A_2(p_1,p_2,p_3,p_4,p_5,p_6)$, defined as

$$A_{\binom{0}{2}}(p_1,p_2,p_3,p_4,p_5,p_6) = |M(g^1_-,g^2_-,g^3_-,g^4_{\binom{+}{-}},g^5_+,g^6_+)|^2, \tag{4}$$

where the r.h.s. implicitly contains the sum over the color indices of all gluons. The square of the modulus of the invariant matrix element for six-gluon process, averaged over initial colors and polarizations, and summed over final colors and polarizations, is given by

$$\begin{aligned}
&|M(g^1,g^2,g^3,g^4,g^5,g^6)|^2 = \\
&2^{1-I}(N^2-1)^{-I}\{\, A_0(123456) + A_0(135426) + A_0(126435) + A_0(156423) \\
&\qquad + A_0(125436) + A_0(136425) + A_0(134256) + A_0(124356) \\
&\qquad + A_0(145236) + A_0(146235) \\
&\qquad + A_2(123456) + A_2(135426) + A_2(123546) + A_2(136425) \\
&\qquad + A_2(156423) + A_2(125436) + A_2(126435) + A_2(563412) \\
&\qquad + A_2(526413) + A_2(123645) + A_2(135624) + A_2(623415) \\
&\qquad + A_2(523416) + A_2(125634) + A_2(523614)\,\},
\end{aligned} \tag{5}$$

where

$$A_{\binom{0}{2}}(ijklmn) = A_{\binom{0}{2}}(p_i,p_j,p_k,p_l,p_m,p_n).$$

Here N is the number of colors (N=3 for QCD), and I is the number of initial gluons. The cross section for the scattering of two gluons with momenta p_1,p_2 into four gluons with momenta p_3,p_4,p_5,p_6 is obtained from eq.(5) by setting I=2 and replacing the momenta p_3, p_4, p_5, p_6 by $-p_3, -p_4, -p_5, -p_6$.

As the result of the computation of two hundred forty Feynman diagrams, we obtain

$$A_{\binom{0}{2}}(p_1, p_2, p_3, p_4, p_5, p_6) =$$

$$(\mathcal{D}^\dagger, \mathcal{D}_\rho^\dagger, \mathcal{D}_\sigma^\dagger, \mathcal{D}_\tau^\dagger)_{\binom{0}{2}} \cdot \begin{pmatrix} K & K_\rho & K_\sigma & K_\tau \\ K_\rho & K & K_\tau & K_\sigma \\ K_\sigma & K_\tau & K & K_\rho \\ K_\tau & K_\sigma & K_\rho & K \end{pmatrix} \cdot \begin{pmatrix} \mathcal{D} \\ \mathcal{D}_\rho \\ \mathcal{D}_\sigma \\ \mathcal{D}_\tau \end{pmatrix}_{\binom{0}{2}} \tag{6}$$

where $\mathcal{D}$, $\mathcal{D}_\rho$, $\mathcal{D}_\sigma$ and $\mathcal{D}_\tau$ are 11-component complex vector functions of the momenta p_1, p_2, p_3, p_4, p_5 and p_6, and K, K_ρ, K_σ and K_τ are constant 11×11 matrices. The vectors $\mathcal{D}_\rho$, $\mathcal{D}_\sigma$ and $\mathcal{D}_\tau$ are obtained from the vector $\mathcal{D}$ by the permutations $(p_2 \leftrightarrow p_3)$, $(p_5 \leftrightarrow p_6)$ and $(p_2 \leftrightarrow p_3\ ,\ p_5 \leftrightarrow p_6)$, respectively, of the momentum variables in $\mathcal{D}$. The individual components of the vector $\mathcal{D}$ represent the sums of all contributions proportional to the appropriately chosen eleven basis color factors. The matrices K, which are the suitable sums over the color indices of products of the color bases, contain two independent structures, proportional to $N^4(N^2-1)$ and $N^2(N^2-1)$, respectively (N is the number of colors, N=3 for QCD):

