Jiro Kodaira (1951—2006)

‘97

‘97

‘97

‘85

Top Quark Pair Production at Colliders

Stephen Parke & Jiro Kodaira ♣

with M. Hori, Y. Kiyo, K. Morii, T. Nasuno

♣ Loops & Legs 2000

DESY Zeuthen

Top Quark Decay: $M_t \approx 170$ GeV

$$\Gamma_t \propto G_F M_t^3 \sim 2\ GeV \gg \Lambda_{QCD} \sim 100\ MeV \gg \frac{\Lambda_{QCD}^2}{M_t} \sim 0.1\ MeV$$

$\Rightarrow$ Top Decays

before it hadronizes.

before it's spin is de-correlated.

Therefore the non-trivial spin correlations produced in production result in angular correlations between the decay products.

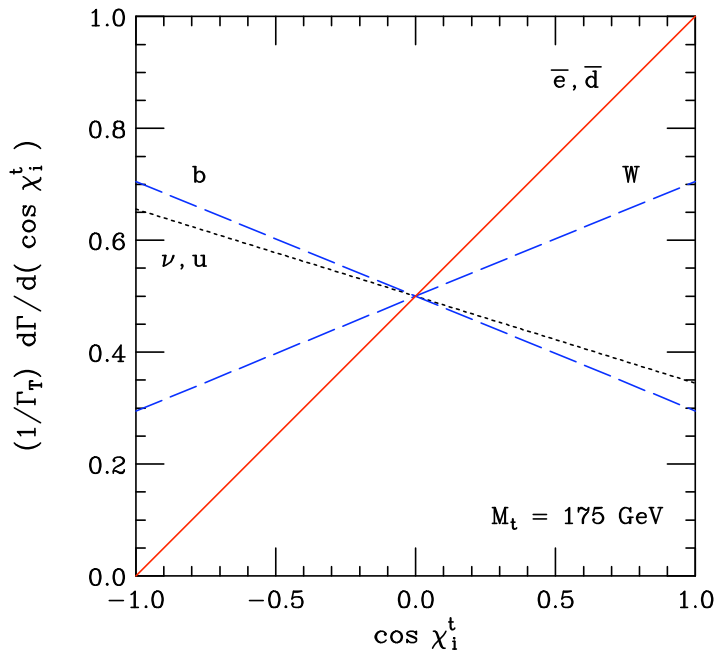
Top is UNIQUE amongst the quarks.

Top Decay Correlations:

Jeżabek and Kühn

$$\frac{1}{\Gamma_T}\,\frac{d\,\Gamma}{d\cos\chi_i^t} \;=\; \frac{1}{2}\left[1+\alpha_i\cos\chi_i^t\right]\,,$$

$$\begin{aligned}
\alpha_{e^+} &= \alpha_{\bar{d}} \equiv 1 \\
\alpha_W &= -\alpha_b = \frac{m_t^2-2m_W^2}{m_t^2+2m_W^2} \approx 0.41 \\
\alpha_\nu &= \alpha_u \approx -0.31
\end{aligned}$$

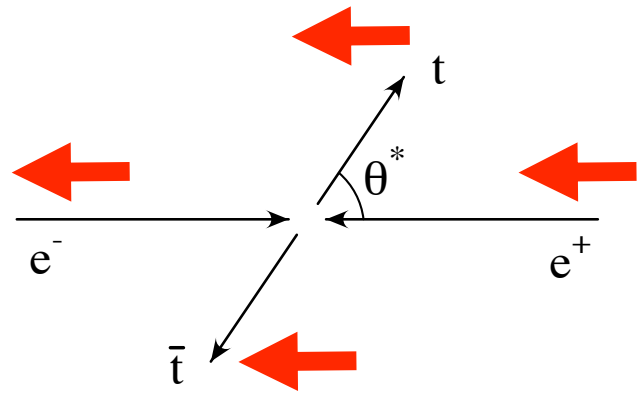


note: charge leptons are more correlated than the W-bosons !!!

Top Production at ILC: $e_R^+ e_L^- \longrightarrow t_s \bar{t}_{\bar{s}}$

Top Production at ILC: $e_R^+ e_L^- \longrightarrow t_s \bar{t}_{\bar{s}}$

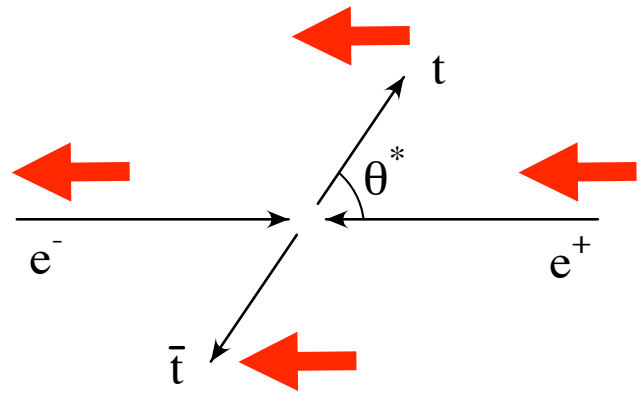
Near Threshold ($\gamma \sim 1$)



top spins aligned along beamline.

