Complementarity of Off-Axis and Reactor Exp'ts:

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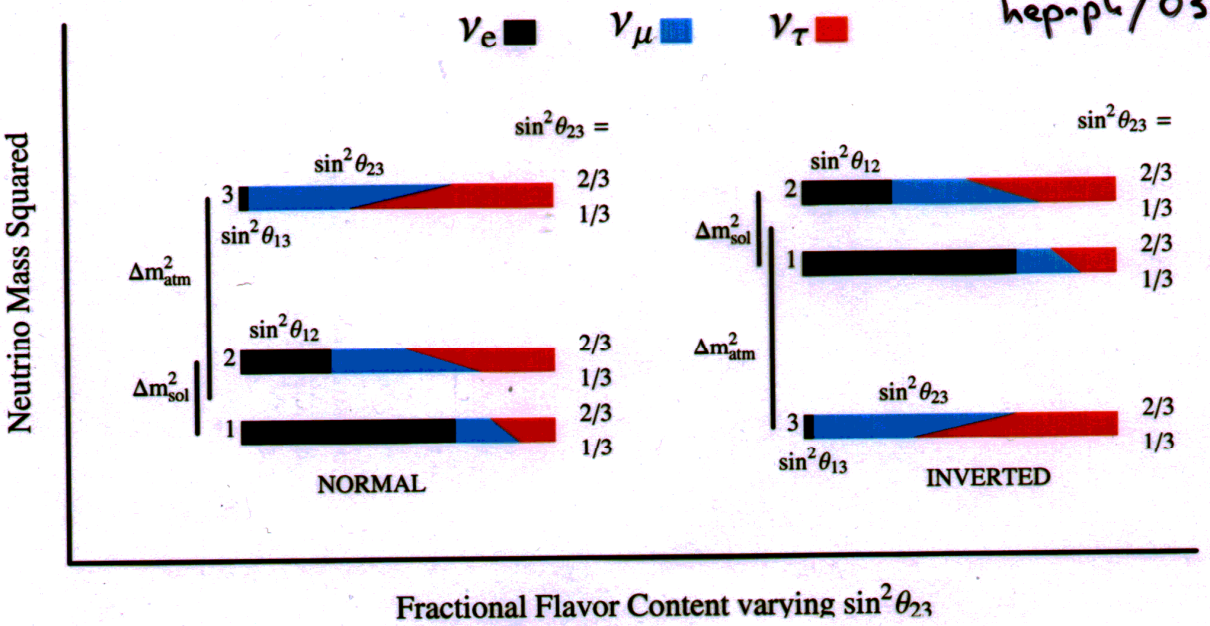
Fermilab

Current Knowledge: 3 active flavors (Nu S.M.)

O. Mena + SP hep-ph/0312131



(21) good

$$+6.6\times10^{-5}\,eV^2 < \delta m^2_{21} < +8.7\times10^{-5}\,eV^2$$

sign known

$$0.25 < \sin^2\theta_{12} < 0.33$$

(32) poor

$$1.3\times10^{-3}\,eV^2 < |\delta m^2_{32}| < 3.0\times10^{-3}\,eV^2$$

$$0.35 < \sin^2\theta_{23} < 0.65$$

(31) unknown

sign δm^2_{31}

$$0 \leq \sin^2\theta_{13} < 0.03\text{-}0.05 \ldots$$

$$-1 \leq \sin\delta \leq 1$$

sign $\cos\delta$

Long Base Line: $\theta_{13}, \theta_{23}, \delta$, sign δm^2_{31}

$$P^{Vac}_{\nu_\mu \to \nu_e} \cong \Big| \underbrace{\sin\theta_{23} \sin 2\theta_{13} \sin\Delta_{31}}_{\text{atm amp.}} e^{-i(\Delta_{32} \pm \delta)} + \underbrace{\cos\theta_{13} \cos\theta_{23} \sin 2\theta_{12} \sin\Delta_{21}}_{\text{solar amp.}} \Big|^2$$

$$= \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2\Delta_{31} \quad \leftarrow P_{ATM}\ \text{Atmos}$$

$$+ J_r \sin\Delta_{21} \sin\Delta_{31} \cos(\Delta_{32} \pm \delta) \quad \leftarrow \text{Interf.}$$

$$+ \cos^2\theta_{13} \cos^2\theta_{23} \sin^2 2\theta_{12} \sin^2\Delta_{21} \quad \leftarrow \text{Solar } P_\odot$$

$$J_r = \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13}$$

Interf.

$$\cos(\Delta_{32} \pm \delta) = \underbrace{\cos\Delta_{32} \cos\delta}_{\leftarrow CPC} \mp \underbrace{\sin\Delta_{32} \sin\delta}_{\leftarrow CPV}$$

Reactor θ_{13}:

Kinematic phase

$$\Delta_\alpha = \frac{\delta m^2_\alpha L}{4E} \simeq 1.27 \frac{\delta m^2_\alpha L}{E}$$

$$1 - P_{\nu_e \to \nu_e} = \sin^2 2\theta_{13} \left\{ \sin^2 \Delta_{atm} + \mathcal{O}\left(\frac{\Delta_{solar}}{\Delta_{atm}}\right) \right\} + \mathcal{O}\left(\frac{\Delta_{solar}}{\Delta_{atm}}\right)^2$$

$\frac{\Delta_{solar}}{\Delta_{atm}} < \frac{1}{30}$

$\left(\frac{\Delta_{solar}}{\Delta_{atm}}\right)^2 < \frac{1}{30^2}$

$\Delta_{atm} \propto \pi/2$

$$\approx 4 \sin^2 \theta_{13}$$

Strength: Measures $\sin^2 \theta_{13}$ independently $\left\{ \begin{array}{l} \text{sign } \delta m^2_{31} \\ \sin\delta \\ \sin^2 \theta_{23} \end{array} \right.$

Weakness: Doesn't measure anything else.

Expected sensitivity down to

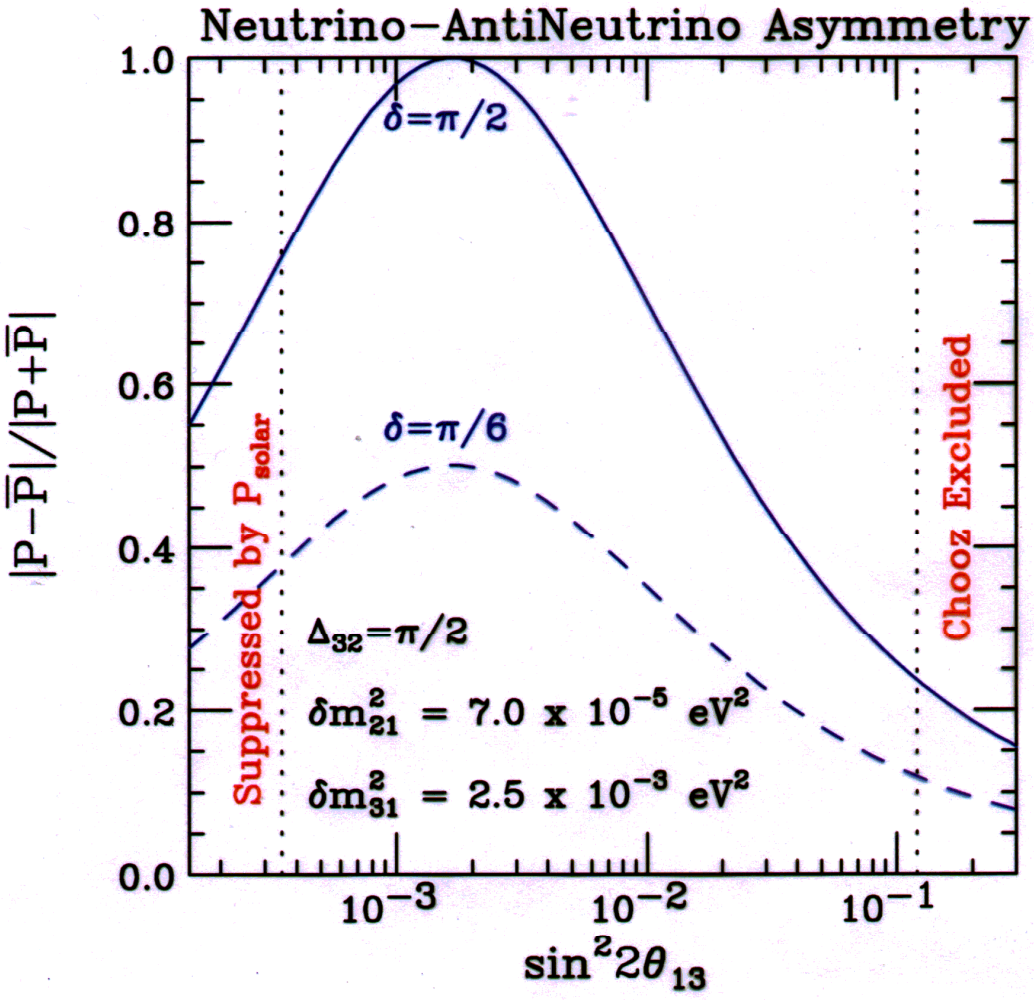
$$\sin^2 \theta_{13} \simeq 0.003$$

(Order of magnitude below Chooz

Premise:

— $\sin^2 \theta_{13}$ has been measured by Reactor Expts. —

At the First Max. of the Atmos. Osc.



