Factorization in Effective Field Theory

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Introduction

New tool for flavor physics:

- factorization theorems for B-decays in the heavy quark limit.
  - $B \to \gamma \ell \nu$
  - $B \to D\pi$
  - $B \to K^* \gamma$
  - $B \to \pi\pi$
  - $\ldots$
- Soft-Collinear Effective Theory
  - factorization proofs
  - perturbative resummations

Overview

1. Factorization in exclusive processes:
   - large-Q^2 pion form factor,
   - Soft-Collinear Effective Theory,
   - factorization theorems for B-decays.
2. Degrees of freedom of SCET:
   - Strategy of Regions
   - processes with light hadrons,
   - B-decays
     $\rightarrow$ soft-collinear messenger mode. TB, Hill, Neubert
3. Construction of the effective theory
   - scalar case
   - gauge theory

1. Factorization in effective theory

- Factorization theorem for the pion form factor
- Soft-Collinear Effective Theory
- Factorization in B-decays.

Factorization

E. g. pion form factor at large $Q^2 = -(p_\pi - p'_\pi)^2 \gg \Lambda_{QCD}$

$$F_\pi(Q^2) = \phi_\pi(\mu) \otimes T(Q,\mu) \otimes \phi_\pi(\mu) + \mathcal{O}(\frac{\Lambda}{Q})$$

Brodsky & Lepage '80
Efremov & Radyushkin '80

- $T(Q,\mu)$: hard-scattering kernel (perturbative).
- $\phi_\pi(\mu)$: pion light-cone wave-function (non-perturbative, process independent).

Wave function and Kernel

$$F_{\pi^+}(Q) = F_\pi^2 \int_0^1 du \int_0^1 dv\, T(u,v,Q,\mu)\phi_\pi(u)\phi_\pi(v) + \mathcal{O}(1/Q^3)$$

Pion light cone distribution amplitude

$$-i\,\bar{n}\cdot p\,\delta^{ab}\,F_\pi\,\phi_\pi(u,\mu) = \int \frac{ds}{(2\pi)} e^{-ius\bar{n}\cdot p} \langle \pi^a(p)|\bar{q}(s\bar{n})\not{\bar{n}}\,\gamma_5\,\frac{\tau^b}{2} q(0)|0\rangle\,.$$

Hard scattering kernel

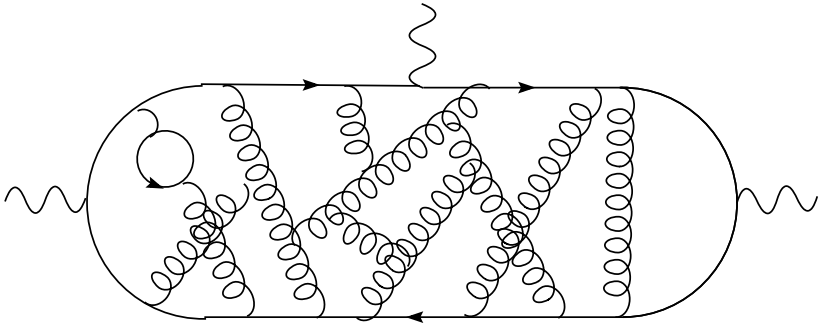
$$T(Q,u,v,\mu=Q) = \frac{4\pi\alpha_s(Q)}{9\,Q^2}\left(\frac{1}{uv} + \frac{1}{(1-u)(1-v)}\right)\,.$$

Perturbative factorization

To prove factorization, analyze all QCD diagrams

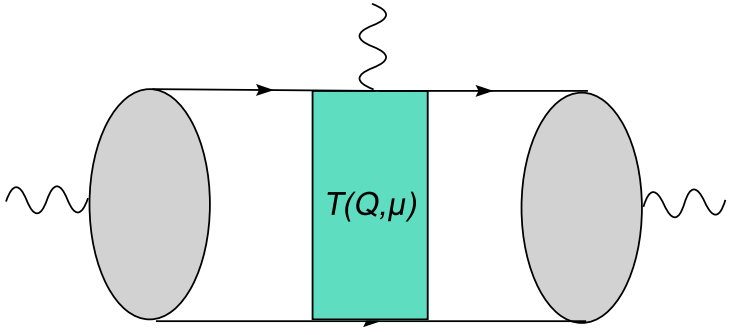
$$Q^2 = -(p_\pi - p'_\pi)^2 \gg \Lambda^2$$

$p_\pi^2 \sim \Lambda^2$



$p_\pi'^2 \sim \Lambda^2$

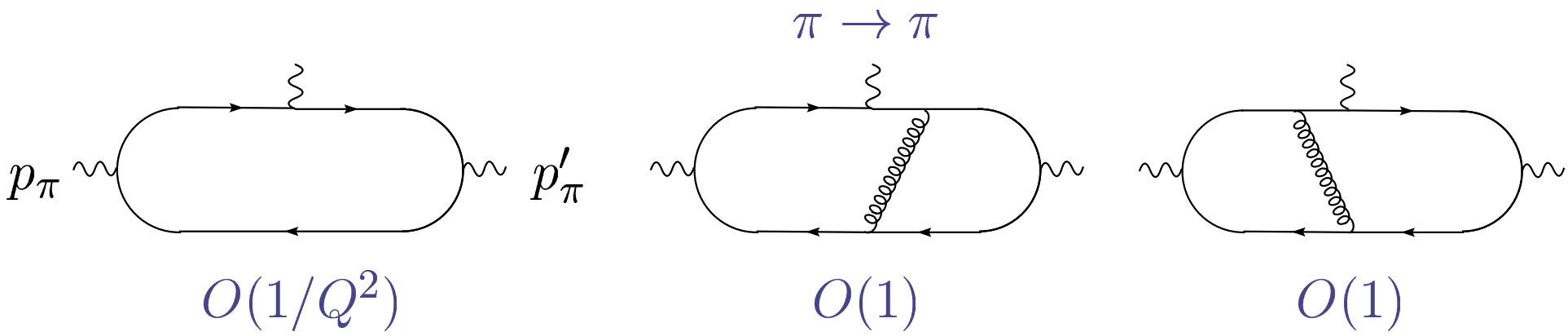
and show that the expansion of Green's function has the form



$$+ \quad \mathcal{O}(\Lambda/Q)\,,$$

and $T(Q,\mu)$ is independent of Λ.

