

Top Polarization at a $\mu^+\mu^-$ Collider¹

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Abstract. The top quark pairs produced at a polarized muon collider are in a (nearly) pure spin configuration. This result holds for all center-of-mass energies, and is insensitive to the next-to-leading order QCD radiative corrections. The decay products of a polarized top quark show strong angular correlations. We describe an interesting interference effect between the left-handed and longitudinally polarized W bosons in top quark decay. This effect is easily observable in the angular distribution of the charged lepton with respect to the beam axis.

With a mass of 173.8 ± 5.2 GeV [], the top quark is by far the heaviest fermion in the Standard Model (SM). Within the SM, the top quark decays very rapidly to a W boson and a b quark. Since the t decay width ($\Gamma_t \sim 1.5$ GeV) is much greater than the spin decorrelation scale ($\Lambda_{\text{QCD}}^2/m_t \sim 230$ keV), a polarized t quark decays before QCD can randomize its spin []. Thus, the decay products of polarized t quarks exhibit strong angular correlations.

Because the focus of this talk is on a muon collider environment, we should begin with a few words about the Higgs boson. Although much has been said about the usefulness of a muon collider as a “Higgs factory” for low-mass Higgs bosons, from the point of view of studying $t\bar{t}$ pair production, the Higgs boson does not play an important role. A light SM Higgs boson shifts the total $t\bar{t}$ production cross section at threshold by only a few percent []. This effect, which is comparable in size to the (incompletely known) 2-loop QCD corrections, becomes even smaller as the machine energy is increased beyond the threshold region. The other case to consider is a Higgs which is heavy enough for the on-shell decay $h \rightarrow t\bar{t}$ to occur, allowing for resonant $t\bar{t}$ production. Assuming SM couplings, a 400 GeV Higgs boson contributes only a fraction of a percent to the total cross section through this channel. Consequently, as far as top quark pair production is concerned, the same analysis applies equally to e^+e^- and $\mu^+\mu^-$ colliders.

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The production of top quark pairs at a muon collider proceeds through the s -channel via an off-shell photon or Z boson. For the purposes of our initial discussion, we will assume that the μ^- beam is 100% polarized in a left-handed helicity state and that the μ^+ beam is 100% polarized in a right-handed helicity state. Details concerning the more general case with arbitrary polarizations of the two beams may be found in Refs. []. In the situation at hand, the matrix element for the production of a $t\bar{t}$ pair whose spins are labelled by λ and $\bar{\lambda}$ takes the form

$$\mathcal{M} \sim \frac{e^2}{s} \left[\bar{v}(\bar{\mu}) \gamma^\mu P_L u(\mu) \right] \left[\bar{u}(t, \lambda) \gamma^\mu \{ f_{LL} P_L + f_{LR} P_R \} v(\bar{t}, \bar{\lambda}) \right], \quad (1)$$

where we have defined the chirality projection operators $P_L \equiv \frac{1}{2}(1-\gamma_5)$ and $P_R \equiv \frac{1}{2}(1+\gamma_5)$. The effective couplings f_{LL} and f_{LR} incorporate the different center-of-mass energy dependence of the photon and Z boson propagators: consequently they are weakly dependent on \sqrt{s} . The ratio f_{LR}/f_{LL} ranges from 0.343 near threshold to 0.365 in the ultra-relativistic limit. Note that since the muon is effectively massless at the center-of-mass energies we are considering, the helicity of the muons is linked to their chirality in the usual fashion. On the other hand, both chiralities contribute to the spin eigenstates of the t quark, even if we choose to work in the helicity basis. Nevertheless, Parke and Shadmi [] have constructed a spin basis, called the *off-diagonal basis*, in which more than 99% of the $t\bar{t}$ pairs are produced in the UD spin state at $\sqrt{s} = 400$ GeV.

In order to motivate the off-diagonal basis, let us imagine that we can turn off the f_{LR} coupling. Then, it is easy to see what happens in the two energy extremes (Fig. 1). At ultrarelativistic energies, the left-handed chirality of the produced $t\bar{t}$ pair translates into left-handed helicity for the t and right-handed helicity for the \bar{t} : we obtain 100% $t_L\bar{t}_R$. Near threshold, however, where there is no orbital angular momentum involved, the t and \bar{t} spins must be parallel to the beam axis, since the initial μ^- and μ^+ spins add up to one unit along that direction. At intermediate energies, it is not surprising to learn that in the lab frame the direction of the t and \bar{t} spins is not entirely along either the beam axis or the $t\bar{t}$ production axis. Instead, when viewed in the t rest frame, the t spin is parallel to the μ^+ momentum. In the \bar{t} rest frame, the \bar{t} spin is antiparallel to the μ^- momentum. These associations (t with μ^+ and \bar{t} with μ^-) may be understood by Fierzing the first term of Eq. (1). Thus, in this basis, the $t\bar{t}$ spin state is 100% UD in the limit $f_{LR} \rightarrow 0$.