Peak occurs when $|a_{atm}| = |a_{sol}|$

$$\sin^2 2\theta_{13} \approx \frac{\sin^2 2\theta_{12}}{\tan^2 \theta_{23}} \left(\frac{\pi}{2} \frac{\delta m^2_{21}}{\delta m^2_{31}} \right)^2$$

$$\approx 0.002$$

Matter Effects:

$$\sin\Delta_{31} \Rightarrow \left(\frac{\Delta_{31}}{\Delta_{31}\mp aL}\right)\sin(\Delta_{31}\mp aL)$$

$$\sin\Delta_{21} \Rightarrow \left(\frac{\Delta_{21}}{\Delta_{21}\mp aL}\right)\sin(\Delta_{21}\mp aL)$$

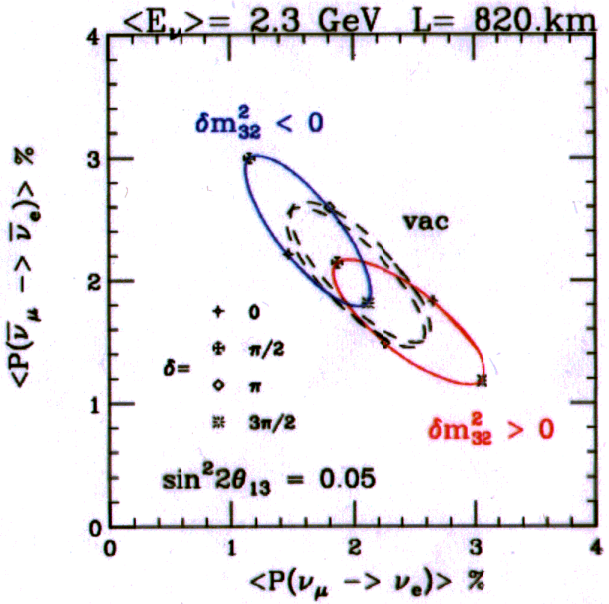
$$\sin\Delta_{32} \Rightarrow \sin\Delta_{32}$$

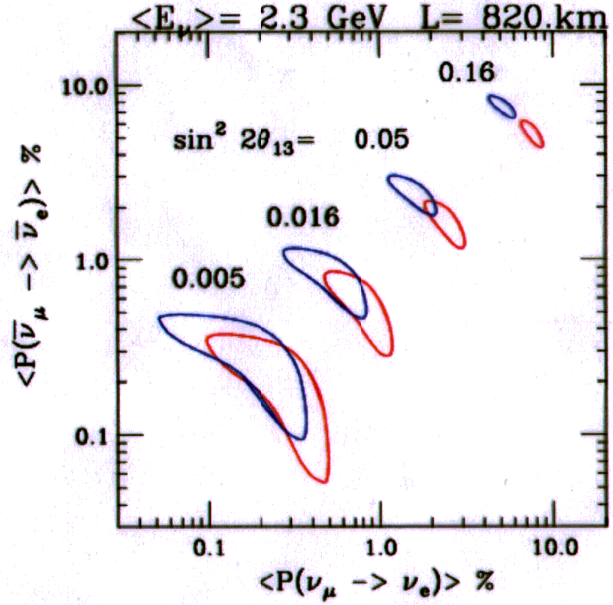
$\{\delta m^2 \sin 2\theta\}$ is invariant