$$K = [g^8N^4(N^2-1)/8]K^{(4)} + [g^8N^2(N^2-1)/2]K^{(2)}. \tag{7}$$

Here g denotes the gauge coupling constant. The matrices $K^{(4)}$ and $K^{(2)}$ are given in Table I. The vector $\mathcal{D}$ is related to the thirty three diagrams D^G(I=1-33) for two gluon to four scalar scattering, eleven diagrams D^F(I=1-11) for two fermion to four scalar scattering and sixteen diagrams D^S(I=1-16) for two scalar to four scalar scattering, in the following way:

$$\mathcal{D}_0 = \frac{2s_{14}}{\sqrt{|s_{15}\,s_{45}\,s_{16}\,s_{46}|}\;s_{23}\,s_{56}} \Big\{ \quad t_{123}^2\, C^G\cdot D_0^G$$

$$- 4 s_{14} t_{123} E(p_5+p_6, p_6)\; C^F\cdot D_0^F$$

$$- 2 s_{14} G(p_5+p_6\,,\ p_5+p_6)\; C^S\cdot D_0^S \quad \Big\} ,$$

$$\mathcal{D}_2 = \frac{s_{56}}{s_{23}}\; C^G\cdot D_2^G\ , \tag{8}$$

where the constant matrices $C^G(11\times33)$, $C^F(11\times11)$ and $C^S(11\times16)$ are given in Table II. The Lorentz invariants s_{ij} and t_{ijk} are defined as $s_{ij} = (p_i+p_j)^2$, $t_{ijk} = (p_i+p_j+p_k)^2$ and the complex functions E and G are given by

$$E(p_i,p_j) = \left[\tfrac{1}{4}(s_{14}s_{ij} - s_{1i}s_{j4} - s_{1j}s_{i4}) + i\varepsilon_{\mu\nu\rho\lambda}p_1^\mu p_i^\nu p_j^\rho p_4^\lambda\right]/s_{14}$$

$$G(p_i,p_j) = E(p_i,p_5)E(p_j,p_6), \tag{9}$$

where ε is the totally antisymmetric tensor, $\varepsilon_{xyzt} = 1$. For the future use, we define one more function,

$$F(p_i,p_j) = (s_{14}s_{ij} + s_{1i}s_{j4} - s_{1j}s_{i4})/2s_{14}. \tag{10}$$

Note that when evaluating A_0 and A_2 at crossed configurations of the momenta, care must be taken with the implicit dependence of the functions E, F and G on the momenta p_1, p_4, p_5, p_6.

The diagrams D_2^G are listed below:

$$D_2^G(1) = \frac{\delta_2}{s_{14}s_{25}s_{36}} \Big\{ [(p_2-p_5)(p_3-p_6)]\cdot[(p_1-p_4)(p_3+p_6)]$$

$$- [(p_2-p_5)(p_3+p_6)]\cdot[(p_1-p_4)(p_3-p_6)]$$

$$+ [(p_2+p_5)(p_3-p_6)]\cdot[(p_1-p_4)(p_2-p_5)] \Big\},$$

$$D_2^G(2) = \frac{1}{s_{25}s_{36}} \Big\{ 2E(p_2-p_5, p_3-p_6) - 2E(p_3-p_6, p_2-p_5)$$

$$+ \delta_2\cdot[(p_2-p_5)(p_3-p_6)] \Big\},$$

$$D_2^G(3) = \frac{4}{s_{25}s_{36}t_{125}} \Big\{ [(p_1+p_2-p_5)(p_4+p_3-p_6)]\cdot E(p_2,p_3)$$

$$-[(p_1+p_2-p_5)(p_4-p_3+p_6)]\cdot E(p_2,p_6)$$

$$-[(p_1-p_2+p_5)(p_4+p_3-p_6)]\cdot E(p_5,p_3)$$

$$+[(p_1-p_2+p_5)(p_4-p_3+p_6)]\cdot E(p_5,p_6)$$

$$-[p_1(p_2-p_5)]\cdot E(p_3-p_6, p_3+p_6)$$

$$-[p_4(p_3-p_6)]\cdot E(p_2+p_5, p_2-p_5)$$

$$+\delta_2\cdot[p_1(p_2-p_5)]\cdot[p_4(p_3-p_6)] \Big\},$$

$$D_2^G(4) = \frac{-2}{s_{36}\, t_{125}} \left\{ E(p_3-p_6\,,\, p_3+p_6) - \delta_2 \cdot [p_4(p_3-p_6)] \right\},$$