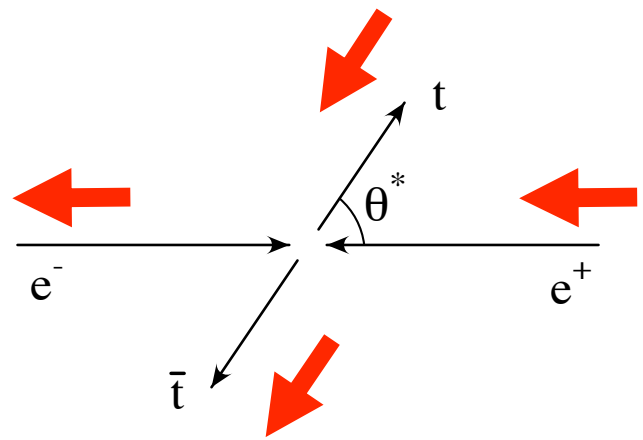
Top Production at ILC: $e_R^+ e_L^- \to t_s \bar{t}_{\bar{s}}$

Near Threshold ($\gamma \sim 1$)



top spins aligned along beamline.

Ultra-High Energies ($\gamma \gg 1$)

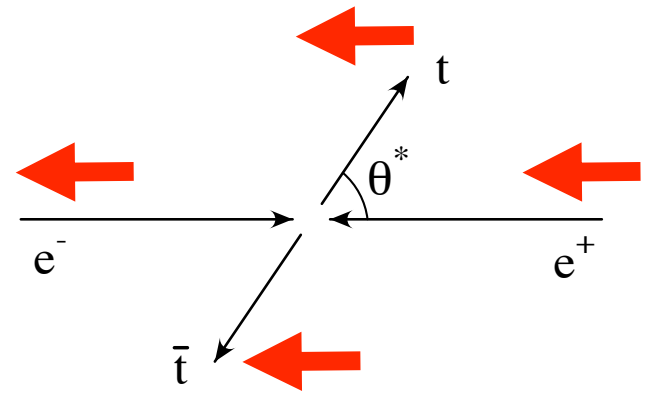


(LR $\gg$ RL)

top spins aligned along direction of motion, helicity.

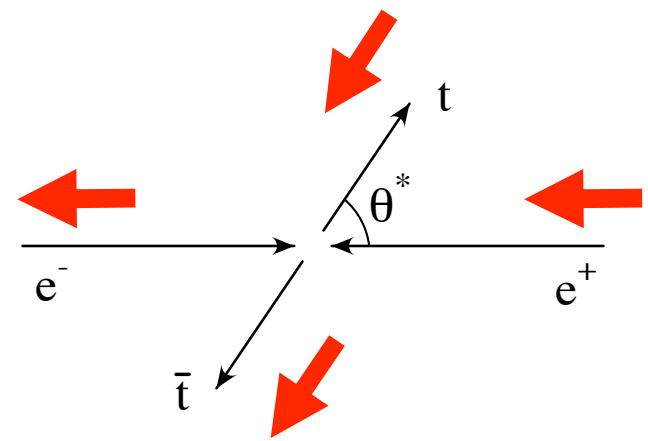
Top Production at ILC: $e^+_R e^-_L \to t_s \bar{t}_{\bar{s}}$

Near Threshold ($\gamma \sim 1$)



top spins aligned along beamline.

Ultra-High Energies ($\gamma \gg 1$)



(LR $\gg$ RL)

top spins aligned along direction of motion, helicity.

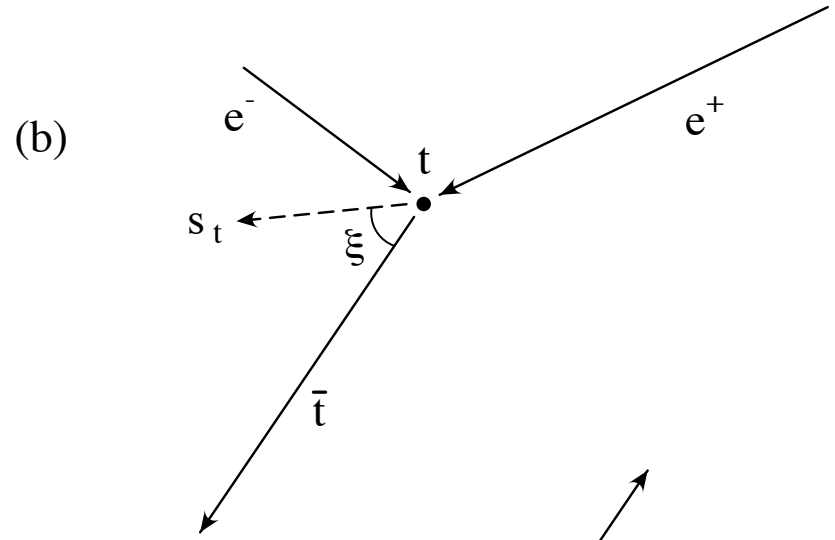
What about intermediate γ's ?

$M_{\rm top} = 175{\rm GeV} \quad \beta = 0.484$ for $\sqrt{s} = 400{\rm GeV}$

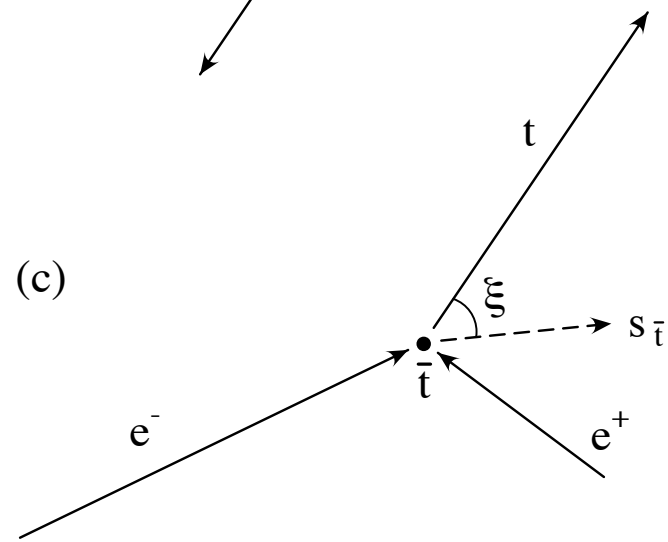
Y. Shadmi & SP hep-ph/9606419

Generalized Spin Basis.

