united nations educational, scientific and cultural organization

international atomic energy agency

the
abdus salam
international centre for theoretical physics

SMR.1508 - 14

SUMMER SCHOOL ON PARTICLE PHYSICS

16 June - 4 July 2003

NEUTRINO PHYSICS

Lectures IV & V

S. PARKE
Fermilab
Batavia, IL
U.S.A.

References:

Past ICTP Summer School Lectures:

Lisi — 2001 (ICTP web)

Akhmedov — 1999 hep-ph/0001264

Pakvasa — 1997 hep-ph/9804426

+ references therein

Recent Reviews

Pakvasa + Valle hep-ph/0301601

Gonzalez - Garcia + Nir hep-ph/0202058

MSW + Solar ν's

Smirnov — hep-ph/0305106

Parke — SLAC Summer School 1986
Fermilab-Conf-86-131-T
available on SPIRES
click Fermilab-Library-Server:

little known

Kamiland:

Expected: 87 ± 6

Observed: 54

Prob $< 0.05\%$

$$\frac{\text{Diff:}}{\sqrt{\text{Exp}}} = \frac{87-54}{\sqrt{87}} = 3.\cancel{5} = \frac{33}{9.3}$$

K2K:

Expected: 80 ± 6

Observed: 56

Prob: $\sim 1.3\%$

$$\frac{\text{Diff:}}{\sqrt{\text{Exp}}} = \frac{80-56}{\sqrt{80}} = \frac{24}{8.9} = 2.7$$

Redo with Poisson Statistics?

Problem:

For 3 flavors:

SHOW

$$2 \sum_{i>j} \mathrm{Im}\left(U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right) \sin 2\Delta_{ij}$$

$$= \pm \sin\delta \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13}$$
$$\sin\Delta_{21} \sin\Delta_{32} \sin\Delta_{31}$$

where $\Delta_{ij} = \frac{\delta m^2_{ij} L}{4E}$

SAME for all $\alpha \neq \beta$ (if $\alpha \neq \beta$)

$-$ or $+$ for $\mu \to e$, $\tau \to \mu$, $e \to \tau$
$+$ or $-$ for $e \to \mu$, $\mu \to \tau$, $\tau \to e$

Hint: $\Delta_{31} - \Delta_{32} - \Delta_{21} \equiv 0$

θ_{13} and Beyond

<u>Reactors</u>: $\bar{\nu}_e \to \bar{\nu}_e$ dissappearance

a la Chooz (KamLAND etc)

$$P_{\nu_e \to \nu_e} \cong 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{31}$$

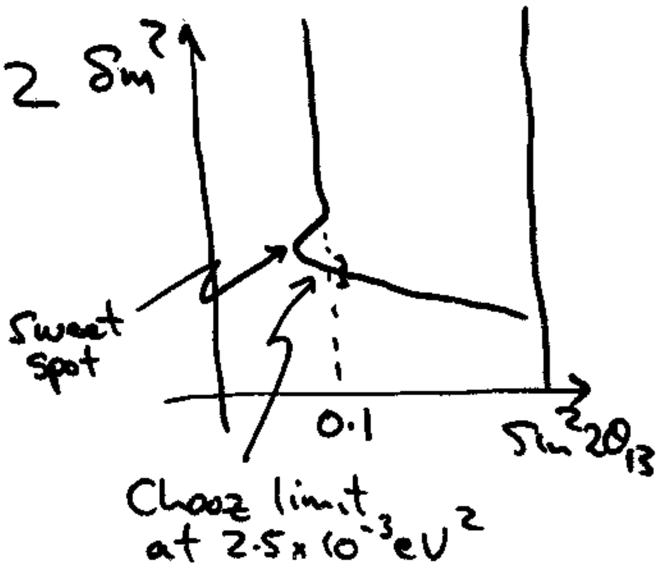
$$\Delta_{31} = \frac{\delta m^2_{31} L}{4E}$$

Currently from Chooz

$\sin^2 2\theta_{13} < 0.05$ at any δm^2

at $2.5 \times 10^{-3}\,eV^2$: $\sin^2 2\theta_{13} < 0.01$
SK δm^2_{atm}

Could get a factor of 2
by working at a better
distance



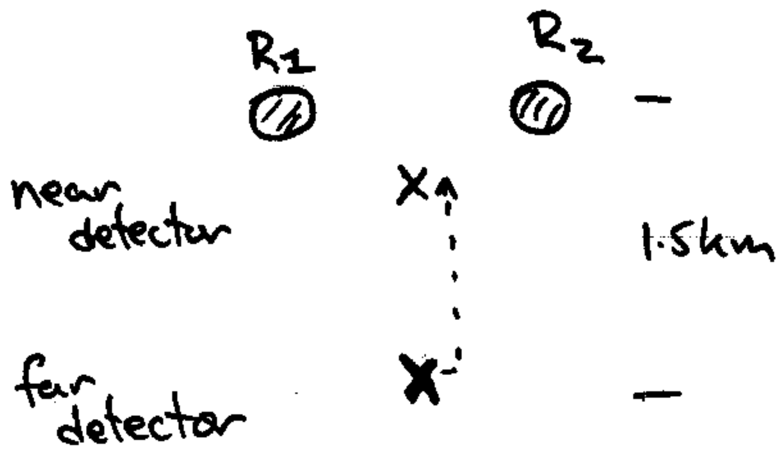
Oscillation Peak (or dip in dissappearance) occurs at

$$L = \frac{4\pi E}{2\,\delta m^2_{atm}} = \frac{\pi E}{2(1.27)\,\delta m^2_{atm}} = \frac{\pi \; 3\times10^{-3}}{2.54 \times 2.5\;10^{-3}}$$

$$\simeq 1.5 \text{ km}$$

Also reduce systematic errors

$\longrightarrow 1\%$



Differential Total Flux measurement

maybe able to get to $\sin^2 2\theta_{13} \sim 0.01$ Shape Lindner et al. '03

- to do better than Chooz, relatively easy
- to get to 0.01 HARD, DIFFERCULT, ...

$$P_{\nu_e\to\nu_e} = \Big| U_{e3}U_{e3}^* e^{-im_3L/2E} + U_{e2}U_{e2}^* e^{-im_2L/2E} + U_{e1}U_{e1}^* e^{-im_1L/2E} \Big|^2$$

$$U_{e1}U_{e1}^* = 1 - U_{e2}U_{e2}^* - U_{e3}U_{e3}^* \quad \text{unitarity}$$

$$P_{\nu_e\to\nu_e} = \Big|1 - 2i\, s_{13}^2 e^{-i\Delta_{31}} \sin\Delta_{31} - 2i\, c_{13}^2 s_{12}^2 e^{-i\Delta_{21}} \sin\Delta_{21}$$

square

$$\left| e^{-i\Delta_{32}} \sin\Delta_{31} + s_{12}^2 \sin\Delta_{21}\right.$$

$$= 1 - \sin^2 2\theta_{13} \big(\sin^2\Delta_{31} + 2 s_{12}^2 \sin\Delta_{31} \cos\Delta_{32} \cdot \sin\Delta_{21} + s_{12}^4 \sin^2\Delta_{21} \big)$$

$$- c_{13}^2 \sin^2 2\theta_{12} \sin^2\Delta_{21}$$

wiggles

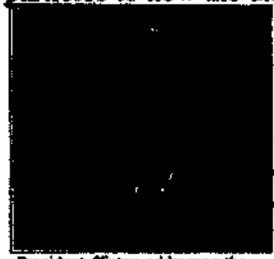
At distances, intermediate between (30km) Chooz and KamLAND, there are wiggles which for $\sin^2 2\theta_{13}$ not too small could be used to determine $\delta m^2_\odot$, $s^2_\odot$, δm^2_{atm}. see Petcov et al June 2003 clinton

Old UCI Page
Announcement
Paper (submitted to Phys.Rev.Lett)
Clinton on Neutrinos
FAQ
Glossary
Links

The President's View

(excerpted from remarks at the MIT commencement, June 6, 1998)

[W]e must help you to ensure that America continues to lead the revolution in science and technology. Growth is a prerequisite for opportunity, and scientific research is a basic prerequisite for growth. Just yesterday in Japan, physicists announced a discovery that tiny neutrinos have mass. Now, that may not mean much to most Americans, but it may change our most fundamental theories -- from the nature of the smallest subatomic particles to how the universe itself works, and indeed how it expands.


President Clinton addresses the graduating class at MIT

This discovery was made, in Japan, yes, but it had the support of the investment of the U.S. Department of Energy. This discovery calls into question the decision made in Washington a couple of years ago to disband the Super-conducting Supercollider, and it reaffirms the importance of the work now being done at the Fermi National Acceleration Facility in Illinois.