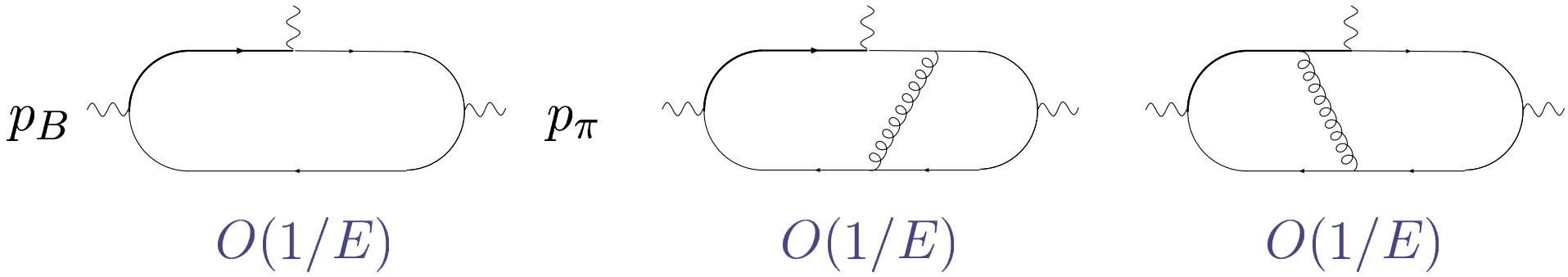
$\pi \to \pi$



- First diagram (end-point configuration) is suppressed.
- Gluon in the second and third diagram is off-shell by Q^2 $\to$ factorization: $F_\pi(Q^2) = \phi_\pi(\mu) \otimes T(Q,\mu) \otimes \phi_\pi(\mu)$.

$B \to \pi$ vs. $\pi \to \pi$

$B \to \pi$ at large recoil



- First diagram unsuppressed.
- Leading contributions from on-shell gluons.
  $B \to \pi$ form factor does not factorize.
- Nonfactorizable part fulfills spin-symmetry relations.

$$F_i^{B\to\pi} = C_i(E,\mu)\,\xi(E,\mu) + \phi_B(\mu) \otimes T_i(E,\mu) \otimes \phi_\pi(\mu)$$

Charles et al., Beneke and Feldmann.

Soft-Collinear Effective Theory

An effective theory for the expansion in Λ/Q:

- Degrees of freedom: momentum modes that give rise to singularities in the limit $\Lambda/Q \to 0$.
  - "Collinear fields" for each direction of large external momentum.
  - "Messenger fields" couple the different collinear sectors.
- Hard scattering kernels: Wilson coefficients of the operators.
- Wave functions: matrix elements of operators in SCET.
- Factorization: no "messenger" contribution to the matrix element.

Advantages of the eff. theory approach

- Simpler to discuss generating functional than graphs.
- Factorization not manifest on the level of individual diagrams, cancellations.
- Power corrections: higher orders in $\frac{\Lambda}{Q}$.
- Can use renormalization group equations in the effective theory to perform resummation of large (single and double) logarithms of $\frac{\Lambda}{Q}$.

Factorization theorems for B-decays

- $B \to DM$ — Bauer, Pirjol, Stewart '01
  no interactions with soft spectator
- $B \to \gamma \ell \nu$ — Lunghi, Pirjol, Wyler '02; Bosch, Hill, Lange, Neubert '03
  no collinear partons
- $B^0 \to D^0 M^0$ — Mantry, Pirjol, Stewart '03
  sub-leading
- $B \to M$ form factor — Bauer, Pirjol, Stewart '02; Beneke, Feldmann '03; Lange, Neubert '03
  sub-leading
- $B \to K^* \gamma$ — TB, Hill, Neubert, hep-ph/soon