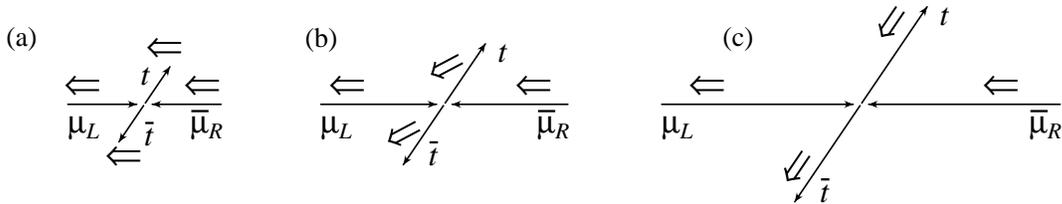


FIGURE 1. The process $\mu_L^- \mu_R^+ \rightarrow t\bar{t}$ in the $f_{LR} \rightarrow 0$ limit (a) near threshold, (b) at intermediate energies, and (c) at ultra-relativistic energies.

Of course, we cannot set $f_{\text{LR}} = 0$ in the real world. However, because $f_{\text{LR}}/f_{\text{LL}}$ is not very large, it is possible to tweak the spin axis choice a bit and retain the overwhelming dominance of the UD spin configuration. The result is the off-diagonal basis of Parke and Shadmi []. In the off-diagonal basis, the UU+DD spin state vanishes identically for $\mu_L^- \mu_R^+$ collisions. The DU spin state is suppressed by $\beta^4(f_{\text{LR}}/f_{\text{LL}})^2$. Since the speed of the produced top quarks is only $\beta \sim 0.5$ at $\sqrt{s} = 400$ GeV, this is a significant suppression, and $\mu_L^- \mu_R^+ \rightarrow t_U \bar{t}_D$ accounts for 99.88% of the tree-level cross section. Fig. 2 shows the contributions to the $t\bar{t}$ cross section as a function of the $t\bar{t}$ production angle in the lab frame, decomposed into the various $t\bar{t}$ spin states for a 400 GeV collider.

All of the preceding discussion has been at tree level. Because the resulting spin state is so pure, we might worry that the next-to-leading-order QCD corrections might spoil this result, especially since the total (unpolarized) cross section grows from 0.63 pb to 0.80 pb at $\sqrt{s} = 400$ GeV, an enhancement of more than 25%. The spin dependence of the NLO QCD corrections has been studied by Kodaira, Nasuno and Parke [], who find that the off-diagonal basis defined at tree level is still the appropriate basis, and that the purity of the $t_U \bar{t}_D$ state produced in $\mu_L^- \mu_R^+$ collisions is still 99.85%. The physics behind this result is easily understood. The large enhancement in the total cross section is mostly attributable to the production of soft gluons. However, soft gluons are incapable of causing spin flips of the top quark. In the soft gluon limit, matrix elements with an extra gluon factorize into matrix elements without the extra gluon times an eikonal factor []. This factorization occurs independently and with the *same* eikonal factor for each spin configuration. Thus, the spin composition of the produced $t\bar{t}$ pairs is altered only slightly by the inclusion of NLO QCD effects.

The decay products of a polarized top quark are strongly correlated with the top quark spin axis. These correlations are most simply viewed in the t rest frame, where the dependence upon the angle χ_i^t between the i th decay product and the t spin axis is of the form