The larger issue is that these kinds of findings have implications that are not limited to the laboratory. They affect the whole of society -- not only our economy, but our very view of life, our understanding of our relations with others, and our place in time.

The full text is of the President's address is also available.

θ_{13} and CP Violation:

Fermion Mass

Leptonic CP and T Violation in Oscillations

$$
\begin{array}{ccccccc}
 & & & \text{CP} & & & \\
 & \nu_\mu \to \nu_e & & \Longleftrightarrow & & \bar\nu_\mu \to \bar\nu_e & \quad \text{Superbeams} \\
\text{T} & \Updownarrow & & \text{CPT} & & \Updownarrow & \text{T} \\
 & \nu_e \to \nu_\mu & & \Longleftrightarrow & & \bar\nu_e \to \bar\nu_\mu & \quad \text{Nu. Fact.} \\
 & & & \text{CP} & & &
\end{array}
$$

IN GENERAL (in vacuum):

CP Violation:

$$\alpha \neq \beta \quad P(\nu_\alpha \to \nu_\beta) \neq P(\bar\nu_\alpha \to \bar\nu_\beta)$$

T Violation:

$$\begin{array}{ll} \alpha \neq \beta & P(\nu_\alpha \to \nu_\beta) \neq P(\nu_\beta \to \nu_\alpha) \\ \text{and} & P(\bar\nu_\alpha \to \bar\nu_\beta) \neq P(\bar\nu_\beta \to \bar\nu_\alpha) \end{array}$$

CPT Violation:

$$\text{any } \alpha,\beta \quad P(\nu_\alpha \to \nu_\beta) \neq P(\bar\nu_\beta \to \bar\nu_\alpha)$$

$P_{\nu_\mu \to \nu_e}$

$$P_{\mu\to e} = \Big| U_{e3}U_{\mu3}^* e^{-im_3^2/2L} + U_{e2}U_{\mu2}^* e^{-im_2^2/2L} + U_{e1}U_{\mu1}^* e^{-im_1^2/2L} \Big|^2$$

multiple by $e^{+im_1^2/2L}$

then $U_{e1}U_{\mu1}^* = -U_{e3}U_{\mu3}^* - U_{e2}U_{\mu2}^*$ $\Big\}$ $\Delta_{ij} = \dfrac{\delta m_{ij}^2 L}{4E}$

$$= \Big| 2U_{e3}U_{\mu3}^* e^{-i\Delta_{31}} \sin\Delta_{31} + 2U_{e2}U_{\mu2}^* e^{-i\Delta_{21}} \sin\Delta_{21} \Big|^2$$

multiple by $e^{+i\Delta_{21}}$

$$= \Big| e^{-i\Delta_{32}}\, 2U_{e3}U_{\mu3}^* \sin\Delta_{31} + 2U_{e2}U_{\mu2}^* \sin\Delta_{21} \Big|^2$$

$$2U_{e3}U_{\mu 3}^{*} = 2S_{13}C_{13}\cdot S_{23}e^{-i\delta}$$

$$2U_{e2}U_{\mu 2}^{*} \simeq 2C_{13}S_{12}\left(C_{23}C_{12} - \cancel{S_{13}S_{12}S_{23}e^{-i\delta}}\right)$$

$$P_{\mu\to e} = \left| e^{-i(\Delta_{32}+\delta)}\sqrt{P_{atm}} + \sqrt{P_{\odot}} \right|^2$$

with

$$P_{atm} \equiv \left(S_{23}\sin^2 2\theta_{13}\sin\Delta_{31}\right)^2$$

$$P_{\odot} = \left(C_{13}C_{23}\sin 2\theta_{12}\sin\Delta_{21}\right)^2$$

Oscillation Max: $\Delta_{31} \approx \Delta_{32} = \pi/2$

Max: CP Violations: $\delta = \pi/2$

$$P_{\mu\to e} = \left| \sqrt{P_{atm}} \pm \sqrt{P_{\odot}} \right|^2$$

$(+\nu)(-\bar{\nu})$

Asym: $\dfrac{P-\bar{P}}{P+\bar{P}} = \dfrac{2\sqrt{P_{atm}}\sqrt{P_{\odot}}}{(P_{atm}+P_{\odot})}$

$P(\nu_\mu \to \nu_e)$

Why Everybody is Excited!

- Maximum Allowed Asymmetry ($\delta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$) for $\nu_\mu \to \nu_e$ at first Oscillation Maximum in vac:

- $P,\ \bar{P} = |a^{atm}_{\mu\to e} + a^{\odot}_{\mu\to e}|^2 \approx (\sin\theta_{23}\sin 2\theta_{13} \pm \sqrt{P_\odot})^2$

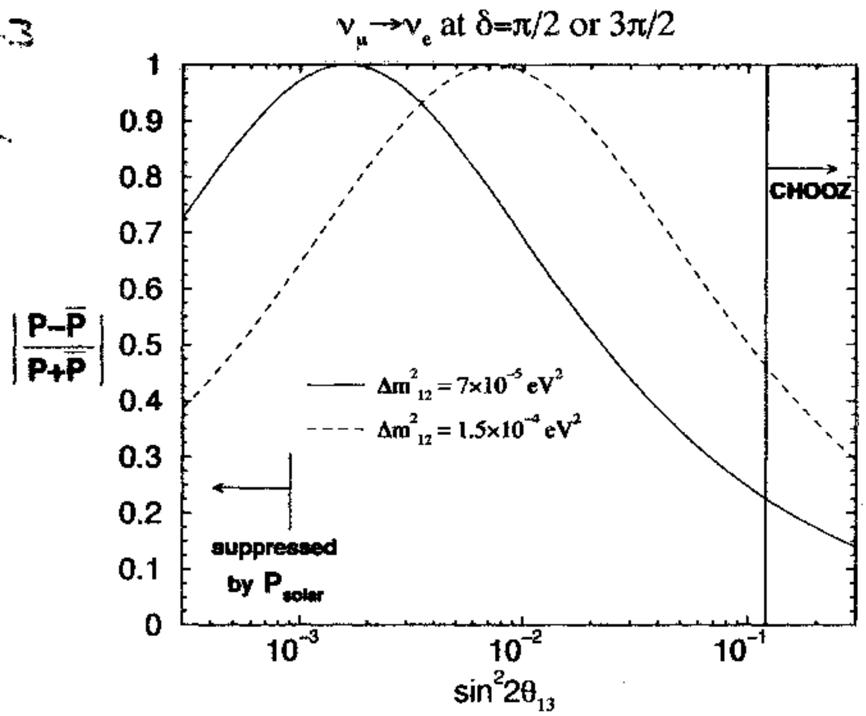
- $|P - \bar{P}| \approx 4\sqrt{P_\odot}\sin\theta_{23}\ \sin 2\theta_{13}$

- $P + \bar{P} \approx 2\sin^2\theta_{23}\ \sin^2 2\theta_{13} + 2P_\odot$

easily generalized
- $\delta \neq \frac{\pi}{2}$
- off O.M.
- matter

$a^{atm}_{\mu\to e} \propto U_{e3}$

$a^{\odot}_{\mu\to e} \propto U_{e2}$



Asymmetry LARGE!

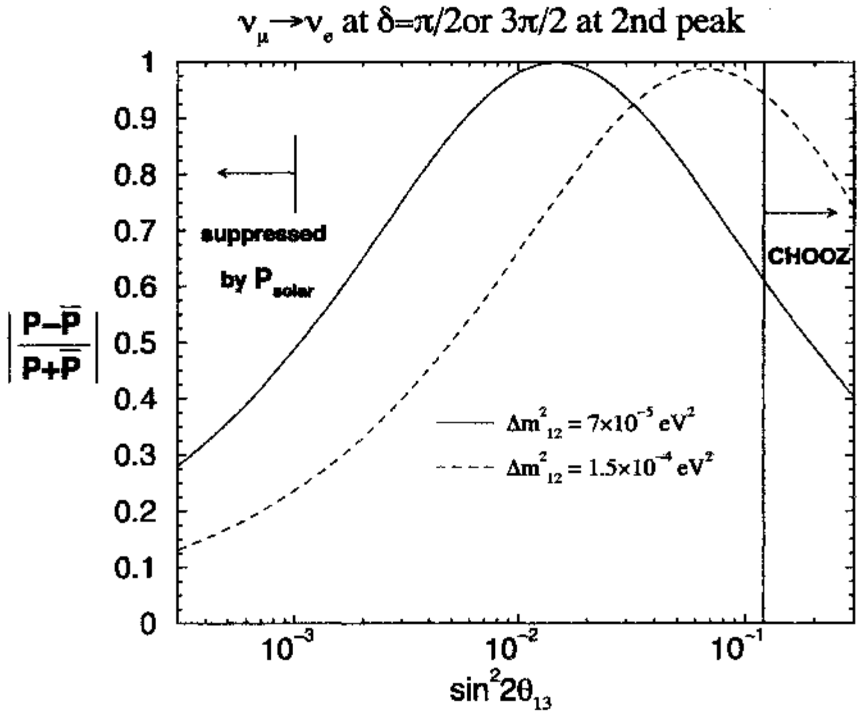
- Peak occurs at

$$\sin^2 2\theta_{13} \approx \frac{\sin^2 2\theta_{12}}{\tan^2\theta_{23}}\left[\frac{\pi}{2}\ \frac{\delta m^2_{12}}{\delta m^2_{13}}\right]^2$$