(Still) no proof for $B \to M_1 M_2$

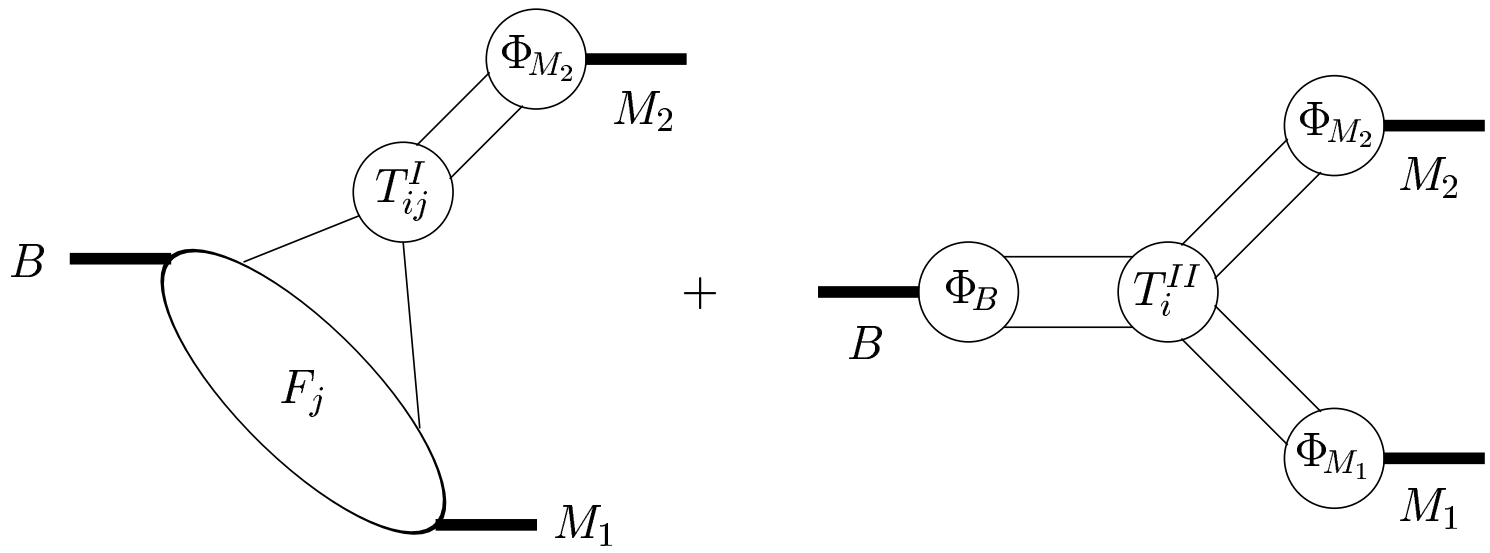
- MANY modes. 29 Br measured,29 constrained, 22 A_{CP}

$B^- \to \pi^- \bar{K}^0$	$B^- \to \pi^0 K^-$	$\bar{B}^0 \to \pi^+ K^-$	$\bar{B}^0 \to \pi^0 \bar{K}^0$	$B^- \to \pi^- \bar{K}^{*0}$
$B^- \to \pi^0 K^{*-}$	$\bar{B}^0 \to \pi^+ K^{*-}$	$\bar{B}^0 \to \pi^0 \bar{K}^{*0}$	$B^- \to \bar{K}^0 \rho^-$	$B^- \to K^- \rho^0$
$\bar{B}^0 \to K^- \rho^+$	$\bar{B}^0 \to \bar{K}^0 \rho^0$	$B^- \to K^- \omega$	$\bar{B}^0 \to \bar{K}^0 \omega$	$B^- \to K^- \phi$
$\bar{B}^0 \to \bar{K}^0 \phi$	$B^- \to K^- K^0$	$\bar{B}^0 \to \bar{K}^0 K^0$	$B^- \to K^- K^{*0}$	$\bar{B}^0 \to \bar{K}^0 K^{*0}$
$B^- \to K^0 K^{*-}$	$\bar{B}^0 \to K^0 \bar{K}^{*0}$	$B^- \to \pi^- \phi$	$\bar{B}^0 \to \pi^0 \phi$	$\bar{B}^0 \to K^- K^+$
$\bar{B}^0 \to K^- K^{*+}$	$\bar{B}^0 \to K^+ K^{*-}$	$\bar{B}^0 \to \pi^0 \bar{K}^{*0}$	$B^- \to K^- \rho^0$	$\bar{B}^0 \to K^- \rho^+$
$B^- \to K^- \phi$	$\bar{B}^0 \to \bar{K}^0 \phi$	$B^- \to K^- K^0$	$B^- \to K^{*-} K^0$	$B^- \to K^- K^{*0}$
$\bar{B}^0 \to K^- K^+$	$\bar{B}^0 \to K^{*-} K^+$	$\bar{B}^0 \to K^- K^{*+}$	$B^- \to \pi^- \pi^0$	$\bar{B}^0 \to \pi^+ \pi^-$
$\bar{B}^0 \to \pi^0 \pi^0$	$B^- \to \pi^- \rho^0$	$B^- \to \pi^0 \rho^-$	$\bar{B}^0 \to \pi^+ \rho^-$	$\bar{B}^0 \to \pi^- \rho^+$
$\bar{B}^0 \to \pi^\pm \rho^\mp$	$\bar{B}^0 \to \pi^0 \rho^0$	$B^- \to \pi^- \omega$	$\bar{B}^0 \to \pi^0 \omega$	$B^- \to \pi^- \pi^0$
$\bar{B}^0 \to \pi^+ \pi^-$	$\bar{B}^0 \to \pi^0 \pi^0$	$B^- \to \pi^- \rho^0$	$B^- \to \pi^0 \rho^-$	$\bar{B}^0 \to \pi^+ \rho^-$
$\bar{B}^0 \to \pi^- \rho^+$	$\bar{B}^0 \to \pi^0 \rho^0$	$B^- \to \pi^- \omega$	$B^- \to \eta K^-$	$\bar{B}^0 \to \eta \bar{K}^0$
$B^- \to \eta' K^-$	$\bar{B}^0 \to \eta' \bar{K}^0$	$B^- \to \eta K^{*-}$	$\bar{B}^0 \to \eta \bar{K}^{*0}$	$B^- \to \eta' K^{*-}$
$\bar{B}^0 \to \eta' \bar{K}^{*0}$	$B^- \to \pi^- \eta$	$\bar{B}^0 \to \pi^0 \eta$	$B^- \to \pi^- \eta'$	$\bar{B}^0 \to \pi^0 \eta'$
$\bar{B}^0 \to \eta \eta$	$\bar{B}^0 \to \eta \eta'$	$\bar{B}^0 \to \eta' \eta'$	$B^- \to \eta \rho^-$	$\bar{B}^0 \to \eta \rho^0$
$B^- \to \eta' \rho^-$	$\bar{B}^0 \to \eta' \rho^0$	$\bar{B}^0 \to \eta \omega$	$\bar{B}^0 \to \eta' \omega$	$\bar{B}^0 \to \eta \phi$
$\bar{B}^0 \to \eta' \phi$	$+B_s$-modes			

- Factorization theorems proposed.

