Physics at the Large Hadron Collider and challenges for perturbative calculations

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New phenomena at TeV energies

- LEP, SLC, Tevatron, muon experiments, B-meson factories, $\ldots$
  - *Discovered all Standard Model particles except the Higgs boson!*
  - *A host of precision measurements, pointing to a light Higgs boson!*
  - *Few inconclusive* ($< 3\sigma$) *deviations:* $(g-2)_\mu$, $sin^2\theta_w$, $\ldots$
- The SM is not the whole story:
  - *Gravity?*
  - *Massive neutrinos.*
  - *Dark matter + dark energy.*
- Revolutionary theoretical possibilities:
  - *Supersymmtery, Extra dimensions.*

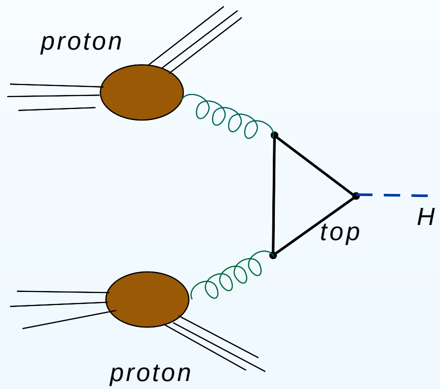
The LHC will give us a unique opportunity for new discoveries.

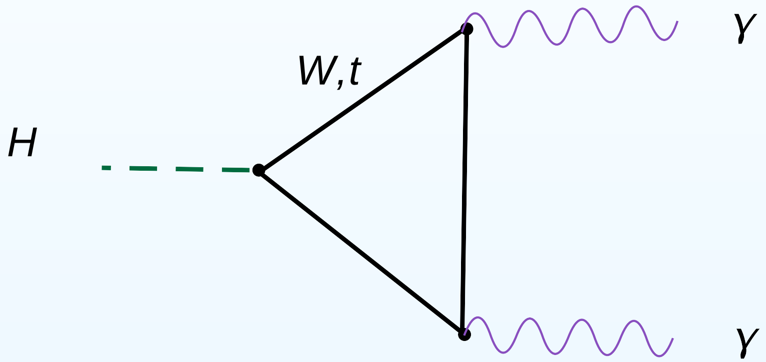
Large Hadron Collider

- Starting operation in 2007
- Collisions of 7 TeV proton beams
- Luminosity 10 - 100 fb^{-1}/year
  - Small statistical uncertainties $1\% - 2\%$ will be easily achieved.
  - Very good detectors. Easier in situ callibration.
- High rates could allow both discoveries and precision studies
- The LHC will put to test our abilities to perform perturbative computations

An example of an “easy” experimental discovery

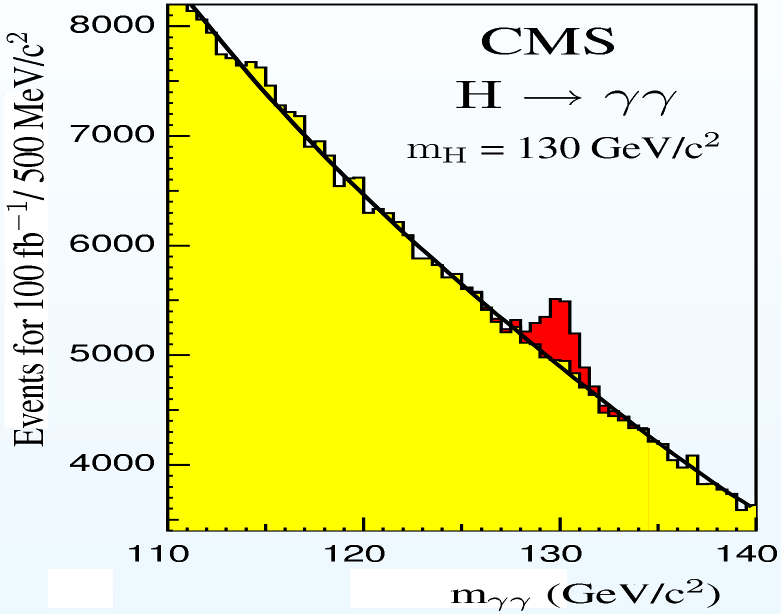
- The SM predicts a significant cross-section for a di-photon signal from a Higgs boson.





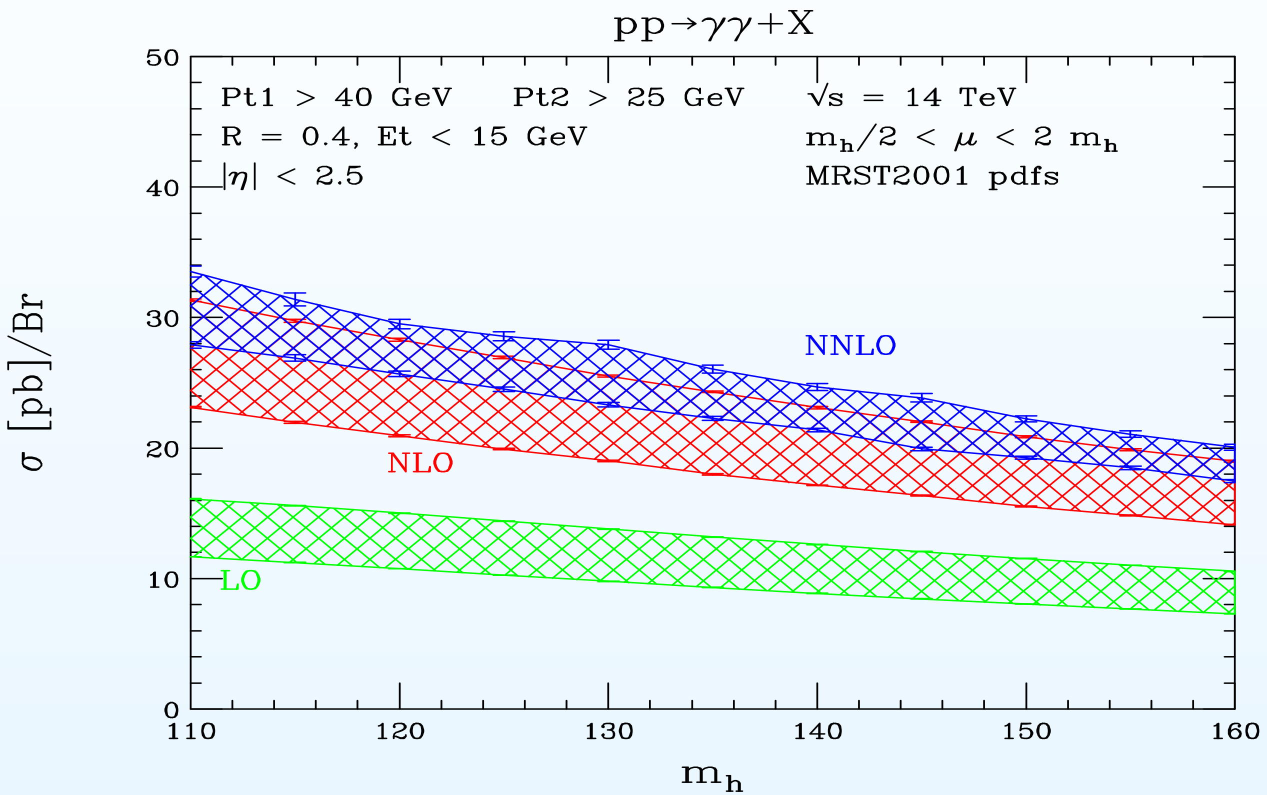
- Discovery of a resonance is a matter of purely (very hard) experimental work and collecting data.