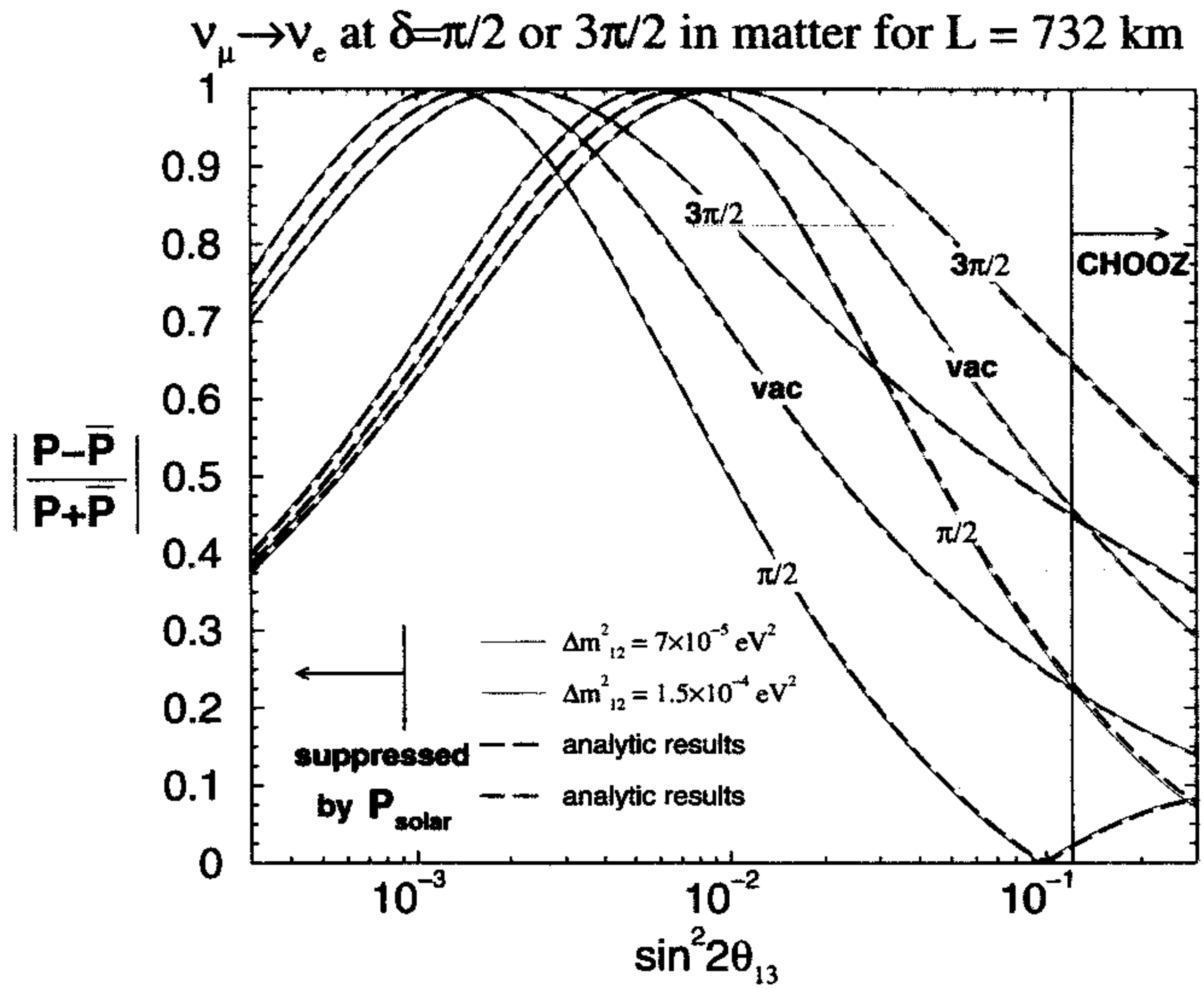
at OM $\sqrt{P_\odot} = \cos\theta_{13}\cos\theta_{23}\sin 2\theta_{12}\sin\left(\frac{\pi}{2}\ \frac{\delta m^2_{12}}{\delta m^2_{13}}\right)$

- For BK

2nd Peak



- Peak occurs at $\sin^2 2\theta_{13} \approx \frac{\sin^2 2\theta_{12}}{\tan^2 \theta_{23}} \left[\frac{3\pi}{2} \; \frac{\delta m^2_{12}}{\delta m^2_{13}}\right]^2$



$\nu_\mu \to \nu_e$ at $\delta=\pi/2$ or $3\pi/2$ in matter for L = 732 km
$\left|\frac{P-\bar{P}}{P+\bar{P}}\right|$
$\sin^2 2\theta_{13}$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10^{-3}
10^{-2}
10^{-1}
$3\pi/2$
$3\pi/2$
vac
vac
$\pi/2$
$\pi/2$
CHOOZ
suppressed by P_{solar}
$\Delta m^2_{12} = 7\times10^{-5}$ eV^2
$\Delta m^2_{12} = 1.5\times10^{-4}$ eV^2
analytic results
analytic results

$$P_{\mu \to e} = P_{atm} + 2\sqrt{P_{atm}}\sqrt{P_\odot}\cos(\Delta_{32} \pm \delta) + P_\odot$$

$$\cos(\Delta_{32} + \delta) = \cos\Delta_{32}\cos\delta + \underline{\sin\Delta_{32}\sin\delta}$$

CP violating term is

$$\sin\delta \cdot \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13}$$
$$* \sin\Delta_{21} \sin\Delta_{32} \sin\Delta_{31}$$

FULL EXPRESSION:

$$P_{\mu\to e} = P_{\otimes} - 2\, s_{23}^2\, s_{2(13)}^2\, s_{12}^2 \sin\Delta_{31} \cos\Delta_{32} \underset{\text{small}}{\underline{\underline{\sin\Delta_{21}}}}$$

$$+ P_{\odot} \left(1 - 2 s_{13} \tan\theta_{23} \tan\theta_{12} \cos\delta + s_{13}^2 \tan^2\theta_{23} \tan^2\theta_{12}\right)$$