$$D_2^G(5) = \frac{-2}{s_{25}\, t_{125}} \left\{ E(p_2+p_5\,,\, p_2-p_5) - \delta_2 \cdot [p_1(p_2-p_5)] \right\},$$

$$D_2^G(6) = \frac{\delta_2}{t_{125}}$$

$$\begin{aligned} D_2^G(7) = \frac{4}{s_{12}\, s_{36}\, t_{125}} \Big\{ & [(p_1+p_2-p_5)(p_4+p_3-p_6)] \cdot E(p_2, p_3) \\ & - [(p_1+p_2-p_5)(p_4-p_3+p_6)] \cdot E(p_2, p_6) \\ & - [p_4(p_3-p_6)] \cdot E(p_2, p_2-p_5) \Big\}, \end{aligned}$$

$$\begin{aligned} D_2^G(8) = \frac{4}{s_{34}\, s_{25}\, t_{125}} \Big\{ & [(p_1+p_2-p_5)(p_4+p_3-p_6)] \cdot E(p_2, p_3) \\ & - [(p_1-p_2+p_5)(p_4+p_3-p_6)] \cdot E(p_5, p_3) \\ & - [p_1(p_2-p_5)] \cdot E(p_3-p_6, p_3) \Big\}, \end{aligned}$$

$$D_2^G(9) = \frac{4}{s_{15}\, s_{36}\, t_{125}} \Big\{ \big[(p_1 - p_2 + p_5)(p_4 + p_3 - p_6) \big] \cdot E(p_5, p_3)$$

$$- \big[(p_1 - p_2 + p_5)(p_4 - p_3 + p_6) \big] \cdot E(p_5, p_6)$$

$$+ \big[p_4 (p_3 - p_6) \big] \cdot E(p_5,\ p_2 - p_5) \Big\} ,$$

$$D_2^G(10) = \frac{4}{s_{25}\, s_{46}\, t_{125}} \Big\{ \big[(p_1 + p_2 - p_5)(p_4 - p_3 + p_6) \big] \cdot E(p_2, p_6)$$

$$- \big[(p_1 - p_2 + p_5)(p_4 - p_3 + p_6) \big] \cdot E(p_5, p_6)$$

$$+ \big[p_1 (p_2 - p_5) \big] \cdot E(p_3 - p_6,\ p_6) \Big\} ,$$

$$D_2^G(11) = \frac{\delta_2}{s_{36}\, t_{124}} \Big\{ s_{35} - s_{56} + s_{36} \Big\} ,$$

$$D_2^G(12) = \frac{-\delta_2}{s_{36}\, t_{145}} \Big\{ s_{23} - s_{26} - s_{36} \Big\} ,$$

$$D_2^G(13) = \frac{\delta_2}{s_{14}\, s_{36}\, t_{124}} \Big\{ \big[s_{12} - s_{24} \big] \cdot \big[s_{35} - s_{56} + s_{36} \big] \Big\} ,$$

$$D_2^G(14) = \frac{\delta_2}{s_{14}\, s_{36}\, t_{145}} \Big\{ \big[s_{15} - s_{45} \big] \cdot \big[s_{23} - s_{26} - s_{36} \big] \Big\} ,$$

$$D_2^G(15) = \frac{\delta_2}{s_{14}\, s_{36}} \left\{ (p_1 - p_4)(p_3 - p_6) \right\} ,$$

$$D_2^G(16) = \frac{-4}{s_{12}\, s_{36}\, t_{124}} \left\{ \left[s_{35} - s_{56} + s_{36} \right] \cdot E(p_2, p_2) \right\} ,$$