Top Rest Frame



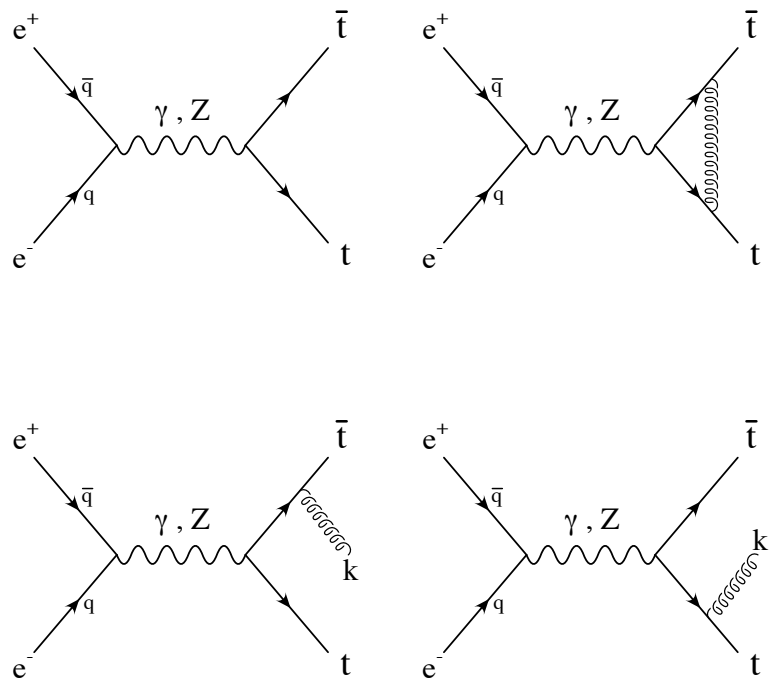
Anti-Top Rest Frame



s_t and $s_{\bar{t}}$ are back to back in ZMF.

♣ Spin Correlation in SM

Diagrams:



♡ Tree-level and Opitmal Spin Basis

$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to t_\uparrow \bar{t}_\uparrow) = \frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to t_\downarrow \bar{t}_\downarrow)$$
$$= \left(\frac{3\pi\alpha^2}{2s}\beta\right) |(A_{LR}\cos\xi - B_{LR}\sin\xi)|^2$$

$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to t_\uparrow \bar{t}_\downarrow \text{ or } t_\downarrow \bar{t}_\uparrow)$$
$$= \left(\frac{3\pi\alpha^2}{2s}\beta\right) |(A_{LR}\sin\xi + B_{LR}\cos\xi \pm D_{LR})|^2$$

Parameters:

$$\begin{aligned}
A_{LR} &= [(f_{LL} + f_{LR})\sqrt{1-\beta^2}\,\sin\theta]/\,2 \\
B_{LR} &= [f_{LL}(\cos\theta + \beta) + f_{LR}(\cos\theta - \beta)]/\,2 \\
D_{LR} &= [f_{LL}(1 + \beta\cos\theta) + f_{LR}(1 - \beta\cos\theta)]/\,2 \\
f_{IJ} &= -Q_t + Q_I^e\, Q_J^t\, \frac{1}{\sin^2\theta_W}\, \frac{s}{s - M_Z^2}
\end{aligned}$$

- Helicity Basis

$$\xi = \pm\pi$$

Nothing Interesting Happens !!

- Off - Diagonal Basis

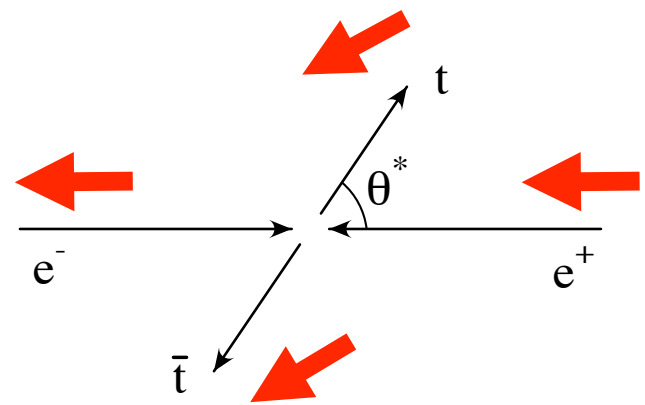
$$\tan\xi = \frac{A_{LR}}{B_{LR}} = \frac{(f_{LL}+f_{LR})\sqrt{1-\beta^2}\,\sin\theta}{f_{LL}(\cos\theta+\beta)+f_{LR}(\cos\theta-\beta)}$$

$$\Downarrow$$

$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to t_\uparrow \bar{t}_\uparrow \text{ and } t_\downarrow \bar{t}_\downarrow) = 0$$

