Ghost Condensation and Infrared Modification of Gravity

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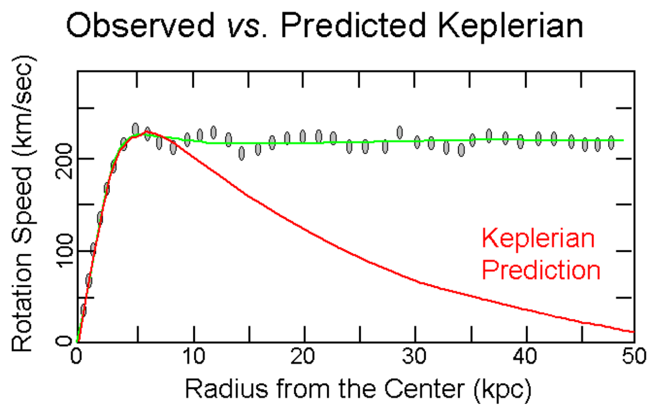
Q: How well do we know about gravity?

Theoretically, general relativity breaks down at the Planck scale $\sim 10^{19}\text{GeV} \sim (10^{-33}\text{cm})^{-1}$.

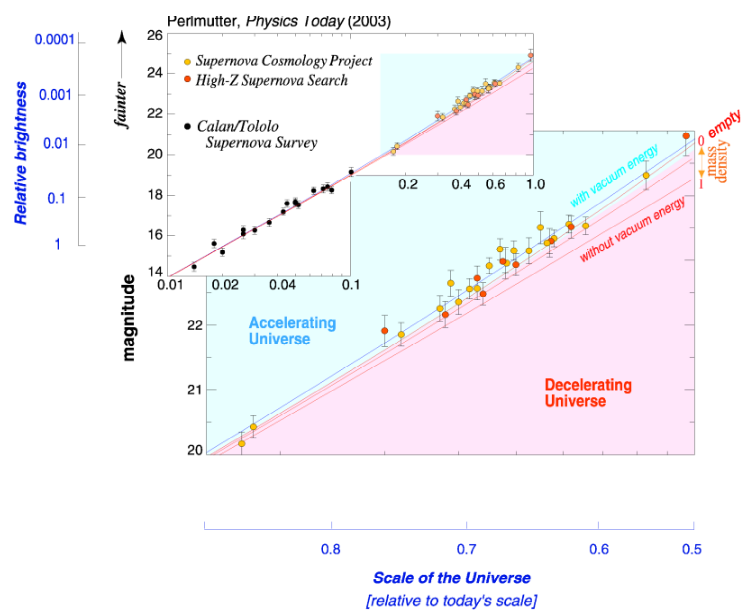
Experimentally or observationally, gravity is observed between 0.2 mm ($\sim 10^{-3}$eV) and $H_0^{-1} \sim 10^{26}$ m ($\sim 10^{-33}$eV). We know gravity very well at distance scales of the solar system and the laboratory (Cavendish-type experiments). However, we know nothing about gravity at distance shorter than 0.2 mm, and we don't know gravity very well at distance of galactic scale or larger. In fact, there are surprises at larger and larger distance scales.

- **Galactic rotation curves**



Standard explanation: dark matter

- **Accelerated expansion of the universe**



Standard explanation: cosmological constant (dark energy)

—"Dark" age of cosmology

It's not the first time in the history that observations do not agree with the theory directly.

In the 1800's, a dark planet "Vulcan" was proposed to explain the precession of perihelion of the Mercury.

The right answer was that Newtonian gravity is modified to General Relativity.

Q: Is it possible to modify gravity at large distances to address some of the problems?

First step: Can gravity be modified in the infrared in a theoretically consistent and experimentally viable way?

There exist proposals which modify gravity at large distances, for example, scalar-tensor theory by adding a scalar coupling to $T_{\mu\nu}$, and its disguises. However, they are not very interesting either theoretically or experimentally.