and

$$a = G_F N_e/\sqrt{2}$$
$$= (4000\ km)^{-1}$$

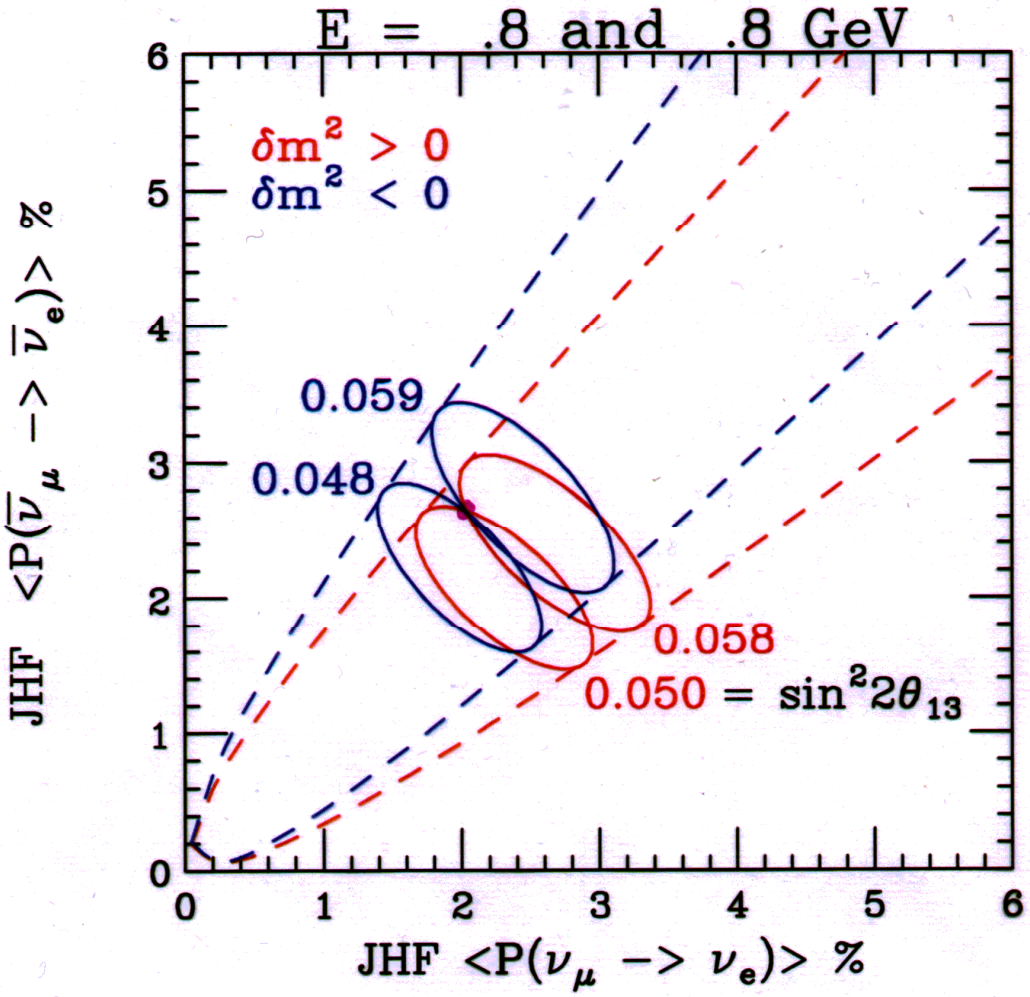
Matter effects are IMPORTANT when $\sin(\Delta \mp aL) \neq (\Delta \mp aL)$.





Matter Effects important for NuMI-OFF-Axis (800 km), less so for JParc (295 km).

Degeneracies:



θ_{13} and δ degeneracy

sign δm^2_{32} degeneracy

$\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$ Degeneracy:

ν_μ disappearance

$$\sin^2 2\theta_{23} = 1 - \epsilon^2$$

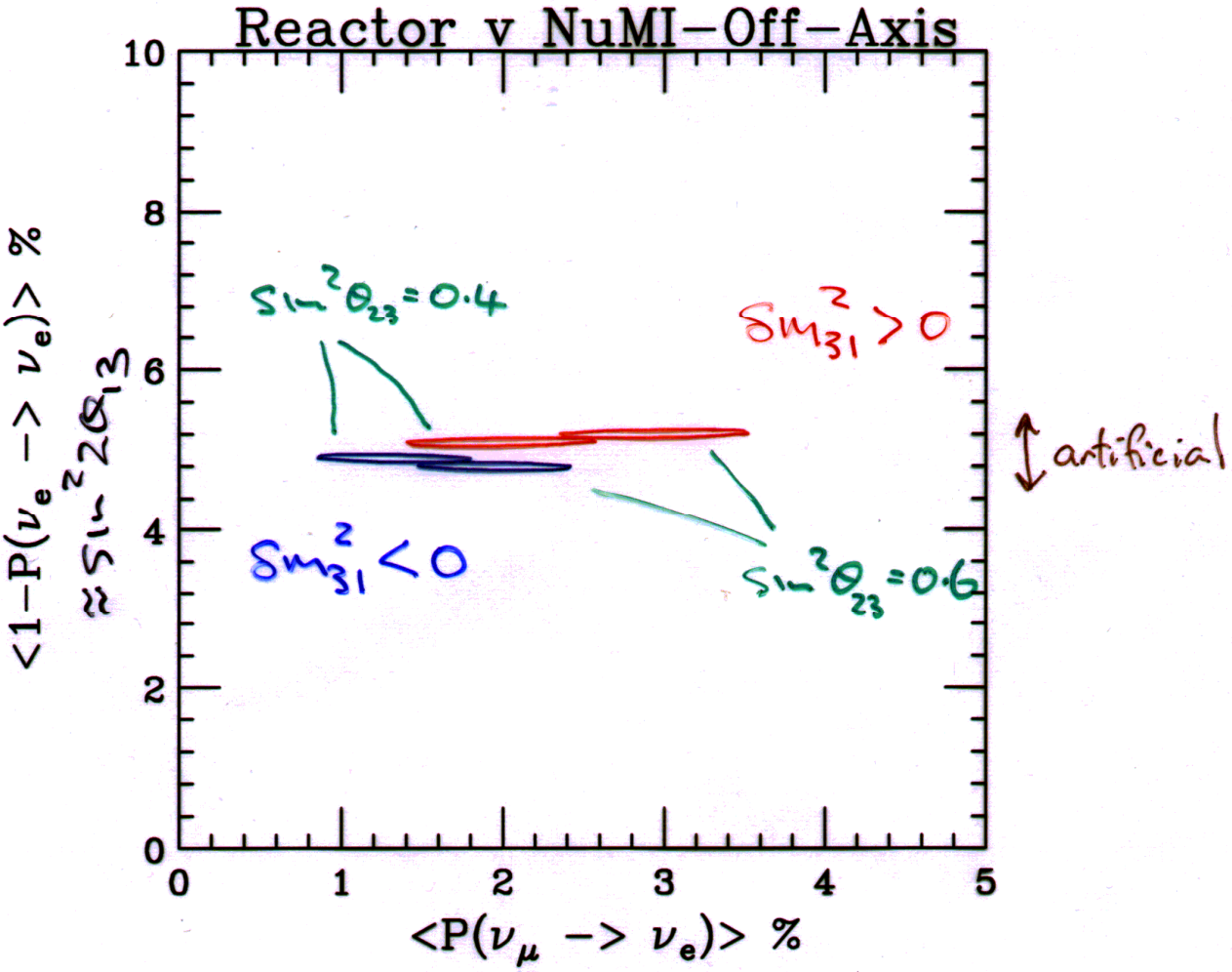
$$\Downarrow$$

$$\sin^2 \theta_{23} = \frac{1 \mp \epsilon}{2}$$

$\sin^2 2\theta_{23}$	$\sin^2 \theta_{23}^{(1)}$	$\sin^2 \theta_{23}^{(2)}$	Error if us $\sin^2 \theta_{23} = \frac{1}{2}$
0.91	0.35	0.65	> 30%
0.96	0.40	0.60	> 20%
0.99	0.45	0.55	> 10%

