2. PERTURBATIONS

$V = 1 + O(\lambda)$

PARAMETRISE $V(\lambda) \rightarrow$

$$V = e^{A}, \quad A = \begin{pmatrix} 0 & \sum_{i=1} a_i \lambda^i & \sum_{i=1} c_i \lambda^i \\ -\sum_{i=1} a_i^* \lambda^i & 0 & \sum_{i=1} b_i \lambda^i \\ -\sum_{i=1} c_i^* \lambda^i & -\sum_{i} b_i^* \lambda^i & 0 \end{pmatrix}$$

[EXAMPLE → WOLFENSTEIN PARAM

$a_1 = 1, \quad b_2 = A, \quad c_3 = A(\rho - \tfrac{1}{2} - i\eta)$

$b_1 = c_1 = c_2 = 0$]

CLASSIFY MODELS ACCORDING TO $O(\lambda)$ EFFECTS:

- SINGLE SHIFTS: 1 $O(\lambda)$ PERTURBATION
  (E.G. $a_1 \neq 0, \quad b_1 = c_1 = 0$)
- DOUBLE SHIFTS: 2 $O(\lambda)$ PERTURBATIONS
- TRIPLE SHIFTS: 3 $O(\lambda)$ PERTURBATIONS

MORE COMPLICATED → "SEQUENTIAL" PERTURBATIONS

$V = V_1 V_2$

- ADDITIONAL DOUBLE · TRIPLE SHIFTS
- POSSIBILITY OF LARGER EFFECTS?

[redacted]

3. SCENARIOS:

RECALL $[W, V(\lambda)] \neq 0 \rightarrow$

DIFFERENT WAYS TO INTRODUCE PERTURBATIONS:

RIGHT SHIFTS: $U_{MNSP} = W\,V(\lambda)$

LEFT SHIFTS: $U_{MNSP} = V(\lambda)\,W$

MIDDLE SHIFTS: $U_{MNSP} = R_1\,V(\lambda)\,R_2\,R_3\,P$

$U_{MNSP} = R_1\,R_2\,V(\lambda)\,R_3\,P$

FOR $V = V_1\,V_2$, ADDITIONAL MIDDLE SHIFTS $\rightarrow$

E.G. $U_{MNSP} = R_1\,V_1(\lambda)\,R_2\,V_2(\lambda)\,R_3\,P$

$\vdots$

FOR SPECIFIC CHOICE OF $V(\lambda)$,

SCENARIOS $\rightarrow$ SPECIFIC PREDICTIONS FOR CABIBBO SHIFTS

MORE PRECISELY →

CABIBBO HAZE :

$$u_{MNSP}(\lambda) = \sum_{n=0}^{\infty} \lambda^n W_n \qquad (W_0 = W)$$

CAN BE PARAMETRIZED AS RIGHT SHIFT →

$$u_{MNSP}(\lambda) = W \sum_{n=0}^{\infty} \lambda^n (W^{-1} W_n) = W V(\lambda)$$

BUT ALSO AS MIDDLE SHIFT (OR LEFT SHIFT) WITH DIFFERENT V →

E.G. $u_{MNSP}(\lambda) = R_1 R_2 R_3 V(\lambda)$

$$= R_1 R_2 R_3 V R_3^{-1} R_3 = R_1 R_2 V'(\lambda) R_3$$

SINCE $V(\lambda) = 1 + \sum_{n=1}^{\infty} \lambda^n V_n$

$$V'(\lambda) = R_3 V R_3^{-1} = 1 + \sum_{n=1}^{\infty} \lambda^n (R_3 V_n R_3^{-1})$$

$$= 1 + \sum_{n=1}^{\infty} \lambda^n V_n'$$

[HERE → ASSUME SIMPLE FORM FOR $V(\lambda)$ + ALLOW RIGHT, LEFT, MIDDLE]

RECIPE FOR PARAMETRIZING U_{MNSP}

SPECIFY BARE MIXINGS $\eta_\odot, \eta_{13}, \eta_\oplus$

SPECIFY LEADING + SUBLEADING TERMS IN $V(\lambda)$

SPECIFY RIGHT, LEFT, MIDDLE SHIFTS

HUGE RANGE OF POSSIBILITIES!

BROAD RANGE OF EXPERIMENTAL DATA

$$\Delta\theta_\odot \sim O(\lambda^2)$$

$$\Delta\theta_\oplus \sim O(\lambda)$$

$$\Delta\theta_{13} \sim O(\lambda)$$

CHALLENGE → SINGLE OUT PARAMETRIZATIONS FROM FLAVOR THEORY

HERE → FOCUS ON REPRESENTATIVE EXAMPLES

(ALSO IN hep-ph/0503222)

CATALOG $O(\lambda)$ EFFECTS

→ GUIDE FOR MODEL BUILDING + EXPTS

hep-ph/0510326

EXAMPLE 1.