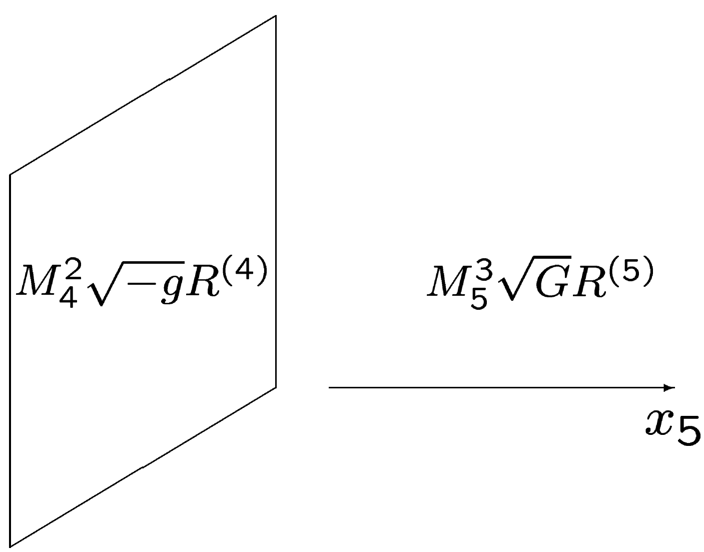
More interesting examples:

- **Massive gravity**

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}M_P^2\left\{R + m_g^2\left(h^2 - h_{\mu\nu}h^{\mu\nu}\right)\right\}, h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}.$$

Yukawa type suppression $e^{-m_g r}$ for $r > m_g^{-1}$.

- **Dvali-Gabadadze-Porrati (DGP) model**:



Gravity becomes 5-dimensional at $r > r_c$ ($r_c \equiv M_4^2/M_5^3$).

However, in both theories, matter couples with gravitational strength to a new scalar degree of freedom ϕ, which becomes strongly coupled at a scale $\Lambda = (m_g^2 M_P)^{1/3}$ $\left[(r_c^{-2} M_P)^{1/3} \text{for DGP}\right]$.

Massive gravity: $\phi \sim$ scalar longitudinal mode of the graviton

(N. Arkani-Hamed, H. Georgi, M. D. Schwartz)

DGP: $\phi \sim$ brane bending mode

(M. Luty, M. Porrati, R. Rattazzi)

For $m_g(r_c^{-1}) \sim H_0$, $\Lambda \sim (1000\text{km})^{-1}$, i.e., effective theory breaks down at distance shorter than 1000km.

General covariance, like all local symmetries, is not a "real symmetry," but a redundancy of description. For any Lagrangian, we can always formally restore general covariance (by including new fields).

The real issue of modifying gravity is not whether the theory is general covariant or not, but rather how many degrees of freedom it describes and how they interact.

The root of the problem:
We couple gravity to a sector which is sick in the absence of gravity (no ϕ kinetic term). Only coupling to gravity makes it tenuously healthy.

$\Rightarrow$ We should look for a sector which is healthy even without gravity (and transforms non-trivially under diffeomorphism).

A such sector can be described by "Ghost condensation."

It can be thought as a theory with a scalar ϕ having a vev, $\langle\phi\rangle = c\,t$, (and it does not redshift like other fluids as the universe expands.)

It is a new kind of fluid which can fill the universe, and it has the equation of state $p = -\rho$, just like a cosmological constant. However, it's not a c.c. as it has a physical scalar excitation,

$$\phi = c\,t + \pi.$$

$*$ There are 2 important properties which account for the new phenomena we found.

1. $\langle\phi\rangle = ct$ breaks t translation and ϕ shift symmetry ($\phi \to \phi + a$) down to the diagonal new " time translational symmetry." $\Rightarrow$ 2 energies, $E_{\text{particle physics}} \neq E_{\text{grav}}$.