Beneke, Buchalla, Neubert, Sachrajda '99
Keum, Li , Sanda, '00

Structure of factorization theorem



- same structure as form factor.
- Keum, Li and Sanda claim first part to be suppressed after perturbative (Sudakov) resummation.

2. Degrees of freedom

- Strategy of regions
- for a simple 1d integral
- for the Sudakov form factor ($\pi \to \pi$ form factor)
- for $B \to \pi$ form factor

Strategy of Regions

Need to expand diagrams in $\lambda = \Lambda/Q$.

- Clearly, expansion does not commute with loop integration, because integrals develop singularities.
- Strategy of regions:
  1. Find integration regions, where integrand becomes singular.
  2. Put cutoffs to separate regions.
  3. Approximate integrand in each region, integrate, sum up different contributions.
  4. Cutoff-dependence drops out in the sum.
- In dim. reg. step 2 can be omitted!

Simple example: cut-off

$$F(a) = 2\int_0^\infty dk\,\frac{\ln(1+k)}{(k+a)^3} = \frac{1}{a} + 1 + \ln a + \ldots$$

Singularities at $|k| = 1$ and $|k| = a$. Choose $a \ll \Lambda \ll 1$:

$$F(a) = \begin{cases} 2\int_0^\Lambda dk\,\frac{1}{(k+a)^3}\left[k - \frac{k^2}{2} + \ldots\right] = +\frac{1}{a} - \frac{2}{\Lambda} + \frac{3}{2} + \ln\frac{a}{\Lambda} + \ldots \\ 2\int_\Lambda^\infty dk\,\frac{\ln(1+k)}{k^3}\left[1 - \frac{3a}{k} + \ldots\right] = +\frac{2}{\Lambda} - \frac{1}{2} + \ln\Lambda + \ldots \end{cases}$$

In the sum the dependence on Λ cancels. Note: First part has UV, second part IR divergences in Λ.

Simple example: dim. reg.

$$F(a) = 2\int_0^\infty dk\,\frac{\ln(1+k)}{(k+a)^3} = \frac{1}{a} + 1 + \ln a + \ldots$$

Use "dim. reg." instead of cut-off:

$$F(a) = \begin{cases} 2\int_0^\infty dk\; k^{-\epsilon}\frac{1}{(k+a)^3}\left[k - \frac{k^2}{2} + \ldots\right] = \frac{1}{a} - \frac{1}{\epsilon} + \frac{3}{2} + \ln a + . \\ 2\int_0^\infty dk\; k^{-\epsilon}\,\frac{\ln(1+k)}{k^3}\left[1 - \frac{3\,a}{k} + \ldots\right] = \frac{1}{\epsilon} - \frac{1}{2}\ldots \end{cases}$$

- Soft part (first line): count $k \sim a$, expand integrand.
- Hard part (second line): count $k \sim 1 \gg a$, expand.
- Soft part has UV, hard part IR divergences. Cancel in the sum.

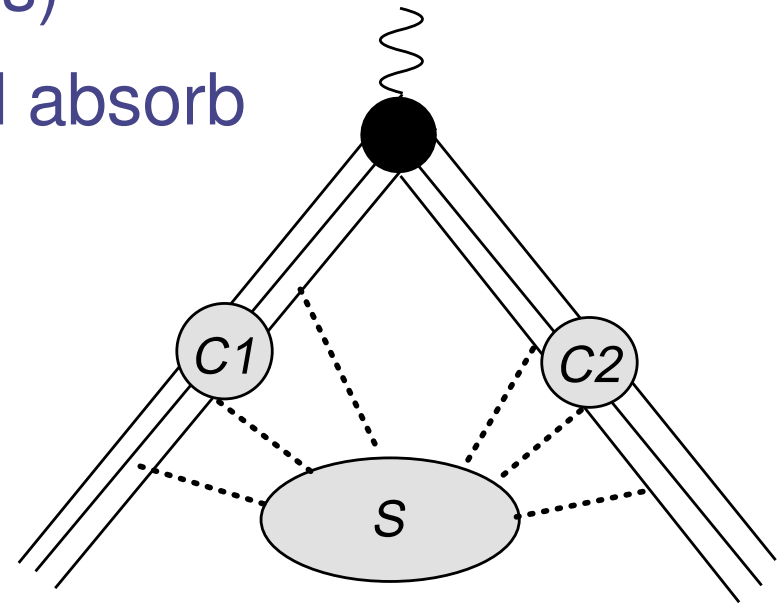
Strategy of Regions in dim. reg.

- Method works for the expansion of multi-loop integrals
  - in large masses, large off-shell momenta
  - around thresholds ($\rightarrow$ NRQCD) Beneke & Smirnov
  - around Sudakov limit Smirnov
  - lattice ints. around continuum limit TB & Melnikov
- Much simpler than working with explicit cut-offs.
- Correspondence to effective theories in dim. reg.

