

The University of Auckland | New Zealand

YEAR  
2002

SCIENCE

# Physics

Undergraduate  
& Graduate  
Handbook

2002



Handbook

# The Neutrino Oscillation Landscape



Mt Cook, 3754m and 20km from ocean

Stephen Parke, Fermilab

$$|\nu_\alpha\rangle_{flavor} = U_{\alpha i} |\nu_i\rangle_{mass}.$$

$$\begin{pmatrix} 1 & & \\ c_{23} & s_{23} & \\ -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix}$$

Atmos. L/E  $\mu \leftrightarrow \tau$

Atmos. L/E  $e \leftrightarrow \mu, \tau$

500km/GeV

$$\begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & e^{i\alpha} \\ & 1 & e^{i\beta} \\ & & \end{pmatrix}$$

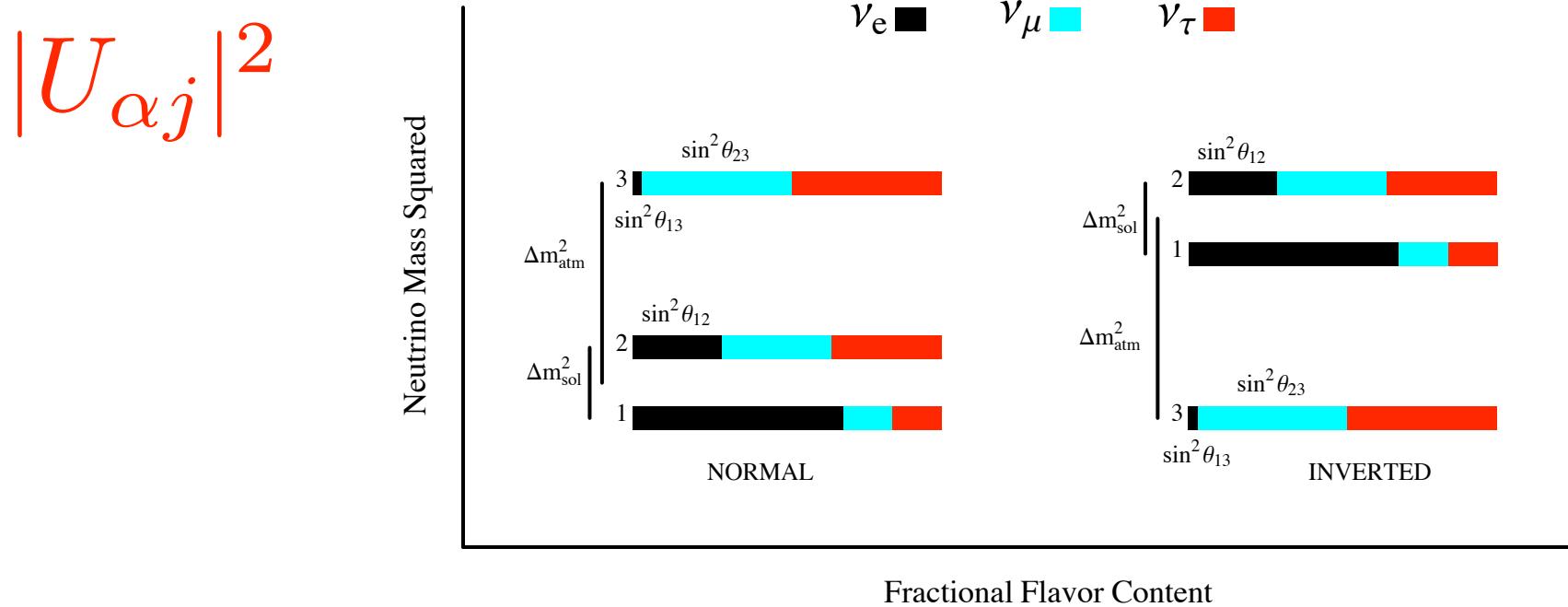
Solar L/E  $e \leftrightarrow \mu, \tau$   $\beta\beta0\nu$  decay

15km/MeV

In oscillation phenomena:

the phases  $\alpha, \beta$  are unobservable,  $U_{\sigma,k}^* U_{\rho,k}$   
and also the value of  $m_{lite}$  is irrelevant,  $\delta m^2$ .

# Solar Sector: {12}



$$\delta m_{21}^2 = 8.0 \pm 0.4 \times 10^{-5} \text{ eV}^2$$

KamLAND + SNO, SK, Ga, Cl

$$\sin^2 \theta_{12} = 0.31 \pm 0.03$$

SNO + SK, Ga, Cl, KamLAND

$$\text{SNO's } \frac{CC}{NC} \approx \sin^2 \theta_{12} \\ (0.35)$$

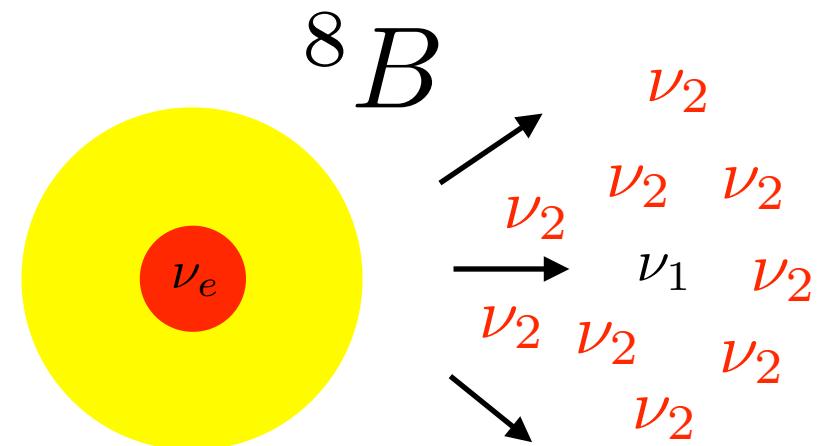
Kinematic phase:  $\Delta \equiv \frac{\delta m_\odot^2 L}{4E} = 10^{7\pm 1}$  for  ${}^8B$  to pp

mass eigenstates are “Effectively Incoherent”

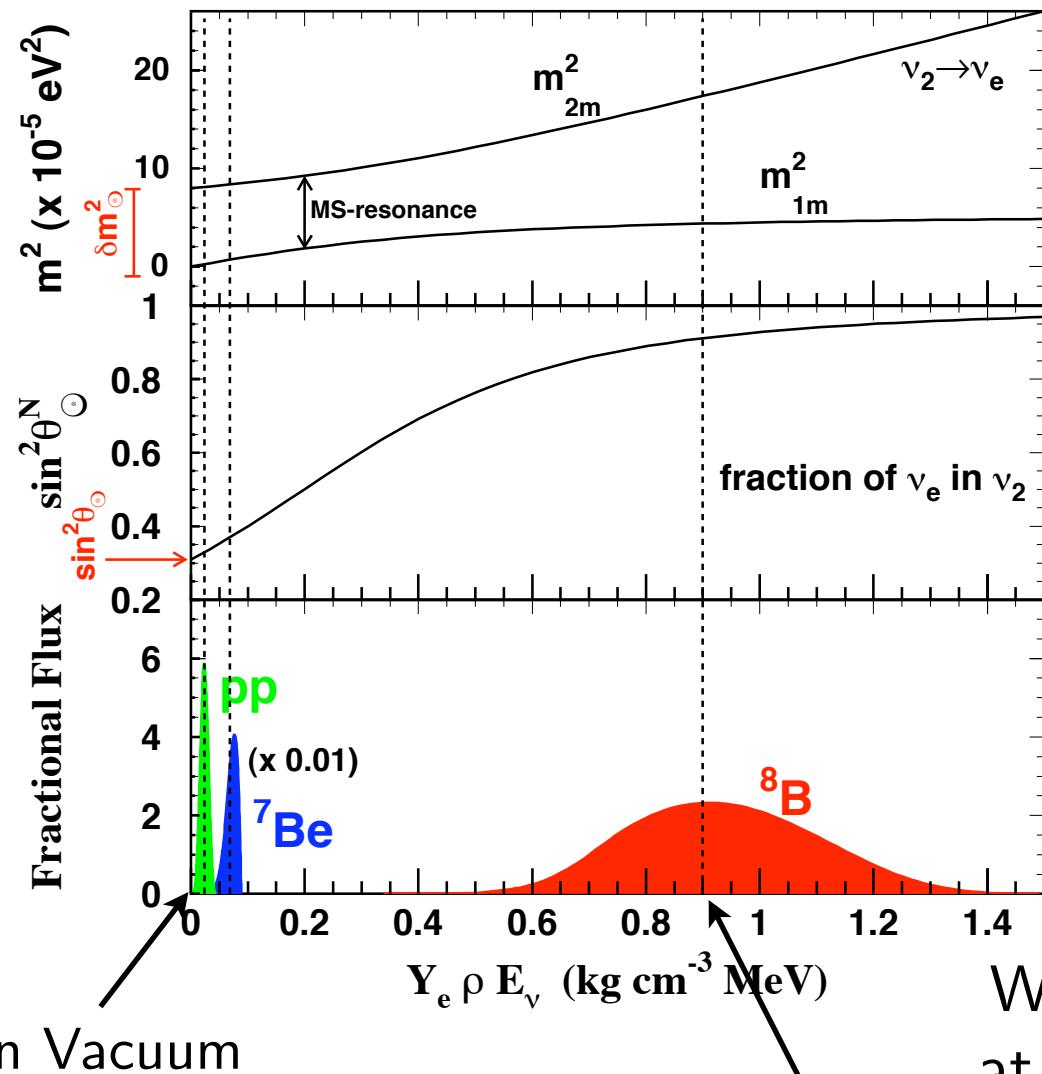
SNO's

$$\begin{aligned}\frac{CC}{NC} = \langle P_{ee} \rangle &= f_1 \cos^2 \theta_\odot + f_2 \sin^2 \theta_\odot \\ &= \sin^2 \theta_\odot + f_1 \cos^2 2\theta_\odot\end{aligned}$$

$$\begin{aligned}f_1 &= \frac{\frac{CC}{NC} - \sin^2 \theta_\odot}{\cos 2\theta_\odot} \\ &= \frac{0.35 - 0.31}{0.4} \approx 10\%\end{aligned}$$



${}^8B$  fraction of  $\nu_2$ 's is 90 % !!!



Weighted by  
detector sensitivity  
(x-section + threshold)

Whereas for  ${}^8\text{B}$   
at center of Sun

$$\begin{aligned}\delta m_N^2 &= 14 \times 10^{-5} \text{ eV}^2 \\ \sin^2 \theta_\odot^N &= 0.91\end{aligned}$$

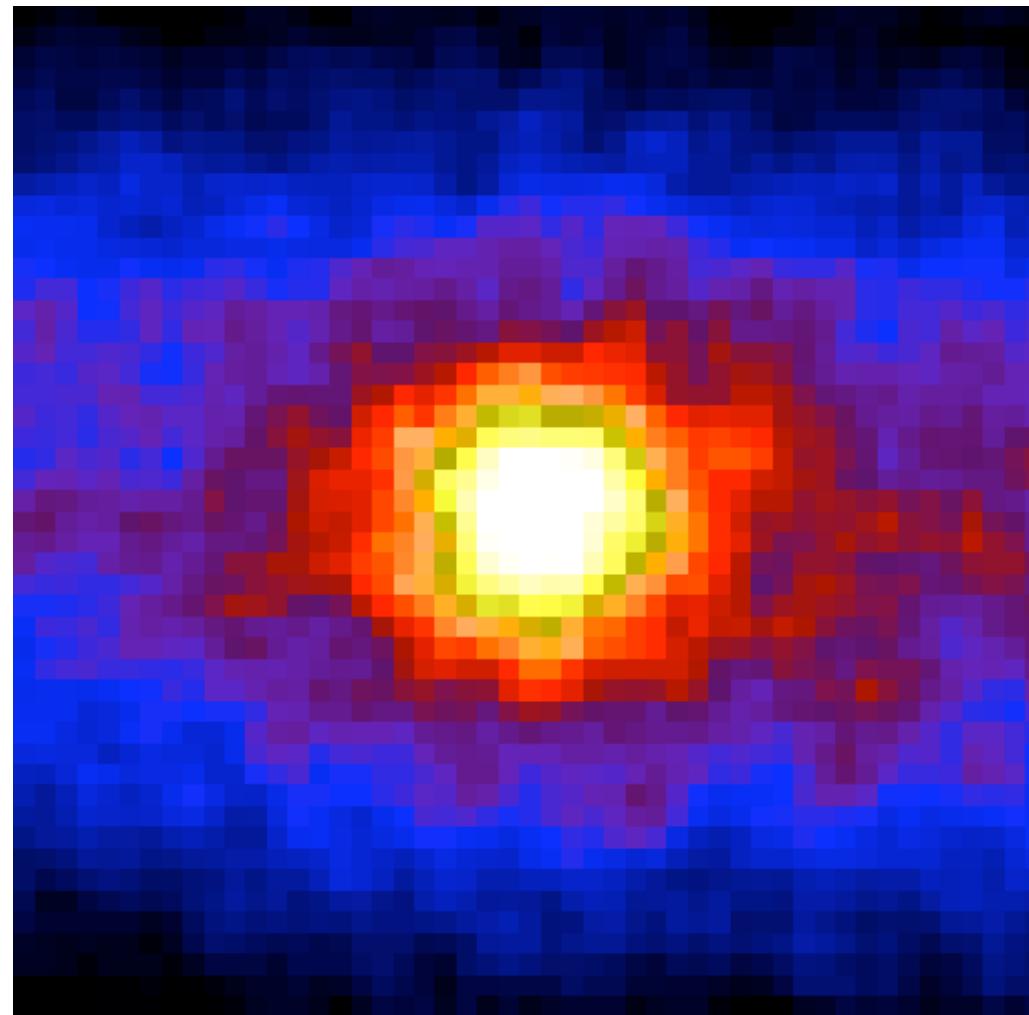
$$\delta m_\odot^2 = 8.0 \pm 0.4 \times 10^{-5} \text{ eV}^2$$

$$\sin^2 \theta_\odot = 0.31 \pm 0.03$$

$$\{\text{3-flavors: } f_3 = \sin^2 \theta_{13} : f_2 = 0.91 \pm 0.02 - \sin^2 \theta_{13} : f_1 = 0.09 \mp 0.02 \}$$

# SuperK

Flavor  
Fraction  
76%  $\nu_e$ 's



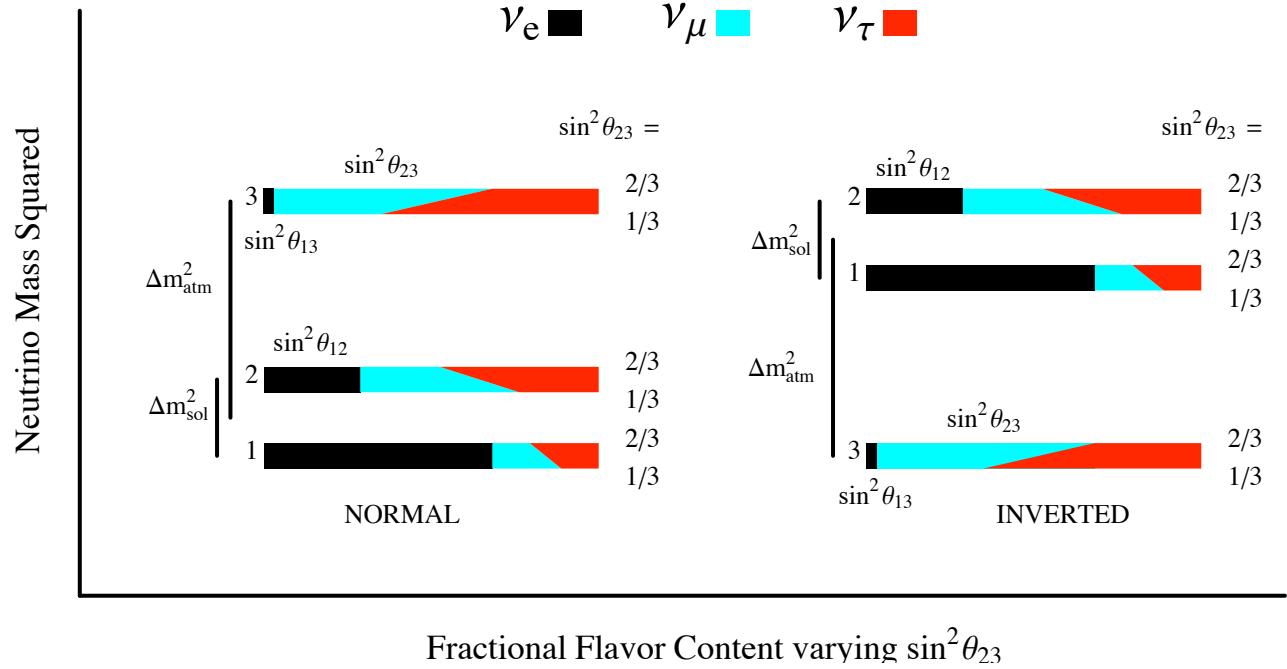
Mass E-state  
Fraction  
84%  $\nu_2$ 's



Which Neutrinos ?