$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to t_\uparrow \bar{t}_\downarrow \text{ or } t_\downarrow \bar{t}_\uparrow)$$

$$= \left(\frac{3\pi\alpha^2}{2s}\beta\right)\left|\left(\sqrt{A_{LR}^2+B_{LR}^2} \mp D_{LR}\right)\right|^2$$



Numerically $|f_{LL}| \gg |f_{LR}|$

$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to t_\downarrow \bar{t}_\uparrow) \simeq 0$$

$UU = DD = 0$ and $DU \gg UD$

♡ QCD One-Loop Calculation

Results are Modified by Two plus one Effects

- Modified Magnetic Moment from Vertex Correction
- Spin Flip Effect from Real Gluon Emission
- Top and Anti-top are not Produced Back-to-Back

Calculate Single Spin Correlation

$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to t_\uparrow X)$$

Numerical Results I

$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to t_\uparrow X)$$
$$= (1+\kappa)\frac{d\sigma^0}{d\cos\theta}(e_L^- e_R^+ \to t_\uparrow X) + \frac{d\sigma^R}{d\cos\theta}(e_L^- e_R^+ \to t_\uparrow X)$$

$\sigma_{\rm tot}$ and K-factor

$\sqrt{s}$ (GeV)	α_s	$\sigma^0_{\rm tot}$ (pb)	$\sigma^1_{\rm tot}$ (pb)	κ
400	0.098	0.871	1.113	0.278
800	0.091	0.353	0.373	0.057

Numerical Results II

$$\frac{\frac{d\sigma^R}{d\cos\theta}(e_L^- e_R^+ \to t_\uparrow X)}{(1+\kappa)\frac{d\sigma^0}{d\cos\theta}(e_L^- e_R^+ \to t_\uparrow X)} < 10\% \sim \text{a few } \%$$

Tiny Spin Flip Effect $\longrightarrow$ Soft Gluon Dominates

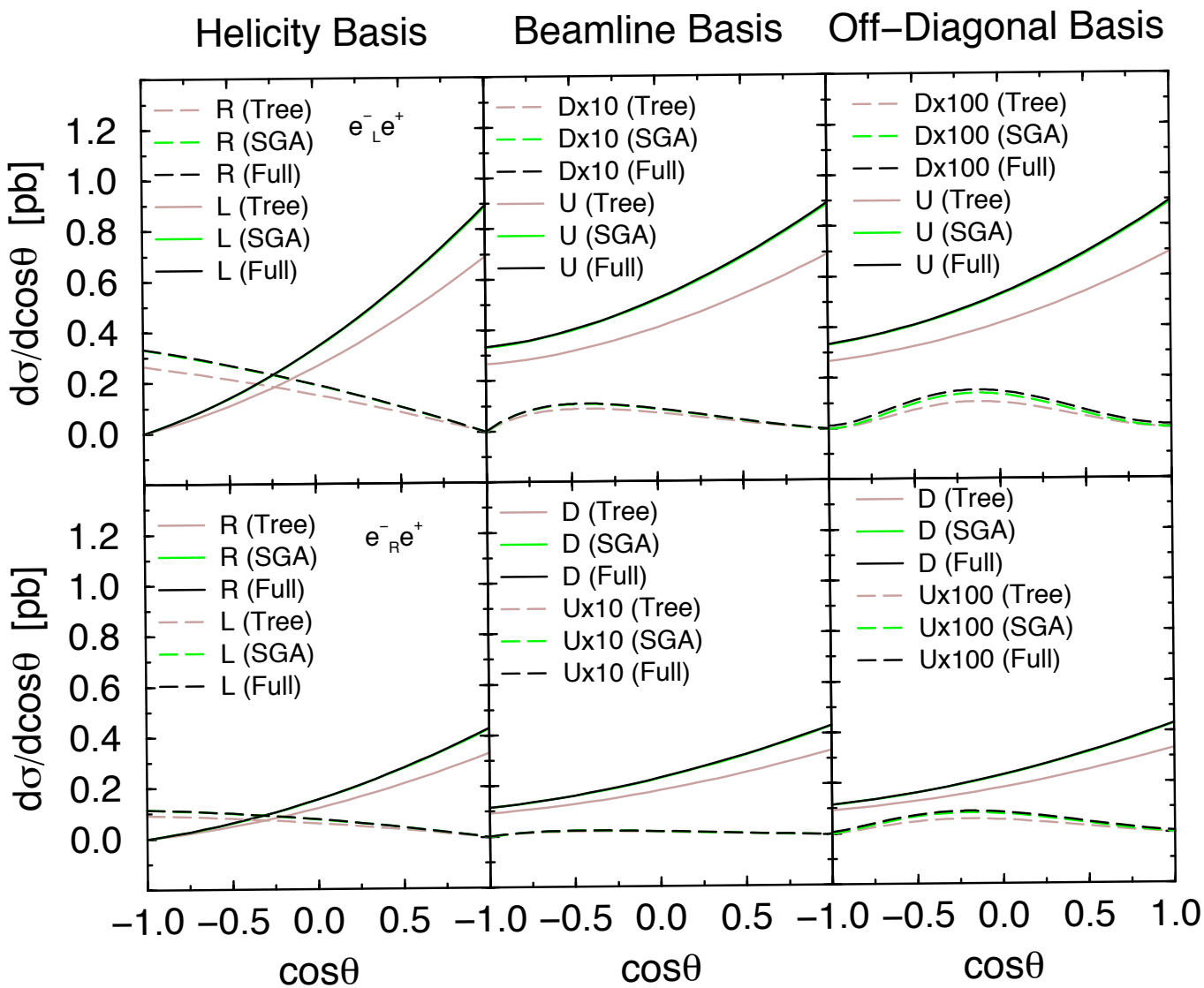
Numerical Results III

Fraction of σ_{tot} in Dominant Spin

$\sqrt{s}$ (GeV)	β	Helicity	Off-Diagonal
400	0.484	0.6636	0.9988
		0.6681	0.9985
800	0.899	0.8318	0.9735
		0.8350	0.9682