$$D_2^G(17) = \frac{4}{s_{36}\, s_{45}\, t_{145}} \left\{ \left[s_{23} - s_{26} - s_{36} \right] \cdot E(p_5, p_5) \right\} ,$$

$$D_2^G(18) = \frac{-4}{s_{12}\, s_{36}\, s_{45}} \left\{ \left[2(p_1 + p_2)(p_3 - p_6) - s_{36} \right] \cdot E(p_2, p_5) \right\} ,$$

$$D_2^G(19) = \frac{-2}{s_{12}\, s_{36}} \cdot E(p_2, p_3 - p_6) ,$$

$$D_2^G(20) = \frac{2}{s_{36}\, s_{45}} \cdot E(p_3 - p_6, p_5) ,$$

$$D_2^G(21) = \frac{-4}{s_{25}\, s_{34}\, t_{134}} \left\{ \left[s_{26} - s_{56} + s_{25} \right] \cdot E(p_3, p_3) \right\} ,$$

$$D_2^G(22) = \frac{4}{s_{16}\, s_{25}\, t_{146}} \left\{ \left[s_{23} - s_{35} - s_{25} \right] \cdot E(p_6, p_6) \right\} ,$$

$$D_2^G(23) = \frac{4}{s_{16}\, s_{25}\, s_{34}} \left\{ \left[2(p_1 + p_6)(p_2 - p_5) + s_{25} \right] \cdot E(p_6, p_3) \right\} ,$$

$$D_2^G(24) = \frac{-2}{s_{25}\, s_{34}} \; E(p_2 - p_5 , p_3) \quad ,$$

$$D_2^G(25) = \frac{2}{s_{16}\, s_{25}} \; E(p_6 , p_2 - p_5) \quad ,$$

$$D_2^G(26) = \frac{-2}{s_{12}\, t_{125}} \; E(p_2 , p_2 - p_5) \quad ,$$

$$D_2^G(27) = \frac{2}{s_{46}\, t_{125}} \; E(p_3 - p_6 , p_6) \quad ,$$

$$D_2^G(28) = \frac{2}{s_{15}\, t_{125}} \; E(p_5 , p_2 - p_5) \quad ,$$

$$D_2^G(29) = \frac{-2}{s_{34}\, t_{125}} \; E(p_3 - p_6 , p_3) \quad ,$$

$$D_2^G(30) = \frac{4}{s_{12}\, s_{34}\, t_{125}} \left\{ \left[(p_1 + p_2 - p_5)(p_4 + p_3 - p_6) - t_{125} \right] E(p_2 , p_3) \right\} ,$$

$$D_2^G(31) = \frac{4}{s_{12}\, s_{46}\, t_{125}} \left\{ \left[(p_1 + p_2 - p_5)(p_4 - p_3 + p_6) + t_{125} \right] E(p_2 , p_6) \right\} ,$$

$$D_2^G(32) = \frac{4}{s_{15}\, s_{34}\, t_{125}} \left\{ \left[(p_1 - p_2 + p_5)(p_4 + p_3 - p_6) + t_{125} \right] E(p_5 , p_3) \right\} ,$$

$$D_2^G(33) = \frac{4}{s_{15}\, s_{46}\, t_{125}} \left\{ \left[(p_1 - p_2 + p_5)(p_4 - p_3 + p_6) - t_{125} \right] E(p_5 , p_6) \right\} , \tag{11}$$

where $\delta_2 = 1$.

The diagrams D_0^G are obtained from D_2^G by replacing δ_2 by $\delta_0 = 0$ and the functions $E(p_i,p_j)$ by $G(p_i,p_j)$.