2. Symmetries determines the low energy dispersion relation for π, $\omega^2 = k^4/M^2$.

Summary of results and surprises:

- A new fluid which behaves like the cosmological constant ($\rho = -p$), but it's not a cosmological constant because it has physical excitations. It can give rise to a deSitter universe even without a real cosmological constant.

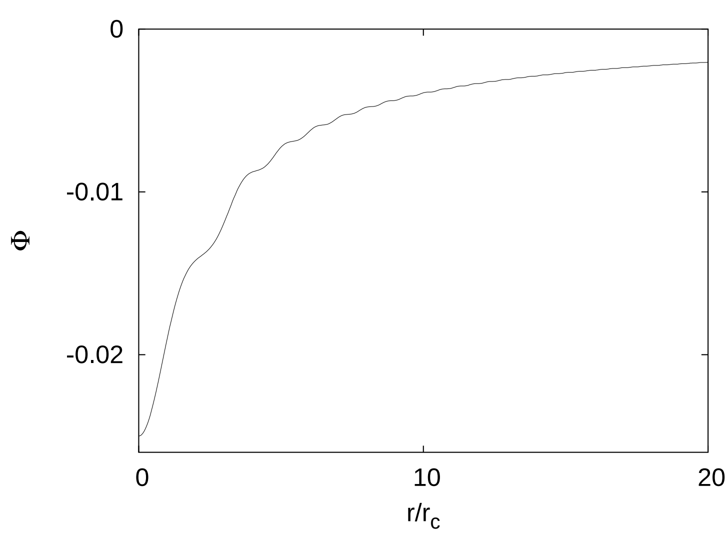
- The excitations have the dispersion relation $\omega^2 \sim \frac{k^4}{M^2}$. Time scale and length scale are not related by c.

- The fluid does not need to couple to ordinary matter directly. If it couples to matter, it generates Lorentz violating effects in the matter (e.g., spin-dependent dispersion relations), and it can also induce $\frac{1}{r^2}$ spin-spin force.

- Antigravity! Two energies. The one which couples to gravity can be negative while the conserved one is positive.

- Jeans-like instability for very long wave length modes in flat space. It produces oscillatory gravitational potential at late time.

- No instability in dS (for $H > \Gamma$), but oscillatory modulation of gravitational potential still happens at late time.



Ghost condensation (without gravity)

Consider a scalar with the wrong sign kinetic term,

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \cdots .$$

The vacuum at $\phi = 0$ is unstable.

Compare it with tachyon,

$$V(\phi) = -\frac{1}{2}m^2\phi^2 + \cdots .$$

Similarly, vacuum at $\phi = 0$ is unstable, but if there are higher order terms in the potential, e.g.,

$$V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \cdots ,$$

there can be a stable vacuum where $\langle\phi\rangle \neq 0$.

Can the ghost also be stabilized by higher order terms, e.g.,

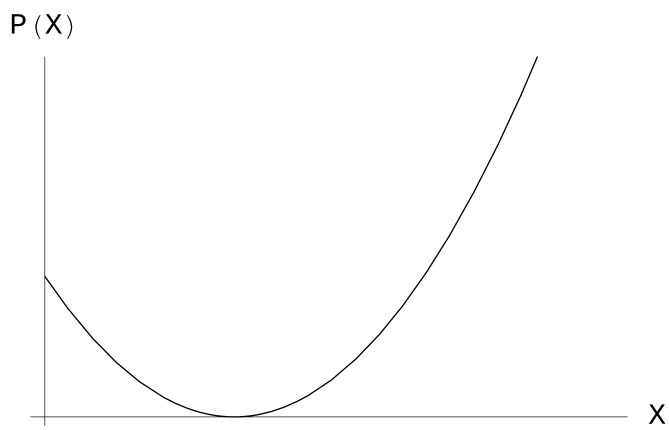
$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{4M^4}(\partial\phi)^4 + \cdots ?$$

$(\partial\phi)^4, \cdots$ are higher dimensional operators, so we should really take

$$\mathcal{L} = P\left((\partial\phi)^2\right) + Q\left((\partial\phi)^2\right) R(\Box\phi) + \cdots,$$

where we assume there is a shift symmetry on ϕ, $\phi \to \phi +$ constant.