Expansion around the Sudakov limit

Find singularities of loop integrals in the limit Λ/Q and approximate integrand in each region:

- Occur when propagators go on shell
- Correspond to classical scattering processes (off-shell propagators $\equiv$ vertices)
- Energetic particle can emit and absorb
  - collinear particles
  - soft "messenger" particles



Light-cone basis vectors

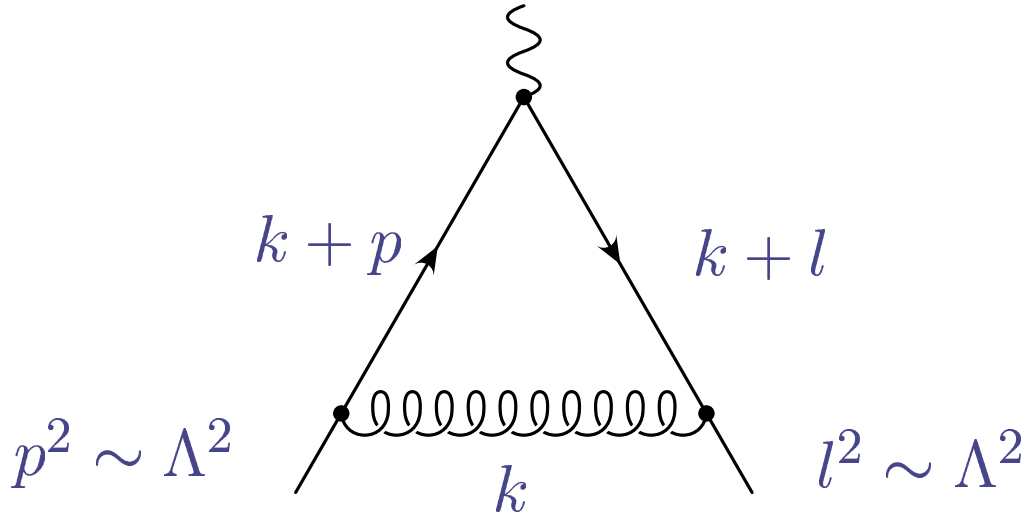
- Different components of the momenta scale different with large scale Q
- Introduce light-like vectors n_μ and $\bar{n}_\nu$

$$n^2 = \bar{n}^2 = 0, n \cdot \bar{n} = 2$$

- E.g. $n_\mu = (1, 0, 0, 1)$, $\bar{n}_\mu = (1, 0, 0, -1)$.
- Split

$$\begin{aligned} p_\mu &= p \cdot n \frac{\bar{n}_\mu}{2} + p \cdot \bar{n} \frac{n_\mu}{2} + p_\mu^\perp \\ &= p_\mu^+ \; + \; p_\mu^- \; + \; p_\mu^\perp \end{aligned}$$

Sudakov form factor



Large momentum transfer $Q^2 = -(p-l)^2 \gg \Lambda^2$

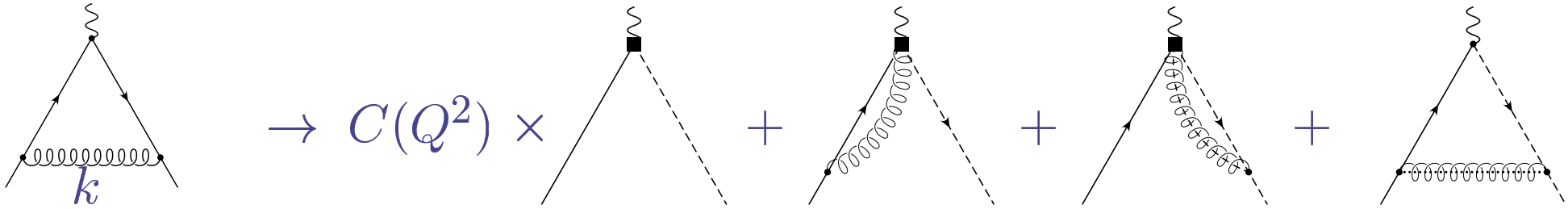
Split: $p_\mu = p \cdot n \frac{\bar{n}_\mu}{2} + p \cdot \bar{n} \frac{n_\mu}{2} + p_\mu^\perp$

$p^2 = p \cdot n \, p \cdot \bar{n} + p_\perp^2$

Counting: $(p \cdot n, p \cdot \bar{n}, p_\perp) \sim Q(\lambda^2, 1, \lambda)$

$(l \cdot n, l \cdot \bar{n}, l_\perp) \sim Q(1, \lambda^2, \lambda)$ with $\lambda = \frac{\Lambda}{Q}$.

Regions in the Sudakov form factor



$(k \cdot n, k \cdot \bar{n}, k_\perp)$	$\sim$	$(1,1,1)$	$(\lambda^2,1,\lambda)$	$(1,\lambda^2,\lambda)$	$(\lambda^2,\lambda^2,\lambda^2)$
		hard	coll.-1	coll.-2	ultra-soft

- All other scalings of k_μ lead to vanishing integrals (either scale-less or no pinch-singularities)
- $k^2 \sim \Lambda^4/Q^2 \ll \Lambda^2$ for the ultra-soft region.

Λ^4/Q^2 ?!?

- In factorization proofs, we want to take limit $\Lambda \to \Lambda_{QCD}$.
- Physics at ultra-soft scale becomes non-perturbative long before $\Lambda \approx \Lambda_{QCD}$!
- "Such [messenger] modes would correspond to color fields extending over large distances of order Q/Λ^2 and do not appear in QCD because of confinement."
- For the above reasons, these modes were *not* introduced in SCET for exclusive processes.

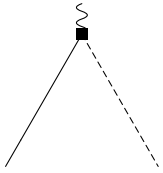
$\to$ No interactions between different collinear sectors.

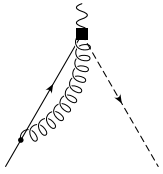
$\to$ Everything factorizes!

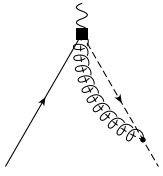
A different look

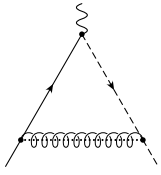
- Cannot “correct” perturbation theory by omitting modes with virtualities below Λ, however, cannot trust perturbative result if they contribute: Factorization breaks down.
- Factorization at leading order $\equiv$ messenger modes do not contribute at leading order.

