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Top Quark Physics at a Polarized Muon Collider [1]

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Abstract. Top quark pair production is presented at a polarized Muon Collider above the threshold region. The off-diagonal spin basis is the natural basis for this discussion as the top quark pairs are produced in an essentially unique spin configuration for 100% polarization. Modest polarization, say 30%, can lead to 90% of all top quark pair events being in one spin configuration. This will lead to sensitive tests on anomalous top quark couplings.

Recently Parke and Shadmi [1] have shown that at a 100% polarized lepton collider that the top and anti-top quark pairs are produced in essentially a unique spin configuration. This spin basis has been called the "off-diagonal" basis and it interpolates between the beam direction at threshold and the top quark direction, i.e. helicity, far above threshold. The differential cross section using this basis is given by

$$\frac{d\sigma}{d\cos\theta^*}\ (\mu_L^-\ \mu_R^+ \to t_\uparrow\ \bar{t}_\uparrow\ or\ t_\downarrow\ \bar{t}_\downarrow) = 0\ ,$$

$$\frac{d\sigma}{d\cos\theta^*}\ (\mu_L^-\ \mu_R^+ \to t_\uparrow\ \bar{t}_\downarrow\ or\ t_\downarrow\ \bar{t}_\uparrow) = \Big(\ \frac{3\pi\alpha^2}{8s}\beta\ \Big)$$

$$\Bigg[\ f_{LL}(1+\beta\cos\theta^*) + f_{LR}(1-\beta\cos\theta^*)$$

$$\pm\sqrt{(f_{LL}(1+\beta\cos\theta^*) - f_{LR}(1-\beta\cos\theta^*))^2 + 4f_{LL}f_{LR}(1-\beta^2)}\ \Bigg]^2 \qquad (1)$$

where the f_{IJ}'s are the sum of the photon and Z-boson couplings corrected for the difference in the propogators. Details of this basis can be found in

[1] Presented at the workshop on "Physics at the First Muon Collider," November 6-7, 1997 at Fermilab, Batavia, IL and at the "Fourth International Conference on the Physics Potential and Development of Muon Collider," Dec 10-12, 1997, San Francisco, CA.

reference [1]. Figure 1 is the spin components for top quark pair production in the off-diagonal basis for both LR and RL incoming lepton helicities for a $\sqrt{s} = 400\ GeV$ collider. The sub-leading terms have been amplified by a factor of 100 so it is clear that the dominant configuration makes up more than 99% of the total cross section.

In the helicity basis the differential cross section is [2]

$$\frac{d\sigma}{d\cos\theta^*}\ (\mu_L^-\ \mu_R^+ \to t_L\ \bar{t}_L\ or\ t_R\ \bar{t}_R) = \left(\ \frac{3\pi\alpha^2}{8s}\beta\ \right)\ \ (1-\beta^2)\sin^2\theta^*\ |f_{LL}+f_{LR}|^2\ ,$$

$$\frac{d\sigma}{d\cos\theta^*}\ (e_L^-\ e_R^+ \to t_R\ \bar{t}_L\ or\ t_L\ \bar{t}_R) = \left(\ \frac{3\pi\alpha^2}{8s}\beta\ \right)\ \ (1\mp\cos\theta^*)^2$$
$$\times|f_{LL}(1\mp\beta)+f_{LR}(1\pm\beta)|^2\ . \qquad (2)$$

Figure 2 is the corresponding plot for the helicity basis. Here the dominant spin configuration is less than 60% of the total.

In figure 3 I have plotted the dominant spin component's fraction of the total as function of the polarization of the beams,

$$\frac{(1+P^-)(1-P^+)\sigma_{LR\to UD}\ +\ (1-P^-)(1+P^+)\sigma_{RL\to UD}}{(1+P^-)(1-P^+)\sigma_{LR}^{tot}\ +\ (1-P^-)(1+P^+)\sigma_{RL}^{tot}} \qquad (3)$$