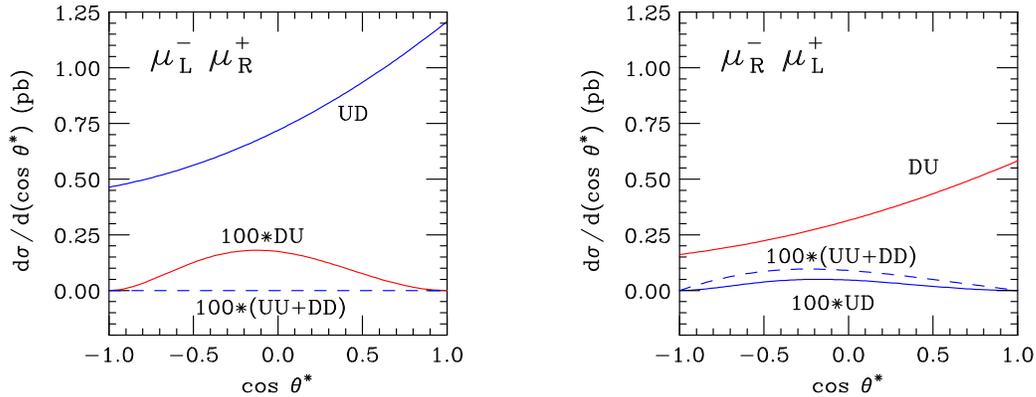


FIGURE 2. Top pair spin configurations as a function of the lab frame production angle using the off-diagonal basis at $\sqrt{s} = 400$ GeV. Note that the sub-dominant cross sections have been multiplied by 100 to make them visible.

$$\frac{1}{\Gamma_T} \frac{d\Gamma}{d(\cos \chi_i^t)} = \frac{1}{2}(1 + \alpha_i \cos \chi_i^t) \quad (2)$$

for a spin up top quark [] (see Fig. 3). The charged lepton or d -type quark from the decaying W boson displays maximal correlation with the top quark spin. The values of the α_i 's for the other decay products depend on the top and W masses (see Ref. [] for a convenient tabulation).

The spin states of the W bosons coming from a spin up top quark decay display interesting interference effects. The simplest description utilizes the W helicity measured in the t rest frame. Then, in the decay of a spin up top quark, the W 's are produced with left-handed or longitudinal helicity only. The ratio of left-handed to longitudinal W 's is $2m_W^2:m_t^2$ ($= 30\%:70\%$), and is reflected in the decay angular distribution

$$\frac{1}{\Gamma_T} \frac{d\Gamma}{d(\cos \chi_\ell^W)} = \frac{3}{4} \frac{m_W^2(1 - \cos \chi_\ell^W)^2 + m_t^2 \sin^2 \chi_\ell^W}{2m_W^2 + m_t^2}. \quad (3)$$

In Eq. (3), we have denoted W rest frame angle between the b quark and charged lepton by $\pi - \chi_\ell^W$.

In Fig. 4 we present a contour plot of the lepton emission angle in the W rest frame versus the W emission angle in the t rest frame. From this plot, we see that the longitudinal W 's are emitted mainly parallel to the top quark spin, while the left-handed W 's are emitted mainly antiparallel to the top quark spin. Near $\cos \chi_W^t = 0$, interference between the left-handed and longitudinal W 's dominates.

The interference between the left-handed and longitudinal W bosons from t quark decay is the explanation for an apparent mystery contained in Fig. 3. Recall that the W bosons are only moderately correlated with the top quark spin: $\alpha_W = 0.40$.

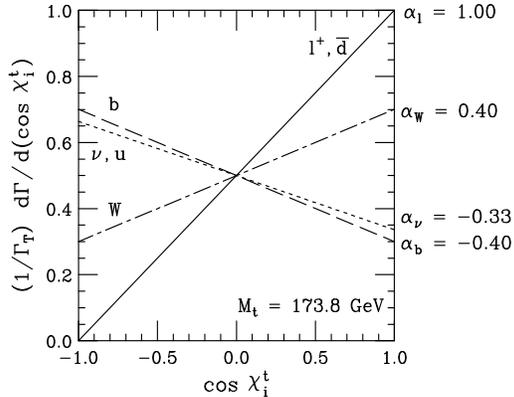


FIGURE 3. Angular correlations in the decay of a spin up top quark. The lines labeled W , b , ℓ^+ , \bar{d} , ν , and u are the angle between the spin axis and the particle in the rest frame of the top quark.

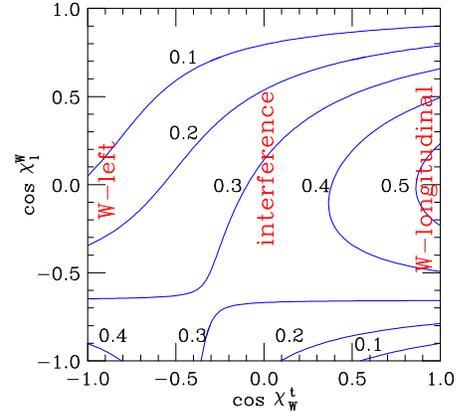


FIGURE 4. Contours of the top quark decay distribution in the $\cos \chi_W^t - \cos \chi_\ell^W$ plane. W bosons emitted in the forward direction are primarily longitudinal, whereas backward-emitted W 's are mostly left-handed.