Structure of the result: Three scales

$C(Q^2) \times$  $= \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{\pi^2}{6}\right) (Q^2)^{-\epsilon}$

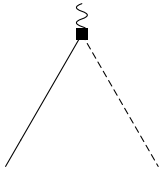
 $= \left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} + 4 - \frac{\pi^2}{6}\right) (-l^2)^{-\epsilon}$

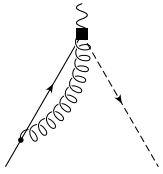
 $= \left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} + 4 - \frac{\pi^2}{6}\right) (-p^2)^{-\epsilon}$

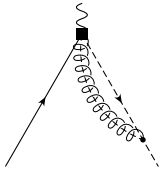
 $= \left(-\frac{2}{\epsilon^2} - \frac{\pi^2}{2}\right) \left(\frac{(-l^2)\,(-p^2)}{Q^2}\right)^{-\epsilon}$

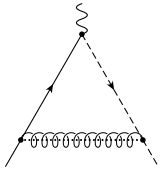
Three scales: $Q^2 \sim O(\lambda^0)$, $p^2 \sim l^2 \sim O(\lambda^2)$, $\frac{p^2\, l^2}{Q^2} \sim O(\lambda^4)$

Structure of the result: Divergent terms

$C(Q^2) \times$  $= \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{\pi^2}{6}\right) (Q^2)^{-\epsilon}$

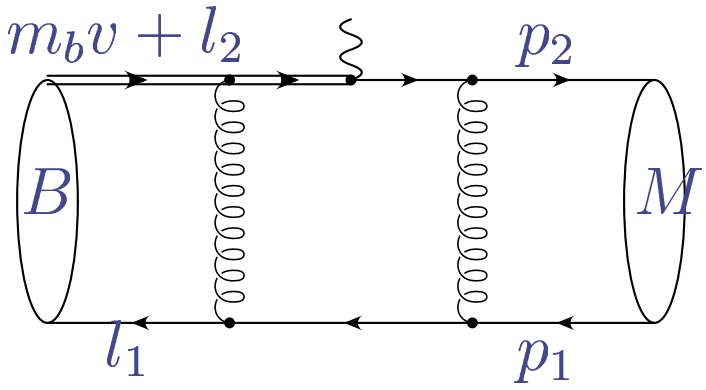
 $= \left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} + 4 - \frac{\pi^2}{6}\right) (-l^2)^{-\epsilon}$

 $= \left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} + 4 - \frac{\pi^2}{6}\right) (-p^2)^{-\epsilon}$

 $= \left(-\frac{2}{\epsilon^2} \qquad - \frac{\pi^2}{2}\right) \left(\frac{(-l^2)\,(-p^2)}{Q^2}\right)^{-\epsilon}$

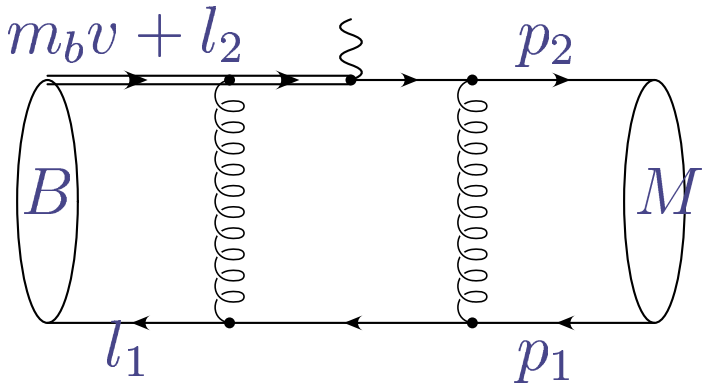
$$\frac{0}{\epsilon^2} + \frac{1}{\epsilon}\left[-\ln Q^2 + \ln(p^2\, l^2) - \ln\frac{p^2\, l^2}{Q^2}\right] + \frac{1}{\epsilon}$$

Regions in heavy-to-light decays:



	$(n \cdot k, \bar{n} \cdot k, k_\perp)$	k^2
Hard	$(\lambda^0, \lambda^0, \lambda^0)$	λ^0
Soft	$(\lambda^1, \lambda^1, \lambda^1)$	λ^2
Collinear	$(\lambda^2, \lambda^0, \lambda^1)$	λ^2

Regions in heavy-to-light decays:



	$(n \cdot k, \bar{n} \cdot k, k_\perp)$	k^2
Hard	$(\lambda^0, \lambda^0, \lambda^0)$	λ^0
Hard-collinear	$(\lambda^1, \lambda^0, \lambda^{\frac{1}{2}})$	λ^1
Soft	$(\lambda^1, \lambda^1, \lambda^1)$	λ^2
Collinear	$(\lambda^2, \lambda^0, \lambda^1)$	λ^2

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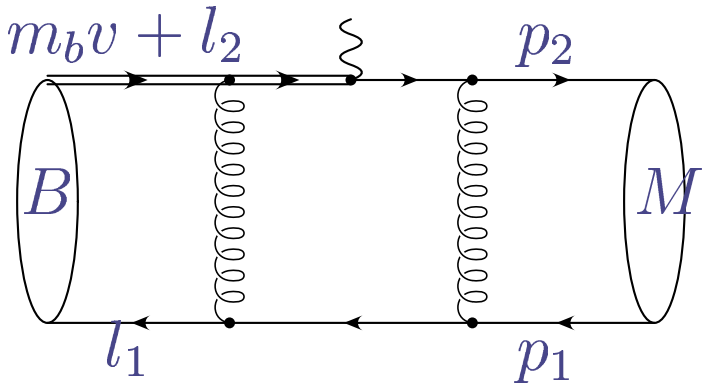
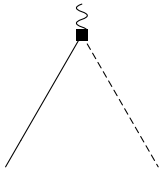
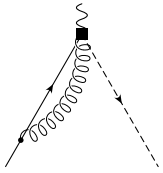