The di-photon signal



- It is not necessarily true that this peak is a SM Higgs boson.
- New physics beyond the SM can change significantly the height of the peak.
- So do higher order QCD corrections

Di-photon signal cross-section



CA, Melnikov, Petriello

- The cross-section at NNLO is 2 times the LO result.
- Scale uncertainty reduces from $\pm 15\%$ (NLO) to $\pm 7\%$ (NNLO).

A global approach to precision calculations

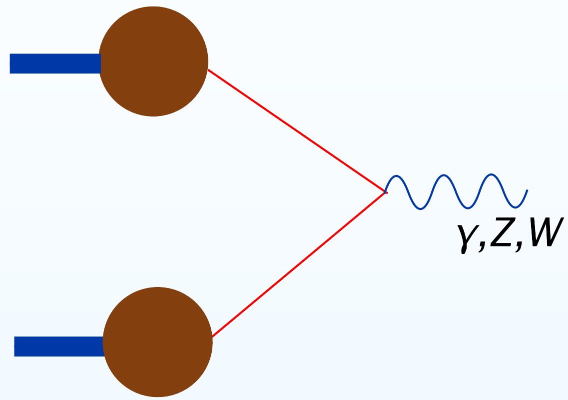
$$N = \mathcal{L} \times \left(\int f_i(x_1) f_j(x_2) \sigma(i+j \to H+X) \right) \times \frac{\Gamma(H \to \gamma\gamma)}{\Gamma_{total}}$$

- The measurement of the Higgs boson cross-section could become a tool for precision studies, if we know accurately:

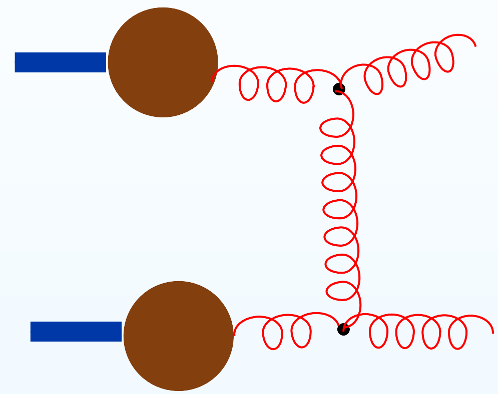
1. *Production cross-section and branching ratio*
2. *Strong coupling*
3. *Parton distribution functions*
4. *Luminosity (or partonic luminosities:* $\mathcal{L}_{ij}(x_1, x_2) = \mathcal{L} f_i(x_1) f_j(x_2)$*)*

ALL of the above require theory input!

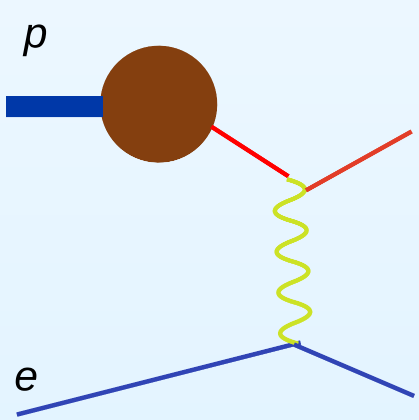
Standard candle processes



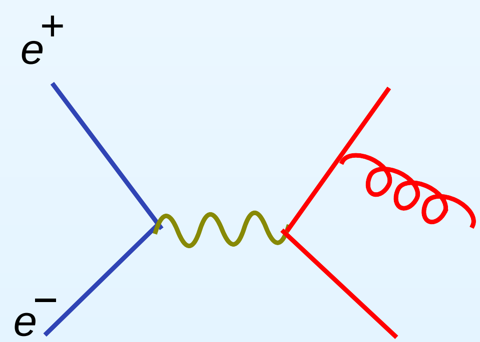
Luminosity measurement
Parton densities
weak mixing angle
W−mass