$$+ 2\sqrt{P_{\otimes} P_{\odot}}\, \cos\left(\Delta_{32} \mp \delta\right)$$

$$P_{\otimes} = s_{23}^2\, s_{2(13)}^2 \sin^2\Delta_{31}$$

$$P_{\odot} = c_{13}^2\, c_{23}^2\, s_{2(12)}^2 \sin^2\Delta_{21}$$

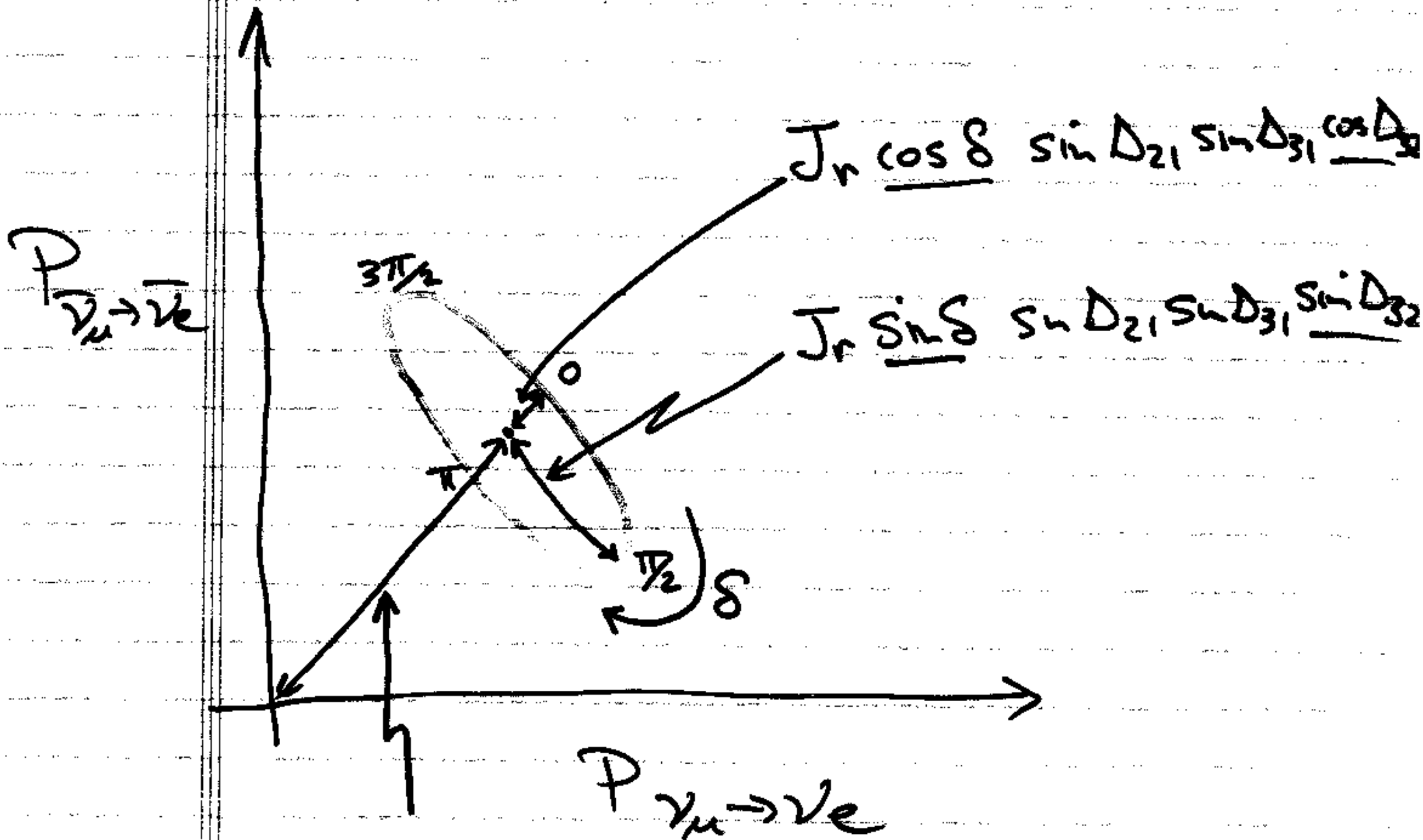
$P_{\odot}$ is irrelevant unless s_{13} is small

where $(1 - 2s_{13} \cdots) \simeq 1$

$$\frac{\Delta_{21}}{\Delta_{31}} = \frac{\delta m_{21}^2}{\delta m_{31}^2} \approx \frac{1}{30} \qquad \text{also } \sin\Delta_{31} \approx 1, \quad \cos\Delta_{32} \approx 0$$

Bi-Probability Plot

$(P, \bar{P})$ varying only δ_{CP}

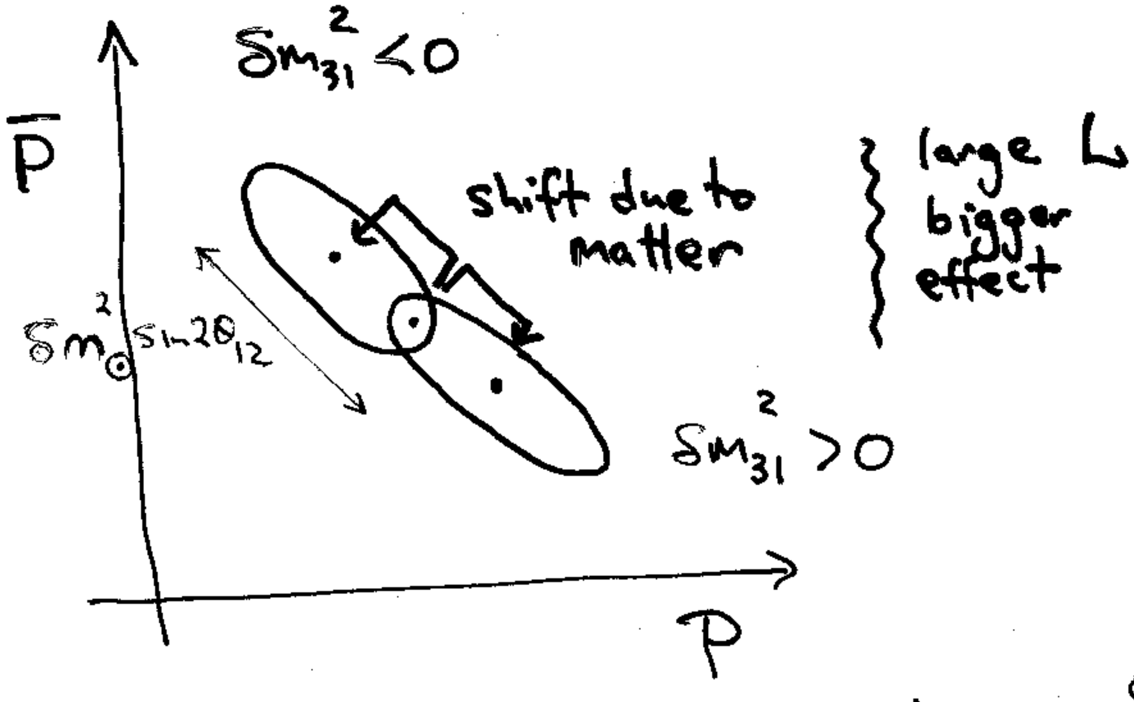


$\sqrt{2} \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2\Delta_{31}$

If SMA or LOW solar sol^n
ellipse collapses to point.

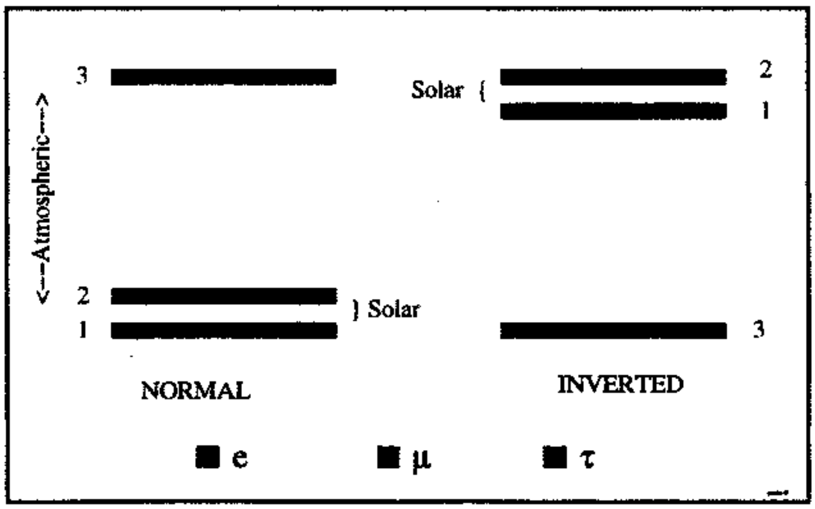
LMA – Wow!!!

Matter Effects:



$$P_{mat}^{center} \approx \left(1 \pm \frac{E_\nu}{6\,\text{GeV}}\right) P_{vac}^{center}$$