# Atmospheric Sector: {23}

$$|U_{\alpha j}|^2$$



$$\sin^2 \theta_{23} = 0.50 \pm 0.14$$

SuperK

$$\delta m_{\text{atm}}^2 = 2.7^{+0.4}_{-0.3} \times 10^{-3} \text{ eV}^2$$

MINOS + SK, K2K

# Hierarchy?

Which  $\delta m_{\text{atm}}^2$  ?

kinematic phase:  
 $\Delta_{ij} \equiv \frac{\delta m_{ij}^2 L}{4E}$

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - 4|U_{\mu 2}|^2|U_{\mu 1}|^2 \sin^2 \Delta_{21} - 4|U_{\mu 3}|^2|U_{\mu 1}|^2 \sin^2 \Delta_{31} - 4|U_{\mu 3}|^2|U_{\mu 2}|^2 \sin^2 \Delta_{32}$$

# Which $\delta m_{atm}^2$ ?

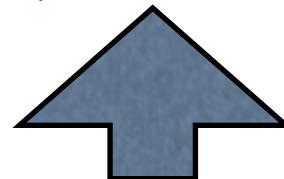
$$P(\nu_\mu \rightarrow \nu_\mu) \approx 1 - 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) \sin^2 \left( \frac{\delta m_{\mu\mu}^2 L}{4E} \right) - \mathcal{O}(\Delta_{21})^2$$

No term linear in  $\Delta_{21}$

$$\begin{aligned} \delta m_{\mu\mu}^2 &= \frac{|U_{\mu 1}|^2 |\delta m_{31}^2| + |U_{\mu 2}|^2 |\delta m_{32}^2|}{|U_{\mu 1}|^2 + |U_{\mu 2}|^2} \\ &\approx \sin^2 \theta_{12} |\delta m_{31}^2| + \cos^2 \theta_{12} |\delta m_{32}^2| \end{aligned}$$

## Physically

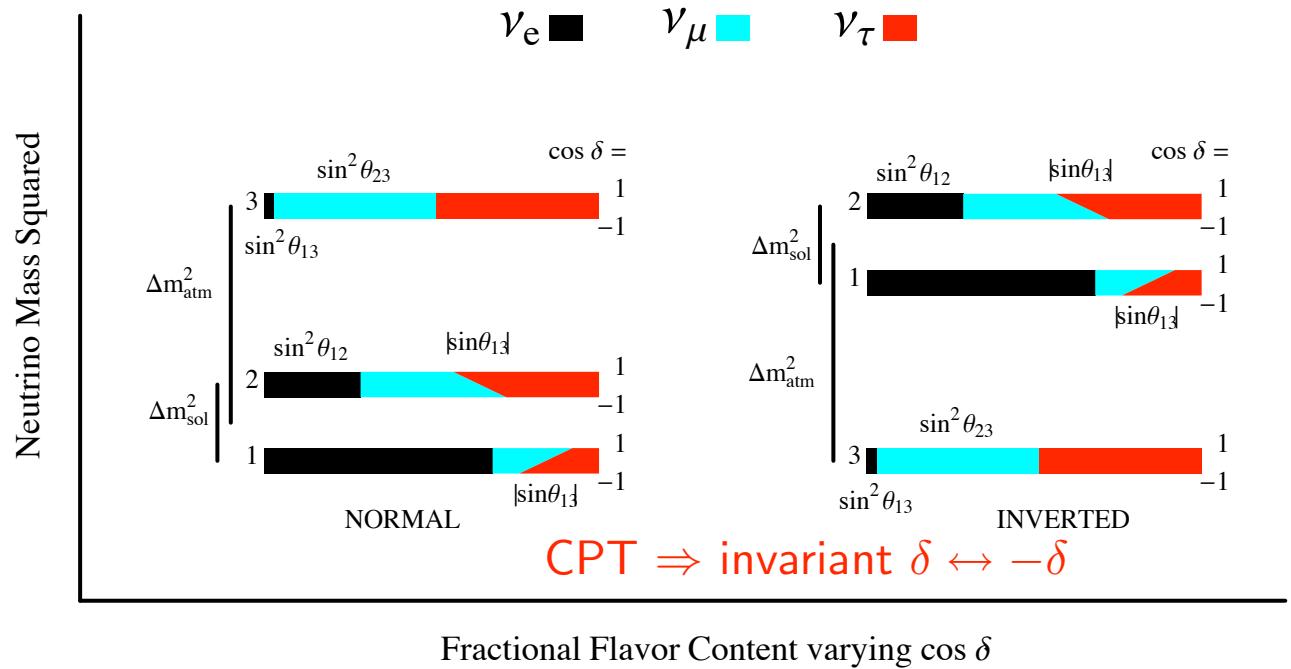
$$\delta m_{\mu\mu}^2 = \left| m_3^2 - \frac{|U_{\mu 1}|^2 m_1^2 + |U_{\mu 2}|^2 m_2^2}{|U_{\mu 1}|^2 + |U_{\mu 2}|^2} \right|$$



$\nu_\mu$  weighted average of  $m_1^2$  and  $m_2^2$

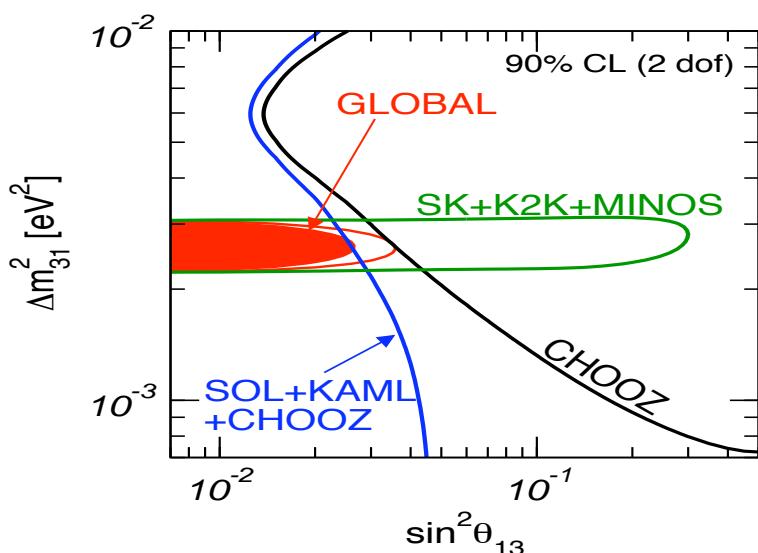
# Reactor/Accelerator Sector: {13}

$|U_{\alpha j}|^2$



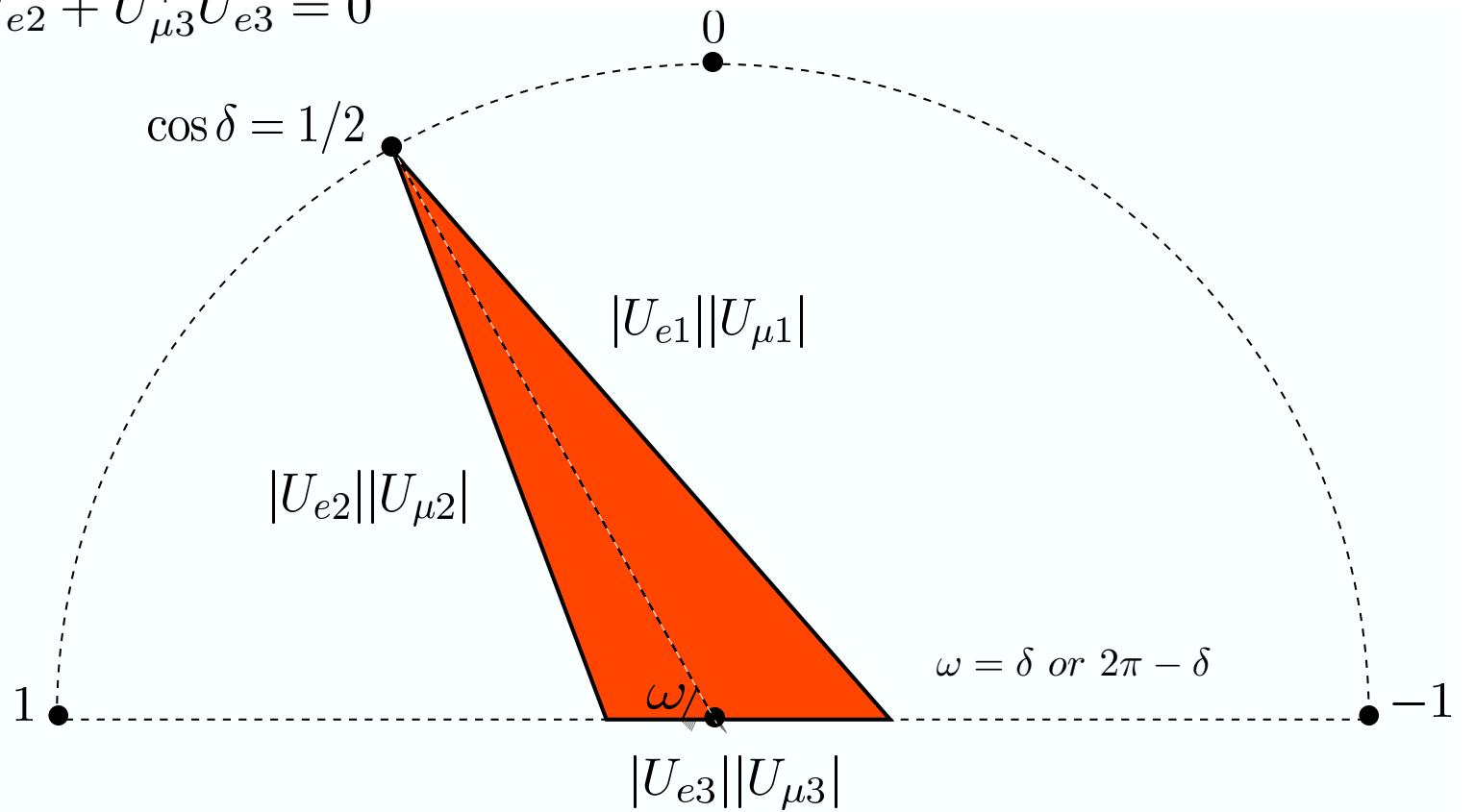
$$\sin^2 \theta_{13} < 0.03$$

$$0 \leq \delta < 2\pi$$



# Unitarity Triangle:

$$U_{\mu 1}^* U_{e1} + U_{\mu 2}^* U_{e2} + U_{\mu 3}^* U_{e3} = 0$$



$$|J| = 2 \times \text{Area}$$

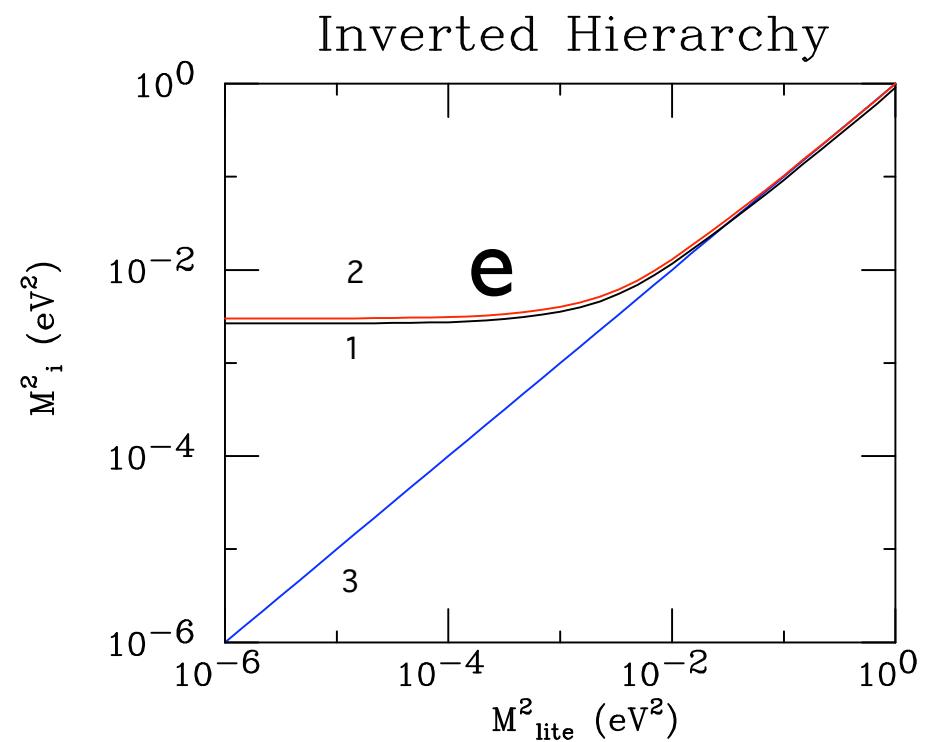
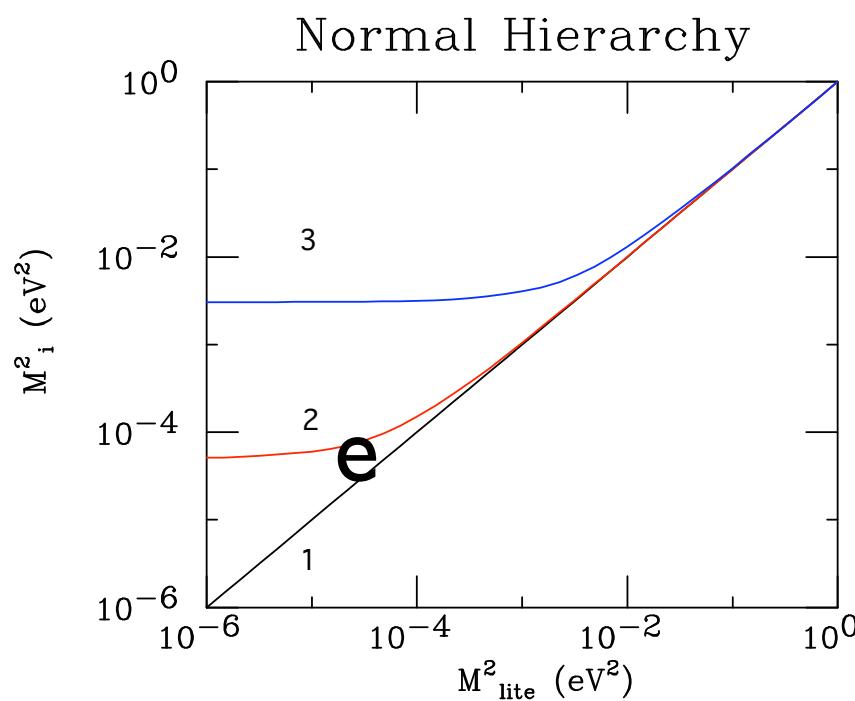
$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2 \sin \delta$$

# Sine/Signs

- $|U_{e3}|^2$ :  $\sin^2 \theta_{13}$
- Hierarchy:  $\text{sign}(\delta m_{31}^2 \text{ or } \delta m_{32}^2)$
- CPV:  $\sin \delta$
- Maximal Mixing:  $\sin^2 \theta_{23} = \frac{1}{2}$
- Quadrant of  $\delta$ :  $\cos \delta = \pm \sqrt{1 - \sin^2 \delta}$
- Unitarity: lite sterile  $\nu$ 's
- New Interactions and Surprises

- Majorana v Dirac
- Absolute mass scale:  $m_{Heavy}$  and  $m_{Lite}$
- New Interactions and Surprises

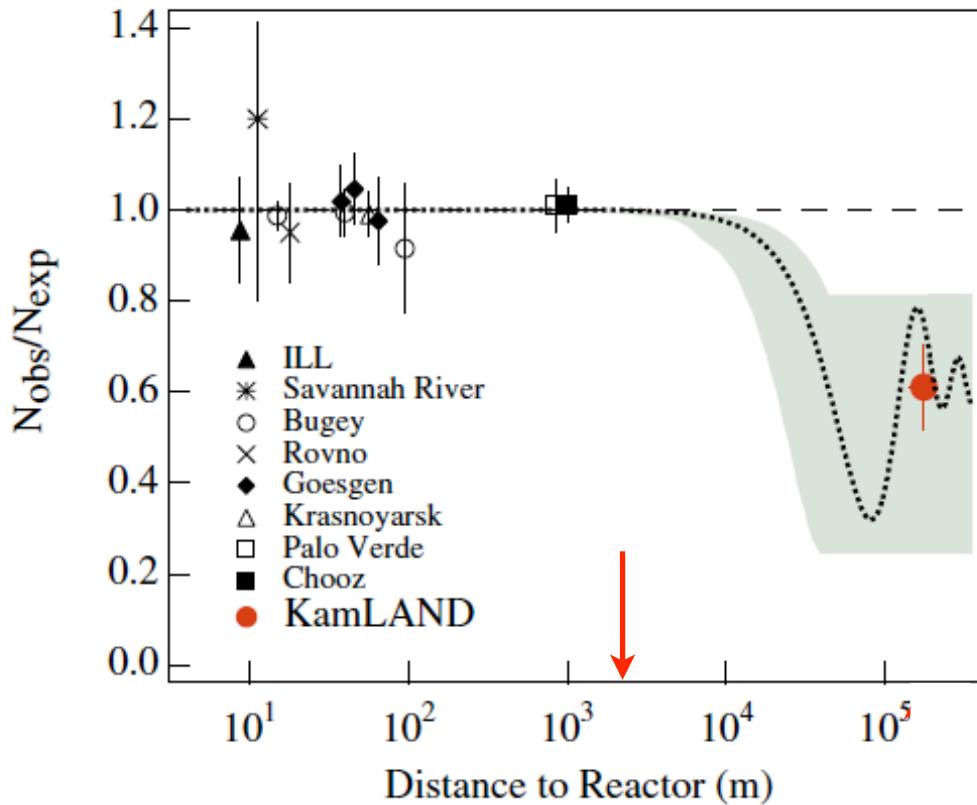
## Masses:



States 1 and 2 are  $\nu_e$  rich.