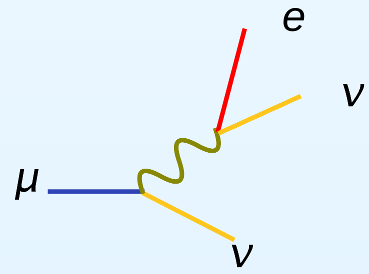
Gluon density



Parton densities

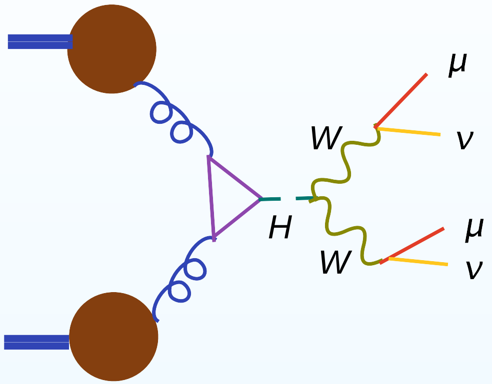


Strong coupling



Fermi constant

Search strategy for $pp \to H \to W^+W^-$

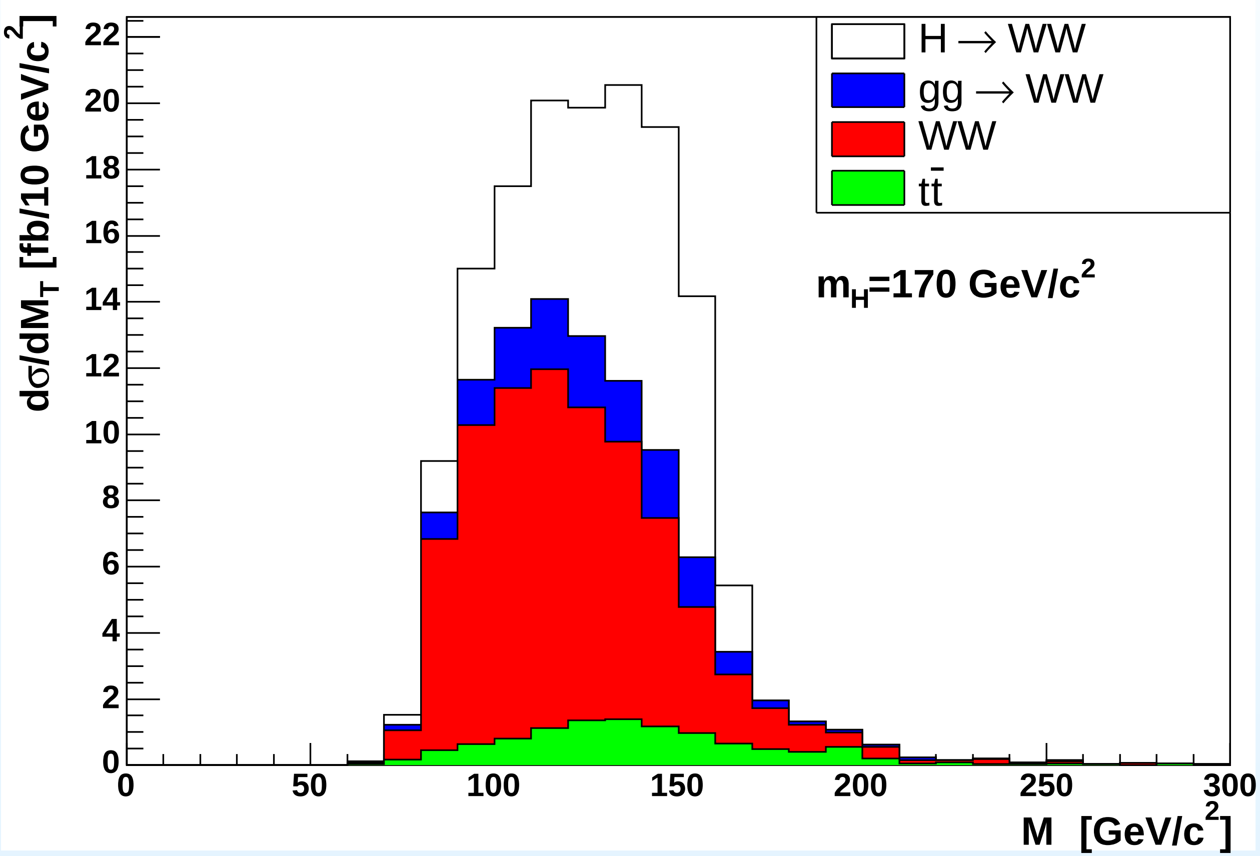


- If $m_h \sim 155 - 180\mathrm{GeV}$ then the Higgs boson decays almost exclusively into W-pairs.
- The reconstruction of the Higgs boson mass is not possible because of the escaping neutrinos
- Theory input is vital: We should have a good quantitative idea about how the SM background and the signal behave.
- For example, $pp \to WW$ production is roughly ten times larger than $pp \to h \to WW$.

Signal and background at leading order

- It is only after we optimize our cuts that we get a signal to backgound ratio of roughly 1 : 1

Dreiner, Dittmar



Dührssen,Jakobs,Marquard,van der Bij

Higher orders could change Signal/Background

- For example, the $gg \to WW$ process is formally NNLO; it increases the background by 30%

  Dührssen et al, Binoth et al.

- $q\bar{q} \to WW$ increases by 70% at NLO if no cuts are applied. With a jet veto, it only increases by about $20-30\%$

  Dixon, Kunszt, Signer.

- The Higgs total cross-section increases by $\sim 100\%$ at NNLO

  Harlander,Kilgore; CA, Melnikov; Ravindran, Smith, van Neerven

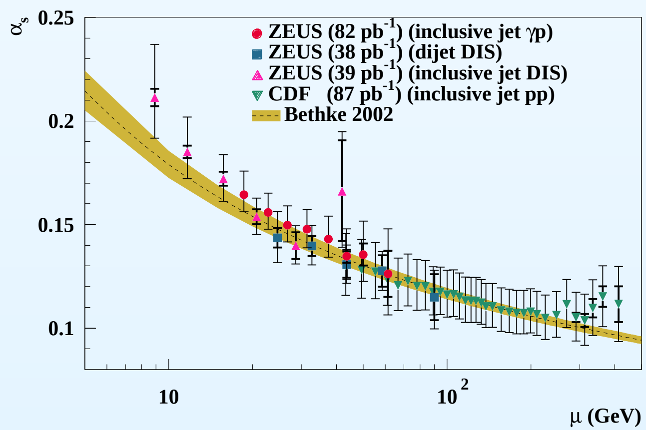
- With a jet-veto it increases by $\sim 85\%$

  Catani, de Florian, Grazzini; Davatz, Dissertori, Dittmar, Grazzini, Pauss; CA, Melnikov, Petriello

- If $m_H \sim 155-180$ GeV then we will be able to find it.
- Higher order corrections change the LO analysis; luckily, the conclusion remains valid!

High multiplicity background processes

- Other vital searches are more complicated. For example, SUSY models with R-parity conservation predict the production of a large number of jets and missing energy.
- Squark and gluino production is uncertain to 100% at leading order, and 30% at NLO. Beenakker, Höpker, Spira, Zerwas
- Standard Model multijet production processes are very sensitive to scale variations.



$$\sigma_{\text{N jets}} \sim \alpha_s^N(\mu) \times (\text{LO} + \ldots)$$

Leading order scale variation for $pp \longrightarrow \nu\bar{\nu} + N$jets

Select high $p_t > 80\,\mathrm{GeV}$, central $|\eta| < 2.5$ jets. Let us assume that a reasonable scale is:

$$\mu^2 = M_Z^2 + \sum_{jet} p_{t,jet}^2$$

and allow to vary: $\mu_R = \mu_F = \mu/2 - 2\mu$

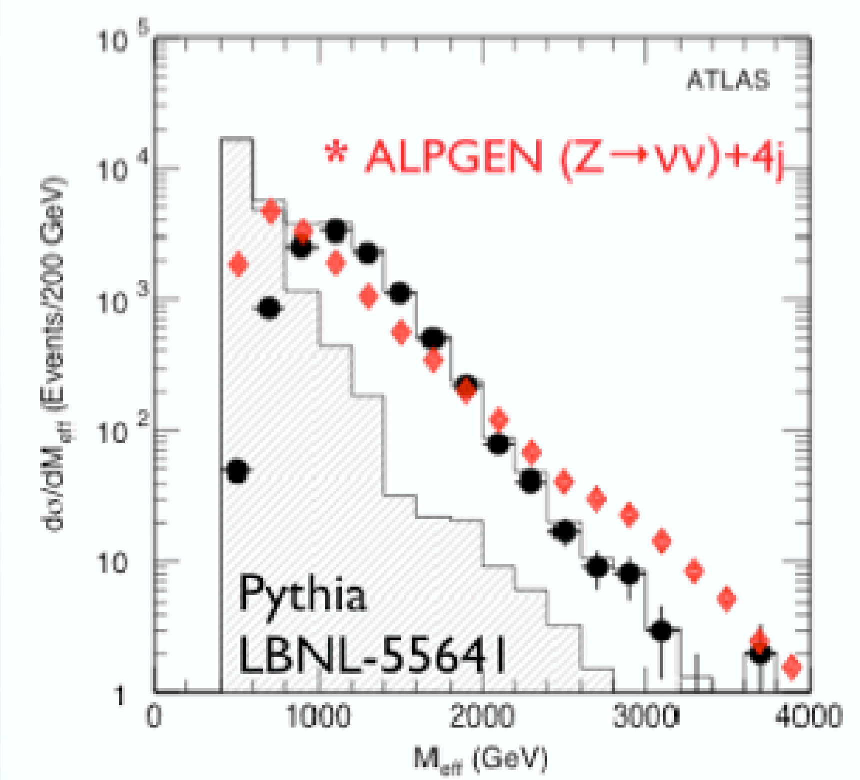
N	$\sigma(2\mu)[pb]$	$\sigma(\mu/2)[pb]$	variation
1	182	216	17%
2	47.1	75.4	46%
3	6.47	13.52	70%
4	0.90	2.48	93%

ALPGEN

$\boxed{\text{For a } 5\sigma \text{ discovery with LO magnitudes: } \rightsquigarrow \text{ Signal} > 2.5 \text{ Background}}$

Can we assess the discovery potential?

- Naive leading order perturbation theory is not sufficient.



Mangano

- We could estimate the background from other measurements ($Z \to ee$ vs $Z \to \nu\bar{\nu}$)! But $W +$ jets?
- NLO calculations will be particularly important

What is needed?

- NLO computations for many interesting analyses at the LHC, involving both signal and background processes.
- NNLO calculations for observables which can be measured very well and be used for high precision studies:
    - *cross-sections for resonances (Higgs boson, W,Z, new gauge bosons, . . .)*
    - *High rate processes, e.g. inclusive jet cross-section, top-quark cross-section, etc*
- Flexibility to adapt and perform fast calculations in new models.