The diagrams D_0^F are listed below:

$$D_0^F(1) = \frac{4}{s_{15}\, s_{34}\, t_{125}} \Big\{ F(p_5,p_6)\cdot E(p_3,p_5) - F(p_5,p_3)\cdot E(p_6,p_5) + \big[F(p_6,p_3) + s_{34}\big]\cdot E(p_5,p_5) \Big\} ,$$

$$D_0^F(2) = \frac{-4}{s_{16}\, s_{25}\, s_{34}} \Big\{ \big[F(p_6,p_2) + \tfrac{s_{16}}{2}\big]\cdot E(p_3,p_5) + \big[F(p_2,p_3) + \tfrac{s_{34}}{2}\big]\cdot E(p_6,p_5) - F(p_6,p_3)\cdot E(p_2,p_5) \Big\} ,$$

$$D_0^F(3) = \frac{4}{s_{15}\, s_{36}\, t_{125}} \Big\{ F(p_5,p_6)\, E(p_3,p_5) - F(p_5,p_3)\cdot E(p_6,p_5) - \big[F(p_3,p_6) - \tfrac{s_{36}}{2} - \tfrac{s_{34}}{2} + \tfrac{s_{46}}{2}\big]\cdot E(p_5,p_5) \Big\} ,$$

$$D_0^F(4) = \frac{4}{s_{25}\, s_{34}\, t_{125}} \Big\{ F(p_2,p_3)\cdot E(p_5,p_5) - F(p_5,p_3)\cdot E(p_2,p_5) + \big[F(p_5,p_2) - \tfrac{s_{25}}{2} - \tfrac{s_{12}}{2} + \tfrac{s_{15}}{2}\big]\cdot E(p_3,p_5) \Big\} ,$$

$$D_0^F(5) = \frac{2}{s_{16}\, s_{25}\, t_{146}} \Big\{ \big[s_{35} - s_{23} + s_{25}\big]\cdot E(p_6,p_5) \Big\} ,$$

$$D_0^F(6) = \frac{2}{s_{25}\, s_{34}\, t_{134}} \left\{ \left[s_{56} - s_{26} - s_{25} \right] \cdot E(p_3, p_5) \right\} ,$$

$$\begin{aligned} D_0^F(7) = \frac{4}{s_{25}\, s_{36}\, t_{125}} \Big\{ & \left[F(p_5, p_2) - \frac{s_{25}}{2} - \frac{s_{12}}{2} + \frac{s_{15}}{2} \right] \cdot E(p_3, p_5) \\ & + \left[F(p_2, p_3) + \frac{t_{125}}{4} \right] E(p_5, p_5) \\ & - \left[F(p_5, p_3) + \frac{t_{125}}{4} \right] E(p_2, p_5) \Big\} , \end{aligned}$$

$$D_0^F(8) = \frac{1}{s_{14}\, s_{36}} \cdot E(p_3 - p_6, p_5) ,$$

$$D_0^F(9) = \frac{2}{s_{14}\, s_{36}\, t_{124}} \left\{ \left[s_{35} - s_{56} + s_{36} \right] \cdot E(p_2, p_5) \right\} ,$$

$$D_0^F(10) = \frac{2}{s_{14}\, s_{36}\, t_{145}} \left\{ \left[s_{23} - s_{26} - s_{36} \right] \cdot E(p_5, p_5) \right\} ,$$

$$\begin{aligned} D_0^F(11) = \frac{1}{2\, s_{14}\, s_{25}\, s_{36}} \Big\{ & \left[s_{23} + s_{35} - s_{26} - s_{56} \right] \cdot E(p_2 - p_5, p_5) \\ & - \left[s_{23} + s_{26} - s_{35} - s_{56} \right] \cdot E(p_3 - p_6, p_5) \\ & - \left[s_{23} + s_{56} - s_{35} - s_{26} \right] \cdot E(p_2 + p_5, p_5) \Big\} . \end{aligned} \tag{12}$$

The diagrams D_0^S are listed below:

$$D_o^s(1) = \frac{1}{s_{25}s_{36}t_{125}}\left[s_{34}-s_{46}+s_{36}\right]\cdot\left[s_{12}-s_{15}-s_{25}\right] ,$$

$$D_o^s(2) = \frac{1}{s_{14}s_{36}t_{124}}\left[s_{12}-s_{24}-s_{14}\right]\cdot\left[s_{35}-s_{56}+s_{36}\right] ,$$

$$D_o^s(3) = \frac{1}{s_{14}s_{36}t_{145}}\left[s_{15}-s_{45}+s_{14}\right]\cdot\left[s_{23}-s_{26}-s_{36}\right] ,$$