We assume that $P(X)$ has a stable minimum,



Keeping P only for the moment, in flat space without gravity the equation of motion is

$$\partial_\mu[P(X)\partial^\mu\phi] = 0.$$

Solution: $\partial_\mu\phi =$ constant. If $\partial_\mu\phi$ is time-like

$$\phi = ct.$$

Consider the small fluctuations around the classical solution

$$\phi = ct + \pi.$$

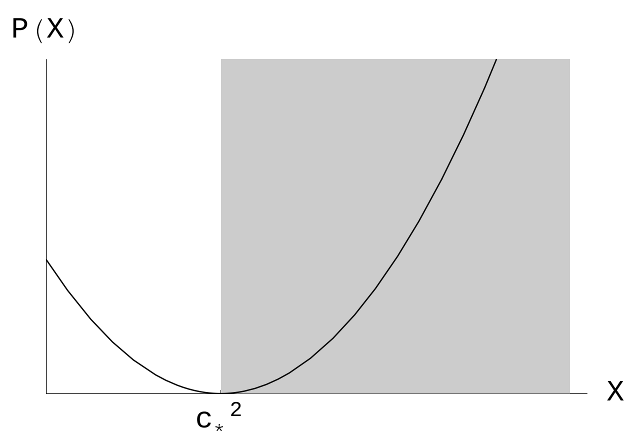
The Lagrangian for π in quadratic order is

$$\mathcal{L}_\pi = \left[P'(c^2) + 2c^2 P''(c^2)\right](\partial_0\pi)^2 - 2P'(c^2)(\nabla\pi)^2.$$

There is a meaningful expansion as long as π fluctuations are small.

The fluctuations are stable provided that c is such that

$$P'(c^2) > 0, \quad P'(c^2) + 2c^2 P''(c^2) > 0.$$



The effect of the expansion of universe

In an expanding universe, the equation of motion becomes

$$\partial_0[a^3(t)P'(X)\partial_0\phi] = 0.$$

We are driven to

$$P'(c_*^2) = 0\,, \qquad \text{(or } c = 0, \text{ but unstable).}$$

The background breaks Lorentz invariance by picking out a preferred frame, similar to any other cosmological fluids (CMBR, matter, rolling inflaton,. . .). However, unlike other fluids, it does not dilute away as universe expands.

$$T_{\mu\nu} = -g_{\mu\nu}P + 2(\partial_\mu\phi\partial_\nu\phi)P'$$

The first term is exactly like a cosmological constant and the second term redshifts like matter ($a^{-3}(t)$).

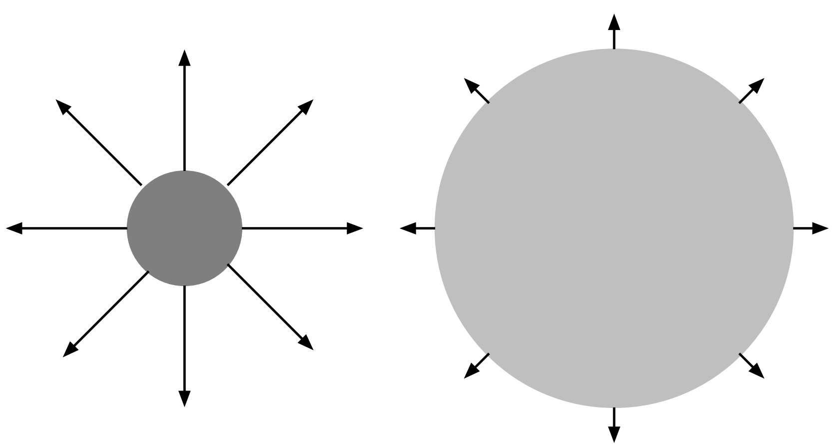
Low energy dispersion relation

At $P' = 0$ there is no quadratic spatial kinetic term for π, the spatial kinetic term has to come from higher derivative terms, e.g.,