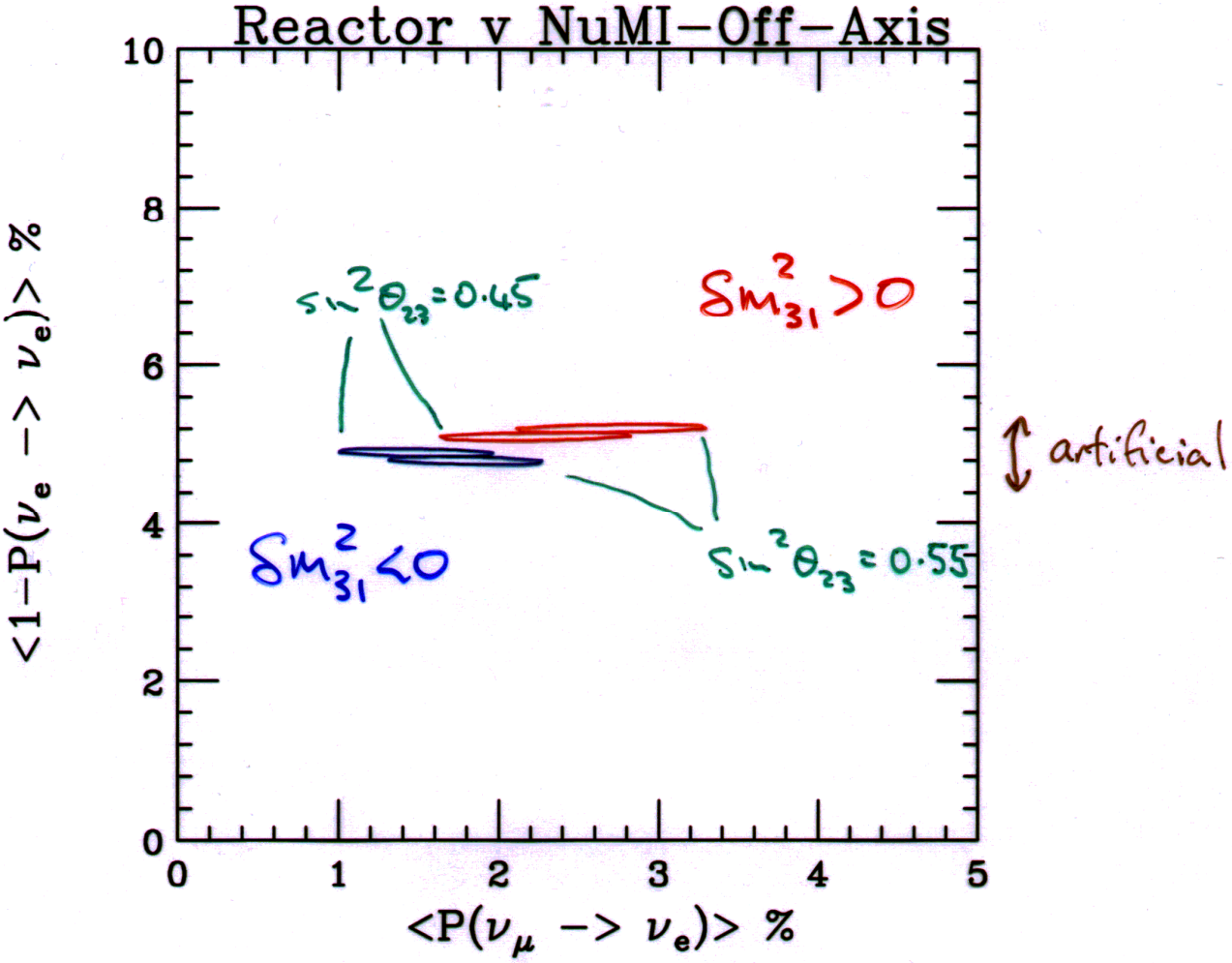
$\sin^2 2\theta_{23} = 0.96$

$\sin^2\theta_{23} = 0.4$ or $\sin^2\theta_{23} = 0.6$



theorists prejudice $\Delta m^2_{31} > 0$ and $\sin^2\theta_{23} < \frac{1}{2}$

$\sin^2 2\theta_{23} > 0.99$

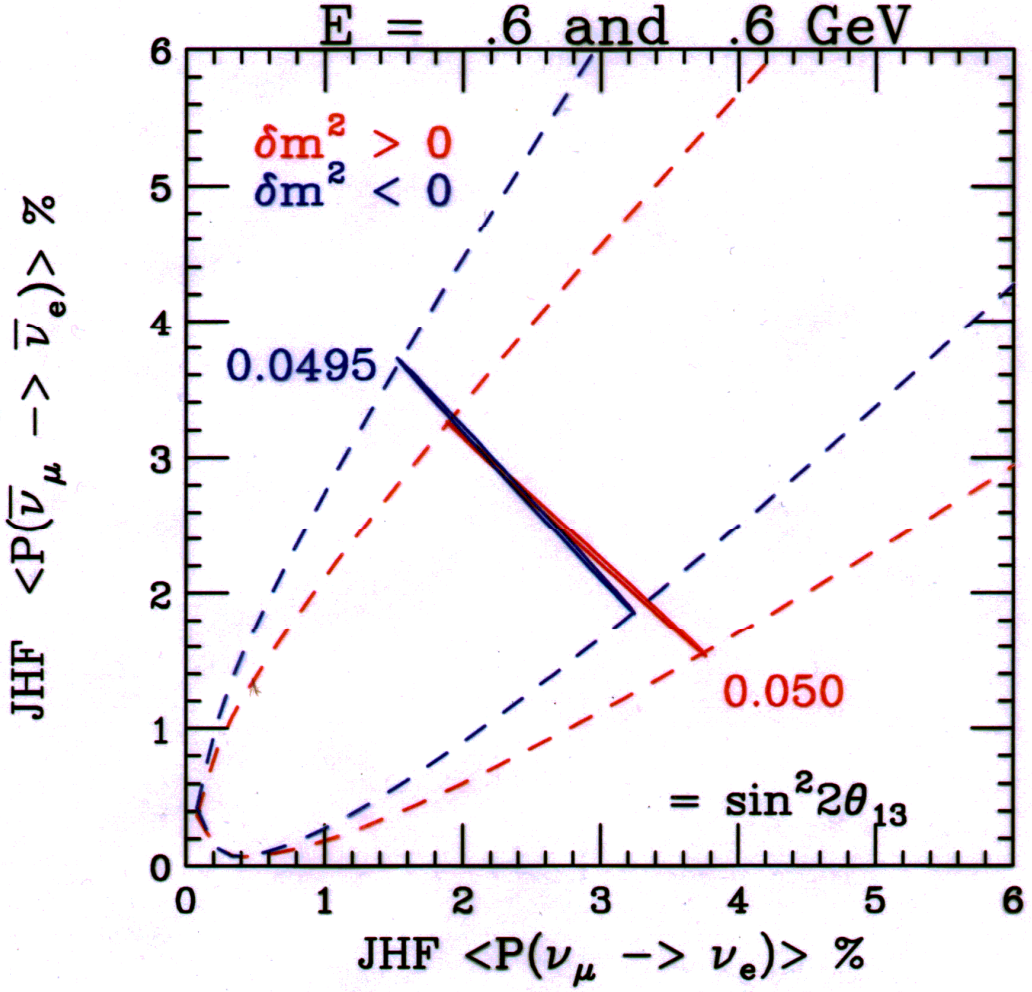


if we're lucky, hierarchy can be determined. for large values of $\sin^2 \theta_{13}$.