- QCD Corrections to Spin Correlation VERY SMALL
  - Main Effect is K-factor
  - Does NOT change Spin Configuration
  - SGA is an Excellent Approximation (See Figs.)
- Off-Diagonal Basis is a Good Choice for $\beta \sim 0.5$
  - Some Spin States are Zero or Negligible

Cross Section at $\sqrt{s} = 400\text{GeV}$



♣ Anomalous Couplings

Assume the Anomalous Couplings for $t - \bar{t} - \gamma/Z$

$$\Gamma^V_\mu = g^V \left\{ \gamma^\mu \left[Q^V_L \omega_- + Q^V_R \omega_+ \right] + \frac{(t-\bar{t})_\mu}{2m} \left[G^V_L \omega_- + G^V_R \omega_+ \right] \right\}$$

- QCD at One Loop (Note SGA is Enough)

$$\begin{aligned} Q^V_{LR} &\sim 1 + \mathcal{O}(\alpha_s) \\ G^V_R + G^V_L &\sim \mathcal{O}(\alpha_s) \quad , \quad G^V_R - G^V_L = \mathcal{O}(\alpha_s^2) \sim 0 \end{aligned}$$

- CP Odd Anomalous Coupling

$$G^V_R - G^V_L = i\, f^V_3 \neq 0 \sim \mathcal{O}(\alpha_s)$$

♣ How to Detect Anomalous Coupling

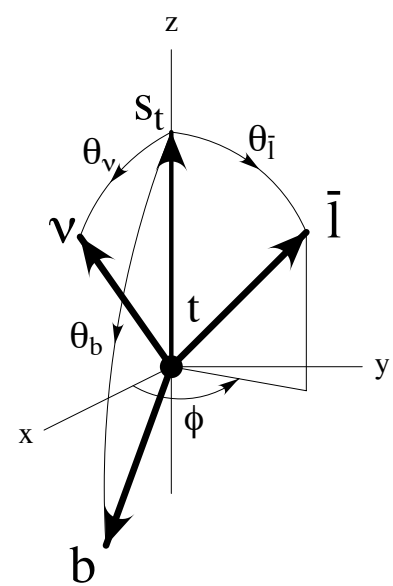
- Cross Section Itself $\leftrightarrow$ No
- Angular Distribution of Decay Product

Decay Density Matrix for Charged Lepton (with NWA)

$$\frac{d\Gamma_{s_t,s_t'}}{\Gamma_T} \propto \begin{pmatrix} 1+\cos\theta_{\bar{l}} & \sin\theta_{\bar{l}} e^{i\phi_l} \\ \sin\theta_{\bar{l}} e^{-i\phi_l} & 1-\cos\theta_{\bar{l}} \end{pmatrix}_{s_t s_t'} \left[\frac{d\cos\theta_{\bar{l}}\, d\phi_l}{4\pi}\right]$$

Consider , *e.g.*, the Chain

$$t \to b\,W \to b\,\bar{l}\,\nu$$



in Top Rest Frame with $x - z$ the Production Plane

♣ Sample Quantities of ϕ Dependence

Combine the Decay with the Production Amplitudes

$\downarrow$

Obtain Angular distributions

- $\theta_{\bar{l}} - \phi$ Correlations
- Partially Integrated Asymmetry

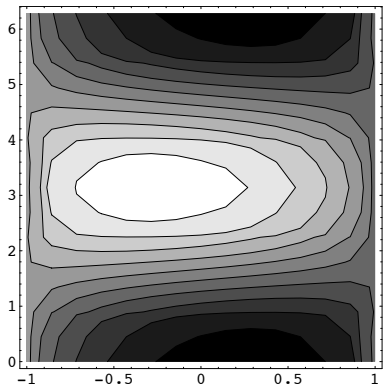
$$\sigma^1 = \int_0^{\pi} d\phi_l \left(\frac{d\sigma}{d\cos\Theta d\phi_l} \right) \ , \ \sigma^2 = \int_{-\pi}^{0} d\phi_{\bar{l}} \left(\frac{d\sigma}{d\cos\Theta d\phi_l} \right)$$

and

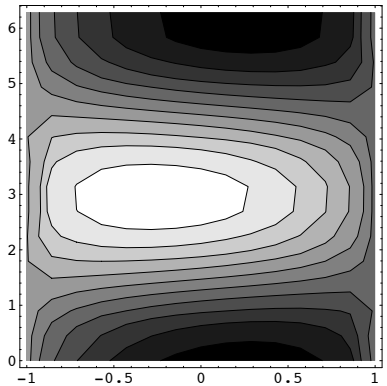
$$\mathcal{A}(\Theta) = \frac{\sigma^2 - \sigma^1}{\sigma^2 + \sigma^1}$$