$$-(\Box\phi)^2 \supset -(\nabla^2\pi)^2,$$

$\Rightarrow$ universal low energy dispersion relation

$$\omega^2 \sim \frac{k^4}{M^2}.$$



Coupling to matter

Unlike other IR modifications of gravity where a scalar mode couples to matter with the gravitational strength, ϕ does not need to couple to matter directly, and coupling through gravity is highly suppressed.

If ϕ couples to matter, it induces Lorentz violations in the matter sector. In addition, it can induces unusual spin-dependent forces. The interaction

$$\frac{1}{F}\overline{\Psi}\gamma^\mu\gamma^5\Psi\,\partial_\mu\phi$$

gives a Lorentz-violating dispersion relation,

$$\omega = \sqrt{(|p| \pm \mu)^2 + m_D^2}, \quad \mu \equiv \frac{M^2}{F},$$

where $\pm$ for $\pm$ helicities, and m_D is the Dirac mass.

It also contains a derivative coupling to π, which in the non-relativistic limit is

$$\frac{1}{F} S_i \partial_i \pi.$$

Exchange of π produces a long-range spin dependent force, as for usual Goldstone bosons. However, the $1/k^4$ propagator gives rise to a $1/r$ potential (in contrast to $1/r^3$ for usual Goldstone bosons),

$$V \sim \frac{M^2}{F^2} \frac{S_1 \cdot S_2 - 3(S_1 \cdot \hat{r})(S_2 \cdot \hat{r})}{r}.$$

However, π does not propagate at speed of light, the retardation effect can be significant.

Two energies and antigravity

The $\langle\phi\rangle = ct$ background breaks the time translational symmetry and ϕ shift symmetry down to the diagonal subgroup. The conserved charge associated with this new translational symmetry is a linear combination of T_{00} and the charge associated with the ϕ shift symmetry,

$$E = T_{00} - cQ.$$

This conserved "energy" is positive for π fluctuations,

$$E \sim (\partial_0 \pi)^2 + (\nabla^2 \pi)^2 + \cdots ,$$

which ensures the stability of the vacuum against small fluctuations.

However, the energy coupled to gravity is T_{00}, not E. It has linear dependence on $\partial_0 \pi$,

$$T_{00} \sim c\,\partial_0 \pi + \cdots ,$$

so it can be either positive or negative. A lump of π with $\partial_0 \pi < 0$ has negative T_{00} and it can antigravitate!

Effective field theory for ghost condensate

We are interested in the vacua

- $\langle \partial_\mu \phi \rangle \neq 0$ and time-like everywhere.

- The background metric is maximally symmetric (flat or dS).

There is a universal low energy theory describing the fluctuations about such a vacuum.

It's convenient to go to the unitary gauge where we use ϕ as the clock,

$$\phi(t,x) \equiv t.$$

No more time diffeomorphism, but we still have the spatial diffeomorphism

$$x \to x'(t,x).$$

First consider the linearized spatial diffeomorphisms in a flat background,

$$\delta h_{00} = 0, \quad \delta h_{0i} = \partial_0 \xi_i, \quad \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i.$$

Invariants:

$$h_{00}, \quad K_{ij} = \frac{1}{2}(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j}).$$

Terms can be added in the Lagrangian:

$$h_{00}^2,\ (\mathrm{Tr}K)^2,\ \mathrm{Tr}(K^2), \cdots.$$

Reintroducing the Goldstone boson,

$$h_{00} \to h_{00} - 2\partial_0 \pi, \quad h_{i0} \to h_{i0} - \partial_i \pi$$

$$h_{00}^2 \supset (\partial_0 \pi)^2, \quad K^2 \supset (\partial_i^2 \pi)^2$$

No invariant contains $(\partial_i \pi)^2$ term, $\Rightarrow \omega^2 \sim k^4$.