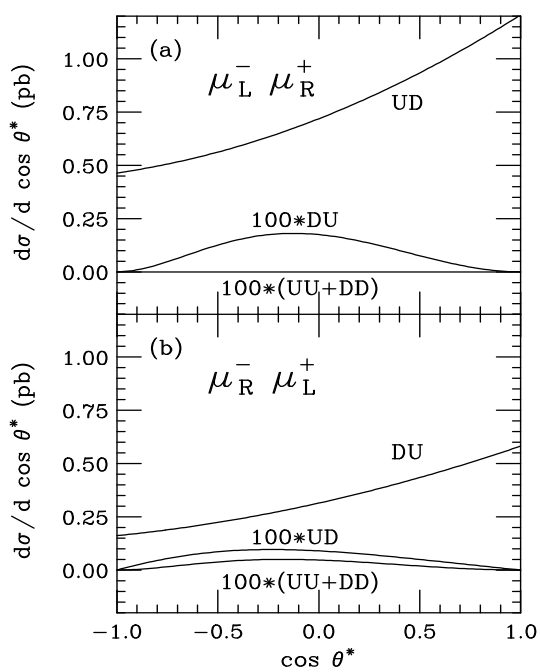

FIGURE 1. The spin configurations using the off-diagonal basis for both $\mu_L^+\ \mu_R^-$ and $\mu_R^+\ \mu_L^-$ for $\sqrt{s} = 400\ GeV$. Note that the sub-leading configurations have been amplified by a factor of 100 in these figures.

for the off-diagonal basis. Here $P \equiv (N_L - N_R)/(N_L + N_R)$ for the μ^+ and μ^- beams. Two different machines are included, the Muon Collider and the NLC. The Muon Collider is assumed to have equal but opposite polarization for the μ^+ and μ^- beams whereas the NLC is an electron-positron collider with only the electron beam polarized. From these curves a modest amount of polarization, say 30%, at a Muon Collider can make the dominant spin configuration close to 90% of the total. Whereas at an electron-positron machine one requires 55% polarization to achieve the same goal.

Figure 4 is the corresponding plot for the helicity configuration. If one uses this spin basis the dominant spin configuration ranges from 41% to 52% of the total. Clearly polarization is not as important here without further cuts.

Since the top-quark pairs are produced in a nearly unique spin configuration, and the electroweak decay products of polarized top-quarks are strongly correlated to the spin axis, the top-quark events at $\mu^+ \mu^-$ collider have a very distinctive topology. Deviations from this topology would signal anomalous couplings. In the Standard Model, the predominant decay mode of the top-quark is $t \to bW^+$, with the W^+ decaying either hadronically or leptonically. For definiteness we consider here the decay $t \to bW^+ \to be^+\nu$. The differential decay width of a polarized top-quark depends non-trivially on three angles. The first is the angle, χ_w^t, between the top-quark spin and the direction of motion of the W-boson in the top-quark rest-frame. Next is the angle between the direction of motion of the b-quark and the positron in the W-boson

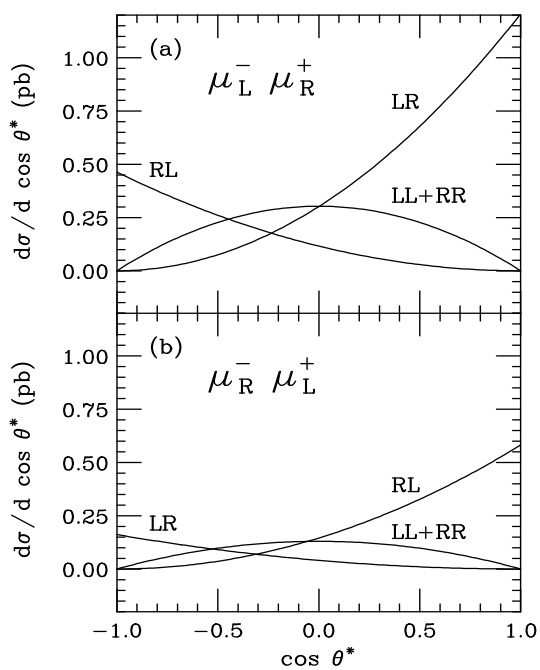

FIGURE 2. The spin configurations using the helicity basis for a $\sqrt{s} = 400\ GeV$ collider.