# $\theta_{13}$ from Reactor Disappearance

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} \\ - \sin^2 2\theta_{13} (\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32})$$



kinematic phase:

$$\Delta_{ij} \equiv \frac{\delta m_{ij}^2 L}{4E}$$

< 0.002

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\delta m_{ee}^2 L}{4E} \right) - \mathcal{O}(\Delta_{21})^2$$

$\nearrow > 0.01$

$$\delta m_{ee}^2 = \cos^2 \theta_{12} |\delta m_{31}^2| + \sin^2 \theta_{12} |\delta m_{32}^2|$$

# Double Chooz:



One nuclear plant & two detectors

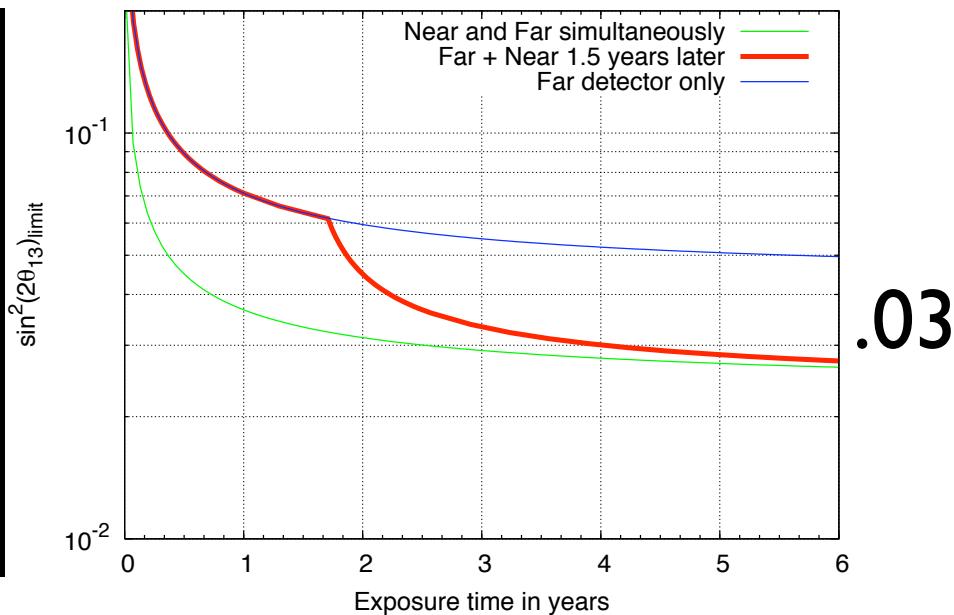
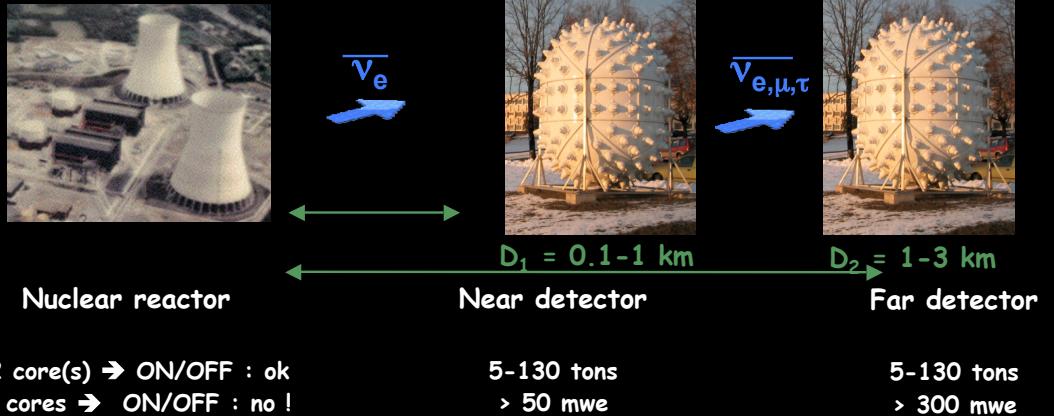
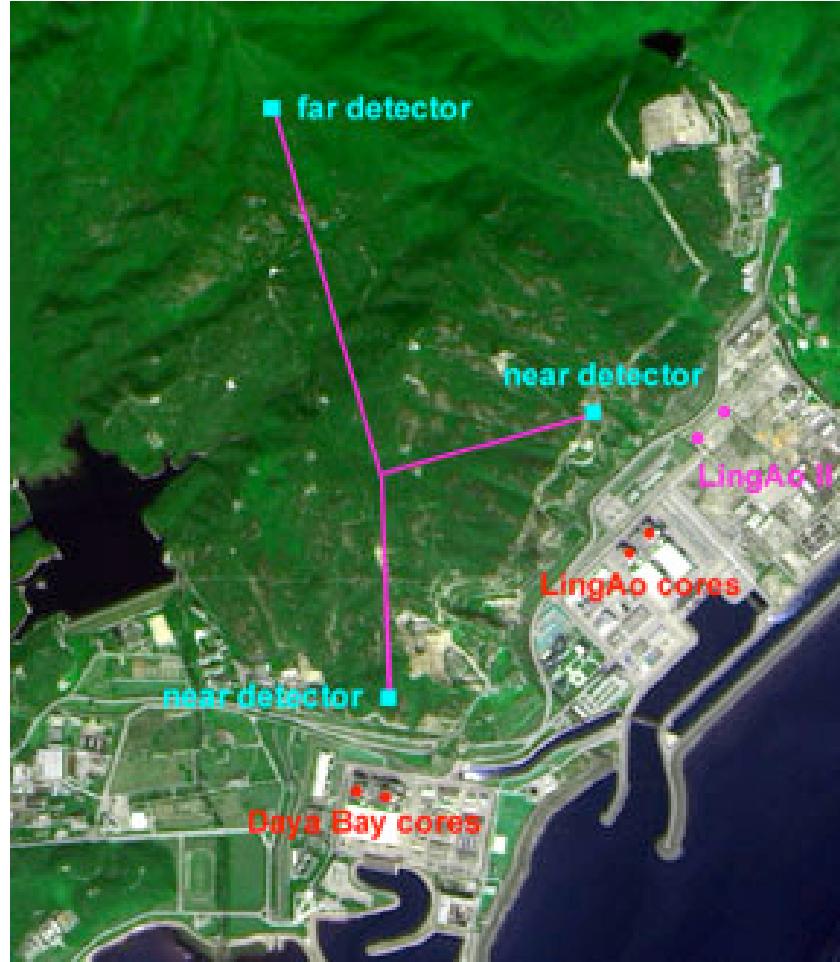


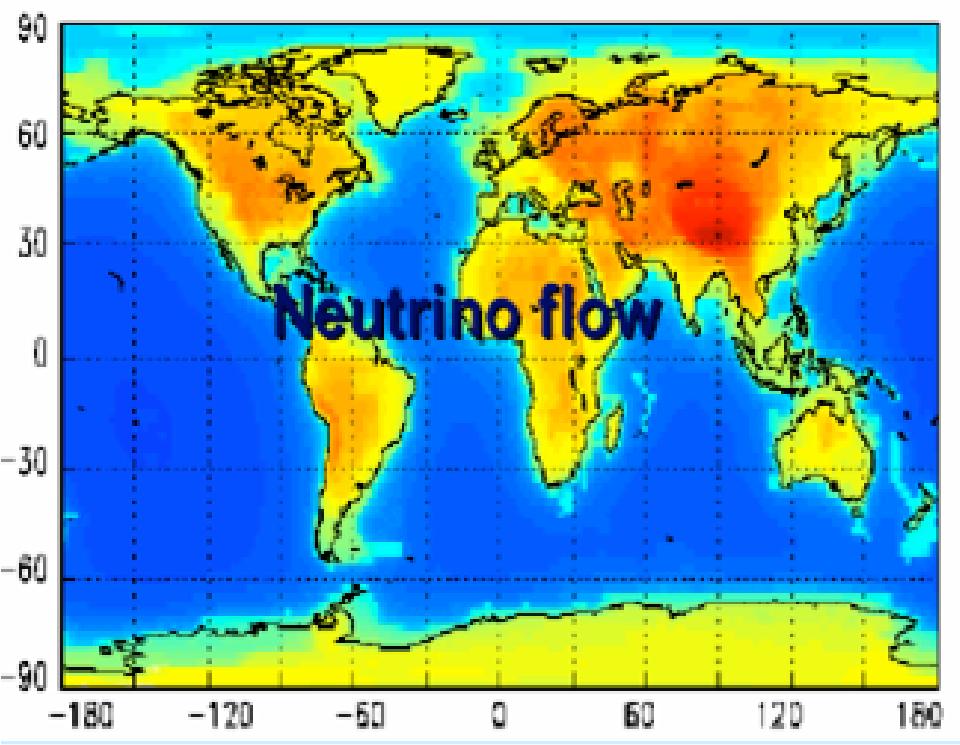
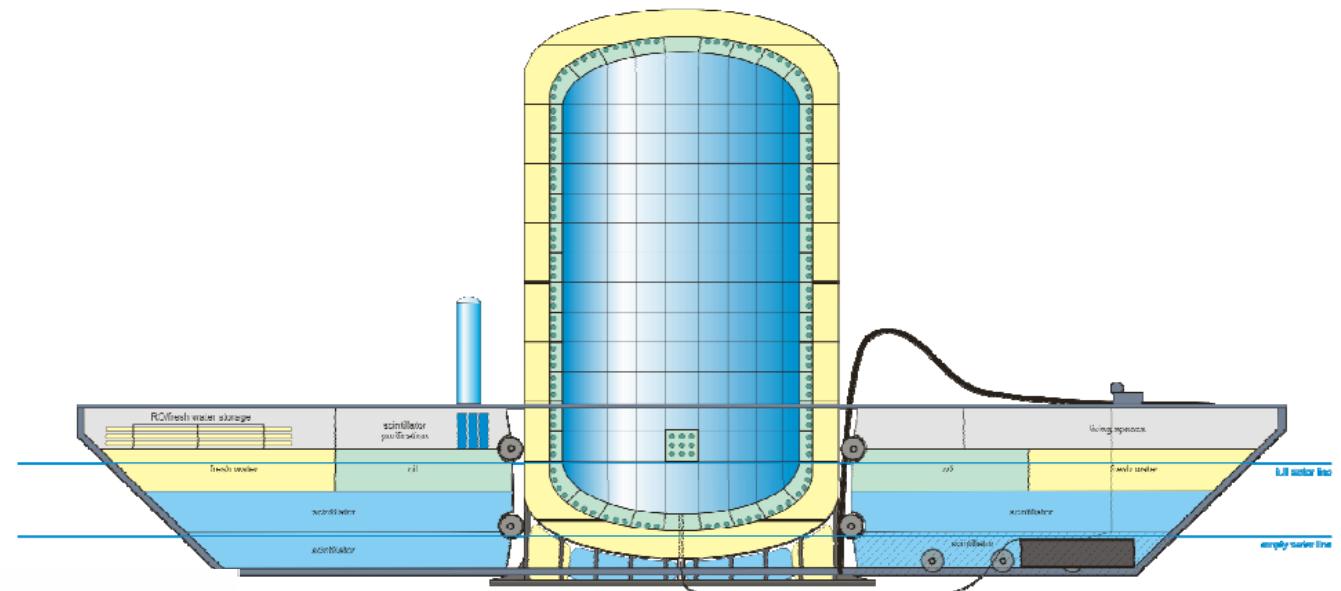
Figure 18:  $\sin^2(2\theta_{13})$  sensitivity limit for the detectors installation scheduled scenario

# Daya Bay



push the limit on  
 $\sin^2 2\theta_{13} < 0.01$

# HanoHano Hierarchy:



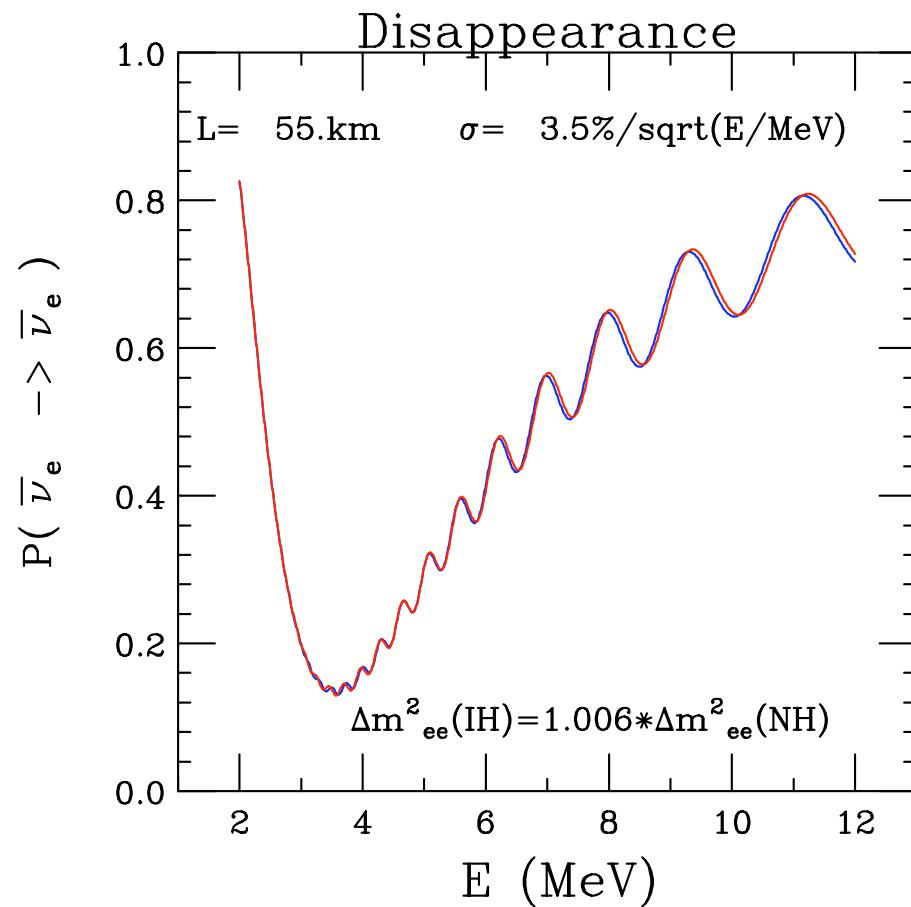
# Fourier Analysis: 4 freq.