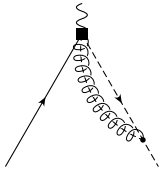
	$(n \cdot k, \bar{n} \cdot k, k_\perp)$	k^2
Hard	$(\lambda^0, \lambda^0, \lambda^0)$	λ^0
Hard-collinear	$(\lambda^1, \lambda^0, \lambda^{\frac{1}{2}})$	λ^1
Soft	$(\lambda^1, \lambda^1, \lambda^1)$	λ^2
Collinear	$(\lambda^2, \lambda^0, \lambda^1)$	λ^2
Soft-collinear	$(\lambda^2, \lambda^1, \lambda^{3/2})$	λ^3

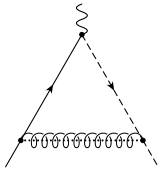
Soft-collinear form factor

Same result as for anti-collinear-collinear case:

$C(Q^2) \times$  $= \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{\pi^2}{6}\right) (Q^2)^{-\epsilon}$

 $= \left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} + 4 - \frac{\pi^2}{6}\right) (-l^2)^{-\epsilon}$

 $= \left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} + 4 - \frac{\pi^2}{6}\right) (-p^2)^{-\epsilon}$

 $= \left(-\frac{2}{\epsilon^2} - \frac{\pi^2}{2}\right) \left(\frac{(-l^2)\,(-p^2)}{Q^2}\right)^{-\epsilon}$

but $Q^2 = -(p-l)^2$ is $O(\Lambda)$ "hard-collinear" instead of hard and $\frac{Q^2}{l^2 p^2}$ is $O(\Lambda^3/E)$ "soft-collinear" instead of ultra-soft.

3. Soft-Collinear Effective Theory

Effective Lagrangian

Construct Lagrangian which gives
the strategy of region Feynman rules.

- Fields for each momentum mode
- Vertices for off-shell propagators
- Expansion of propagators in small momentum components
  (“label formalism”, Bauer et. al.)
  $\leftrightarrow$ derivative expansion of the fields
  (“Multipole expansion”, Beneke et. al.)

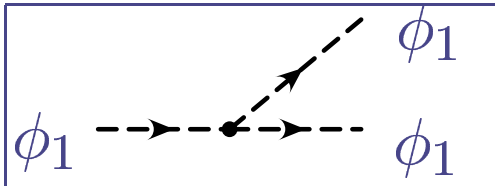
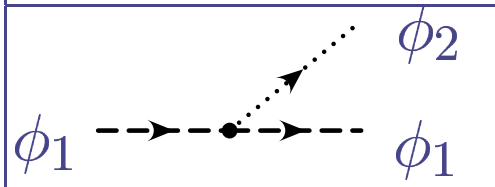
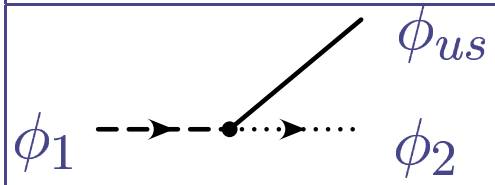
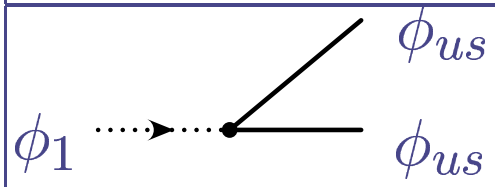
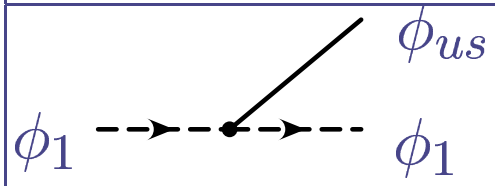
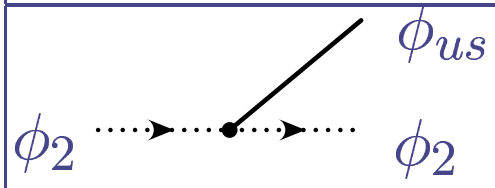
Scalar ϕ^3

$$\mathcal{L}_{\phi^3} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\lambda}{3!}\phi^3$$

Split $\phi = \phi_1 + \phi_2 + \phi_{us}$ and expand:

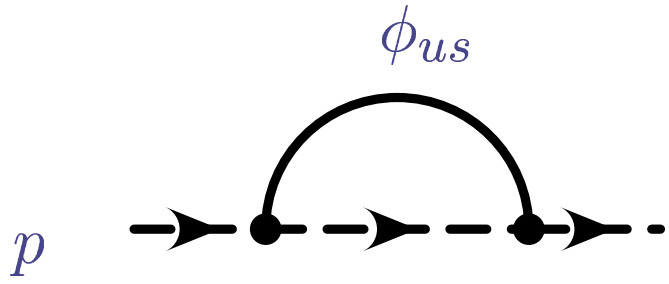
- $\mathcal{L}_{\phi^3} = \mathcal{L}_{\phi_1^3} + \mathcal{L}_{\phi_2^3} + \mathcal{L}_{\phi_{us}^3} + \mathcal{L}_{\mathrm{Int}}$
- Without $\mathcal{L}_{\mathrm{Int}}$ each of the three sectors is equivalent to the full theory.
- Only the external kinematics (sources for the three fields) makes them different.
- Drop kinematically forbidden terms.
- Multipole expand interactions.