At full non-linear level,

$$h_{00} \to -(g^{00} - 1), \quad K_{ij} = \text{extrinsic curvature}$$

Terms that are quadratic in π:

$$(g^{00} - 1)^2, \quad (\mathrm{Tr}K)^2, \quad \mathrm{Tr}(K^2).$$

The leading terms in the low energy effective theory are

$$\mathcal{S} = \int d^4x \sqrt{-g} \left\{ \frac{M_p^2}{2} R + \left[\frac{M^4}{8} (g^{00} - 1)^2 - \frac{\tilde{M}^2}{2} (\mathrm{Tr}K)^2 - \frac{\tilde{M}'^2}{2} \mathrm{Tr}(K^2) \right] + \cdots \right\}.$$

$\Rightarrow$ A theory with 3 degrees of freedom.

The effects of gravity on ghost condensate

Recall gravity effects on ordinary fluids.
In flat space,

$$\omega^2 = v_s^2 k^2 - \omega_J^2, \text{ where } \omega_J^2 = \frac{\rho}{2M_P^2}.$$

Jeans instability for long wavelength modes.
(Goes away in expanding universe for $H > \omega_J$.)

For ghost condensate (in flat space),

$$\omega^2 = \epsilon^2 \left(\frac{k^4}{M^2} - \frac{M^2}{2M_P^2} k^2 \right), \text{ where } \epsilon^2 \sim \frac{\tilde{M}^2}{M^2}.$$

Instability for modes with

$$k < \frac{M^2}{\sqrt{2} M_P} (\equiv m),$$

and the instability time scale Γ^{-1} is given by

$$\Gamma \sim \epsilon \frac{M^3}{4 M_P^2}.$$

Some sample parameters (for $\epsilon \sim 1$):

$$M \sim 10^{-3}\text{eV}, \;\; m \sim H_0, \;\; \Gamma \sim 10^{-30} H_0;$$

$$M \sim 10\,\text{MeV}, \;\; m \sim (1000\,\text{km})^{-1}, \;\; \Gamma \sim H_0.$$

In de Sitter space,

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2, \qquad a(t) = e^{Ht},$$

the equation of motion is modified to

$$\partial_0^2 \pi + H \left(\frac{m^2 - 3\frac{k^2}{a^2}}{m^2 - \frac{k^2}{a^2}} \right) \partial_0 \pi - \frac{4\Gamma^2}{m^4} \frac{k^2}{a^2} \left(m^2 - \frac{k^2}{a^2} \right) \pi = 0.$$

For $H > \Gamma$, the friction term dominates and the instability goes away.

Modification of gravity by G.C.

In flat space, the Newtonian potential from a matter source is modified to (in Fourier space)