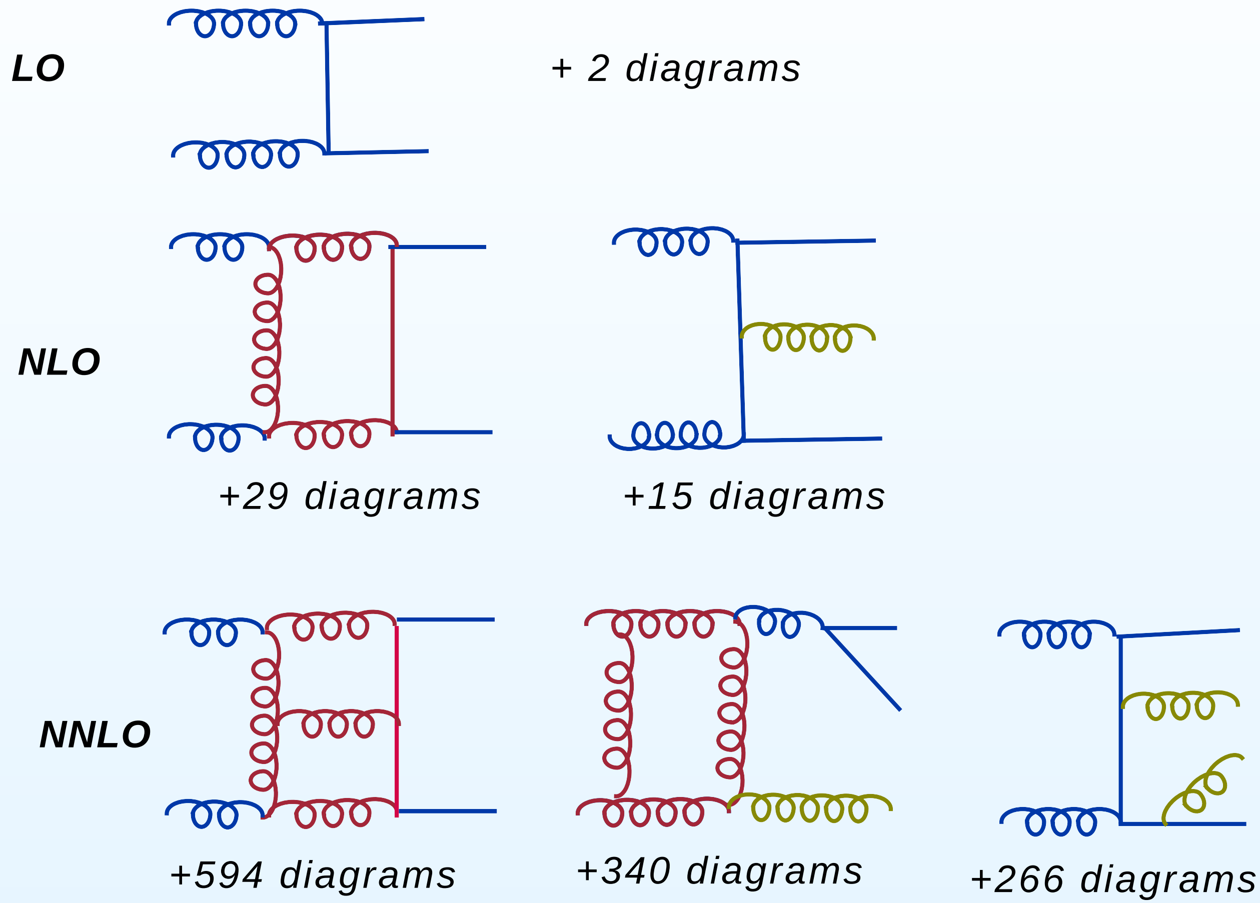
What is available

- Many NLO calculations for $2 \to 2$ and few $2 \to 3$ processes
  - *No results for $2 \to 4$ processes at hadron colliders (recent result by Denner et al on $e^+e^- \to 4$ fermions).*
- NNLO results:
  - *Drell-Yan total cross-section* — *Matsuura, Hamberg, van Neerven (1991)*; *Harlander, Kilgore (2002)*
  - *Higgs boson (h,A) total cross-section* — *Harlander, Kilgore (2002)*; *CA, Melnikov (2002)*; *Ravindran, Smith, van Neerven (2003)*
  - *Drell-Yan rapidity distribution* — *CA, Dixon, Melnikov, Petriello (2003)*
  - *Splitting functions* — *Moch, Vogt, Vermaseren (2004)*
  - *Higgs boson fully differential cross-section* — *CA, Melnikov, Petriello (2004)*

What is available?

Many new techniques!

Structure of perturbative corrections



Total number of diagrams for p p −> jets: 48723

Complications of perturbative corrections

- Virtual radiation in loops:
  - *Size*
  - *Infrared and ultraviolet singularities*
  - *Analytic structure: Thresholds and branch cuts*
- Real particle emission:
  - *Size*
  - *Infrared singularities*
  - *Complicated integration region due to experimental cuts and acceptance: Inclusive vs Differential calculations.*

Algebraic explosion!

- The number of diagrams increases very rapidly at higher orders.
- Each diagram requires a large number of integrals to be computed. Recall:
  - *Feynman rules in gauge theory*

$$\mathcal{V}_{ggg} = f^{abc}\left[g_{\mu_1\mu_2}(p_1-p_2)^{\mu_3} + g_{\mu_2\mu_3}(p_2-p_3)^{\mu_1} + g_{\mu_3\mu_1}(p_3-p_1)^{\mu_2}\right]$$

  - *Algebra of γ matrices, colour algrebra, etc.*

$$Tr(\gamma^{\mu_1}\gamma^{\mu_2}) = \text{1 term}$$

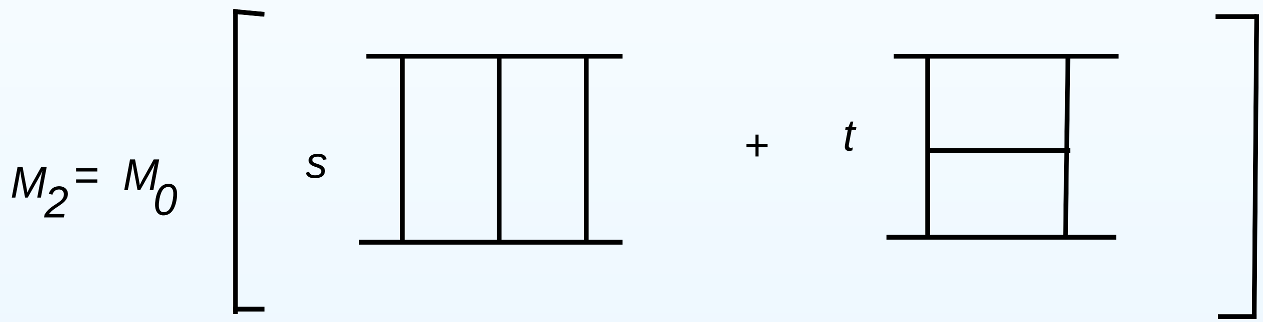
$$Tr(\gamma^{\mu_1}\cdots\gamma^{\mu_8}) = \text{105 terms}$$

$$Tr(\gamma^{\mu_1}\cdots\gamma^{\mu_{14}}) = \text{26931 terms}$$

- It is usually believed that the large algebraic complexity of the problem is the only problem. This is a misconception!