$$D_o^s(4) = \frac{1}{s_{15}s_{36}t_{125}}\left[s_{15}+s_{25}-s_{12}\right]\cdot\left[s_{34}-s_{46}+s_{36}\right] ,$$

$$D_o^s(5) = \frac{1}{s_{15}s_{34}t_{156}}\left[s_{56}-s_{15}-s_{16}\right]\cdot\left[s_{23}-s_{24}-s_{34}\right] ,$$

$$D_o^s(6) = \frac{1}{s_{15}s_{34}t_{125}}\left[s_{46}-s_{34}-s_{36}\right]\cdot\left[s_{12}-s_{25}-s_{15}\right] ,$$

$$D_o^s(7) = \frac{1}{s_{25}s_{34}t_{125}}\left[s_{36}-s_{46}+s_{34}\right]\cdot\left[s_{12}-s_{15}-s_{25}\right] ,$$

$$D_o^s(8) = \frac{1}{s_{16}s_{25}t_{146}}\left[s_{25}+s_{35}-s_{23}\right]\cdot\left[s_{14}-s_{46}+s_{16}\right] ,$$

$$D_o^s(9) = \frac{1}{s_{25}s_{34}t_{134}}\left[s_{14}+s_{34}-s_{13}\right]\cdot\left[s_{26}-s_{56}+s_{25}\right] ,$$

$$D_o^s(10) = \frac{1}{s_{25}\, s_{36}} \, (p_2 - p_5)(p_3 - p_6) \quad ,$$

$$D_o^s(11) = \frac{1}{s_{14}\, s_{36}} \, (p_1 - p_4)(p_3 - p_6) \quad ,$$

$$D_o^s(12) = \frac{1}{s_{16}\, s_{25}} \, (p_6 - p_1)(p_2 - p_5) \quad ,$$

$$D_o^s(13) = \frac{1}{s_{15}\, s_{34}} \, (p_5 - p_1)(p_3 - p_4) \quad ,$$

$$D_o^s(14) = \frac{1}{s_{25}\, s_{34}} \, (p_2 - p_5)(p_3 - p_4) \quad ,$$

$$\begin{aligned} D_o^s(15) = \frac{1}{s_{14}\, s_{25}\, s_{36}} \Big\{ & \big[(p_2+p_5)(p_3-p_6)\big]\cdot\big[(p_1-p_4)(p_2-p_5)\big] \\ & + \big[(p_2-p_5)(p_3-p_6)\big]\cdot\big[(p_1-p_4)(p_3+p_6)\big] \\ & + \big[(p_1+p_4)(p_2-p_5)\big]\big[(p_1-p_4)(p_3-p_6)\big] \Big\} \quad , \end{aligned}$$

$$\begin{aligned} D_o^s(16) = \frac{2}{s_{16}\, s_{34}\, s_{25}} \Big\{ & \big[(p_2-p_5)(p_3+p_4)\big]\cdot\big[(p_1-p_6)(p_3-p_4)\big] \\ & + \big[(p_1+p_6)(p_3-p_4)\big]\cdot\big[(p_1-p_6)(p_2-p_5)\big] \\ & + \big[(p_1-p_6)(p_2+p_5)\big]\cdot\big[(p_3-p_4)(p_2-p_5)\big] \Big\} \quad . \end{aligned} \tag{13}$$

The preceding list completes the result. Let us recapitulate now the numerical procedure of calculating the full cross section. First the diagrams D are calculated by using eqs. (11,12,13). The result is substituted to eq. (8) to obtain the vectors $\mathcal{D}_0$ and $\mathcal{D}_2$. After generating the vectors $\mathcal{D}_{0_\rho}$, $\mathcal{D}_{0_\sigma}$, $\mathcal{D}_{0_\tau}$, $\mathcal{D}_{2_\rho}$, $\mathcal{D}_{2_\sigma}$ and $\mathcal{D}_{2_\tau}$ by the appropriate permutations of momenta, eq. (6) is used to obtain the functions A_0 and A_2. Finally, the total cross section is calculated by using eq. (5). The FORTRAN5 program based on such a scheme generates one Monte-Carlo point in less than a second on the heterotic CDC CYBER 175/875.