O.M. for small E_ν

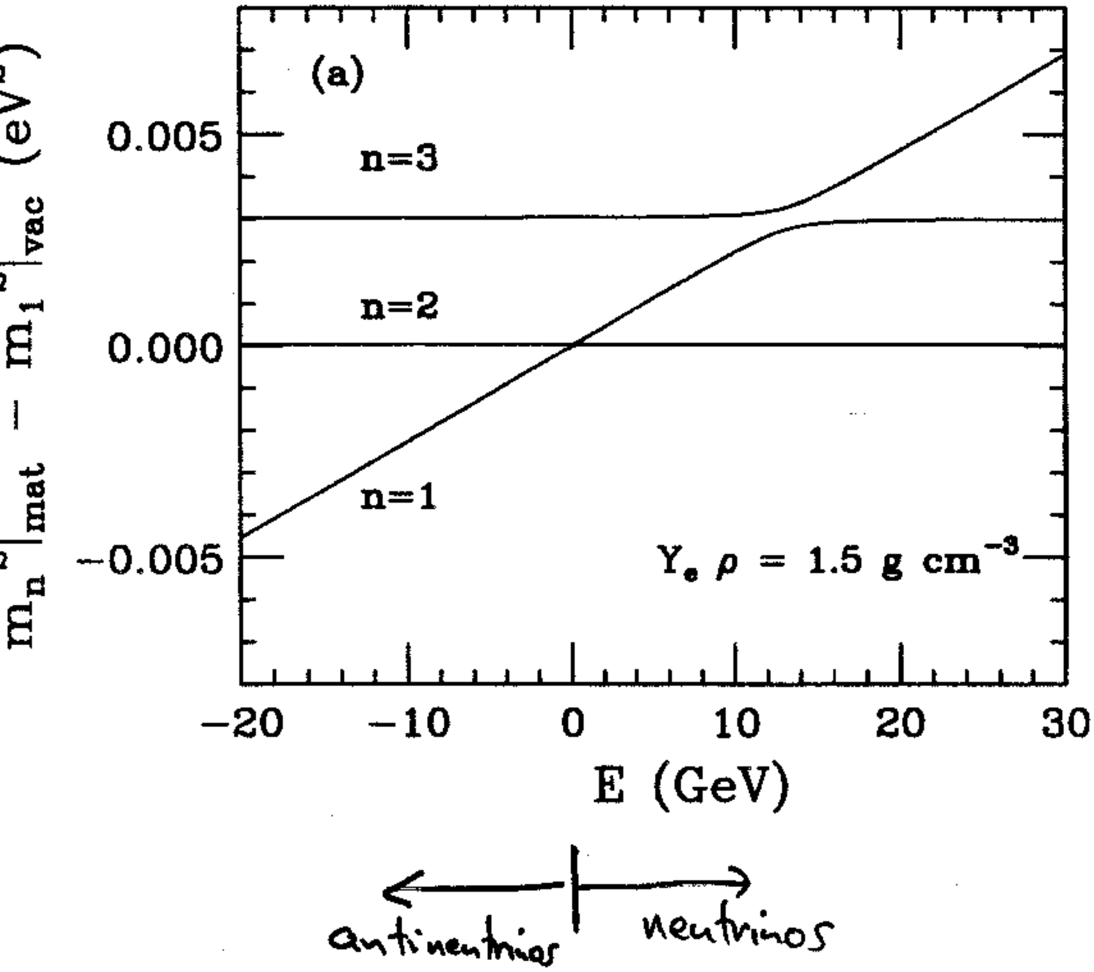


$\delta m_{31}^2 > 0$ $\delta m_{31}^2 < 0$

$$\left(\sin 2\theta_{13}\, \delta m^2_{31}\right)_{\text{matter}} = \left(\sin 2\theta_{13}\, \delta m^2_{31}\right)_{\text{VAC}}$$

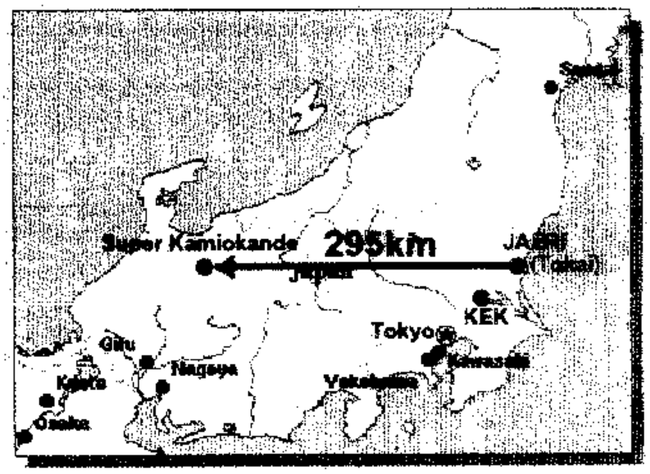
and

$$\left(\sin 2\theta_{12}\, \delta m^2_{21}\right)_{\text{MATTER}} = \left(\sin 2\theta_{12}\, \delta m^2_{21}\right)_{\text{VAC}}$$



JHF → Super-Kamiokande

- 295 km baseline
- Super-Kamiokande:
  - 22.5 kton fiducial
  - Excellent e/μ ID
  - Additional π^0/e ID
- Hyper-Kamiokande
  - 20× fiducial mass of SuperK
- Matter effects small
- Study using fully simulated and reconstructed data



Requires New Beamline:

~~http://www.off-axis.fnal.gov/~~

LOI: hep-ex/0106019

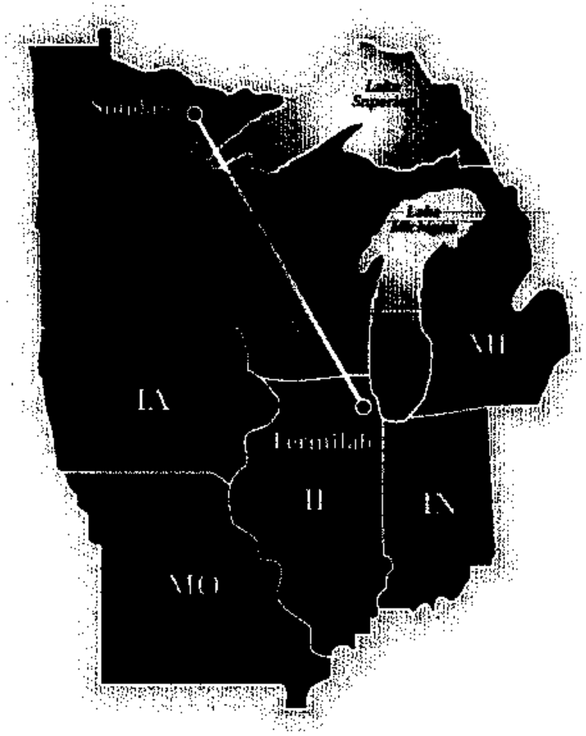
http://www-nu.kek.jp/jhfnu/

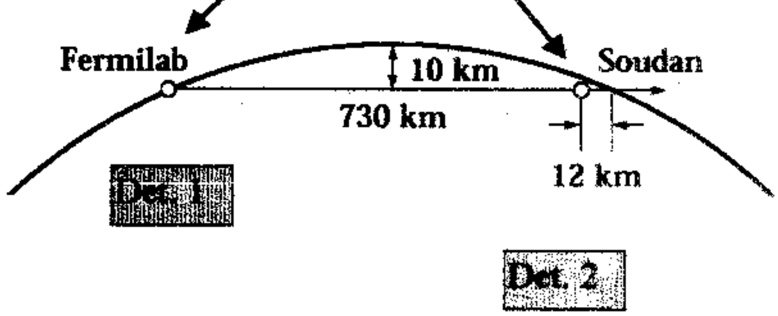
$\overline{E}_\nu = 0.6$ to 1.0 GeV

~20% spread

$L = 295$ km

The NUMI Beamline



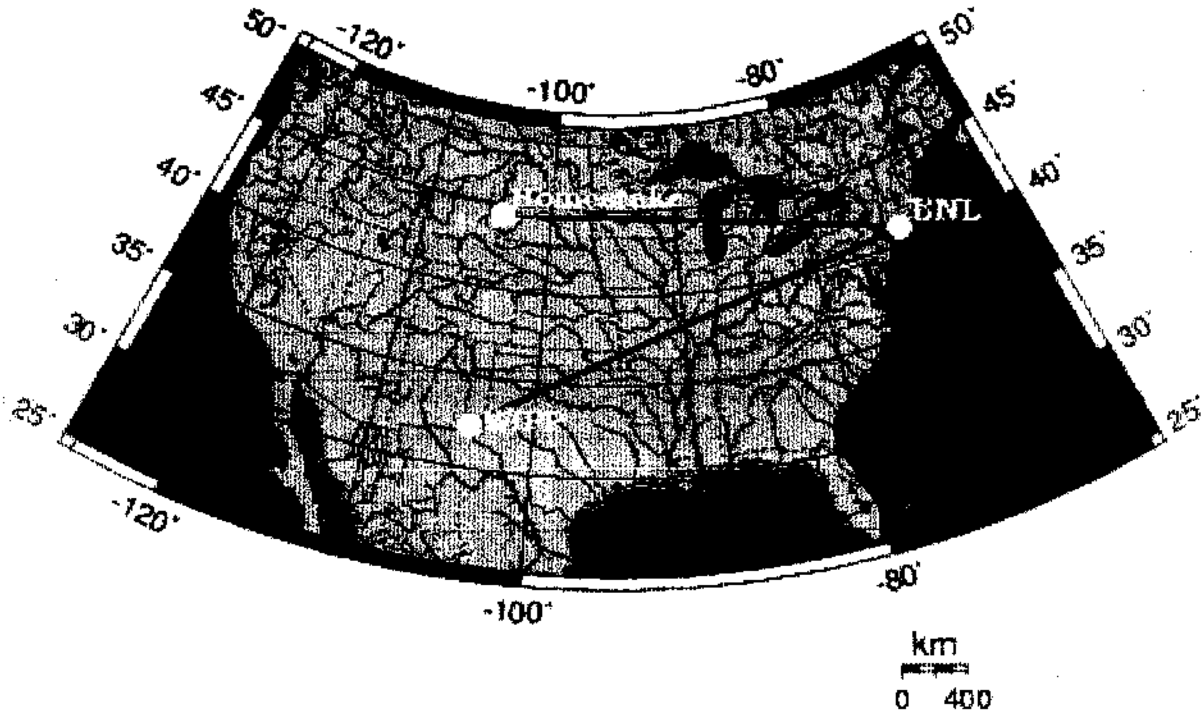


New Detector Required:
http://www-off-axis.fnal.gov/
LOI: hep-ex/0210005

$E \sim 2\,\mathrm{GeV}$

$L \sim 732\,\mathrm{km}$
$\pm 200\,\mathrm{km}$

Brookhaven to Homestake OR WIPP



L= 2540 km or 2880 km

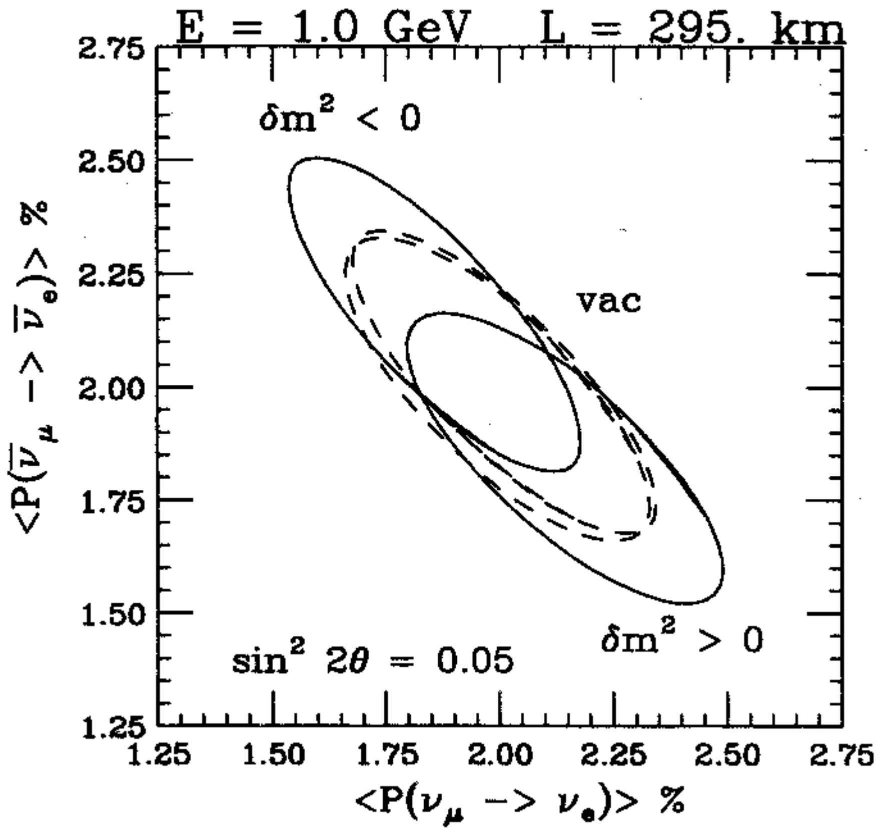
2^{nd} peak 2 GeV:

New Beamline, New Detector:

http://www.neutrino.bnl.gov/

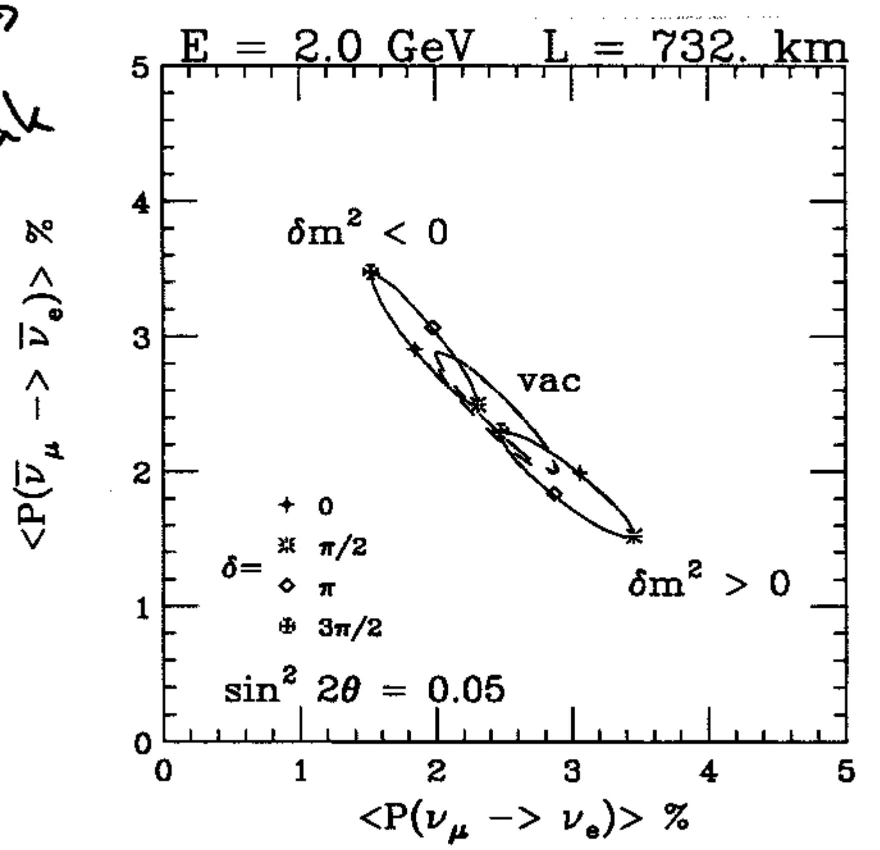
LOI: hep-ex/0205040

JHF → SK
1st peak



20% Spread

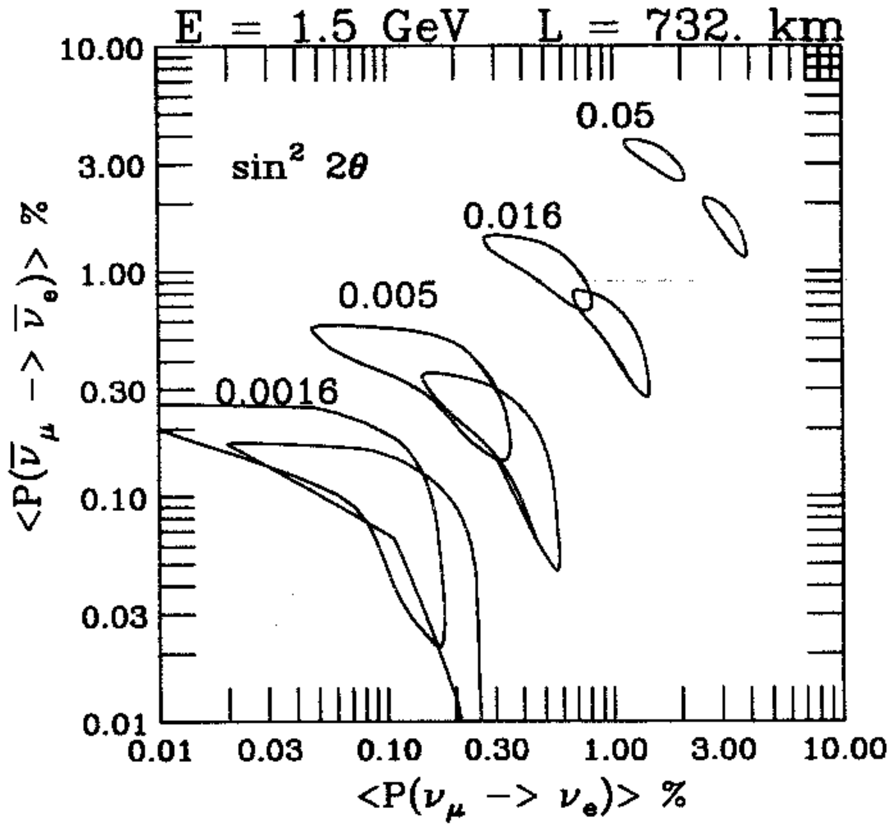
NuMI →
1st peak



20% Spread.