2-loop 4-point amplitudes in $\mathcal{N} = 4$ super Yang-Mills

- These amplitudes can be expressed in terms of one only integral in the planar limit. This was known already in 97.
  Bern, Rosowsky, Yan



- The same integral enters the expression for QCD amplitudes, together with another $\sim 10,000$.
- Many people tried and failed to compute it.
- In a breakthrough, Smirnov solved the problem in 1999.
- So how do we solve $10^3 - 10^5$ integrals in QCD if one takes so much effort?

Loop integral relations

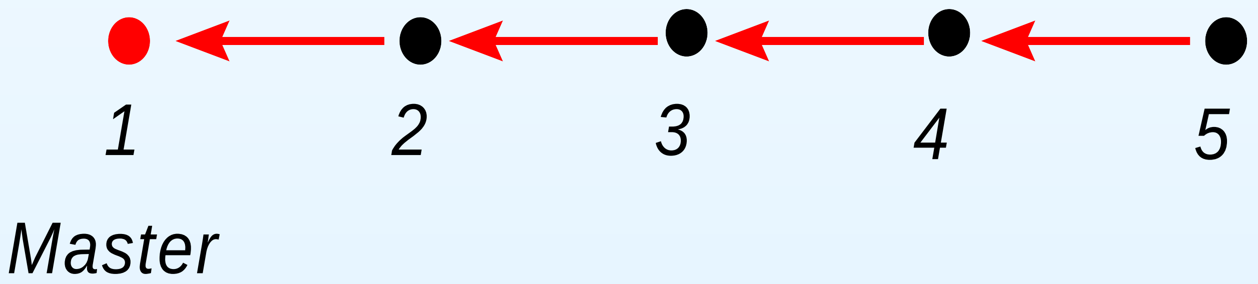
- Loop integrals are not independent:

$$\int d^d k \frac{\partial}{\partial k_\mu} \frac{k_\mu}{k^2 - M^2} = 0$$

Chetyrkin, Tkachov

$$M^2 \int d^d k \frac{1}{(k^2 - M^2)^2} + \left(\frac{d}{2} - 1\right) \int d^d k \frac{1}{(k^2 - M^2)^1} = 0$$

- We need to compute less!



Evaluation of master integrals

- With the reduction, we have made the problem much easier.
- For example, we finally need only 10 integrals for the evaluation of all diagrams in massless $2 \to 2$ two-loop scattering processes.
- Evaluating singular loop-integrals
  - Analytic evaluation methods:
    - Feynman parameters — Feynman, textbooks
    - Differential Equations — Kotikov; Gehrmann, Remiddi
    - Mellin-Barnes representations — Smirnov, Tausk
  - Automated numerical evaluation:
    - Sector decomposition — Binoth, Heinrich
    - Mellin-Barnes representations — CA, Daleo; Czakon

Many new two-loop amplitudes

- $e^+e^- \to e^+e^-$ in QED — Bern, Dixon, Ghinculov (2000)
- $2 \to 2$ in massless QCD — CA, Glover, Oleari, Tejeda-Yeomans (2000-2001)
  Bern, De Freitas, Dixon (2001-2002)
- $e^+e^- \to \gamma\gamma$, $q\bar{q} \to g\gamma$, $q\bar{q} \to \gamma\gamma$, — CA, Glover, Tejeda-Yeomans (2002)
- $gg \to \gamma\gamma$ — Bern, De Freitas, Dixon (2001)
- $e^+e^- \to$ 3 partons — Garland, Gehrmann,Glover, Koukoutsakis, Remiddi (2002)
- Two and three-loop corrections for electroweak parameters
  Awramik, Czakon, Freitas, Weiglein
  Boughezal, Tausk
- Two-loop massive form factors
  Berneuther, Bonciani, Gehrmann, Heinesch, Mastrolia, Remiddi (2004-2005)
  Birthwright, Glover, Marquard (2004)
- $\ldots$

Total cross-sections and simple distributions

- Phase-space integrals over real radiation look very different than loop integrals.

$$\text{Virtual} \rightarrow \int d^d k \frac{i}{k^2 - m^2} \qquad \text{Real} \rightarrow \int d^d k \delta^+(k^2 - m^2)$$

  - *Particles in loops can propagate unrestricted*
  - *Real particles must be on-shell*

- It seems that real radiation integrals are not amenable to reduction algorithms

Duality of Real and Virtual radiation

CA, Melnikov (2002)

- On-shell conditions ($\delta(k^2 - m^2)$) are equivalent to propagators:

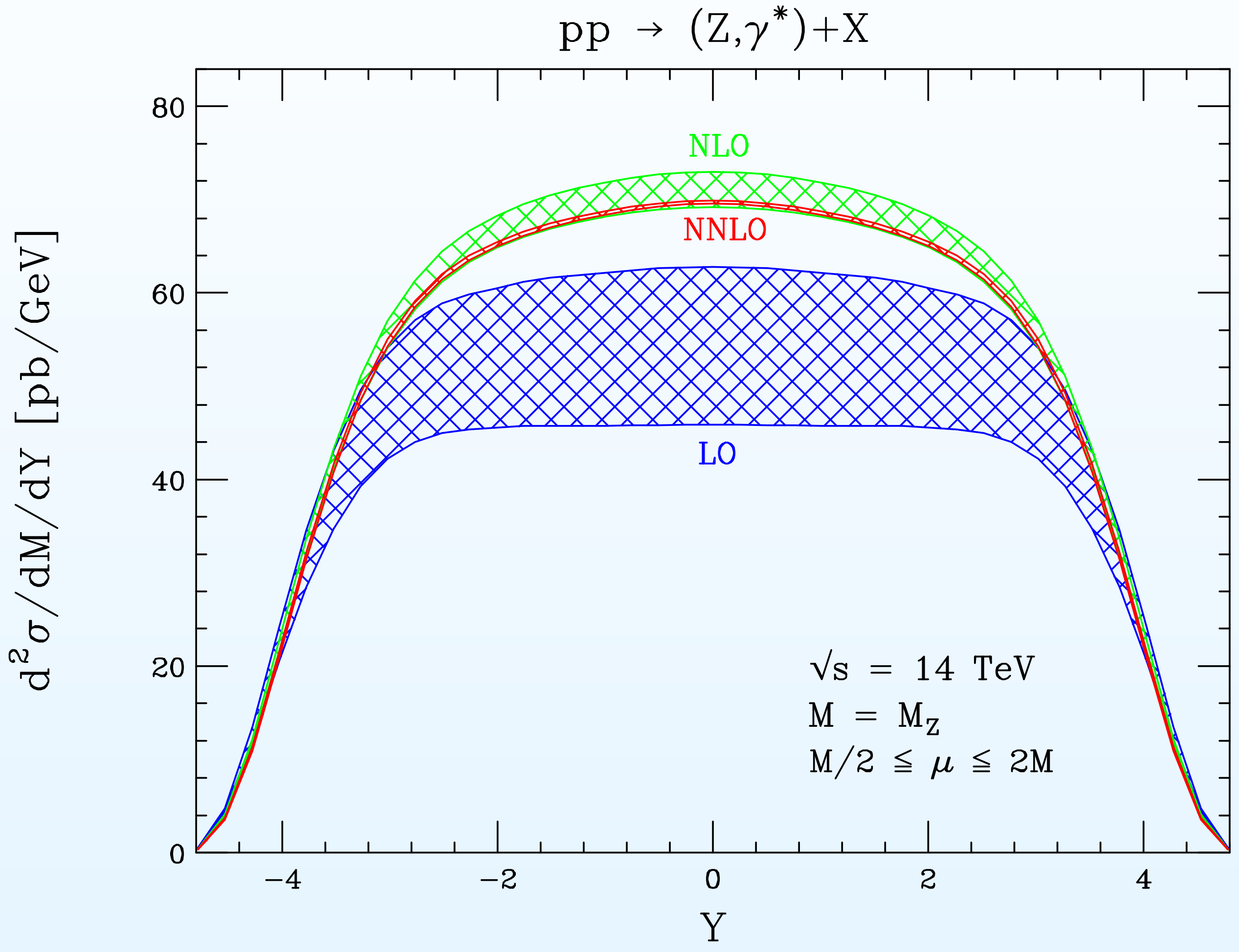
$$2\pi\delta(x) = \frac{i}{x - i0} - \frac{i}{x + i0}$$

- While the left side cannot be treated with reduction algorithms, it is easy to perform the reduction on the right side.
- Kinematic constraints can also be represented with 'fictitious' particles in loops using RV-duality!