$\mathcal{L}_{\text{Int}}$

	$\phi_1(x)^3$
	kin. forbidden
	kin. forbidden
	kin. forbidden
	$\phi_1(x)^2\left[\phi_{us}(x_-) + x_\perp^\mu \partial_\mu \phi_{us}(x_-) + \ldots\right]$
	$\phi_2(x)^2\left[\phi_{us}(x_+) + x_\perp^\mu \partial_\mu \phi_{us}(x_+) + \ldots\right]$

No double counting

- Ultra-soft messenger mode contributes only in graphs which involve both types of collinear fields.
  E.g.

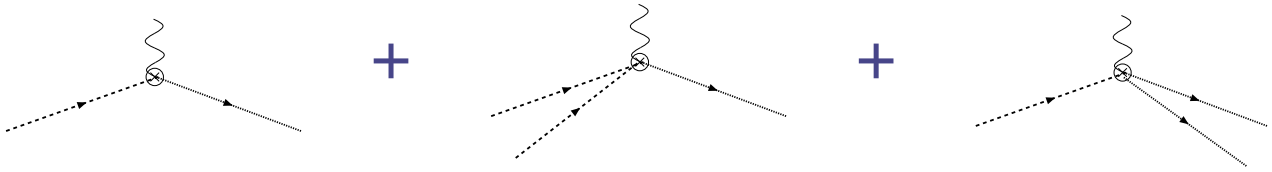


$$= \int d^d k \, \frac{1}{k \cdot k} \frac{1}{p_- \cdot k_+} = 0$$

since the integral is scaleless.

Suggested further playing:

- Introduce collinear$_1$-collinear$_2$ current in scalar SCET.

$J =$  $+\mathcal{O}(Q^{-2})$

- Determine the order of the various terms in $\mathcal{L}_{\text{eff}}$.
- Derive Feynman rules. (Note: Momentum is not conserved at the vertices)
- Calculate scalar Sudakov form factor. (One finds that, exactly as in QCD, all regions contribute at leading order.)

Non-scalar SCET:

- Gauge invariance complicates construction of the effective theory.
- Separate gauge transformations for the different modes.
- Constraints:
  - Sum of the fields has to transform as the QCD field
  - Gauge transformations must respect power counting: Messenger modes do not transform under collinear gauge transformations.
- Can redefine fields, so that $\mathcal{L}$ is invariant under gauge transformations, that are homogeneous in λ.

Beneke et. al. , '02

Quark Lagrangian

$$\mathcal{L}_s = \bar{\mathcal{Q}}_s \, i\not{D}_s \, \mathcal{Q}_s$$

$$\mathcal{L}_c = \bar{\mathcal{X}} \, i\bar{n}\cdot\mathcal{D}_c \, \mathcal{X} + \mathcal{X}\frac{\hbar}{2} i\not{D}_{c\perp} \, \frac{1}{i\bar{n}\cdot\partial} \, i\not{D}_{c\perp} \, \mathcal{X}$$

$$\mathcal{L}_{sc} = \bar{\theta} \, \frac{\hbar}{2} \, in\cdot D_{sc} \, \theta + \bar{\theta} \, \frac{\hbar}{2} \, i\not{D}_{sc\perp} \, \frac{1}{i\bar{n}\cdot D_{sc}} \, i\not{D}_{sc\perp} \, \theta \, .$$

$$\mathcal{L}^{\rm int}_{s+sc} = \bar{\mathcal{Q}}_s \, \frac{\hbar}{2} \, W^\dagger_{sc} \, x_{\perp\mu}\bar{n}_\nu \, g_s G^{\mu\nu}_{sc} \, W_{sc} \, \mathcal{Q}_s + O(\lambda^5)$$
$$+\left\{ \bar{\mathcal{Q}}_s \, \not{\mathcal{A}}_{s\perp} \, W^\dagger_{sc} \left[(1 + x_\perp\cdot D_{sc}) \, \theta + \sigma \right] + \bar{\mathcal{Q}}_s \, \frac{\hbar}{2} \, \bar{n}\cdot\mathcal{A}_s \, W^\dagger_{sc} \, \sigma + \text{h.c.} + O(\lambda^{11/2}) \right\} .$$

$$\mathcal{L}^{\rm int}_{c+sc} = \bar{\mathcal{X}} \, \frac{\hbar}{2} \, S^\dagger_{sc} \, x_{\perp\mu} n_\nu \, g_s G^{\mu\nu}_{sc} \, S_{sc} \, \mathcal{X} + O(\lambda^5)$$
$$+\left\{ \bar{\mathcal{X}} \, \not{\mathcal{A}}_{c\perp} \, S^\dagger_{sc} \, (1 + x_\perp\cdot D_{sc}) \, \sigma - \bar{\mathcal{X}} \, i\not{D}_{c\perp} \, \frac{\hbar}{2} \, \frac{1}{i\bar{n}\cdot\partial} \, \not{\mathcal{A}}_{c\perp} \, S^\dagger_{sc} \, \theta + \bar{\mathcal{X}} \, \frac{\hbar}{2} \, n\cdot\mathcal{A}_c \, S^\dagger_{sc} \, \theta \right.$$
$$\left. + \text{h.c.} + O(\lambda^{11/2}) \right\} .$$

TB, Hill, Neubert, hep-ph/0308122

SCET for exclusive B-decays

- Identified degrees of freedom; constructed effective Lagrangian to sub-leading order. TB, Hill, Neubert '03
- Anomalous dimensions of leading-order currents and four-quark operators. TB, Hill, Lange, Neubert '03
- Framework has been used to analyze $B \to \pi$ form factor:
  - nonfactorizable part of the form factor evolves like leading order current
  - no perturbative (Sudakov) suppression of the non-factorizable part.

Lange, Neubert '03

Summary

- Formalism for analyzing B-decays into energetic light particles is in place.
- $B \to \pi$ form factor analyzed. $B \to M_1 M_2$ is next...

Conclusion

- Don’t kill the messenger.