Magic of Vac. Osc. Max. ($\Delta_{32} = \pi/2$)

Kayser, Murakata, Nunokawa

$\sin^2\theta_{23} \equiv \frac{1}{2}$



$\nu + \bar{\nu}$ Can Measure

$\sin^2 2\theta_{13}$ { almost independent of δm^2_{31}

$\sin\delta$ { two sol^n dependent on hierachy

$$\cos^2\theta_{13} \approx 1$$

$$P^{VAC}_{\nu_e \to \nu_e} = 4\left(\sin\theta_{23}\sin\theta_{13}\right)^2 \sin^2\Delta_{31}$$

$$+ 8\sin\theta_{12}\cos\theta_{12}\left(\sin\theta_{23}\sin\theta_{13}\right)$$

$$\left\{\cos\Delta_{32}\left(\cos\theta_{23}\cos\delta\right) \mp \sin\Delta_{32}\left(\cos\theta_{23}\sin\delta\right)\right\}$$

$$+ \underline{\underline{P_\odot}} \quad \text{tiny}$$

Scaled variables:

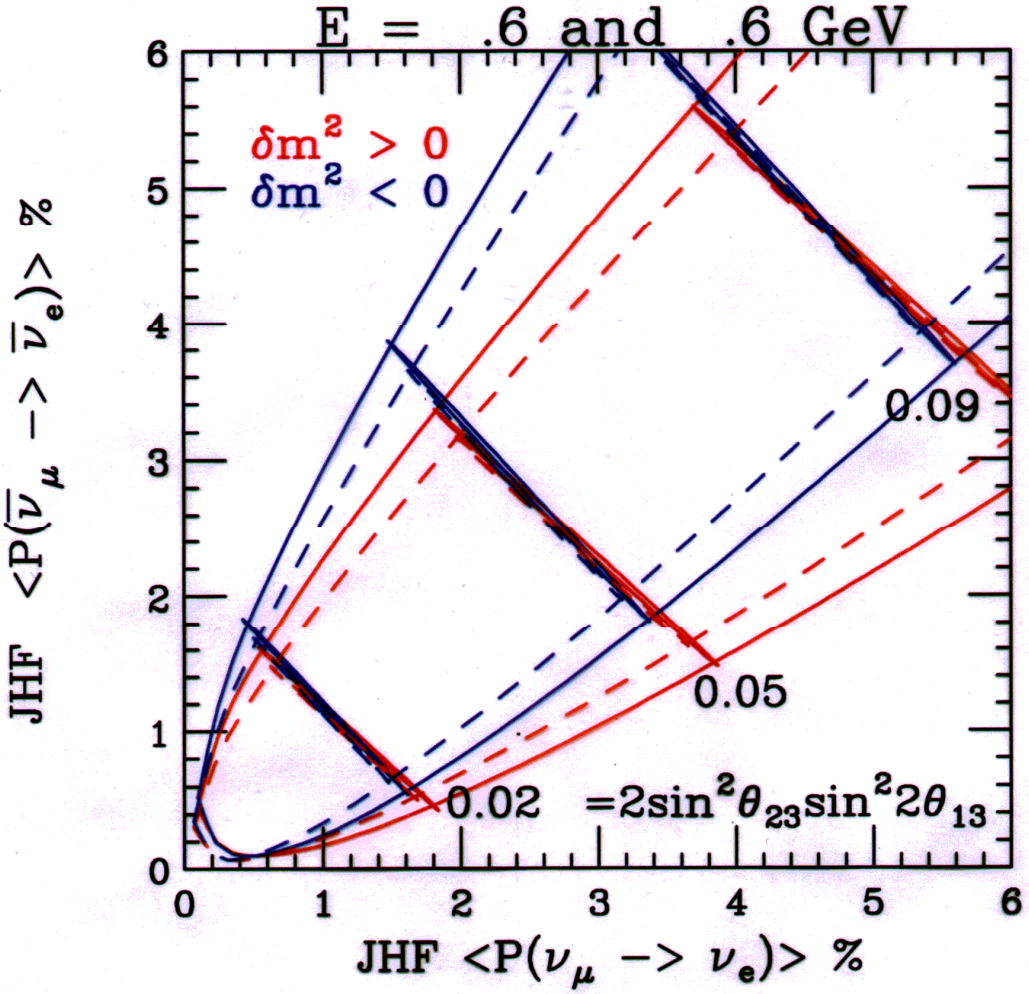
$$\sin\theta_{23}\sin\theta_{13}$$

$$\cos\theta_{23}\cos\delta$$

$$\cos\theta_{23}\sin\delta$$

Vac. Osc. Max.

$\sin^2 2\theta_{23} = 0.96$ —— $\sin^2\theta_{23} = 0.4$
---- $\sin^2\theta_{23} = 0.6$



Scaled Variables:

Consistency Reactor & LBL?