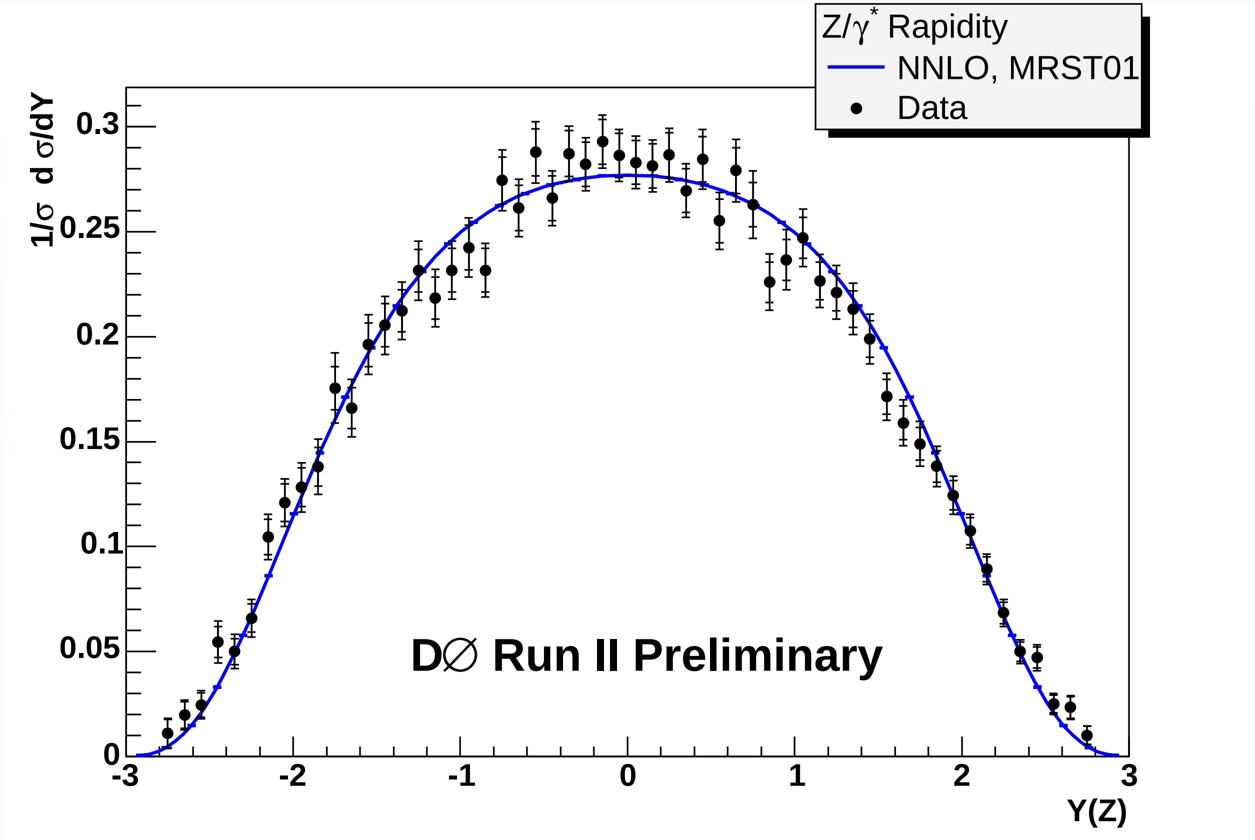
CA,Dixon,Melnikov,Petriello (2003)

$$\delta\left(u - \frac{2k\cdot p_1}{2k\cdot p_2}\right) \rightarrow \frac{k\cdot p_2}{k\cdot(p_1 - up_2)}$$

Z-boson rapidity distribution at the LHC



Z-boson rapidity distribution at the Tevatron



- *Errors will be much smaller in a couple of years (more Tevatron data, LHC)*

R-V duality NNLO applications

- Higgs boson total cross-sections at NNLO CA, Melnikov (2002)
- Drell-Yan rapidity distribution at NNLO CA, Dixon, Melnikov, Petriello (2003)
- Heavy quark fragmentation function Melnikov, Mitov; Mitov (2004)
- Photon energy distribution in $B \to X_s \gamma$, Melnikov, Mitov(2005)

Limited to kinematic variables with simple infrared limits. A different approach is required for observables with arbitrary experimental constraints.

A new method for fully differential cross-sections

C.A., Melnikov, Petriello

- New methods are under development based on infrared factorization properties of QCD:

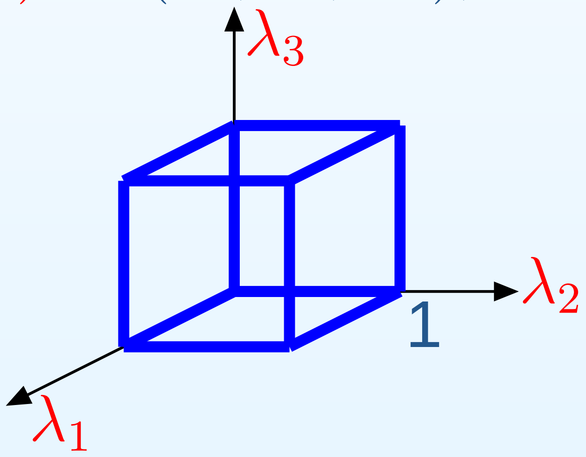
  (Weinzierl; Kosower; Gehrmann de-Ridder, Gehrmann, Glover, Heinrich; Kilgore; Frixione, Grazzini; Somogyi, Trocsanyi, del Duca)
- Automated subtraction of divergences
- Produces an epsilon expansion for real radiation graphs

$$\sigma_{\text{real}} = \frac{A_4\left[Obs\right]}{\epsilon^4} + \frac{A_3\left[Obs\right]}{\epsilon^3} + \ldots + A_0\left[Obs\right]$$

Singularities in a form amenable to algorithms

- Singularities have a very complicated form in momentum space (beyond NLO)
- Map phase-space volume to the unit hypercube

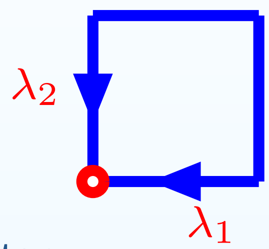
$$(E, p_x, p_y, p_z) \to (\lambda_1, \lambda_2, \ldots), \quad 0 \le \lambda_i \le 1$$



  - Simple geometry $\rightsquigarrow$ (automatization)
  - Easy to spot singular regions $\rightsquigarrow$ the edges!