NH:  $|\Delta m_{31}^2| > |\Delta m_{32}^2|$

|H:  $|\Delta m_{31}^2| < |\Delta m_{32}^2|$

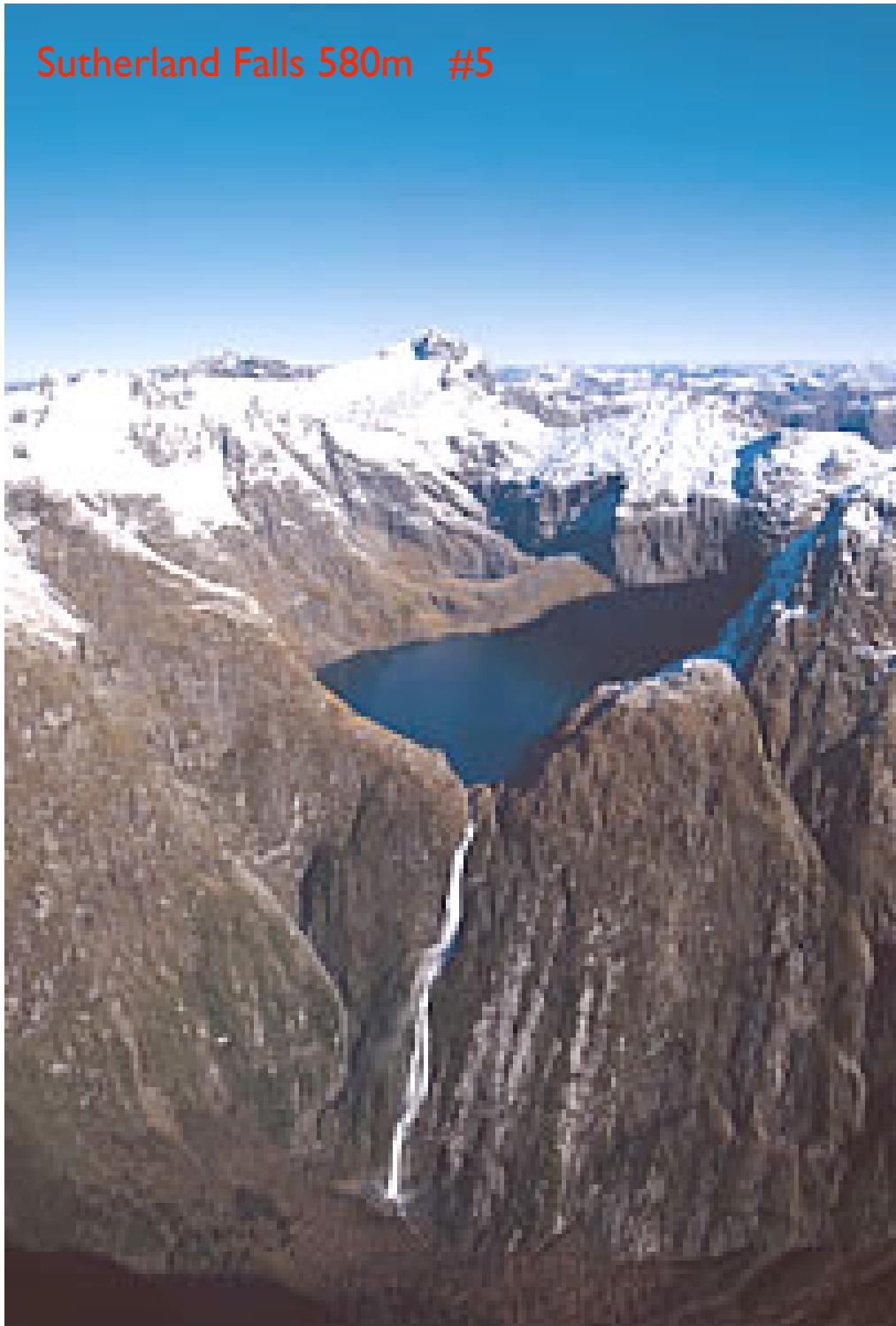
Power:  $\cos^4 \theta_{12} : \sin^4 \theta_{12}$

# Neutrinos from 2-8 MeV:



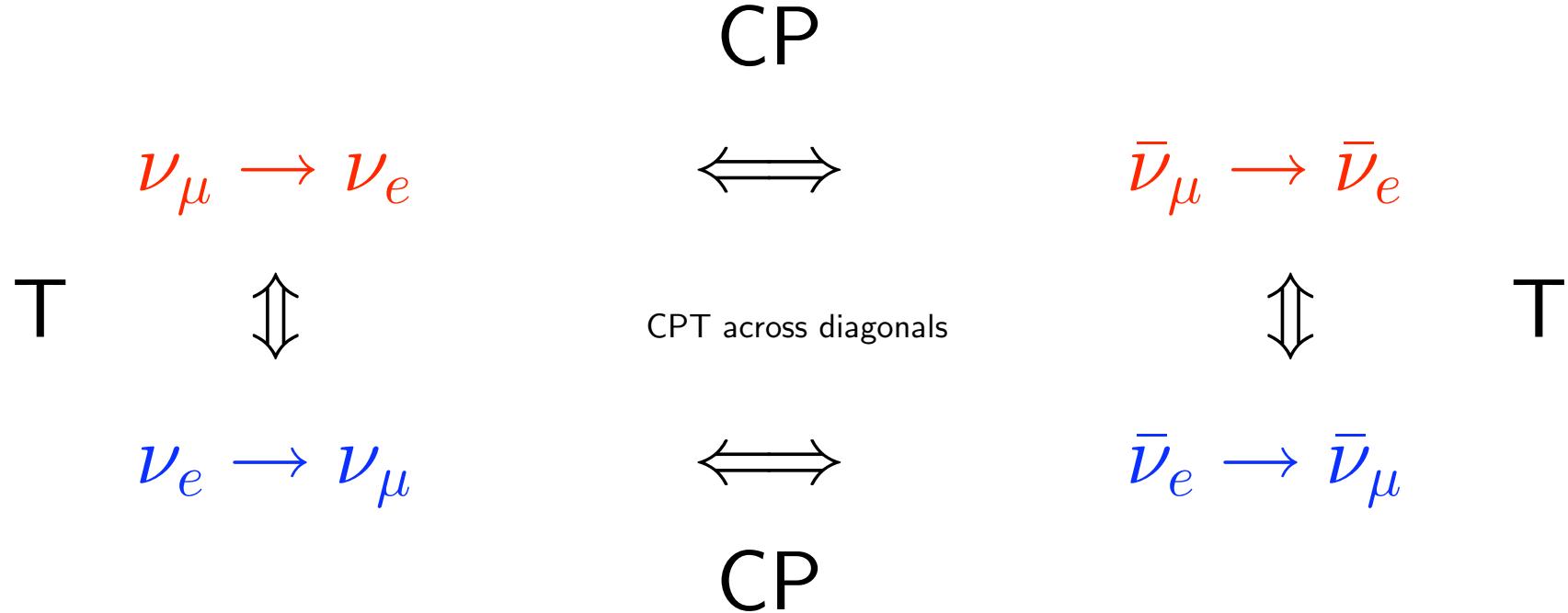
% diff < 2% !!!

Sutherland Falls 580m #5



$$\nu_\mu \rightarrow \nu_e$$

and related processes:



- First Row: Superbeams where  $\nu_e$  contamination  $\sim 1\%$
- Second Row:  $\nu$ -Factory or  $\beta$ -Beams, no beam contamination

Even in matter, a vestige of CPT exists:  
Instead of switch matter to anti-matter, switch neutrino hierarchy !!!

$$\nu_\mu \rightarrow \nu_e$$

$$| U_{\mu 3}^* e^{-im_3^2 L/2E} U_{e3} + U_{\mu 2}^* e^{-im_2^2 L/2E} U_{e2} + U_{\mu 1}^* e^{-im_1^2 L/2E} U_{e1} |^2$$

use unitarity to eliminate  $U_{\mu 1}^* U_{e1}$  term:

$$P(\nu_\mu \rightarrow \nu_e) = |2U_{\mu 3}^* U_{e3} \sin \Delta_{31} e^{-i\Delta_{32}} + 2U_{\mu 2}^* U_{e2} \sin \Delta_{21}|^2$$

↑
Atmospheric  $\delta m^2$ 
↑
Solar  $\delta m^2$

$$2U_{\mu 3}^* U_{e3} = \sin \theta_{23} \sin 2\theta_{13} e^{-i\delta}$$

$$2U_{\mu 2}^* U_{e2} \approx \cos \theta_{23} \sin 2\theta_{12}$$

Vacuum LBL:

$$\nu_\mu \rightarrow \nu_e$$

$$P_{\mu \rightarrow e} \approx | \sqrt{P_{atm}} e^{-i(\Delta_{32} \pm \delta)} + \sqrt{P_{sol}} |^2$$



$$\Delta_{ij} = |\delta m_{ij}^2|L/4E$$

CP violation !!!

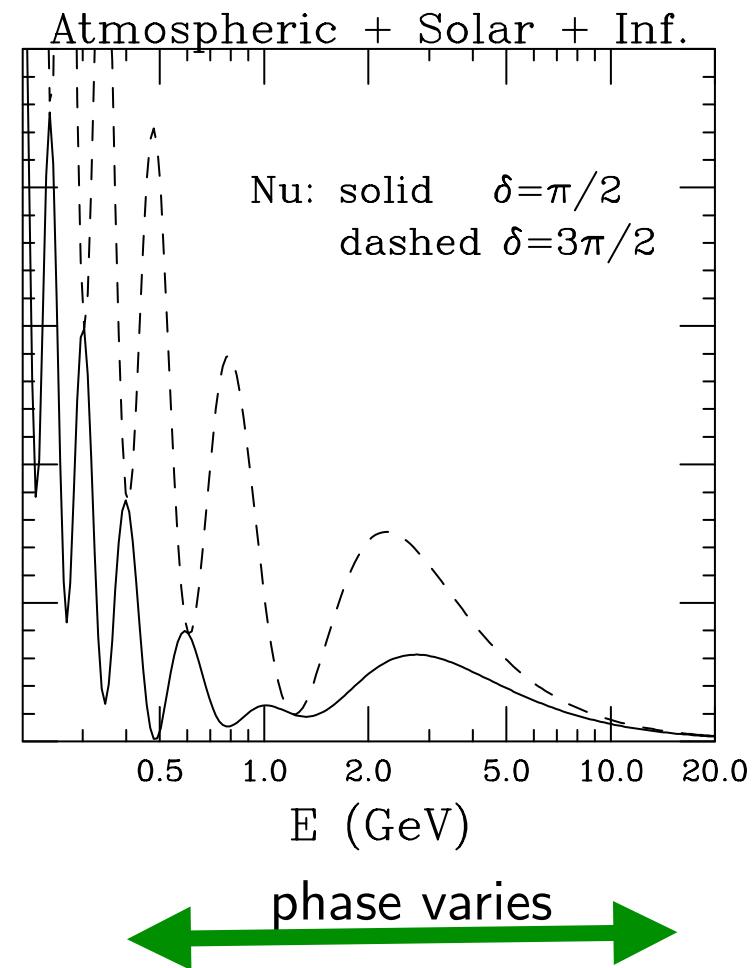
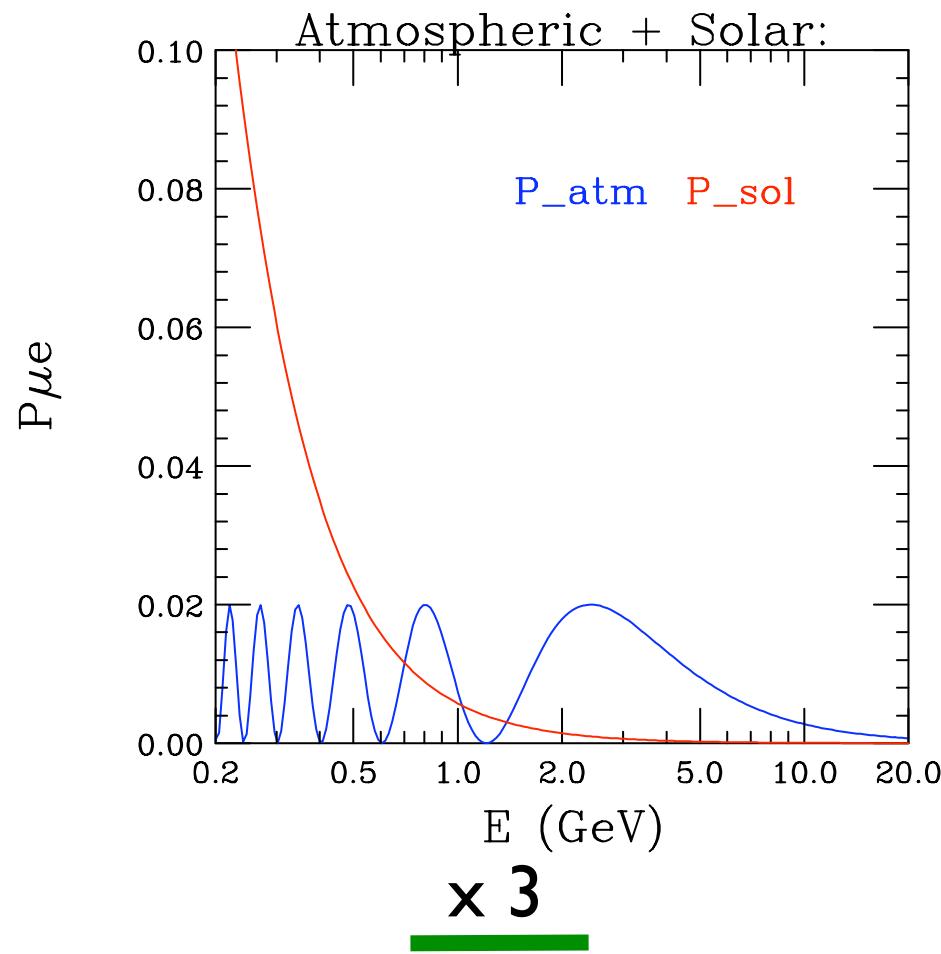
where  $\sqrt{P_{atm}} = \sin \theta_{23} \sin 2\theta_{13} \sin \Delta_{31}$

and  $\sqrt{P_{sol}} = \cos \theta_{23} \sin 2\theta_{12} \sin \Delta_{21}$

$$P \approx P_{atm} + 2\sqrt{P_{atm}P_{sol}} \cos(\Delta_{32} \pm \delta) + P_{sol}$$

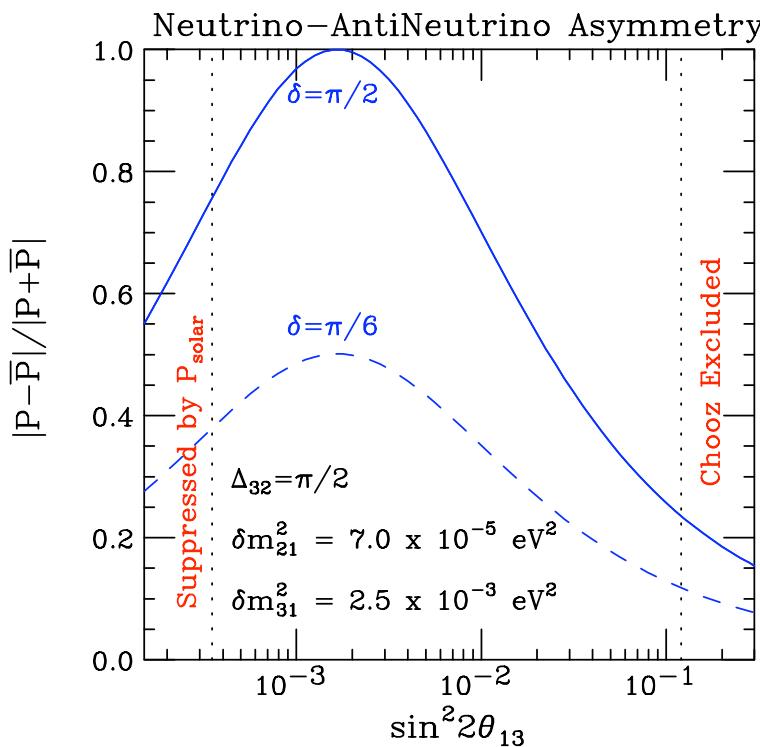
$$P(\nu_\mu \rightarrow \nu_e) \approx |\sqrt{P_{atm}} e^{-i(\Delta_{32} + \delta)} + \sqrt{P_{sol}}|^2$$

For  $L = 1200 \text{ km}$   
and  $\sin^2 2\theta_{13} = 0.04$



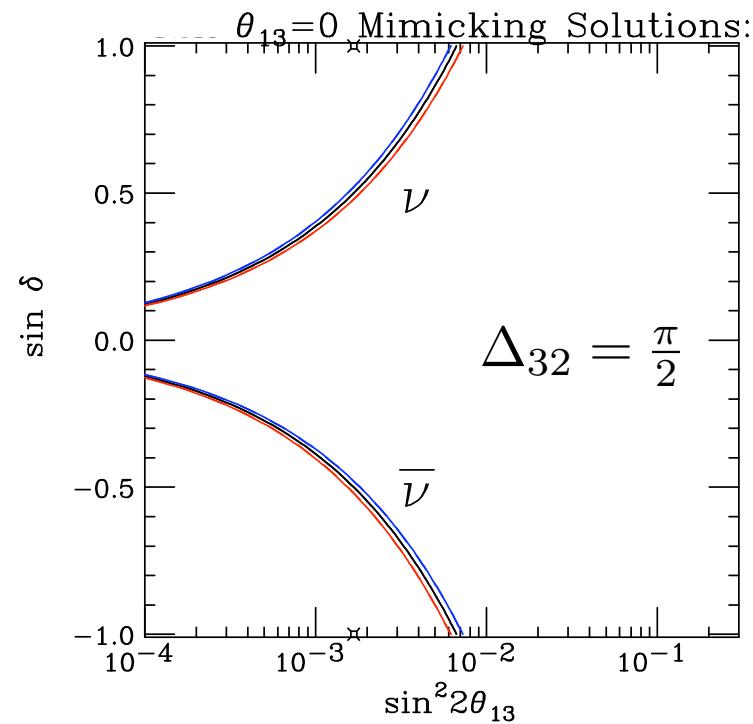
$$P_{\mu \rightarrow e} \approx | \sqrt{P_{atm}} e^{-i(\Delta_{32} \pm \delta)} + \sqrt{P_{sol}} |^2$$

## Asymmetry Peaks:



$$\sqrt{P_{atm}} = \sqrt{P_{sol}}$$

## Zero Mimicking Solutions:



$$\sqrt{P_{atm}} = -2\sqrt{P_{sol}} \cos(\Delta_{32} \pm \delta)$$

$$P = P_{sol}$$

# Two Flavors: Vacuum and Uniform Matter

Vacuum:  $P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta_0 \sin^2 \Delta_0$

Matter:  $P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta_N \sin^2 \Delta_N$

BUT  $\delta m_0^2 \sin 2\theta_0 = \delta m_N^2 \sin 2\theta_N \Rightarrow \sin^2 2\theta_N = \left( \frac{\delta m_0^2}{\delta m_N^2} \right)^2 \sin^2 2\theta_0$