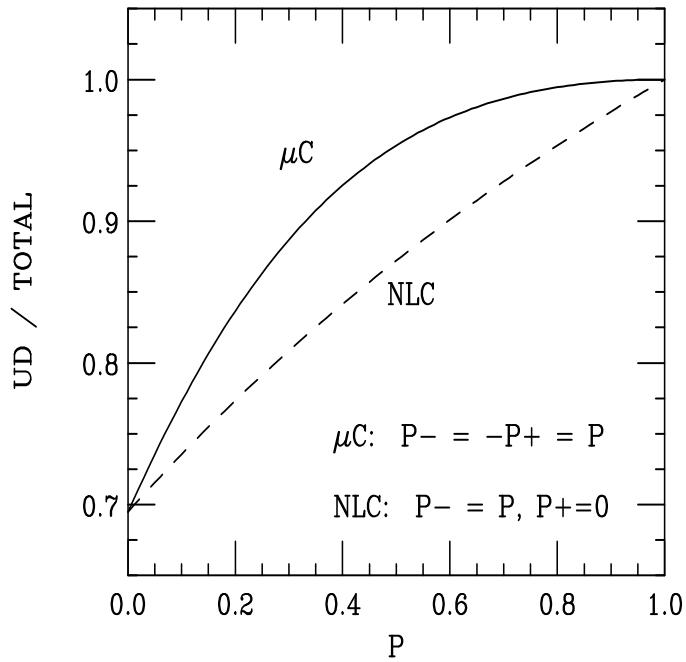

FIGURE 3. Fraction of the total cross section in the off-diagonal basis' Up-Down spin configuration as a function of the polarization. Both beams are assumed to be polarized for the Muon Collider (μC) but only one beam for the NLC.

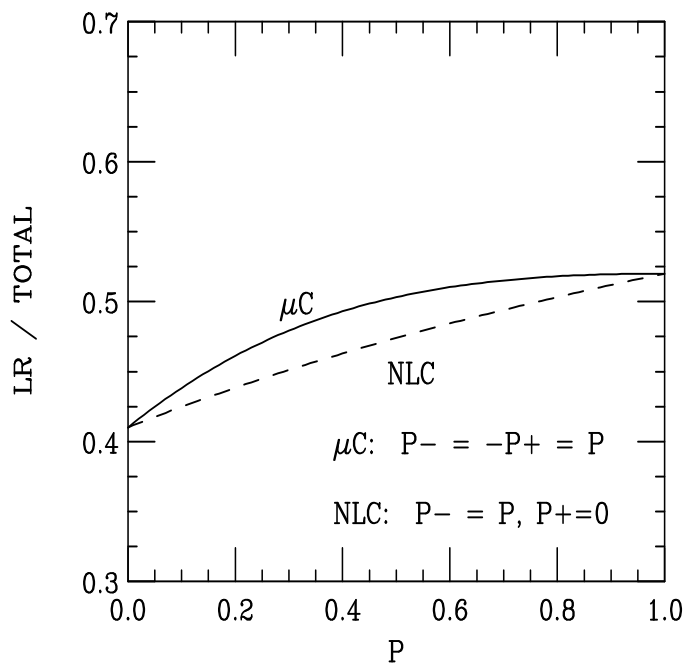

FIGURE 4. Same as Fig. 3 but the helicity basis is used.

rest-frame. We call this angle $\pi - \chi_e^w$. Finally, in the top-quark rest-frame, we have the azimuthal angle, Φ, between the positron direction of motion and the top-quark spin around the direction of motion of the W-boson.

The differential polarized top-quark decay distribution in terms of these three angles is given by

$$\frac{1}{\Gamma_T}\,\frac{d^3\,\Gamma}{d\cos\chi_w^t\;d\cos\chi_e^w\;d\Phi} = \frac{3}{8\;(m_t^2+2m_W^2)}$$
$$\Big[m_t^2(1+\cos\chi_w^t)\sin^2\chi_e^w + m_W^2(1-\cos\chi_w^t)(1-\cos\chi_e^w)^2$$
$$+\,2m_t m_W(1-\cos\chi_e^w)\sin\chi_e^w\sin\chi_w^t\cos\Phi\Big]\;, \qquad (4)$$

where m_t is the top-quark mass, m_W is the W mass, and Γ_T is the total decay width (we neglect the b-quark mass). The first and second terms in (4) give the contributions of longitudinal and transverse W-bosons respectively. The interference term, given by the third term in (4), does not contribute to the total width, but its effects on the angular distribution of the top-quark decay products are sizable. Fig. 5 shows contour plots of the differential angular decay distribution in the $\chi_e^w - \chi_w^t$ plane after integrating over the azimuthal angle Φ. The peak at the center of the right hand side of this figure is due to