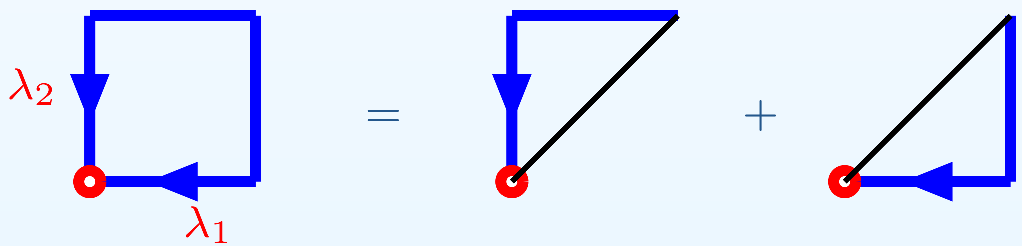
Overlapping singularities

- Singularity when two (or more) variables reach the same corner

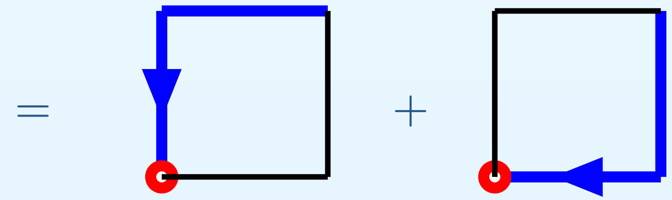


$$: \quad \frac{\lambda_1^\epsilon \lambda_2^\epsilon}{(\lambda_1+\lambda_2)^2} f(\lambda_1,\lambda_2; Obs(\lambda_1,\lambda_2))$$

- Split into sectors

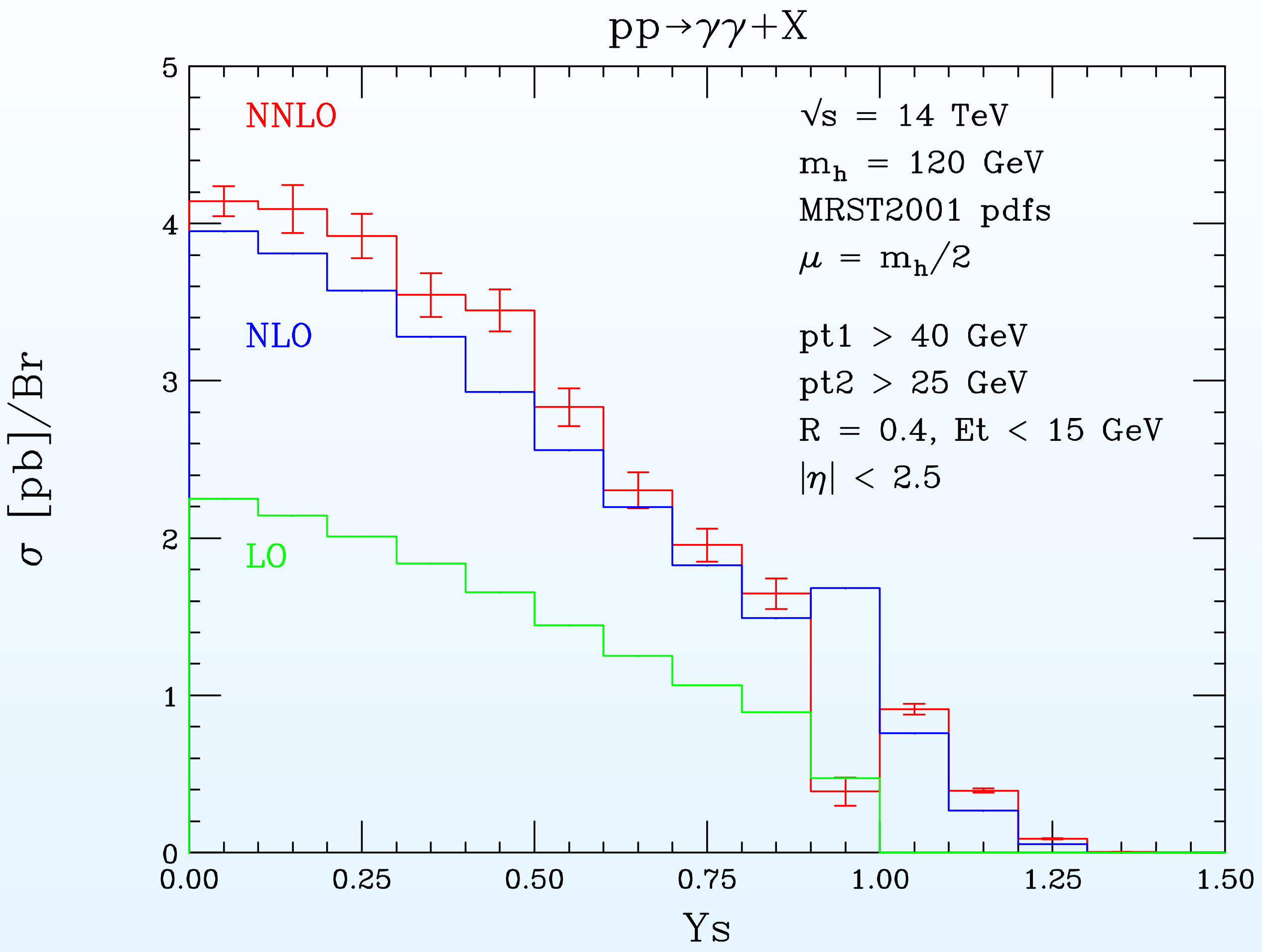


- map each sector to $[0,1]$



- Repeat until singularities are fully factorized in all phase-space variables.

Fully differential Higgs production



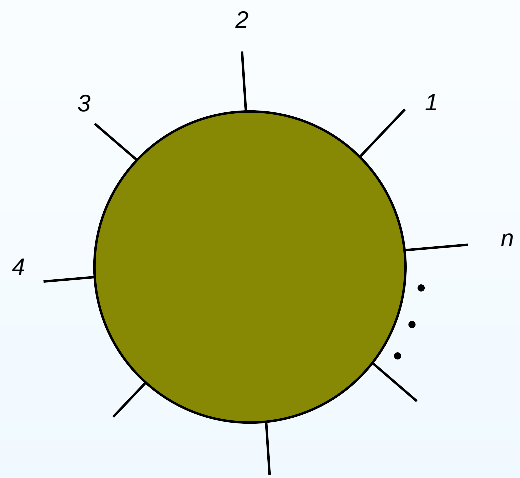
Fully differential NNLO cross-sections

CA, Melnikov, Petriello

- $e^+e^- \to 2$ jets (2004),
- NNLO Monte-Carlo (first) for Higgs production at the LHC $h_1 h_2 \to H + X$ (2004)
- NNLO Monte-Carlo for the differential decay rate in muon $\mu \to e\nu\bar{\nu} + X$. Direct comparison with the TWIST experiment which puts limits on the existence of right-handed weak interactions. (2005)

The first multi-scale two loop amplitude which is evaluated purely numerically.

A humbling problem!



- We have a very solid formalism and methods from the 90's on how to calculate NLO cross-sections:

  Bern,Catani,Denner, Dittmaer,

  Campbell,Dixon,Ellis,Frixione,Glover,Kosower,Kunszt, Nagy,Seymour,Signer,

  Trocsanyi, . . .
- But we are stuck at $n \geq 5$: algebra of Gigabyte sized expressions.