Therefore:  $P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta_0 \left( \frac{\sin^2 \Delta_N}{\Delta_N^2} \right) \Delta_0^2$

$$\begin{aligned}\delta m_N^2 &= \sqrt{(\delta m_0^2 \cos 2\theta_0 - 2\sqrt{2}G_F N_e E)^2 + (\delta m_0^2 \sin 2\theta_0)^2} \\ &\approx \delta m_0^2 - 2\sqrt{2}G_F N_e E \quad \text{for small } \theta_0.\end{aligned}$$

$$\Delta_N = \Delta - aL \text{ where } a = G_F N_e / \sqrt{2}$$

Therefore in Matter:  $P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta_0 \left( \frac{\sin \Delta_0 \mp aL}{\Delta_0 \mp aL} \right)^2 \Delta_0^2$

$\nu_\mu \rightarrow \nu_e$   
 with MATTER

$$P_{\mu \rightarrow e} \approx | \sqrt{P_{atm}} e^{-i(\Delta_{32} \pm \delta)} + \sqrt{P_{sol}} |^2$$

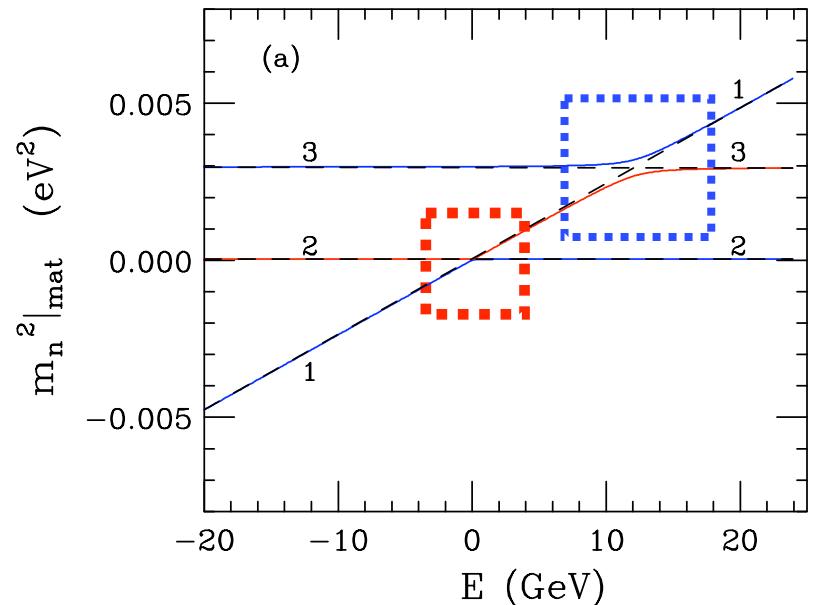
where  $\sqrt{P_{atm}} = \sin \theta_{23} \sin 2\theta_{13} \frac{\sin(\Delta_{31} \mp aL)}{(\Delta_{31} \mp aL)} \Delta_{31}$

and  $\sqrt{P_{sol}} = \cos \theta_{23} \sin 2\theta_{12} \frac{\sin(aL)}{(aL)} \Delta_{21}$

$$a = G_F N_e / \sqrt{2} = (4000 \text{ km})^{-1},$$

$$\pm = sign(\delta m_{31}^2) \quad \Delta_{ij} = |\delta m_{ij}^2| L / 4E$$

$\{\delta m^2 \sin 2\theta\}$  is invariant

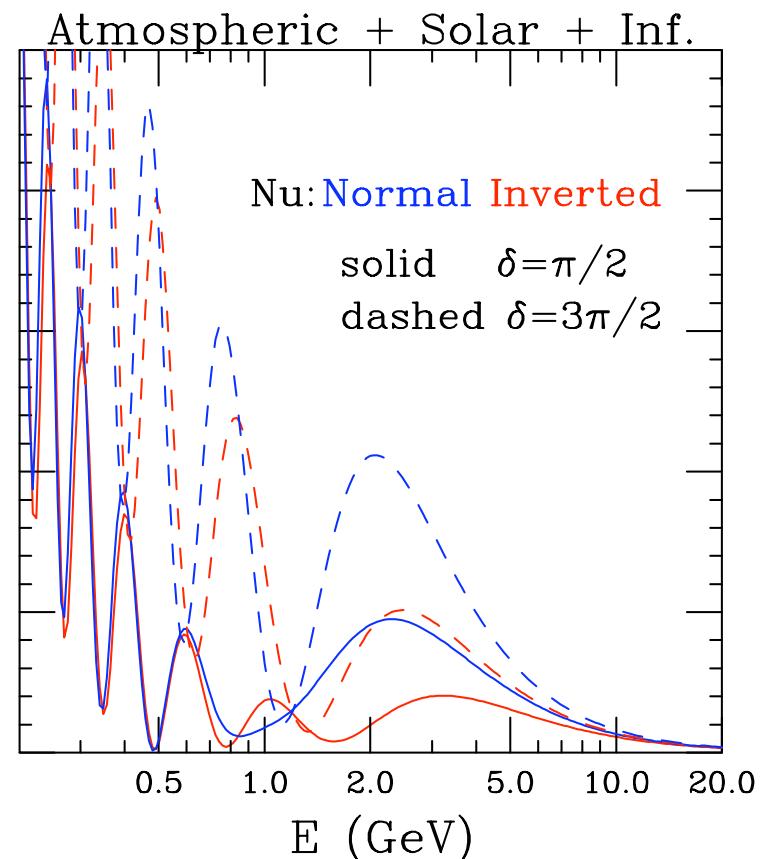
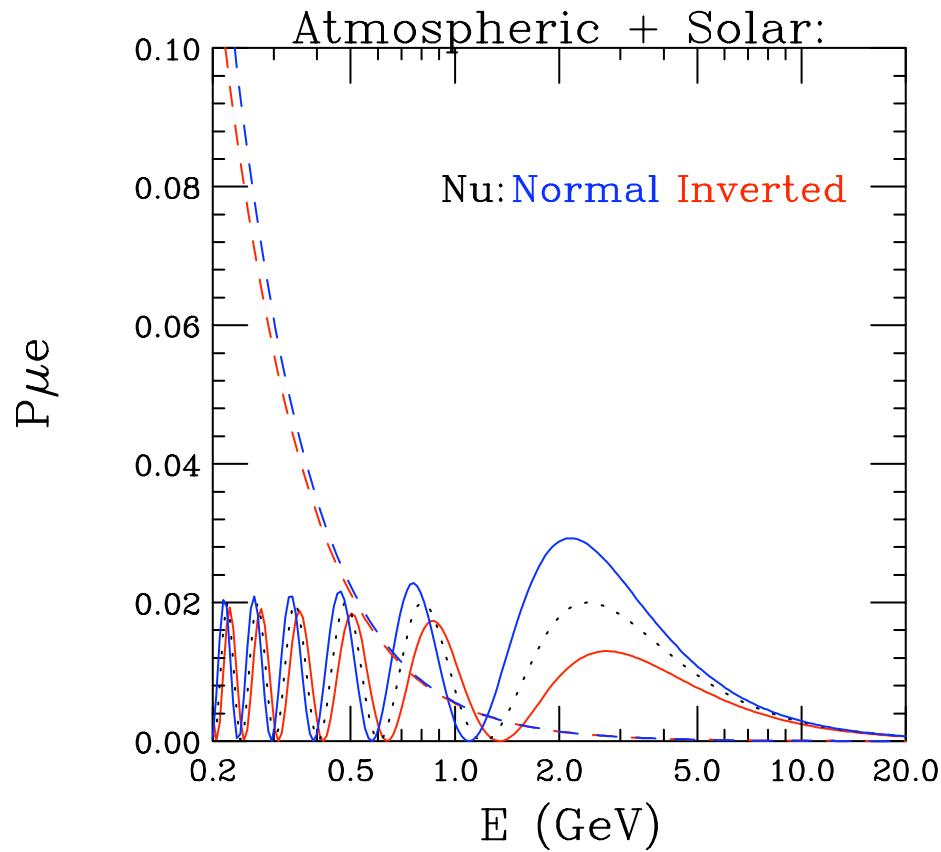


In Matter:

$$P_{\mu \rightarrow e} \approx | \sqrt{P_{atm}} e^{-i(\Delta_{32} \pm \delta)} + \sqrt{P_{sol}} |^2$$

For  $L = 1200 \text{ km}$

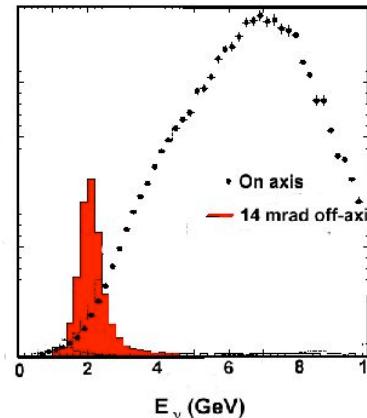
and  $\sin^2 2\theta_{13} = 0.04$



Anti-Nu: Normal Inverted  
dashes  $\delta = \pi/2$   
solid  $\delta = 3\pi/2$

# Off-Axis Beams

BNL 1994



**T2K**

**JHF → Super-Kamiokande**

- ✓ 295 km baseline
- ✓ Super-Kamiokande:
  - 22.5 kton fiducial
  - Excellent e/μ ID
  - Additional  $\pi^0/e$  ID
- ✓ Hyper-Kamiokande
  - 20× fiducial mass of SuperK
- ✓ Matter effects small
- ✓ Study using fully simulated and reconstructed data

A map of Japan showing the 295 km baseline between JHF (Japan Proton Accelerator Research Center) and Super-Kamiokande. Other locations like KEK, JAERI, and Tokai are also marked.

**NOVA**

A map of the Northern United States showing the Ash River and Fermilab sites. An inset map shows the location of the NOVA experiment relative to Lake Superior, Duluth, and Ash River.

$$E_\nu = \frac{0.43 \gamma m_\pi}{1 + \gamma^2 \theta^2}$$

$L=295$  km and

Energy at Vac. Osc. Max. (vom)

$$E_{vom} = 0.6 \text{ GeV} \left\{ \frac{\delta m_{32}^2}{2.5 \times 10^{-3} \text{ eV}^2} \right\}$$

0.75 upgrade to 4 MW

$L=700 - 1000$  km and

Energy near 2 GeV

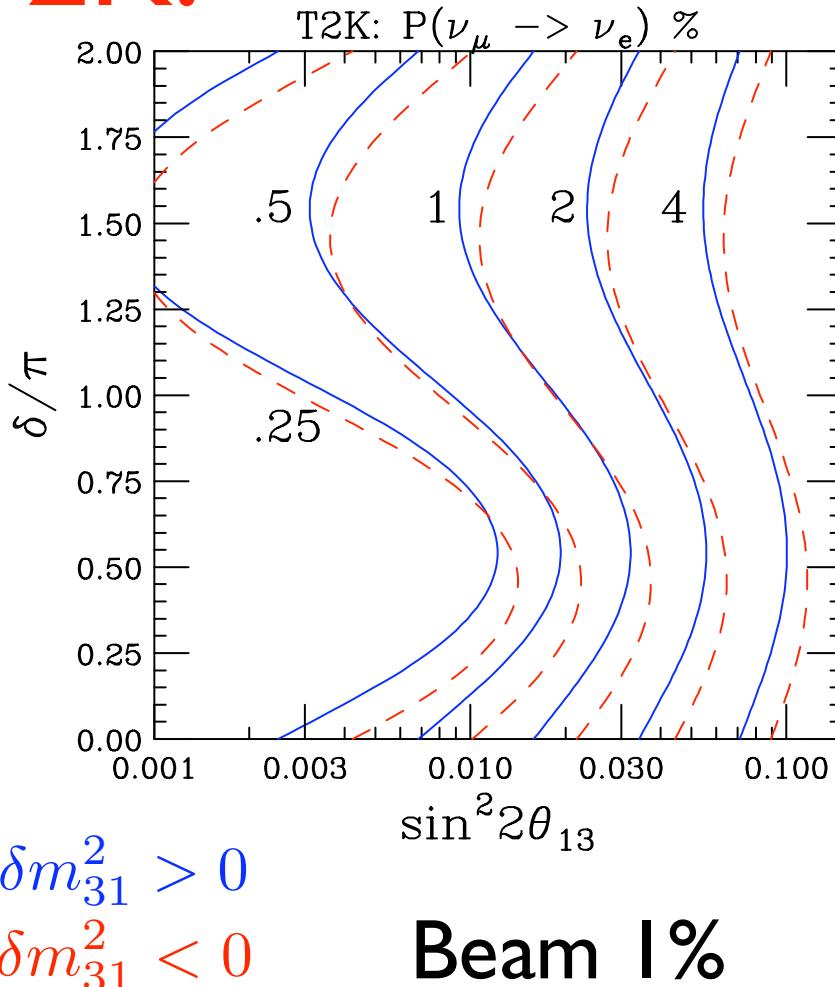
$$E_{vom} = 1.8 \text{ GeV} \left\{ \frac{\delta m_{32}^2}{2.5 \times 10^{-3} \text{ eV}^2} \right\} \times \left\{ \frac{L}{820 \text{ km}} \right\}$$

0.4 upgrade to 2 MW

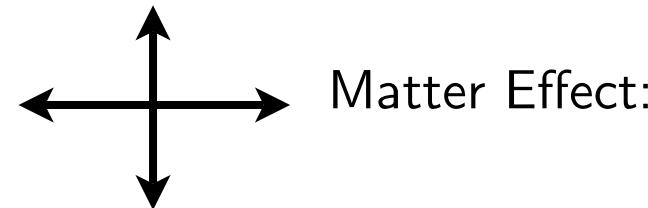
# Sensitivity to $\sin^2 2\theta_{13}$

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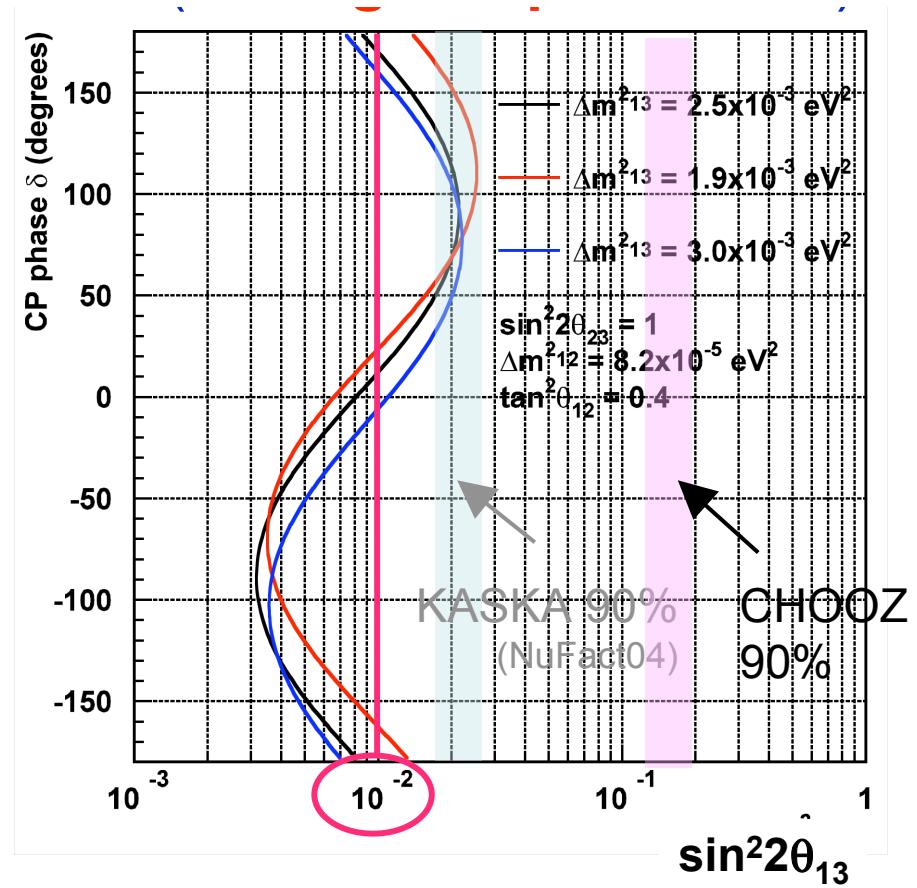
# T2K:



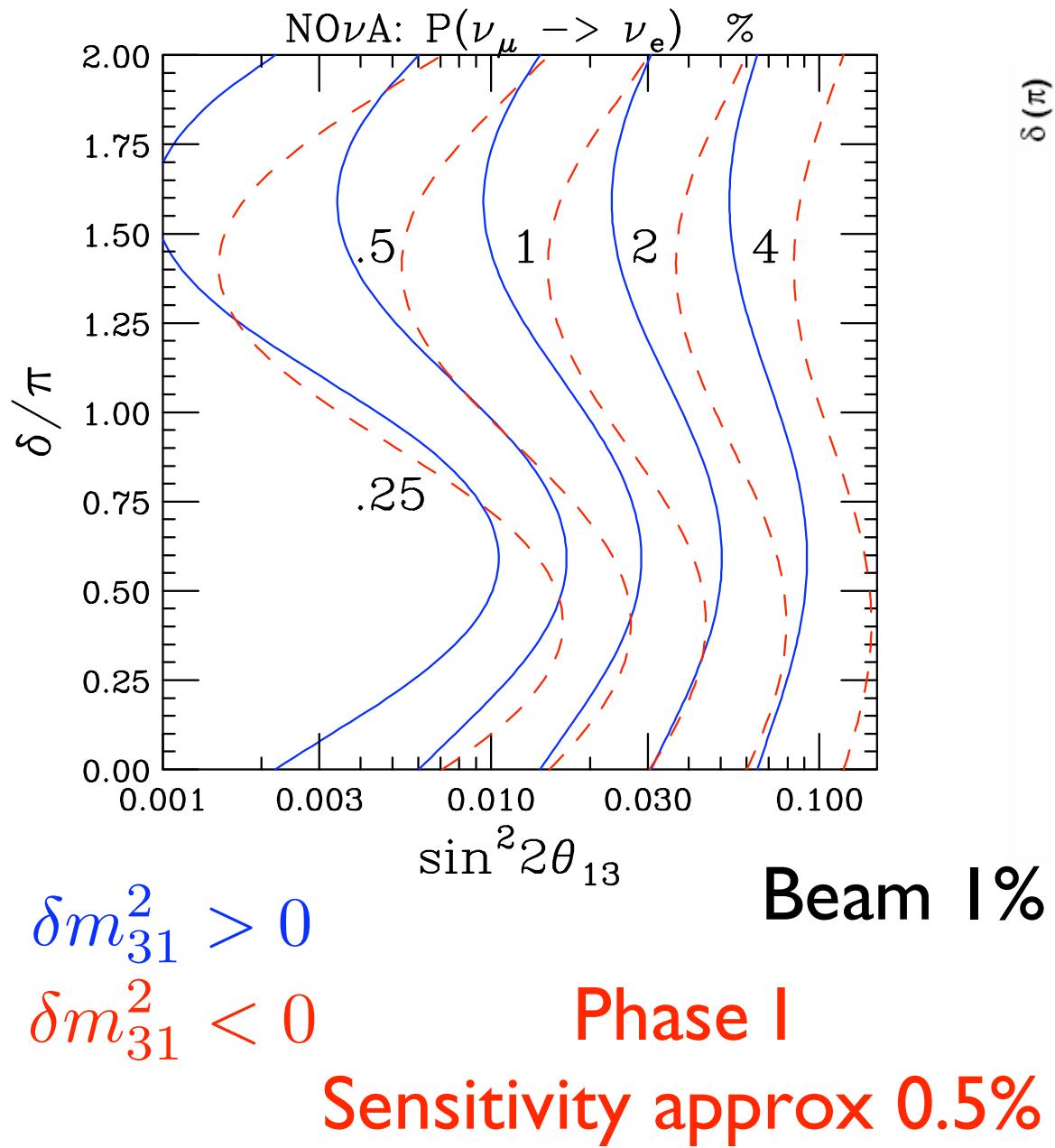
VOM:  $\Delta_{31} \neq \pi/2$



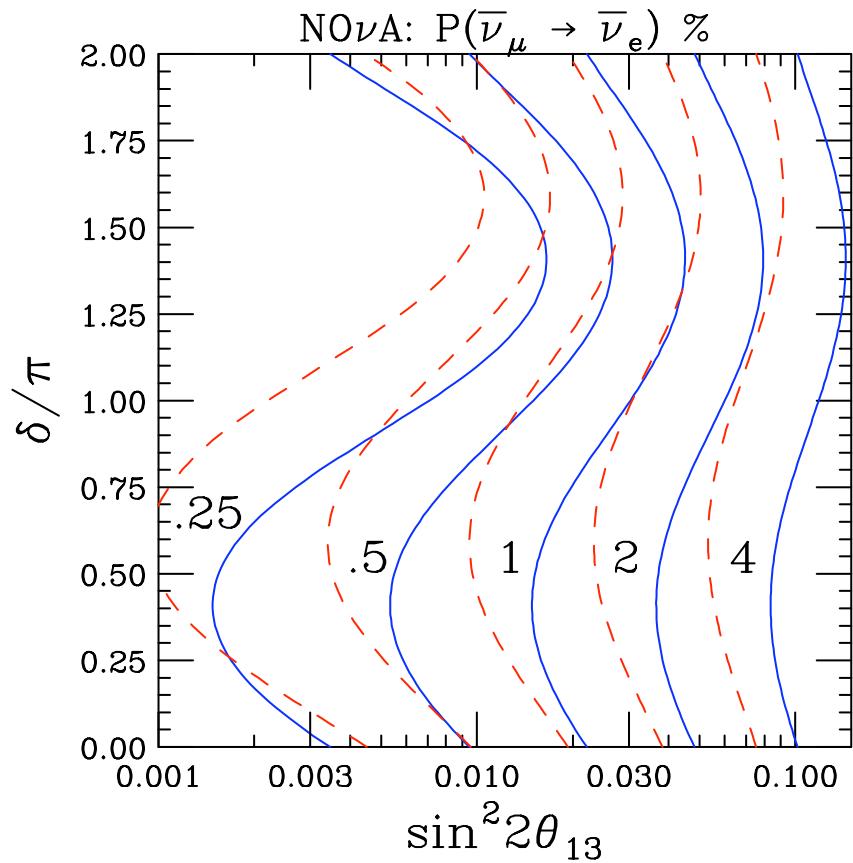
## Aihara for T2K, P5 talk



# NOvA:



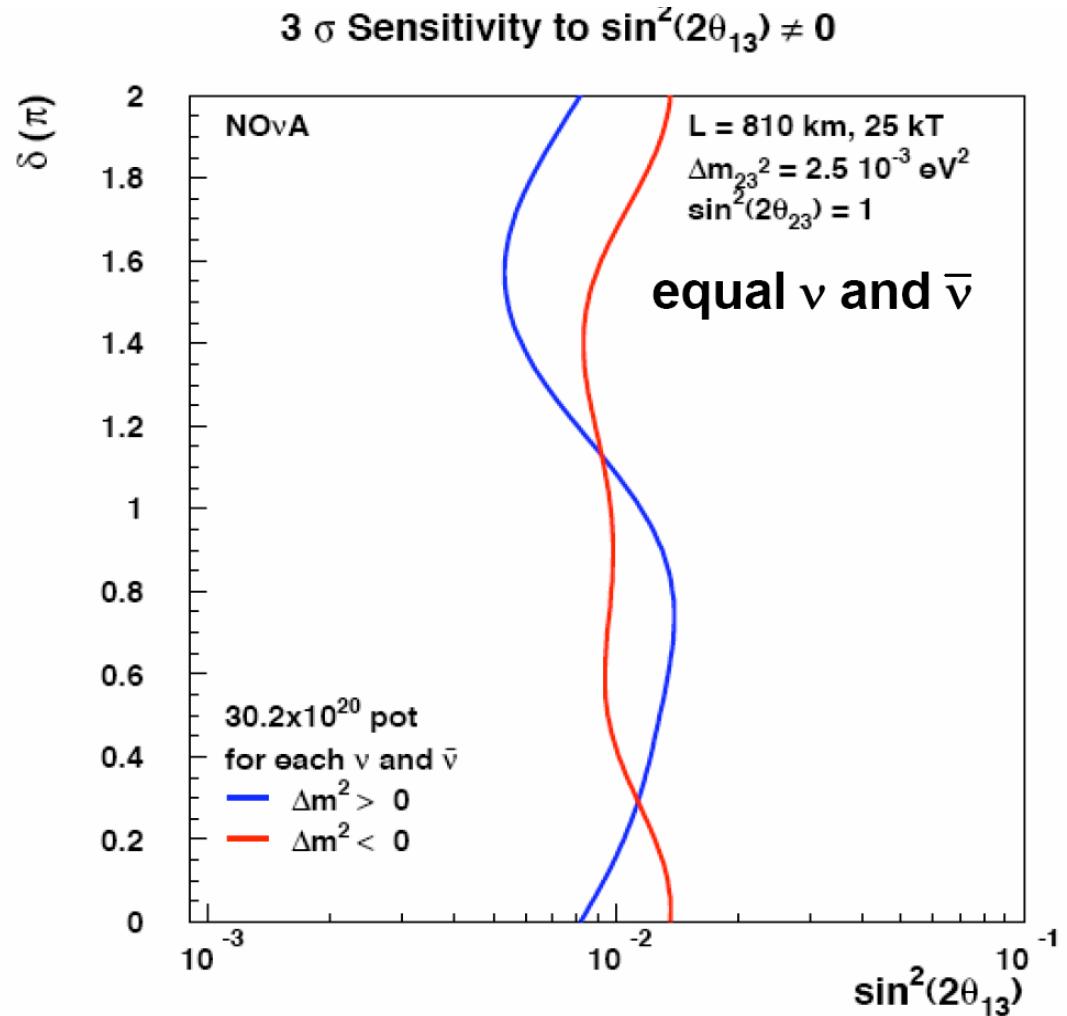
# NO $\nu$ A:



Beam > 1%

$$\delta m_{31}^2 > 0$$

$$\delta m_{31}^2 < 0$$

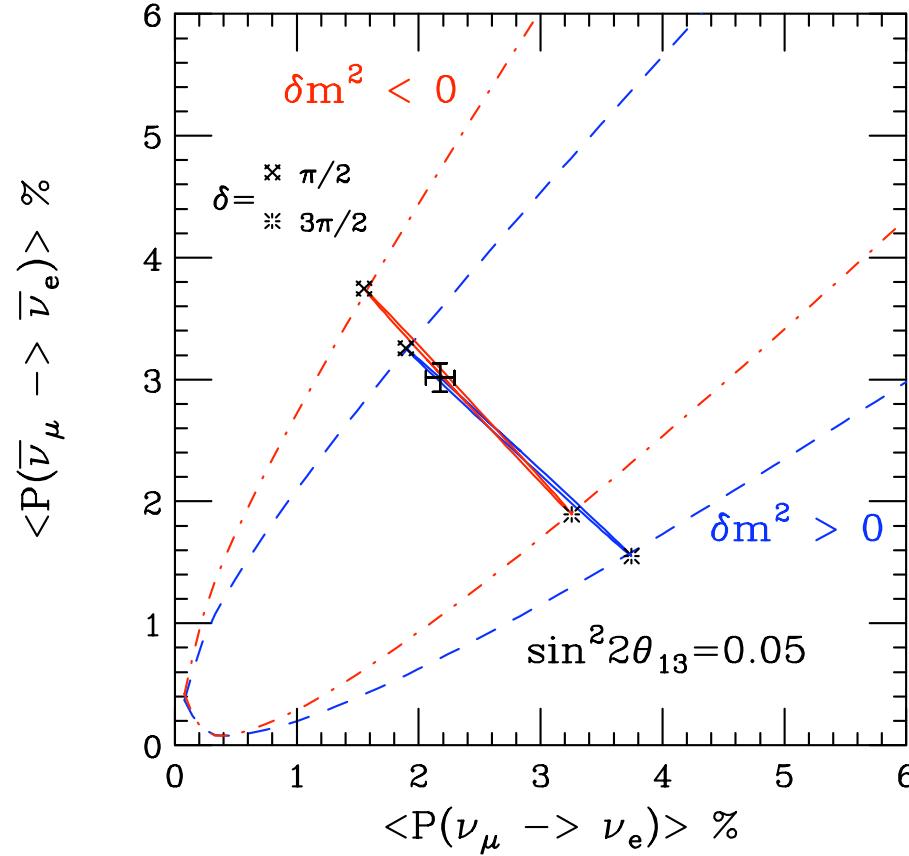


Sensitivity to Hierarchy: *sign*  $\delta m_{31}^2$

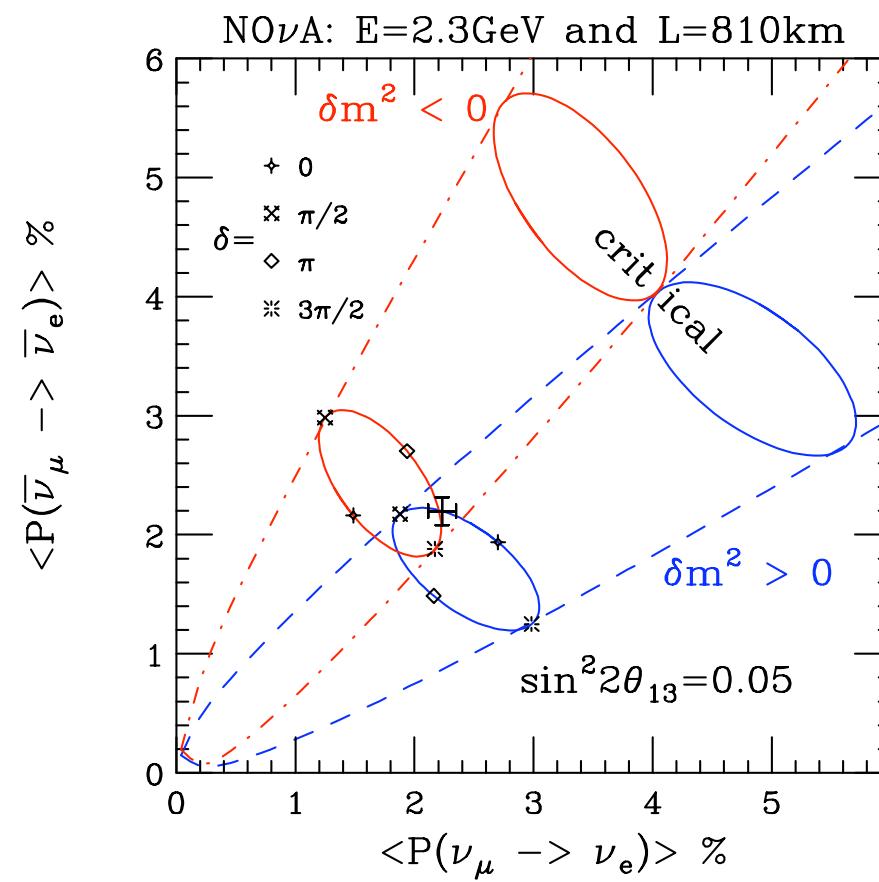
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# T2K:

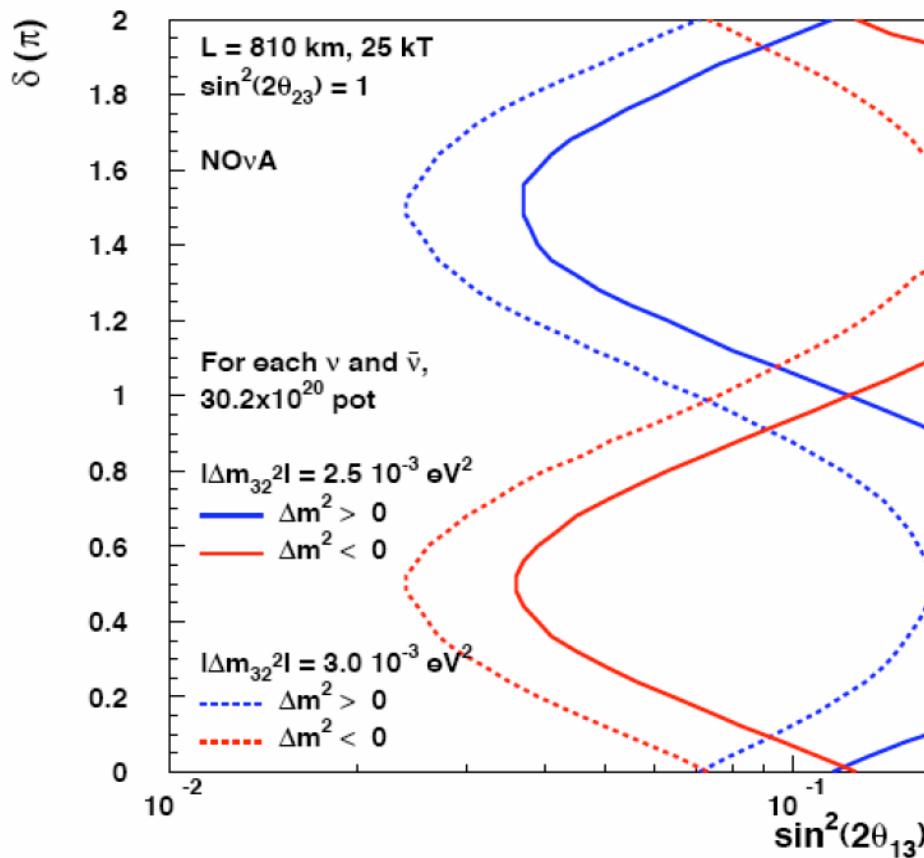
T2K: E=0.6GeV and L=295km

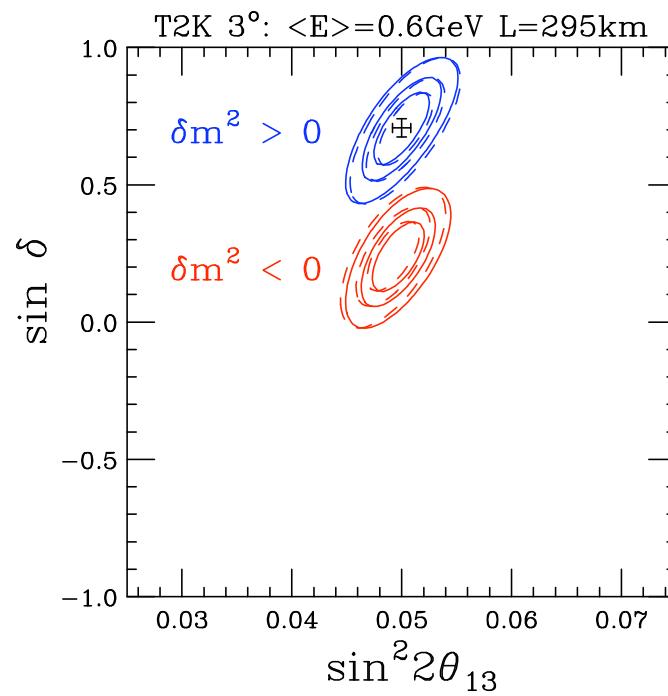
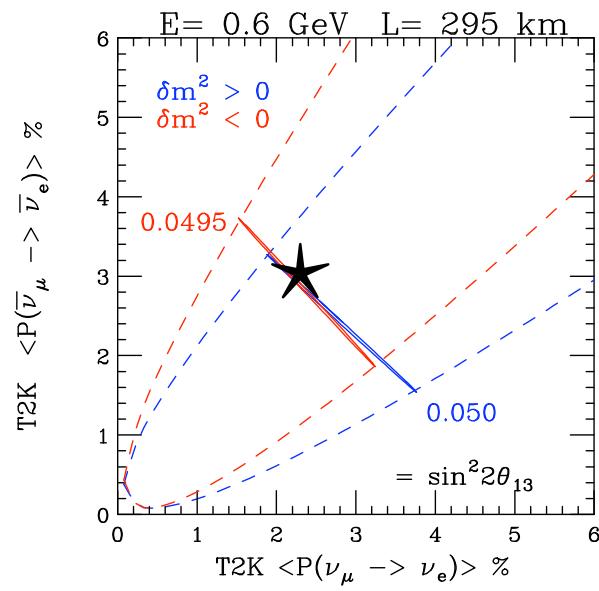