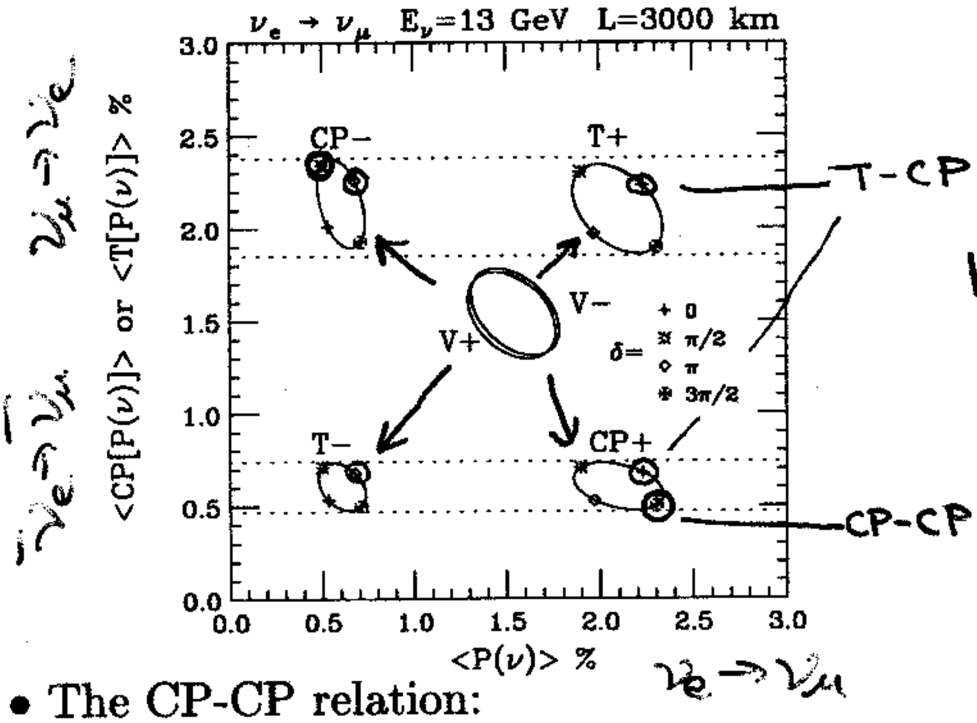
Matter Effects separate $\delta m^2 > 0$ from $\delta m^2 < 0$

As $\sin^2 2\theta_{13}$ Varies:



NuMI OffAxis LOI
hep-ex/0210005

Anatomy of the Bi-Probability Plot:



Matter Effects

- The CP-CP relation:

$$\begin{aligned} & P(\nu_e \to \nu_\mu; \Delta m^2_{31}, \Delta m^2_{21}, \delta, a) \\ = \quad & P(\bar\nu_e \to \bar\nu_\mu; -\Delta m^2_{31}, -\Delta m^2_{21}, \delta, a) \\ \approx \quad & P(\bar\nu_e \to \bar\nu_\mu; -\Delta m^2_{31}, +\Delta m^2_{21}, \pi + \delta, a) \end{aligned}$$

evolution eqn.

- The T-CP relation:

$$\begin{aligned} & P(\nu_\mu \to \nu_e; \Delta m^2_{31}, \Delta m^2_{21}, \delta, a) \\ = \quad & P(\bar\nu_e \to \bar\nu_\mu; -\Delta m^2_{31}, -\Delta m^2_{21}, -\delta, a) \\ \approx \quad & P(\bar\nu_e \to \bar\nu_\mu; -\Delta m^2_{31}, +\Delta m^2_{21}, \pi - \delta, a) \end{aligned}$$

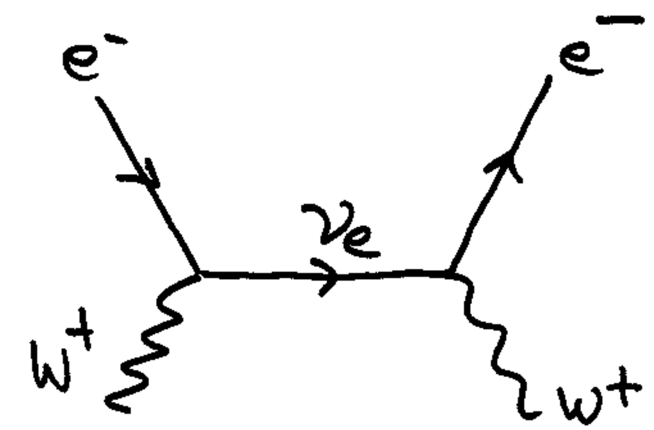
assuming symmetry matter distribution

- $\approx$ trade sign of δm^2_{12} for shift by π of δ:

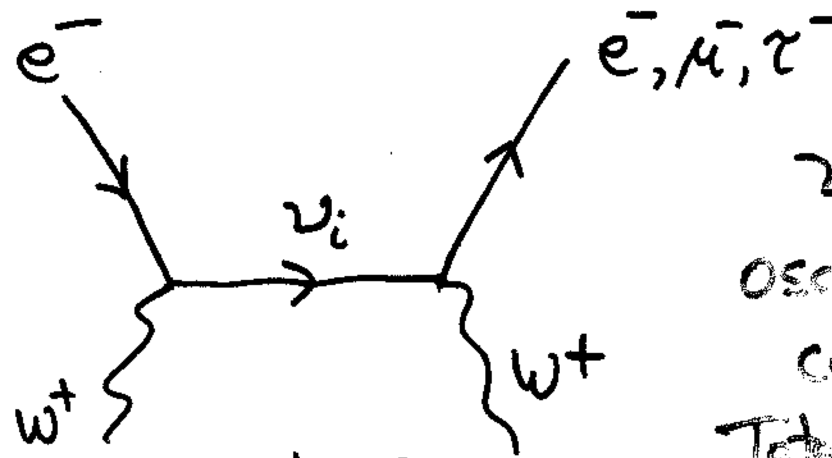
$$(\cdots) + \delta m^2_{12}[(\cdots)\cos\delta + (\cdots)\sin\delta]$$

small correction here

Days of Weinberg + Salam

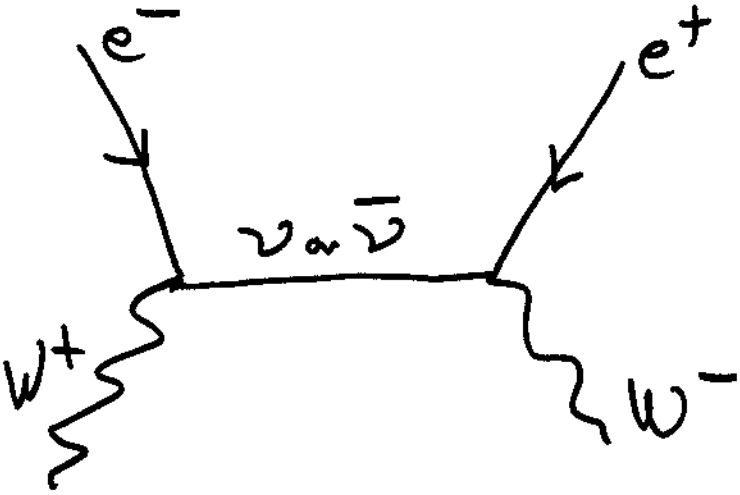


S.M.
flavor diagonal!
conserves
L_e, L_μ, L_τ



L/E large

ν mass
oscillations ~~(L_e, L_μ, L_τ)~~
conserves
Total Lepton #



Majorana Mass
Lepton # Violating
(highly suppressed
Why???)

Fermion Mass: (cartoon:)

- Consider a massless electron:

$$e^-_L \qquad e^-_R$$

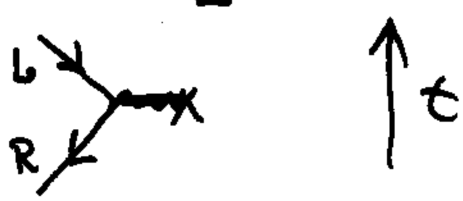
$$e^+_R \qquad e^+_L$$

- Four States: $2e^- + 2e^+$

- Mass is a coupling between Left and Right. (chiral states)