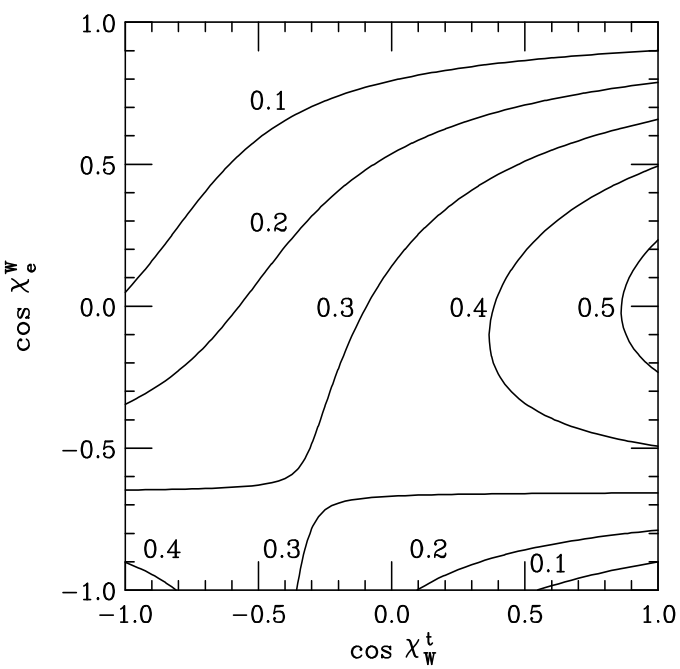

FIGURE 5. Correlations between the W-boson and the top spin direction in the top rest frame ($\cos\chi_W^t$) and the positron and the minus b-quark direction in the W-boson rest frame ($\cos\chi_e^W$).

the longitudinal W-bosons whereas the peak at the bottom left hand corner is caused by the transverse W-bosons.

There are also significant correlations of the angle between the top-quark spin and the momentum of the i-th decay product, χ_i^t, measured in the top-quark rest-frame, see figure 6. The differential decay rate of the top-quark is given by

$$\frac{1}{\Gamma_T}\,\frac{d\,\Gamma}{d\cos\chi_i^t} = \frac{1}{2}\left[1+\alpha_i\cos\chi_i^t\right]\,, \tag{5}$$

where $\alpha_b = -\alpha_W = -0.41$, $\alpha_\nu = -0.31$ and $\alpha_{e^+} = 1$, for $m_t = 175$ GeV, see ref. [3]. The interference between the longitudinal and transverse W-bosons is very importnat in determining these correlations. Note, the positron is more highly correlated with the spin of the top quark than its parent the W-boson!

Given that we know the spin configuration of the top quark pairs and the correlations of the top quark decay products there are many correlations studies that can be performed in top quark pair product at a lepton collider looking for anomalous couplings of the top quarks. These studies have been performed for the helicity basis [4] but need to be redone using the superior off-diagonal spin basis.

QCD effects modify this picture in only a minor fashion. The reason being that soft gluons cannot flip the spin of the heavy top quarks. Detailed studies of the effects of one loop calculations show that the dominant spin

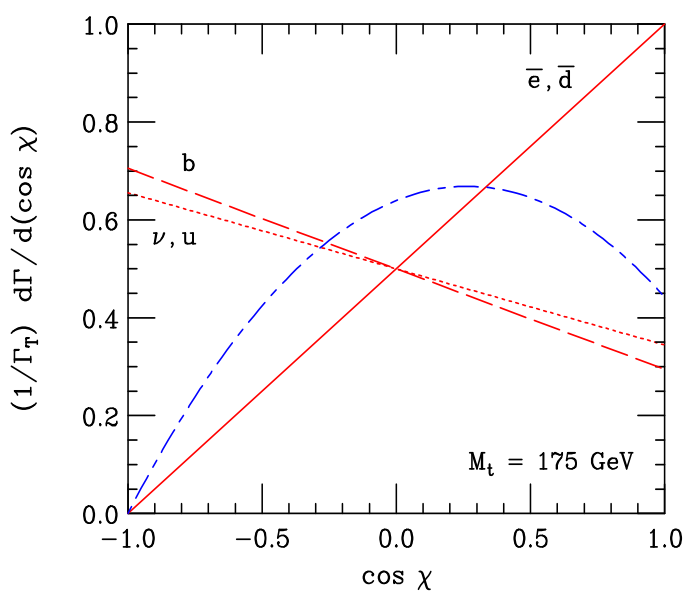

FIGURE 6. The straight lines are the correlations between the top quark decay products and the top quark spin direction in the rest frame of the top ($\cos\chi_i^t$). Whereas the curved line is the correlation between the b-quark and the positron or d-quark in the rest frame of the W-boson ($-\cos\chi_e^W$).

configuration is still more than 99% of the total even when QCD corrections are included [5].

In conclusion top quark pairs above the threshold region at Muon Colliders are a great place to search for anomalous couplings of the top quark. For $\sqrt{s} < 1\,TeV$ the off-diagonal basis is superior to the helicity basis in describing the events in the simplest possible terms. Polarization of the incoming beams enhances this effect. Detailed studies of the one loop QCD corrections have been recently completed, showing no qualitative difference than the tree level analysis. An extensive analysis of the sensitivity to anomalous couplings of the top quark, using the off-diagonal basis, is now needed.

ACKNOWLEDGMENTS

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