with and without Anomalous Couplings

Helicity Basis at $\cos\Theta = 0.5$

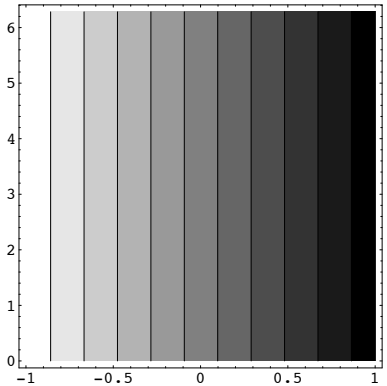


QCD 1 - Loop

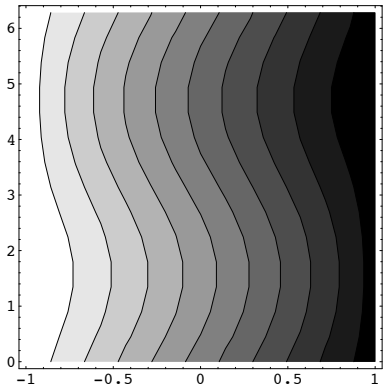


QCD 1 - Loop + Anomalous Coupling $f_3^\gamma = -f_3^Z = 0.2$

Off- Diagonal Basis at $\cos\Theta = 0.5$

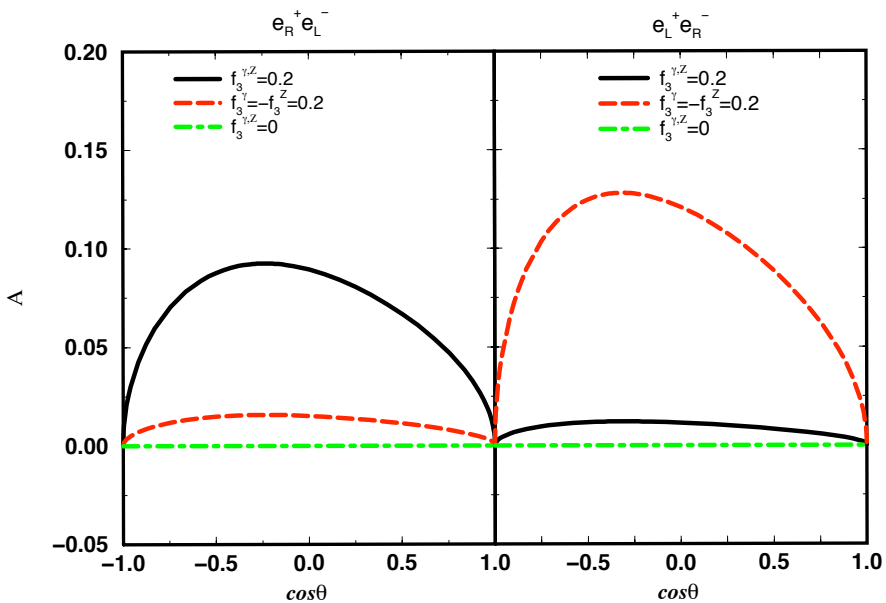


QCD 1 - Loop



QCD 1 - Loop + Anomalous Coupling $f_3^\gamma = -f_3^Z = 0.2$

Partially Integrated Asymmetry



- QCD Corrections to Spin Correlation VERY SMALL
  - Main Effect is K-factor
  - Does NOT change Spin Configuration
- Helicity Basis is a Poor Choice for $\beta \sim 0.5$
  - All Spin States Contribute
  - Correlations between Decay Products and Top Spin Very POOR
    Significant Interference between Intermediate Spin States
- Off-Diagonal Basis is a GOOD Choice for $\beta \sim 0.5$
  - Some Spin States are Zero or Negligible
  - Correlations between Decay Products and Top Spin Very STRONG
    Much Smaller Interference between Intermediate Spin States

‘95

PHYSICAL REVIEW D, VOLUME 58, 014005

Optimal spin basis in polarized photon linear colliders

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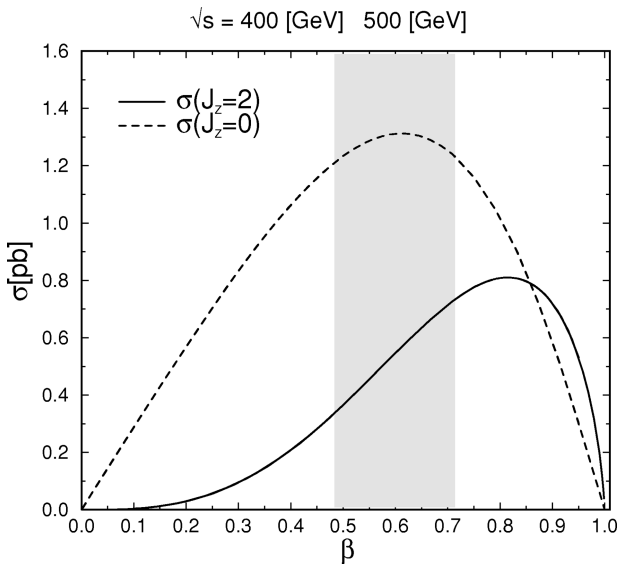
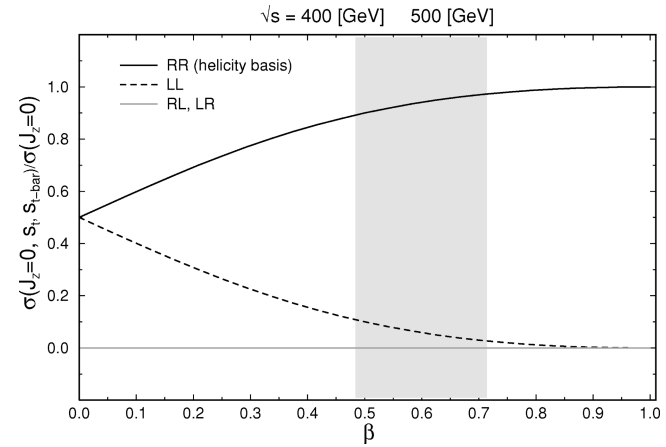
(Received 17 December 1997; published 20 May 1998)