$-2\sin^2\theta_{23}\sin^2 2\theta_{13}$ { ν independent of hierachy

$\sqrt{2}\cos\theta_{23}\sin\delta$ { dependent on hierachy 2 Soln

Sign $\cos\delta$ cannot be determined at Vac. Osc. Max

Degeneracy Tasting:

sign δm^2_{32}

At Vac. Osc. Max., $\Delta_{32} = \frac{\pi}{2}$

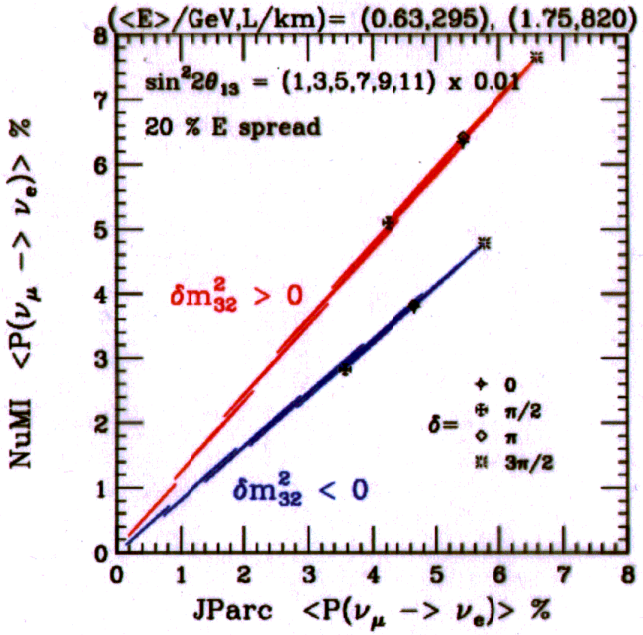
$$P_{mat} = \left(1 \pm 2\frac{E}{E_R}\right) P_{vac}$$

where $E_R \simeq 12$ GeV.

Therefore, if NuMI and JParc both run Neutrinos at Vac. Osc. Max.

$$P_N = \left(1 \pm 2\frac{(E^N - E^J)}{E_R}\right) P_J$$

i.e. $P_N \approx (1.2 \text{ or } 0.8) P_J$



Need about 100 events each:

Separation degraded for $E^N > E_{vom}$.

AND ellipses become FAT

Minakata, Nunokawa and SP – hep-ph/0301210

sign δm^2_{32} conti.

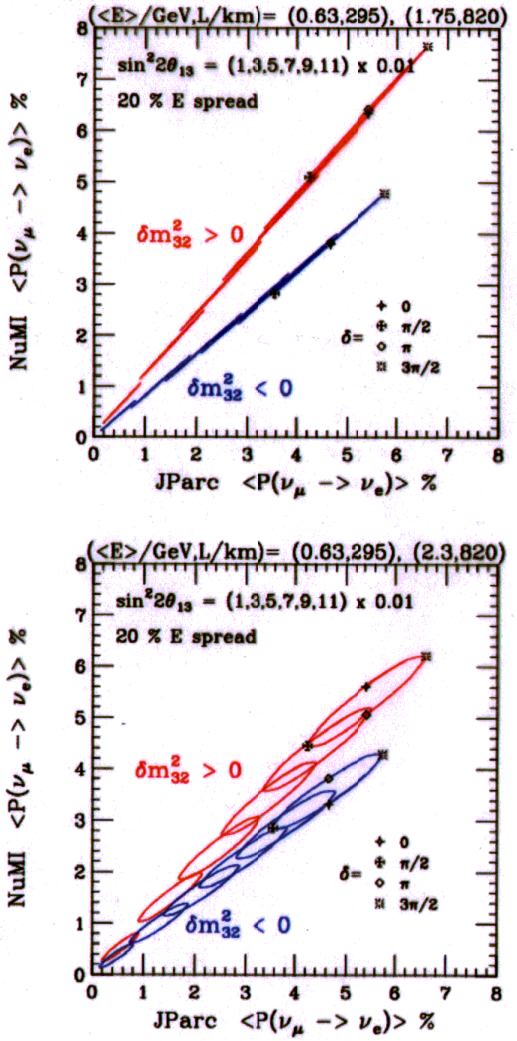
For NuMI above Vac. Oscillation Maximum, $E = E_{vom} + \Delta E$

$$P_{mat} = \left(1 \pm 2\frac{E_{vom}}{E_R} \mp \frac{(\pi^2-4)}{2}\,\frac{\Delta E}{E_R}\right) P_{vac}$$

where $E_R \simeq 12$ GeV.

This extra factor $\sim 3\,\frac{\Delta E}{E_R}$ degrades the separation of the center of the ellipses.

Also the ellipses become FAT, compare bottom to top figures.



SUMMARY & CONCLUSIONS:

A measurement of $\sin^2\theta_{13}$ by Reactor Expts would allow LBL Off-Axis Expts to concentrate on the fun stuff:

Leptonic CPV ($\sin\delta$)

determination of

Mass Hierarchy ($\text{sign}\,\delta m^2_{31}$)

$\sin^2\theta_{23}$ ($\theta_{23} \Leftrightarrow \pi/2 - \theta_{23}$)

for generic values of $\sin\delta$ best done at Vac. Osc. Maximum. ($\frac{\delta m^2_{ATM} L}{4E} = \frac{\pi}{2}$)

However there are special values near

$$\sin\delta = -(\text{sign}\,\delta m^2_{31})\,1.$$

The final degeneracy $\text{sign}\cos\delta$ requires an experiment above Vac. Osc. Max. with energy ν