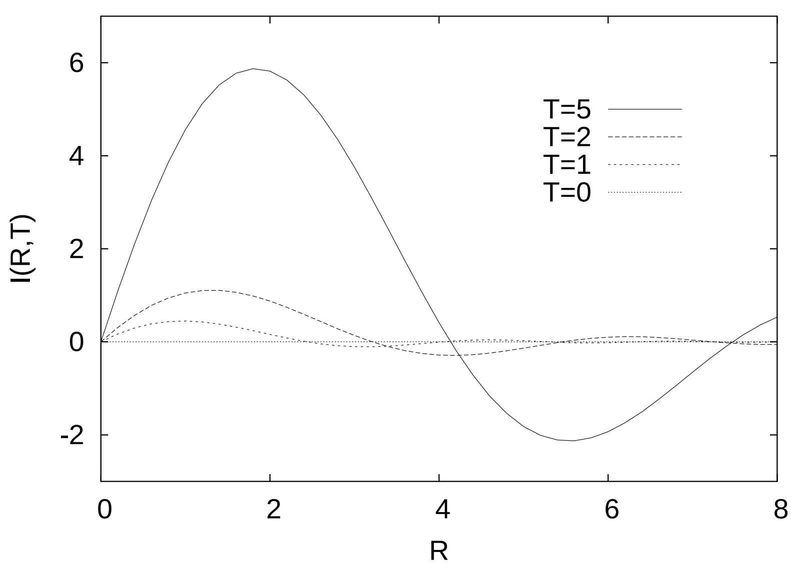
$$\Phi_{\omega,k} = \left(1 - \frac{\epsilon^2 m^2 k^2}{M^2\omega^2 - \epsilon^2 k^4 + \epsilon^2 k^2 m^2}\right) \frac{4\pi G_N \rho_{\omega,k}}{-k^2}$$

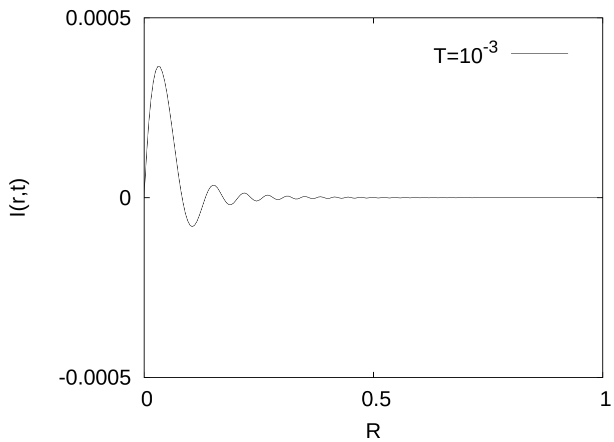
For a point source which turns on at the origin at time $t = 0$,

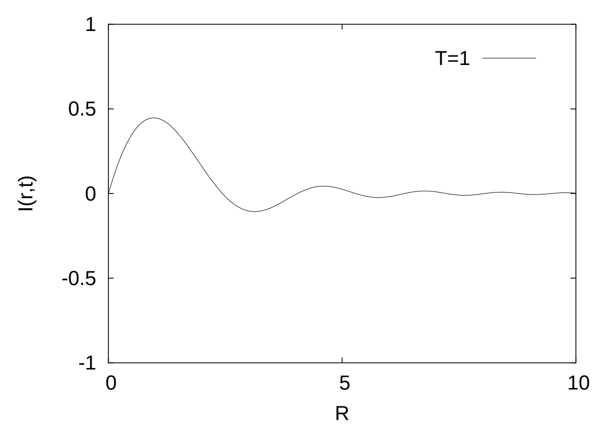
$$\rho(r,t) = \delta^3(r)\theta(t),$$

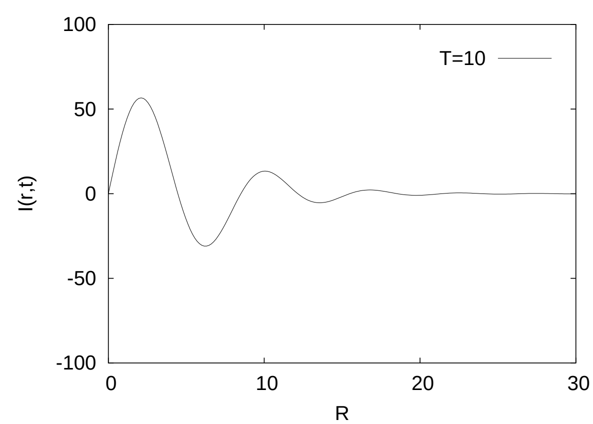
The resulting gravitational potential is,