Given the complexity of the final result, it is very important to have some reliable testing procedures available for numerical calculations. Usually in QCD, the multi-gluon amplitudes are tested by checking the gauge invariance. Due to the specifics of our calculation, the most powerful test does not rely on the gauge symmetry, but on the appropriate permutation symmetries. The function $A_0(p_1,p_2,p_3,p_4,p_5,p_6)$ must be symmetric under arbitrary permutations of the momenta (p_1,p_2,p_3) and separately, (p_4,p_5,p_6), whereas the function $A_2(p_1,p_2,p_3,p_4,p_5,p_6)$ must be symmetric under the permutations of (p_1,p_2,p_3,p_4) and separately, (p_5,p_6). This test is extremely powerful, because the required permutation symmetries are hidden in our supersymmetry relations, eqs. (1) and (3), and in the structure of amplitudes involving different species of particles. Another, very important test relies on the absence of the double poles of the form $(s_{ij})^{-2}$ in the cross section, as required by general arguments based on the helicity conservation. Further, in the leading $(s_{ij})^{-1}$ pole approximation, the answer should reduce to the two goes to three cross section [3,4], convoluted with the appropriate Altarelli-Parisi probabilities [5].

Our result has succesfully passed both these numerical checks.

Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

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REFERENCES

[1]E. Eichten, I. Hinchliffe, K. Lane and C. Quigg, Rev. Mod. Phys. 56 (1984) 579.

[2]Z. Kunszt, Nucl. Phys. B247 (1984) 339.

[3]S.J. Parke and T.R. Taylor, Phys. Lett. 157B (1985) 81.

[4]T. Gottschalk and D. Sivers, Phys. Rev. D21 (1980) 102; F.A. Berends, R. Kleiss, P. de Causmaecker, R. Gastmans and T.T. Wu, Phys. Lett. 103B (1981) 124.

[5]G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298.

Table I. Matrices K(I,J) [I=1-11,J=1-11]. Matrices K are symmetric.

Matrix $K^{(4)}$

8	4	-2	2	-1	2	0	1	0	0	-1
4	8	-1	1	-1	0	2	1	0	1	-1
-2	-1	8	4	4	1	1	2	2	1	2
2	1	4	8	2	-1	-1	4	1	1	1
-1	-1	4	2	8	1	2	4	-2	-1	4
2	0	1	-1	1	8	4	-1	0	1	0
0	2	1	-1	2	4	8	-2	0	0	0
1	1	2	4	4	-1	-2	8	-1	-1	2
0	0	2	1	-2	0	0	-1	8	4	-2
0	1	1	1	-1	1	0	-1	4	8	-1
-1	-1	2	1	4	0	0	2	-2	-1	8

Matrix $K^{(2)}$

0	0	0	0	0	0	0	0	3	3	-3
0	0	0	0	0	0	0	0	3	3	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	3	3	0
0	0	0	0	0	0	0	0	3	3	0
0	0	0	0	0	0	0	0	0	0	0
3	3	0	0	0	3	3	0	0	0	0
3	3	0	0	0	3	3	0	0	0	0
-3	0	0	0	0	0	0	0	0	0	0

Table I continued

Matrix $K_\rho^{(4)}$

0	0	0	0	1	1	0	1	1	0	-1
0	0	0	0	2	0	1	1	2	1	-2
0	0	0	0	0	1	1	1	0	1	1
0	0	0	0	1	0	0	2	0	1	0
1	2	0	1	0	1	2	2	0	0	2
1	0	1	0	1	4	2	0	0	0	-1
0	1	1	0	2	2	4	0	0	0	-2
1	1	1	2	2	0	0	4	0	0	0
1	2	0	0	0	0	0	0	0	2	-1
0	1	1	1	0	0	0	0	2	4	0
-1	-2	1	0	2	-1	-2	0	-1	0	4