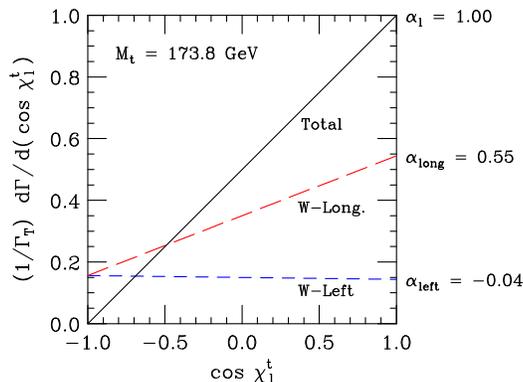


FIGURE 5. Interference effects between the left-handed and longitudinal W bosons from the decay of a spin up top quark. Plotted is the angle between the charged lepton and the t spin axis, broken down into decays through left-handed and longitudinal W bosons. The solid line represents the quantum mechanical sum of the two contributions.

However, one of the W decay products (the charged lepton or d -type quark) is maximally correlated with the top quark spin: $\alpha_\ell = 1!$ In Fig. 5, we show the angular distribution of the charged leptons coming from left-handed and longitudinal W decays separately,² for the left-handed as well as their quantum mechanical sum, including interference effects. We see that at $\cos \chi_\ell^t = -1$ there is total destructive interference. For $\cos \chi_\ell^t > 0$, the interference is, on average, constructive. Thus, the interference term allows the charged lepton or d -type quark to be more correlated with the top quark spin than the parent W .

This interference effect also leaves a visible imprint in the lab frame. In Fig. 6 we show the distribution of the angle between the charged lepton and the direction of the μ^- beam, again broken down into the contributions from left-handed and longitudinal W bosons. The effect is particularly striking for $\mu_L^- \mu_R^+$ collisions, as the shape of the quantum mechanical sum of the two contributions is completely dominated by presence of the interference term. The dotted line in Fig. 6 shows the top quark production angle. Even though the t quark is predominantly produced in the forward direction in $\mu_L^- \mu_R^+$ collisions, the charged lepton from its decay tends to move backward in the lab frame. This is a consequence of the interference between the left-handed and longitudinal W bosons.

²⁾ The α_i 's of Eq. (2) for the left-handed and longitudinal W bosons are given by

$$\alpha_{\text{left}} = -\frac{\xi^3 + 8\xi^2 - 4\xi + 1}{(\xi - 1)^3} + \frac{6\xi^3 \ln \xi}{(\xi - 1)^4}$$

and

$$\alpha_{\text{long}} = \frac{(\xi + 1)(\xi^2 - 8\xi + 1)}{(\xi - 1)^3} + \frac{12\xi^2 \ln \xi}{(\xi - 1)^4}$$

where $\xi \equiv (m_t/m_W)^2$. For $m_t = 173.8$ GeV and $m_W = 80.4$ GeV these expressions yield $\alpha_{\text{left}} = -0.041$ and $\alpha_{\text{long}} = 0.55$.

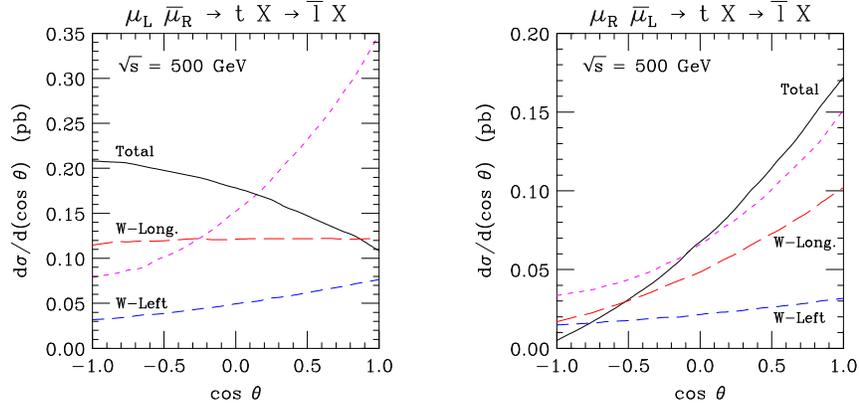


FIGURE 6. Lab frame charged lepton angular distributions which are affected by the interference effects between the left-handed and longitudinal W bosons. Plotted is the angle between the charged lepton and the μ^- beam direction in the lab frame for the two W -boson helicity states (dashed lines) as well as their quantum-mechanical sum (solid line). The dotted line indicates the top quark production angular distribution.

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