$$\Phi(r,t) = -\frac{G_N}{r}\left(1 + I(r,t)\right).$$

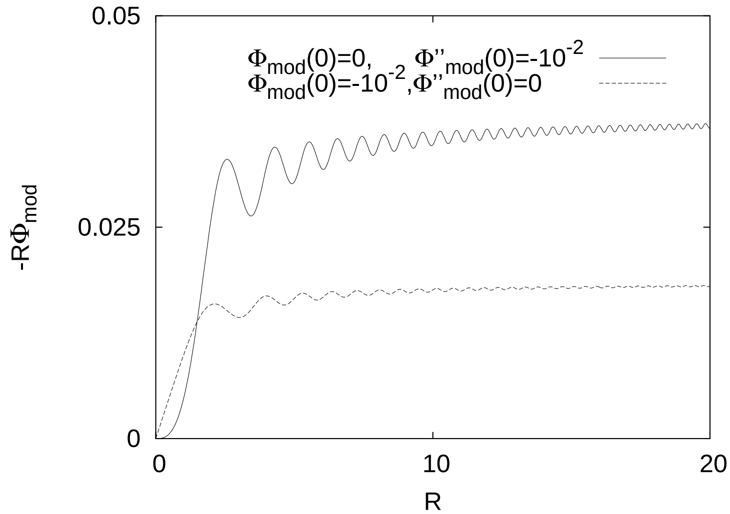


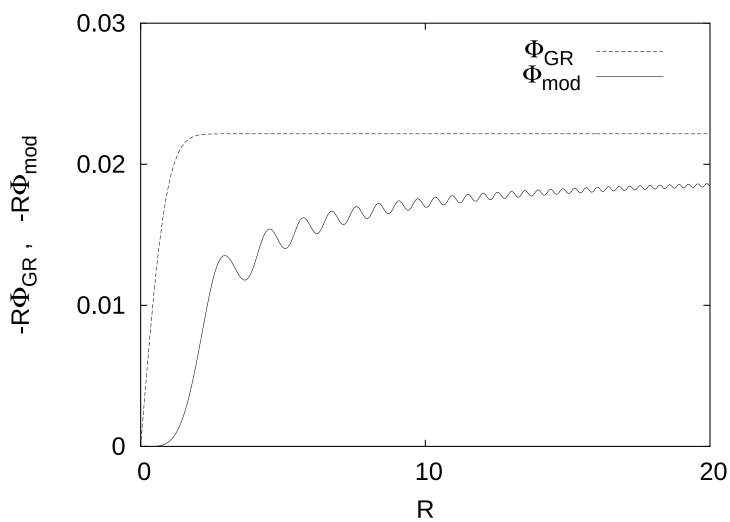

T=10^{-3}
I(r,t)
0.0005
0
-0.0005
0
0.5
1
R


T=1
I(r,t)
1
0.5
0
-0.5
-1
0
5
10
R


T=10
I(r,t)
100
50
0
-50
-100
0
10
20
30
R

In de Sitter space, modulation of Newtonian potential also happens at late time. There is no instability for $H > \Gamma$. Instead, there are stationary solutions where the oscillatory potentials are frozen by the expansion of the universe.





Conclusions and outlook

- A consistent framework for IR modification of gravity — controllable, calculable, and can still be consistent with experimental observations.

- Many striking results and possible signals associated with the new ghost condensate fluid,
  —antigravity,
  —oscillatory potentials,
  —time scale $\neq$ distance scale,
  and if it couples directly to Standard Model particles:
  —Lorentz violations,
  —$1/r^2$ spin-dependent forces..

There are also many directions and open questions waiting to be explored:

- Application to cosmology:
  —Inflation without Λ,
  —dark energy,
  —dark matter,
  . . . .

- What are the current bounds on the model? (New forces, modification of gravity, Lorentz violations, . . .)

- What about nonlinear effects, black holes?

- Possible UV completions?

- Can it provide new ways to think about the cosmological constant problem?