e^-_L and e^-_R

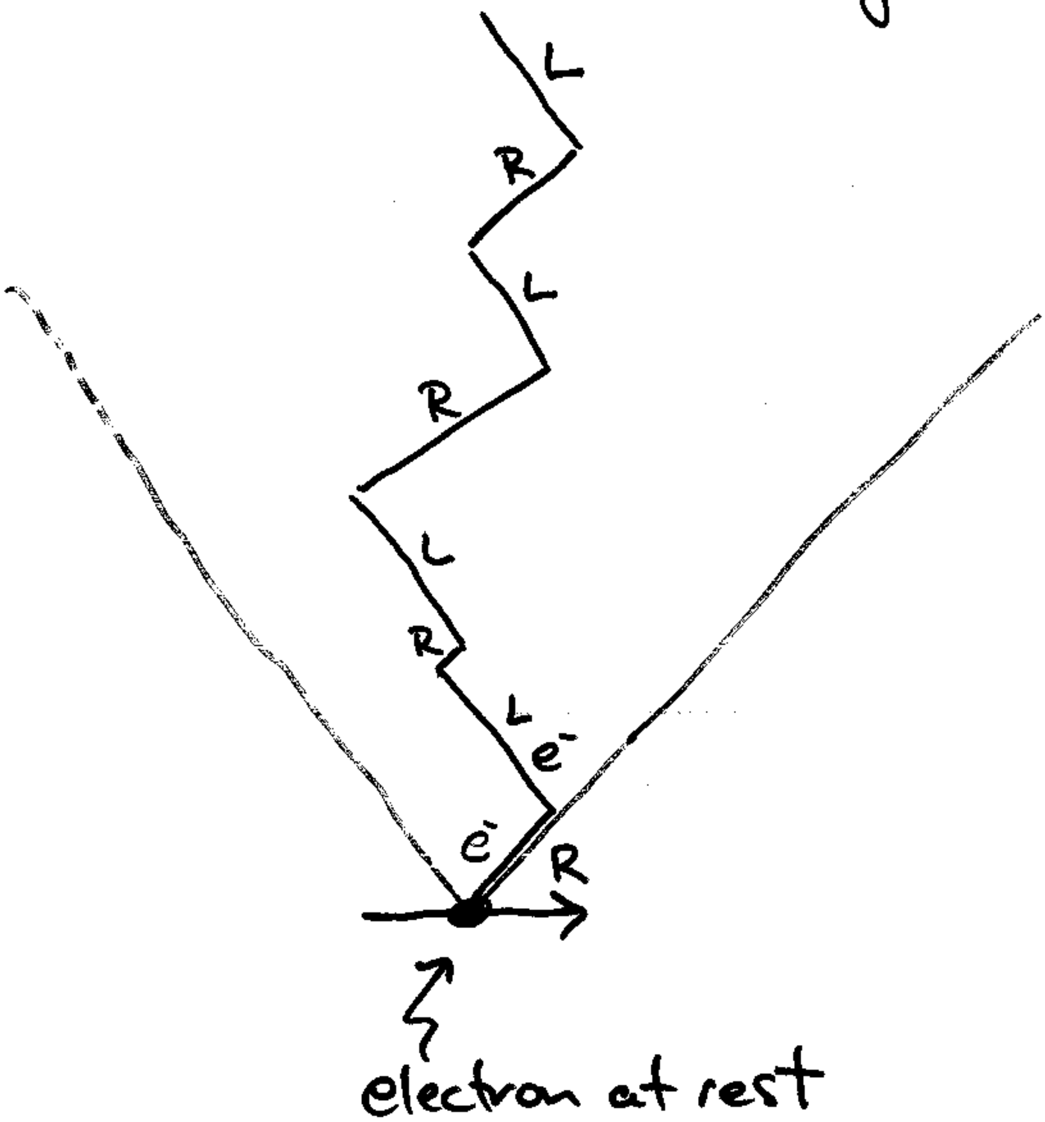
e^+_R and e^+_L



CANNOT couple

e^-_L to e^+_R

because of electric charge conservation.



Massive Fermion with given Spin:

$$P^2 = M^2 \qquad S^2 = -1 \qquad P\cdot S = 0$$

e.g. { at rest $P = (M,0,0,0)$ $S = (0,0,0,1)$
spin in z direction

$$P = \frac{P+MS}{2} + \frac{P-MS}{2}$$

$$\frac{P+MS}{2} \quad \text{and} \quad \frac{P-MS}{2} \qquad \underline{\text{Massless}}$$

$$\left(\frac{P \pm MS}{2}\right)^2 = \frac{P^2 \pm 2MP\cdot S + M^2S^2}{4}$$

$$= \frac{M^2 \pm 2M.0 + M^2(-1)}{4}$$

Sol^n Dirac Eqn
Massive, Spin

$$= 0$$

Massless
Spinors

$$U(P,S) = \frac{1+\gamma_5}{2} u\left(\frac{P+MS}{2}\right) + e^{i\phi} \frac{1-\gamma_5}{2} u\left(\frac{P-MS}{2}\right)$$

chiral
projections

$$u(p,s) = \frac{1+\gamma_5 \not{s}}{2}\, u(p)$$

$$= \frac{1+\gamma_5}{2} \underbrace{\frac{1+\not{s}}{2}\, u(p)}_{Q_+} + \frac{1-\gamma_5}{2} \underbrace{\frac{1-\not{s}}{2}\, u(p)}_{Q_-}$$

$$Q_\pm \overline{Q_\pm} = \frac{\not{p} \pm M\not{s}}{2}$$

and $\frac{\not{p} \pm M\not{s}}{2}\, Q_\pm = 0$

therefore $Q_\pm = u\left(\frac{p \pm Ms}{2}\right)$

Relative phase determined by

$$\bar{u}(p,s)\, u(p,s) = 2M$$

Massive Particle at Rest, Spin in z direction:

$$P = \frac{P + MS}{2} + \frac{P - MS}{2}$$

$$(M,0,0,0) = \frac{M}{2}(1,0,0,1) + \frac{M}{2}(1,0,0,-1)$$

BIG BOOST IN Z-DIRECTION $\begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix}$

$$\gamma M(1,0,0,\beta) = \frac{M}{2}(\gamma+\beta\gamma)(1,0,0,1) + \frac{M}{2}(\gamma-\beta\gamma)(1,0,0,-1)$$

as $\beta \to 1$ $\quad (\gamma+\beta\gamma) \to 2\gamma \quad (\gamma-\beta\gamma) \to \frac{1}{2\gamma}$

where $\gamma = E/M$

$$E(1,0,0,\beta) \simeq E(1,0,0,1) + \frac{M^2}{E}(1,0,0,-1)$$

$$U(P,S) = \frac{1+\gamma_5}{2}\, U\left(\frac{P+MS}{2}\right) + \frac{1-\gamma_5}{2}\, U\left(\frac{P-MS}{2}\right)$$

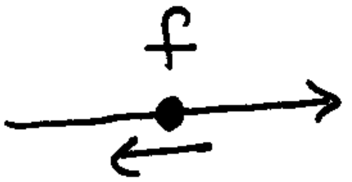
HELICITY STATE $\qquad$ RIGHT $\qquad$ LEFT

CHIRAL PROJECTIONS

HELICITY $\neq$ CHIRALITY

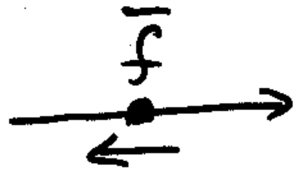
for massive particles

Massless Fermion:

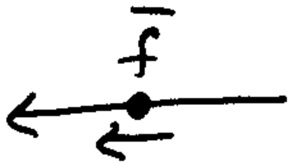


f_L

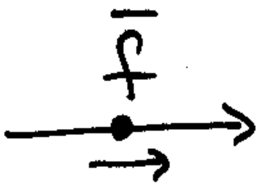
Charge Conjugation (C)



Parity (P)



Time Reversal (T)



$\bar{f}_R$

CPT $\quad f_L \longleftrightarrow \bar{f}_R$

At a minimum by CPT

$f_L, \bar{f}_R$

For Neutral Fermions (neutrinos)

ν_L	ν_R	Dirac Mass.
$\bar{\nu}_R$	$\bar{\nu}_L$	

required by weak interaction

Majorana Mass
(Violates L#)

See-Saw Mechanism gives

ν_L	and	N_R
$\overset{(-)}{\nu}_R$		$\overset{(-)}{N}_L$
light $\frac{m^2}{M}$		M

Neutrinos being Majorana is the minimal solution to giving ν mass.

$\nu = \bar{\nu}$ two states ν_L and ν_R

If Dirac then there is a global symmetry (lepton #) which needs to be explained.

$\nu \neq \bar{\nu}$ and four states

ν_L and $\bar{\nu}_R$

weak

ν_R and $\bar{\nu}_L$

Suppression factor:

$$\left(\frac{\overline{M}_\nu}{E}\right)^2$$

chirality $\neq$ helicity for massive particles

$\overline{M}_\nu \sim 0.1\,\text{eV}$ $\quad E = 1\,\text{GeV}$ $\quad \left(\frac{M}{E}\right)^2 \approx \underline{\underline{10^{-20}}}$

BUT $\quad \cancel{\nu}\beta\beta$ decay

Same Nucleus also has $2\nu\beta\beta$ decay

→ end point:

WHAT WE DON'T KNOW:

- Majorana OR Dirac
- Absolute mass of lighest neutrino.
  (except $< \sim 1\,\mathrm{eV}$)
- Size of θ_{13}: (ν_e in the "3" state.)
  $\sin^2\theta_{13} < 0.03$
- Is $\theta_{23} = $ or $\lessgtr \pi/4$ the $\mu \leftrightarrow \tau$ symmetric point.
  (maximal mixing)
  $0.35 < \sin^2\theta_{23} < 0.65$
- Sign of δm^2_{atm} (or)
  type of spectrum
- phase δ $\Leftarrow$ if $\neq 0$ leads to CP violation
- Number of light Neutrinos: 3 or are there more than 3