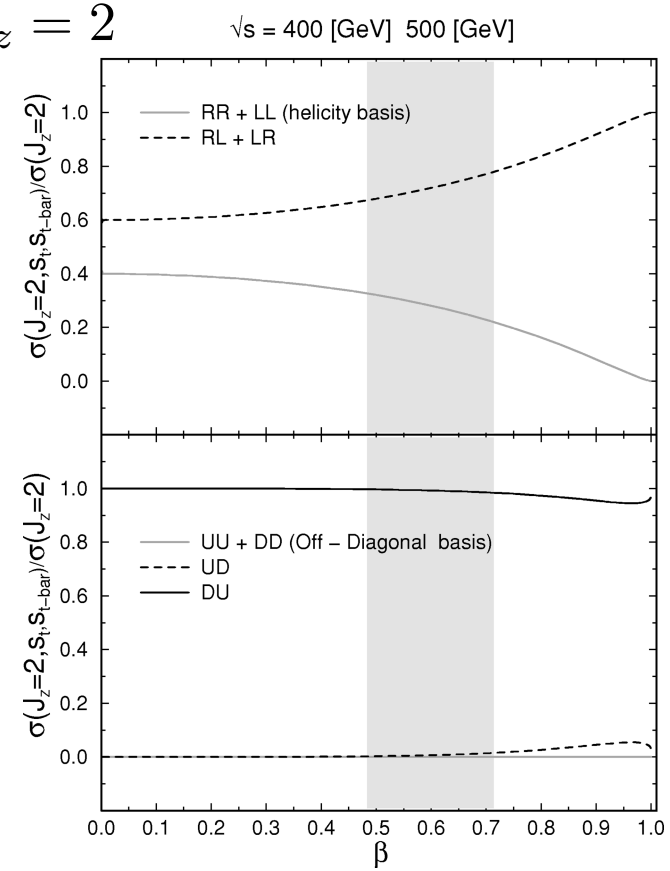


$J_z = 0$



RR

$J_z = 2$



DU

Now since $d\sigma(gg \to t\bar{t}) = \mathcal{Y}(\beta, \theta^*)\ d\sigma(\gamma\gamma \to t\bar{t})$

$\Rightarrow$ LHC !!!

SPINOR IDENTITY:

Mass couples L to R:

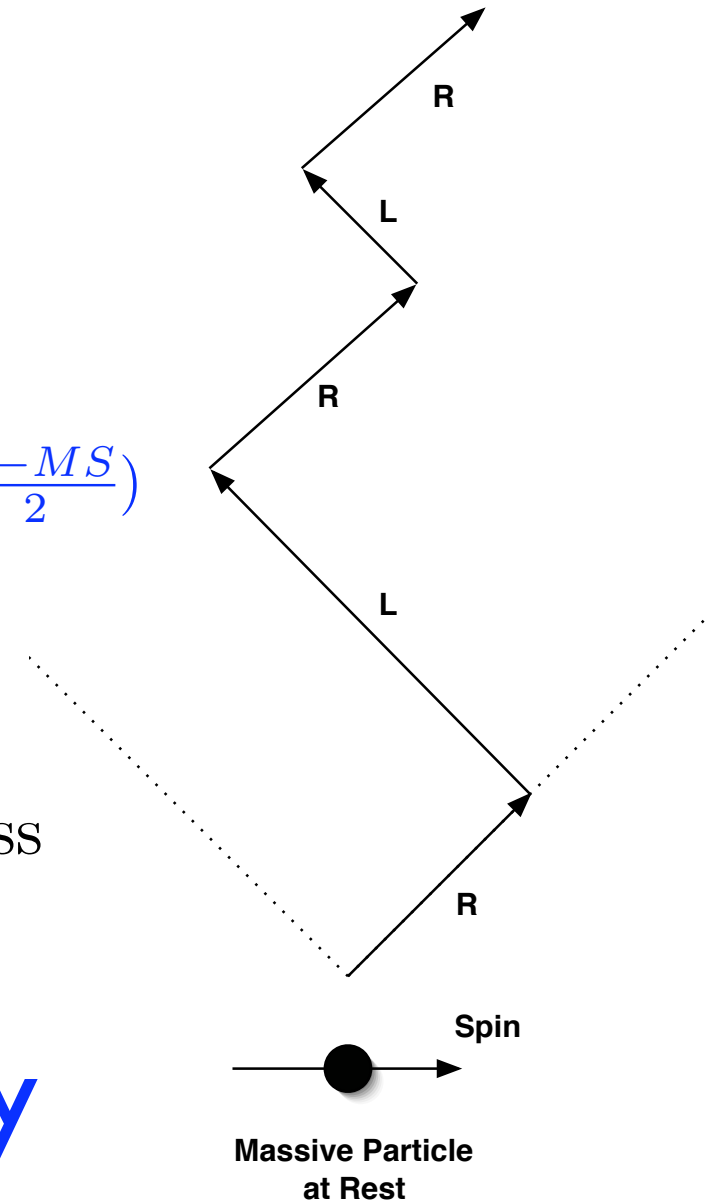
$$P^2 = M^2, \quad P \cdot S = 0 \quad and \quad S^2 = -1$$

therefore $(P \pm MS)^2 = 0$

$$u(P,S) = \frac{(1+\gamma_5)}{2}\, u\left(\frac{P+MS}{2}\right) \quad + \quad e^{-i\phi}\, \frac{(1-\gamma_5)}{2}\, u\left(\frac{P-MS}{2}\right)$$

right massless

left massless



simple, elegant proof by Kodaira-san:

Kodaira-san < July 13, 1998

Spinor Helicity Method for Massive Spinor

Massive Spinor Can be written in terms of
Massless Chiral Spinor

For Spinor with Spin *Bjorken and Drell*

$$s^2 = -1 \quad , \quad s \cdot p = 0$$

$$\begin{aligned} u(p) &= u_\uparrow(p) + u_\downarrow(p) \\ &= \frac{1+\gamma_5 \not{s}}{2} u(p) + \frac{1-\gamma_5 \not{s}}{2} u(p) \end{aligned}$$

e.g. for $u_\uparrow(p)$

$$\begin{aligned} u_\uparrow(p) &= \frac{1+\gamma_5}{2} \frac{1+\gamma_5 \not{s}}{2} u(p) + \frac{1-\gamma_5}{2} \frac{1+\gamma_5 \not{s}}{2} u(p) \\ &= \frac{1+\gamma_5}{2} \frac{1+\not{s}}{2} u(p) + \frac{1-\gamma_5}{2} \frac{1-\not{s}}{2} u(p) \end{aligned}$$

now

$$Q(\pm) \equiv \frac{1 \pm \not{s}}{2} u(p)$$

are Massless Spinor with Momentum t_1 , t_2

$$t_{1,2} = \frac{p \pm m\, s}{2} \quad , \quad t_{1,2}^2 = 0$$

$\hookleftarrow$

$$\begin{aligned} Q(\pm)\, \overline{Q}(\pm) &= \frac{1 \pm \not{s}}{2} u(p)\, \bar{u}(p) \frac{1 \pm \not{s}}{2} \\ &= \frac{1 \pm \not{s}}{2} (\not{p} + m) \frac{1 \pm \not{s}}{2} = \frac{\not{p} \pm m \not{s}}{2} \end{aligned}$$

with

$$\frac{\not{p} \pm m \not{s}}{2} Q(\pm) = 0$$

Massive Spinor Decomposition

Above Consideration Tells us

$$\begin{aligned} u_\uparrow(p) &= \frac{1+\gamma_5}{2} Q(t_1) + e^{i\phi} \frac{1-\gamma_5}{2} Q(t_2) \\ &\equiv |t_1 +\rangle + e^{i\phi} |t_2 -\rangle \end{aligned}$$

ϕ is Determined by Normalization

$$\bar{u}_\uparrow(p)\, u_\uparrow(p) = 2\, m$$

$\hookrightarrow$

$$e^{i\phi} = \frac{\langle t_2 - | t_1 +\rangle}{m}$$

with

$$|\langle t_2 - | t_1 +\rangle|^2 = 2\, t_1 \cdot t_2 = m^2$$

$$\begin{aligned} u_\uparrow(p) &= |t_1 +\rangle + |t_2 -\rangle \frac{\langle t_2 - | t_1 +\rangle}{m} \\ u_\downarrow(p) &= |t_1 -\rangle + |t_2 +\rangle \frac{\langle t_2 + | t_1 -\rangle}{m} \\ \bar{u}_\uparrow(p) &= \langle t_1 + | + \frac{\langle t_1 + | t_2 -\rangle}{m} \langle t_2 - | \\ \bar{u}_\downarrow(p) &= \langle t_1 - | + \frac{\langle t_1 - | t_2 +\rangle}{m} \langle t_2 + | \\ v_\uparrow(\bar{p}) &= |\bar{t}_1 -\rangle - |\bar{t}_2 +\rangle \frac{\langle \bar{t}_2 + | \bar{t}_1 -\rangle}{m} \\ v_\downarrow(\bar{p}) &= |\bar{t}_1 +\rangle - |\bar{t}_2 -\rangle \frac{\langle \bar{t}_2 - | \bar{t}_1 +\rangle}{m} \\ \bar{v}_\uparrow(\bar{p}) &= \langle \bar{t}_1 - | - \frac{\langle \bar{t}_1 - | \bar{t}_2 +\rangle}{m} \langle \bar{t}_2 + | \\ \bar{v}_\downarrow(\bar{p}) &= \langle \bar{t}_1 + | - \frac{\langle \bar{t}_1 + | \bar{t}_2 -\rangle}{m} \langle \bar{t}_2 - | \end{aligned}$$

SID DRELL

1) **Qcd Higher Order Effects In Polarized Electroproduction: Flavor Singlet Coefficient Functions.**
Jiro Kodaira (Kyoto U., Yukawa Inst., Kyoto) . RIFP-381, Oct 1979. 24pp.
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2) **Qcd Higher Order Effects In Spin Dependent Deep Inelastic Electroproduction.**
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Published in **Phys.Lett.B112:66,1982**.

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