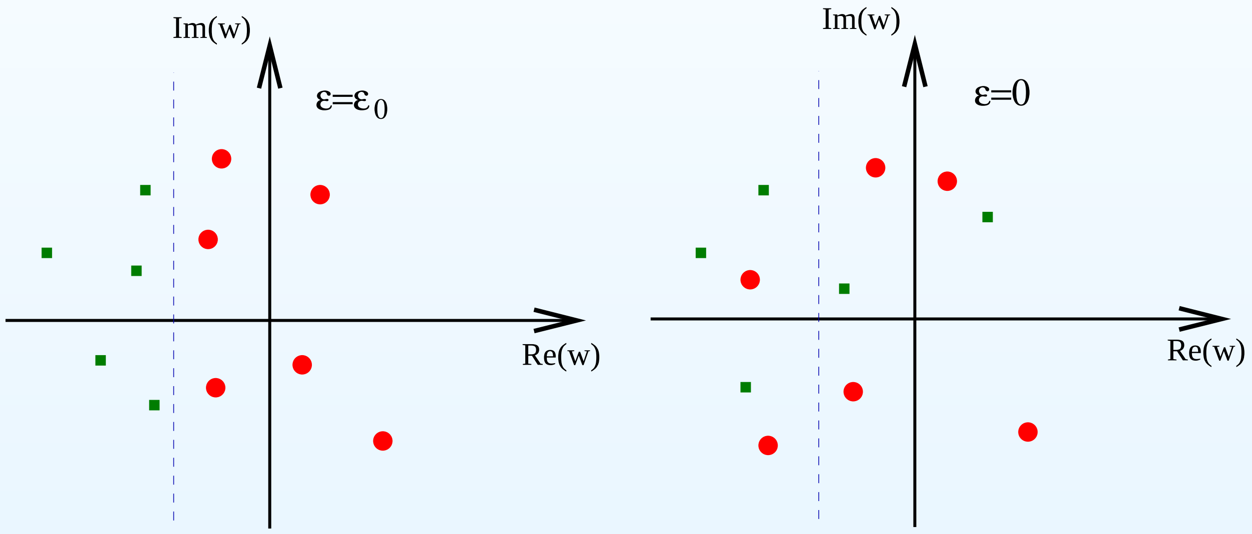
Back to the drawing board

- Improved numerical and semi-analytic reductions
  Denner, Dittmaer; Giele, Glover; Ellis, Giele, Glover, Zanderighi; Binoth,Guillet,Heinrich,Pilon,Schubert; . . .
- Unitarity method + recursive techniques
  Bern,Berger,Dixon,Forde,Kosower; Britto, Cachazo, Feng, Mastrolia; Brandhuber,Spence,Travaglini;Bjerum-Bohr,Dunbar,Ita;. . .
- Infrared subtractions Nagy, Soper

Numerical evaluation of arbitrary loop integrals

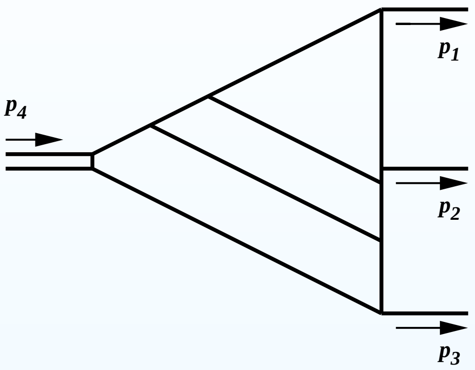
CA, Daleo

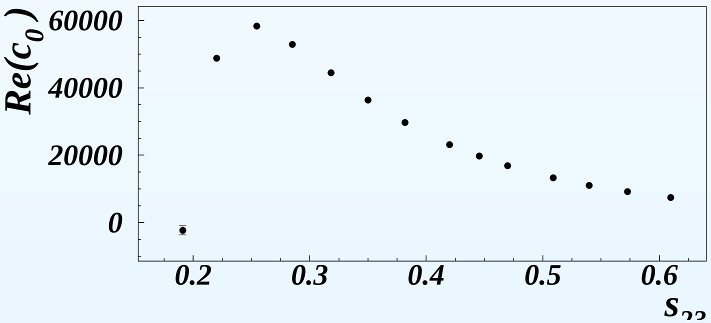
- All loop integrals can be written as complex contour (Mellin-Barnes) integrals

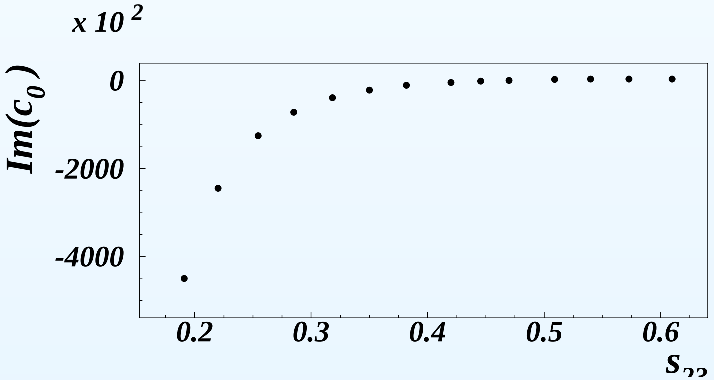


- The infrared divergences are localized on poles that can be extracted automatically with the Cauchy theorem
- Numerical integration on the complex contour!

The most difficult integral ever computed!







In half an hour!

Unexploited properties of gauge theories

- We have made enormous progress in perturbative computations.
- We know, however, that our methods are primitive!
- The results seem to be disproportionally simpler than our efforts to compute them.
- For example, we know that multi-loop amplitudes factorize simply in their infrared limit. *Catani; Sterman, Tejeda-Yeomans*
- Still, we have not figured how to exploit this property for the full or easier evaluation of the amplitudes.

Finding simplicity: $\mathcal{N} = 4$ supersymmetric Yang-Mills

- The full 2-loop 4-point MHV amplitudes obey the same factorization as the infrared limit

$$M_4^{(2)}(\epsilon) = \frac{1}{2}\left(M_4^{(1)}(\epsilon)\right)^2 + f^{(2)}(\epsilon) M_4^{(1)}(2\epsilon) - \frac{5}{4}\zeta_4.$$

Unlikely to be an accident

CA, Bern, Dixon, Kosower

- All two-loop amplitudes obey the same relation *collinear factorization*

$$M_n^{(2)}(\epsilon) = \frac{1}{2}\left(M_n^{(1)}(\epsilon)\right)^2 + f^{(2)}(\epsilon) M_n^{(1)}(2\epsilon) - \frac{5}{4}\zeta_4.$$

- Are multi-loop amplitudes polynomials of the one-loop amplitude?

3-loop amplitudes in the planar limit

In a tour de force calculation, Bern, Dixon and Smirnov proved:

$$M_4^{(3)}(\epsilon) = -\frac{1}{3}\left(M_4^{(1)}(\epsilon)\right)^3 + M_4^{(1)}(\epsilon)M_4^{(2)}(\epsilon) + f^{(3)}(\epsilon)M_3^{(1)}(3\epsilon) + C.$$

and proposed the ansatz:

$$M_n(\epsilon) = \exp\left(\sum_{l=0}^{\infty} a^l \left[f^{(l)}(\epsilon)M_n^{(1)}(l\epsilon) + h^{(l)}\right]\right)$$

NEW: Cachazo, Spradlin, Volovic proved with Mellin-Barnes integrations that the parity even part of two-loop 5-point MHV amplitudes satisfy the conjecture.

Is the perturbative expansion solvable?

Conclusions

- The LHC physics programs puts to test our methods.
- Enormous progress in the last years
- New methods for NNLO calculations.
- New methods for NLO multileg processes and very promising ideas.
- Room for new ideas!