Matrix $K_\rho^{(2)}$

3	3	0	3	0	0	0	3	0	0	0
3	3	0	0	0	0	0	0	0	0	0
0	0	3	0	0	3	0	0	3	3	0
3	0	0	0	0	3	0	0	0	0	0
0	0	0	0	0	0	0	0	3	3	0
0	0	3	3	0	0	0	0	3	3	0
0	0	0	0	0	0	0	0	0	3	0
3	0	0	0	0	0	0	0	3	0	0
0	0	3	0	3	3	0	3	0	0	0
0	0	3	0	3	3	3	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

Table I continued

Matrix $K_{\sigma}^{(4)}$

4	2	0	2	0	1	0	1	0	0	0
2	4	0	1	0	0	1	1	0	1	0
0	0	4	2	2	1	1	1	2	1	0
2	1	2	0	1	2	1	0	1	0	0
0	0	2	1	0	0	0	0	4	2	2
1	0	1	2	0	0	0	0	1	2	0
0	1	1	1	0	0	0	0	2	4	0
1	1	1	0	0	0	0	0	2	0	1
0	0	2	1	4	1	2	2	0	0	-4
0	1	1	0	2	2	4	0	0	0	-2
0	0	0	0	2	0	0	1	-4	-2	4

Matrix $K_{\sigma}^{(2)}$

0	0	0	0	0	0	0	0	3	0	-3
0	0	0	0	3	0	0	3	0	0	0
0	0	0	0	0	0	3	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	3	0	0	0	0	3	0	0	0	0
0	0	0	0	0	3	3	0	0	0	-3
0	0	3	0	3	3	3	0	0	0	0
0	3	0	0	0	0	0	3	0	0	0
3	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	3	0
-3	0	0	0	0	-3	0	0	0	0	0

Table I continued

Matrix $K_\tau^{(4)}$

0	1	−1	−1	1	1	0	1	2	0	0
1	0	−2	−1	2	0	1	1	4	2	0
−1	−2	0	0	0	1	1	1	−1	1	0
−1	−1	0	1	0	2	1	0	1	−1	0
1	2	0	0	1	−1	−1	0	−2	2	1
1	0	1	2	−1	0	1	−2	2	4	−1
0	1	1	1	−1	1	0	−1	4	8	−1
1	1	1	0	0	−2	−1	0	2	−2	0
2	4	−1	1	−2	2	4	2	1	0	−2
0	2	1	−1	2	4	8	−2	0	0	0
0	0	0	0	1	−1	−1	0	−2	0	2

Matrix $K_\tau^{(2)}$

3	3	0	0	0	3	3	0	0	0	0
3	3	0	0	0	3	3	0	0	0	0
0	0	3	3	3	0	0	3	0	0	0
0	0	3	3	3	0	0	3	0	0	3
0	0	3	3	3	0	0	3	0	0	0
3	3	0	0	0	3	3	0	0	0	−3
3	3	0	0	0	3	3	0	0	0	0
0	0	3	3	3	0	0	3	0	0	0
0	0	0	0	0	0	0	0	3	3	0
0	0	0	0	0	0	0	0	3	3	0
0	0	0	3	0	−3	0	0	0	0	0

Table II. Matrices $C^G(I,J)$ [I=1-11,J=1-33], $C^F(I,J)$ [I=1-11,J=1-11] and $C^S(I,J)$ [I=1-11,J=1-16]. Indices I and J specify row numbers and column numbers, respectively.

Matrix C^G

1	0	0	0	0	0	0	0	0	0	1	0	1	0	1	1	0	1	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	0	0	0	0	0	0	0	0	0	1	0	-1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	-1	0	0	-1	0	1	0	0	1	1	0	0	0
0	-1	-1	-1	1	1	-1	0	0	1	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	1	1	0	0	0	1	0	0
0	-1	-1	1	-1	1	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	1	0	0	1	0
-1	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	1	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	-1	-1	1	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	2	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table II continued

Matrix C^F

$$
\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & -1 \\
0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 1 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}
$$

Matrix C^S

$$
\begin{matrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 2 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 & 0 & 1 & -2 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{matrix}
$$