# NO $\nu$ A:

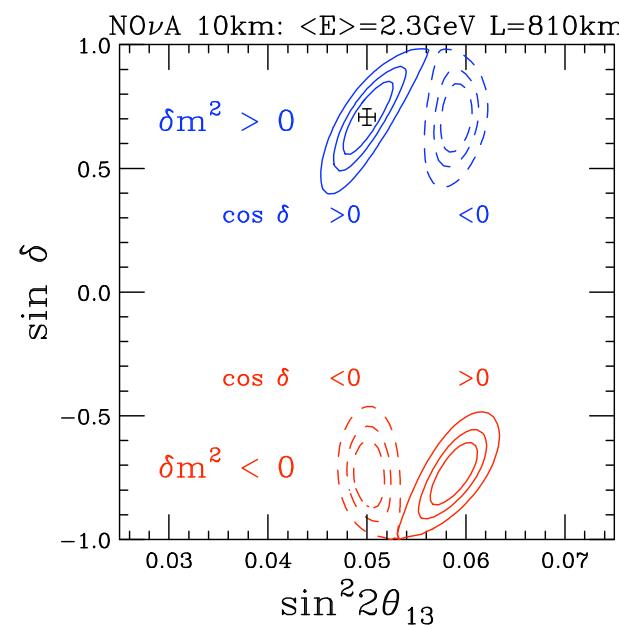
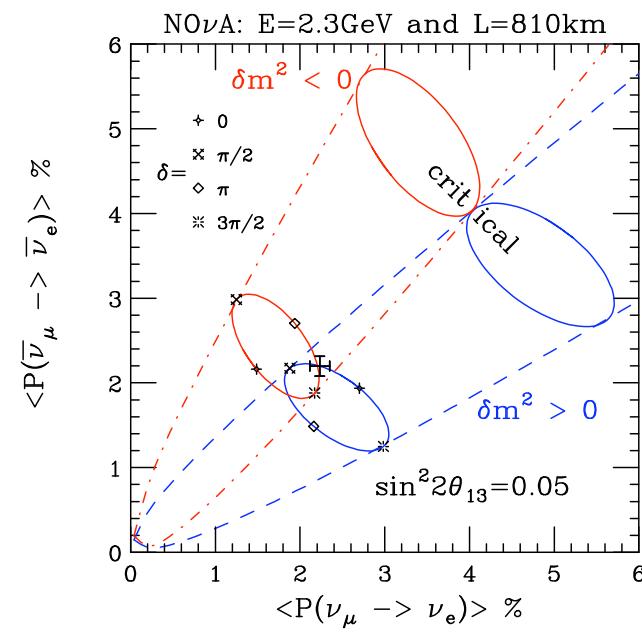


### 95% CL Resolution of the Mass Hierarchy

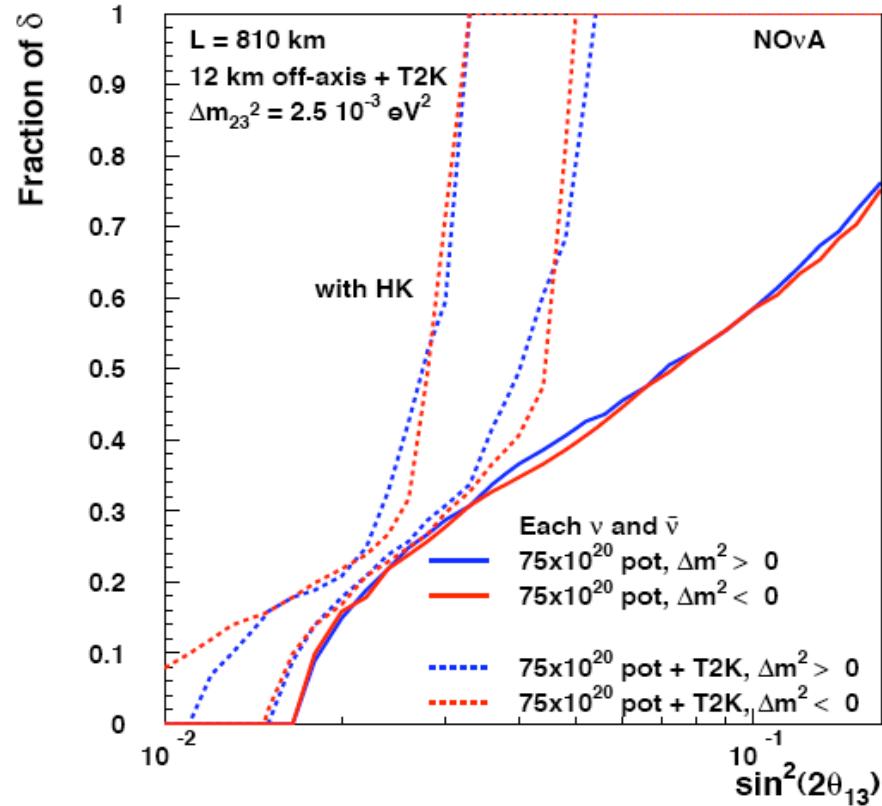




$$\langle \sin \delta \rangle_+ - \langle \sin \delta \rangle_- \approx 0.47 \sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}$$



$$\langle \sin \delta \rangle_+ - \langle \sin \delta \rangle_- \approx 1.4 \sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}$$



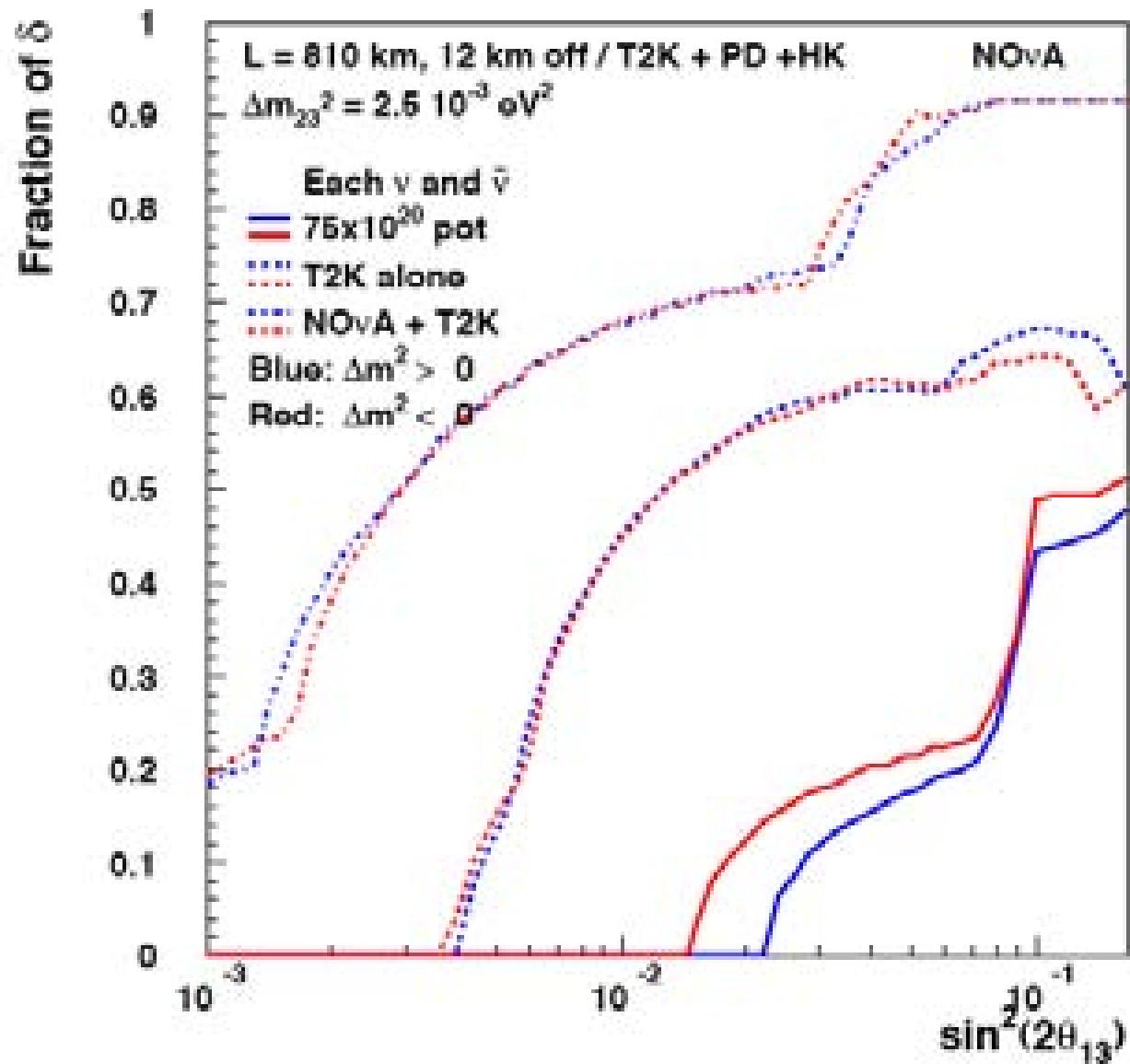
**NOvA/PD with T2K Phase 2**

**95% CL Resolution of  
the Mass Ordering**

Sensitivity to CP violation:  $\sin \delta$

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### 3 $\sigma$ Determination of CP Violation



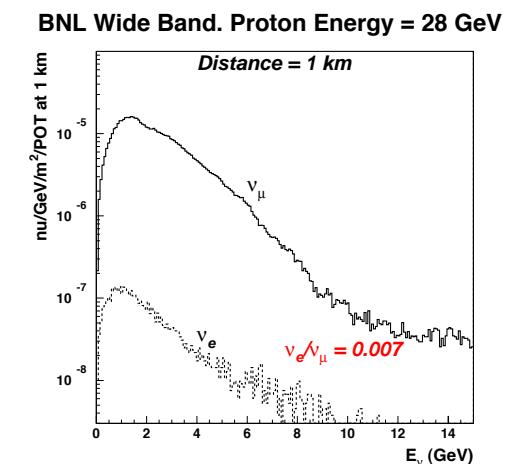
## Beyond the First Oscillation Maximum:

## On Axis Beams:



- 28 GeV protons. 1 MW beam power. Horn focussed
- 500 kT water Cherenkov detector.
- baseline > 2500 km. WIPP, Henderson, Homestake

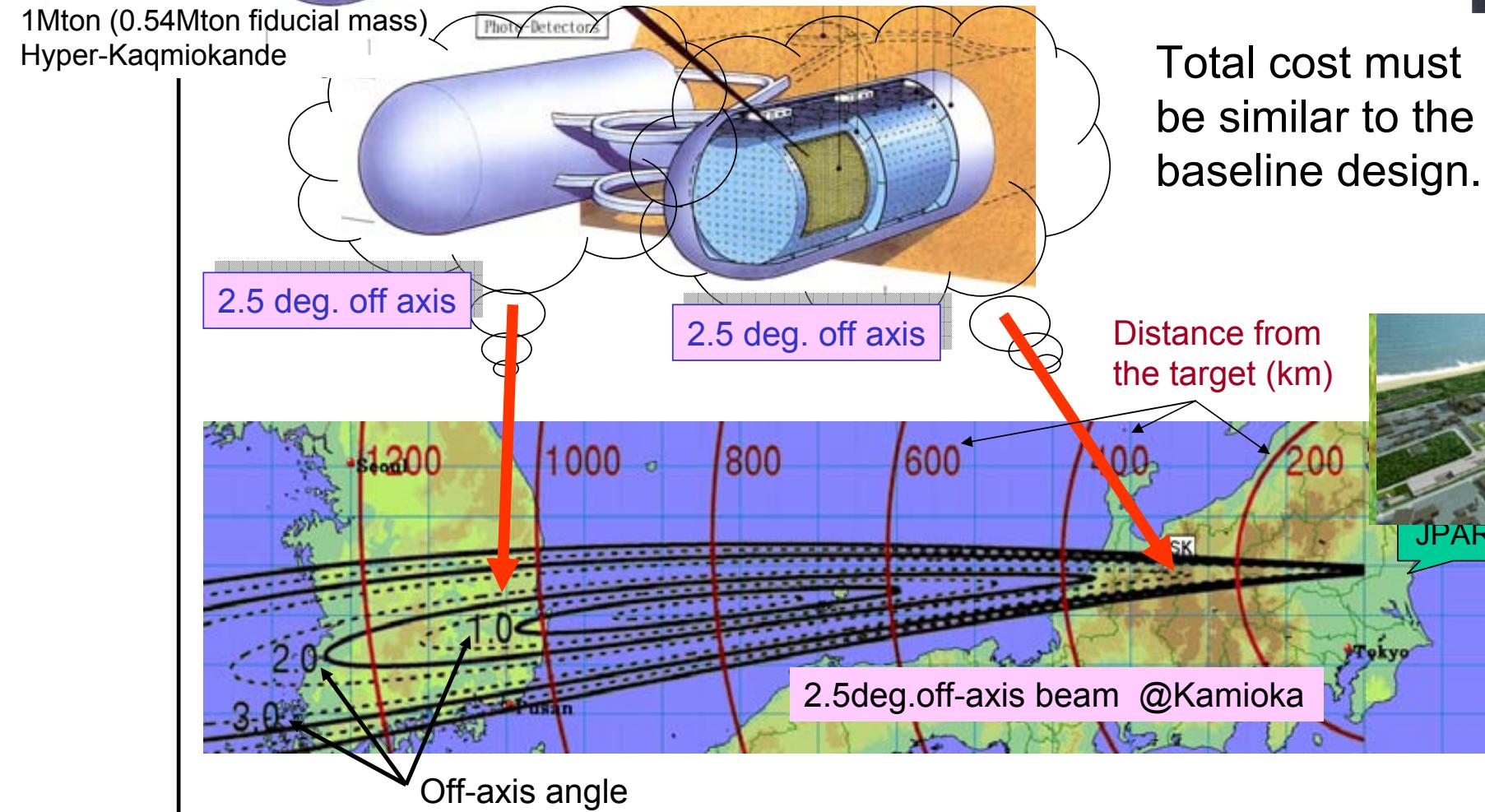
Brookhaven Proposal



# Off Axis:



*Some recent progress: detector in Korea*



see Kajita talk:

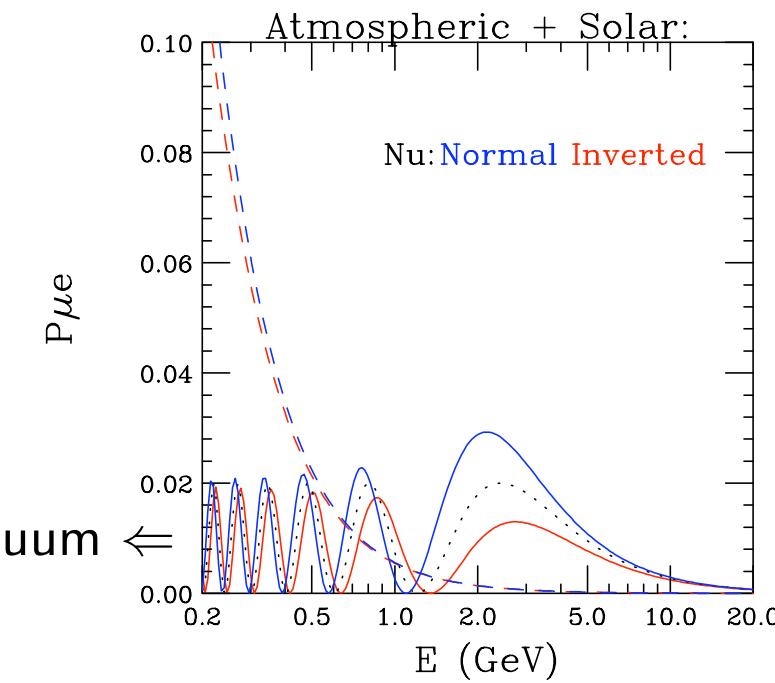
Broadband Beam: Same L, Lower E Fermilab to DUSEL

Narrow Band Beam: Same E, Longer L T2KK

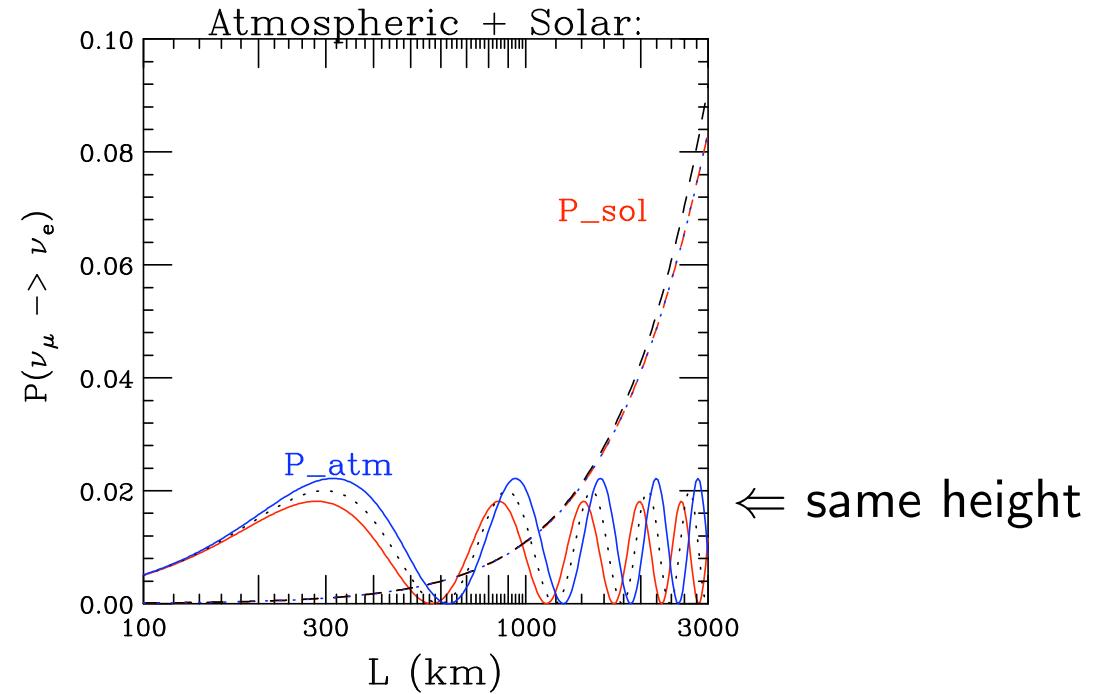
In VACUUM the SAME but NOT in MATTER

$$\sin^2 2\theta_{13} = 0.04$$

L=1200km



E=0.6 GeV



$$P_{\mu \rightarrow e} \approx | \sqrt{P_{atm}} e^{-i(\Delta_{32} \pm \delta)} + \sqrt{P_{sol}} |^2$$

where  $\sqrt{P_{atm}} = \sin \theta_{23} \sin 2\theta_{13} \frac{\sin(\Delta_{31} \mp aL)}{(\Delta_{31} \mp aL)} \Delta_{31}$

and  $\sqrt{P_{sol}} = \cos \theta_{23} \sin 2\theta_{12} \frac{\sin(aL)}{(aL)} \Delta_{21}$

$$P_{\mu \rightarrow e} \approx | \sqrt{P_{atm}} e^{-i(\Delta_{32} \pm \delta)} + \sqrt{P_{sol}} |^2$$

depends on  $\theta_{13}$   
amplification or suppression  
by matter (E)

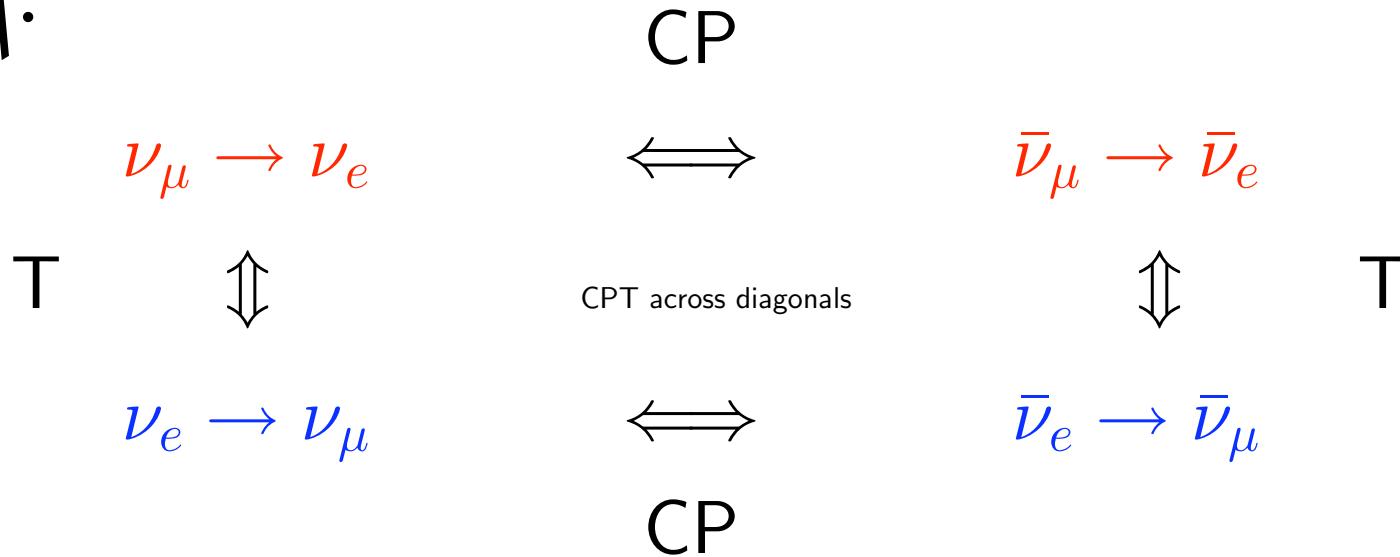
Suppression  $\geq$  Enhancement

independent of  $\theta_{13}$   
 $\approx$  independent of  
matter effect

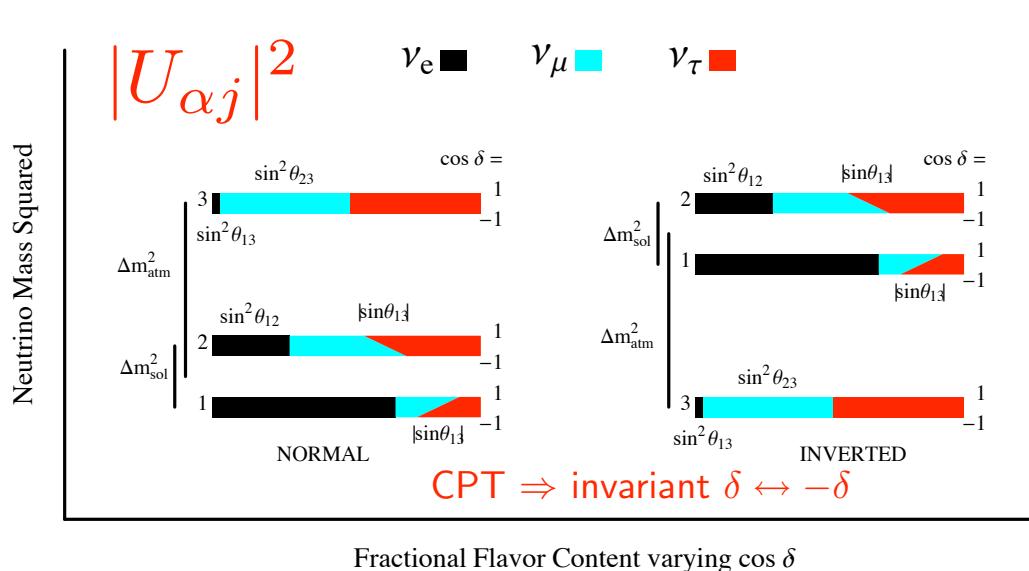
$L/E \geq$  significant fraction of 500 km/GeV

Event rate:  $E(E/L)^2$

# Summary:



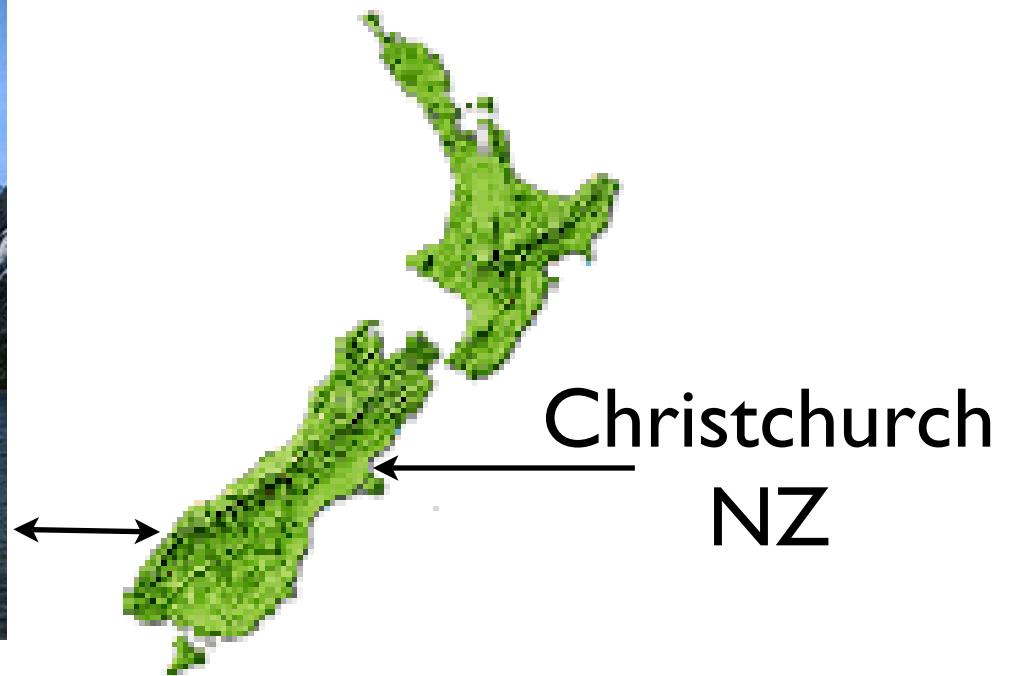
- First Row: Superbeams where  $\nu_e$  contamination  $\sim 1\%$
  - Second Row:  $\nu$ -Factory or  $\beta$ -Beams, no beam contamination



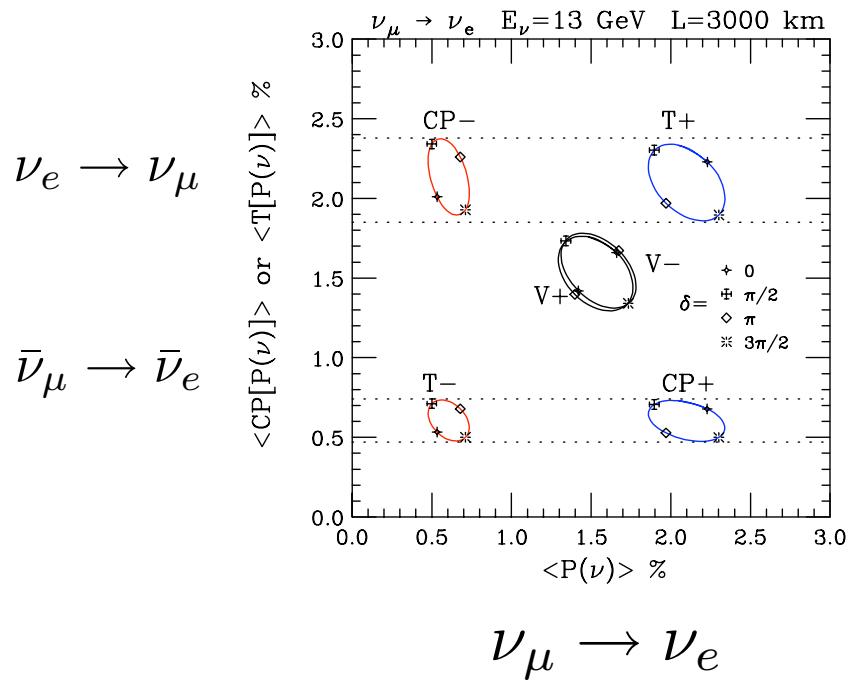
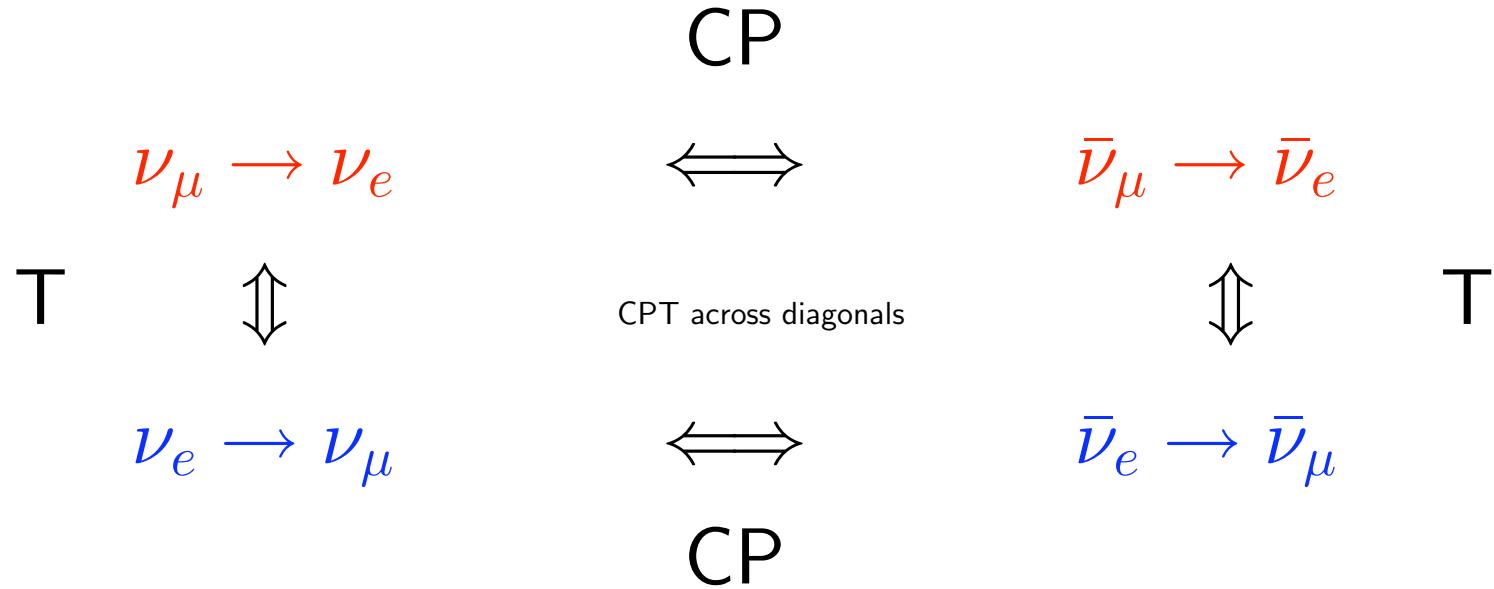
- Size of  $|U_{e3}|^2$
  - Hierarchy ?
  - CPV ?
  - Maximal {23} Mixing ?
  - .....
  - New Interactions and Surprises !!!

# Neutrino 2008

## May 26-31



[www.neutrino2008.co.nz](http://www.neutrino2008.co.nz)



$$\begin{aligned}
 & P(\nu_e \rightarrow \nu_\mu, \text{matter}) \\
 &= P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e, \text{anti-matter}) \\
 &\Downarrow \\
 & P(\nu_e \rightarrow \nu_\mu, \delta m_{31}^2, \delta) \\
 &\approx P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e, -\delta m_{31}^2, \pi - \delta)
 \end{aligned}$$