CASE 1. 2 LARGE ANGLES

$$W = R_1(\eta_\oplus)\, R_3(\eta_\odot)\, P$$

$$V(\lambda) \sim U_{CKM} \sim \begin{pmatrix} 1 & \lambda & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \mathcal{O}(\lambda^2)$$

PROTOTYPE →

$$U_{MNSP} = U^{+}_{CKM}\, \mathcal{F}$$

- LEFT SHIFT MODEL →

$$\theta_\odot = \eta_\odot - \cos\eta_\oplus\, \lambda \quad \leftarrow \eta_\oplus\text{-DEPENDENT SHIFT}$$

$$\theta_\oplus = \eta_\oplus + \mathcal{O}(\lambda^2)$$

$$\theta_{13} = \lambda \sin\eta_\oplus \quad \leftarrow \mathcal{O}(\lambda) \text{ CHOOZ}$$

BUT SUPPOSE PERTURBATIONS ENTER AS

- RIGHT SHIFT →

$$\theta_\odot = \eta_\odot + \lambda^{\cos\alpha_{12}} \quad \leftarrow \text{"FULL STRENGTH" SHIFT (QLC-TYPE)}$$

$$\theta_\oplus = \eta_\oplus + \mathcal{O}(\lambda^2)$$

$$\theta_{13} = \mathcal{O}(\lambda^2) \quad \leftarrow \text{SUPPRESSED CHOOZ}$$

OR

- MIDDLE SHIFT

$$\theta_\odot = \eta_\odot + \lambda$$

$$\theta_\oplus = \eta_\oplus + \mathcal{O}(\lambda^2)$$

$$\theta_{13} = \mathcal{O}(\lambda^3)$$

OF COURSE, MANY OTHER EXAMPLES

CP VIOLATION → "DIRAC" v. "MAJORANA"

- DIRAC CP →

$$J_{CP} \sim \mathrm{Im}\left(U_{i\alpha} U_{j\beta} U^{*}_{i\beta} U^{*}_{j\alpha} \right)$$

JARLSKOG 85
DUNIETZ et al.

3 FAMILIES →

$$J \sim \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \sin\delta$$

1 PHASE δ → δ_{CKM} OR δ_{MNSP}

- MAJORANA CP →

PRESENT FOR MAJORANA FERMIONS

↳ LEPTON MIXING FOR ν MAJORANA

3 FAMILIES →

2 PHASES

$$U = R_1 R_2 R_3 \underset{\sim}{P}$$

INVARIANTS →

$$S_{CP} \sim \mathrm{Im}\left(U_{\alpha i} U^{*}_{\alpha j} \xi_j^{*} \xi_i \right)$$

$$\left(C \bar{\nu}_j^{T} = \xi_j^{*2} \nu_j \right)$$

NIEVES, PAL
PETROV

MAJORANA PHASES → ENTER IN $\not{L}$ PROCESSES:

$0\nu\beta\beta$

$\nu \leftrightarrow \bar{\nu}$ OSCILLATIONS

(HELICITY SUPPRESSED)

$\sim \frac{m_i}{E}$ ↳ DIFFICULT TO MEASURE

QUESTIONS →

HOW DOES $\not{CP}$ ENTER U_{MNSP} ?

PREDICTIONS FOR J (δ_{MNSP}), $\{s\}$?

FOCUS ON DIRAC $\not{CP}$ → (3 NEED NONVANISHING ANGLES)

X • BARE DIRAC PHASES →

3 ANGLES IN W → $J \not{\delta}_{MNSP} \sim O(1)$

1,2 " " " → $J \not{\delta}_{MNSP} \lesssim O(\lambda)$

→ • BARE MAJORANA PHASES →

MAY SOURCE DIRAC $\not{CP}$ AT $O(\lambda)$

→ • PHASES IN $V(\lambda)$

QUESTION → DOES $\not{CP}$ ENTER AT

LEADING OR HIGHER ORDER IN λ ?

MAJORANA ($\lambda \to 0$) $\longrightarrow$ DIRAC $O(1)$

BARE MAJORANA PHASES CAN SOURCE δ_{MNSP} $\rightarrow$

MUST GENERATE / HAVE 3 NONZERO MIXING ANGLES

+ NONTRIVIAL DIRAC ~~CP~~

ANSWER $\rightarrow$

RIGHT SHIFTS CAN DO THIS

$$U_{MNSP} = R_1 R_2 R_3 P \nu$$

$$= R_1 R_2 R_3 P \nu P^{-1} P$$

$$= R_1 R_2 R_3 \nu_M^* P$$

LEFT SHIFTS, MIDDLE SHIFTS CAN'T

$$U_{MNSP} = R_1 .. \nu R_{1,...} P$$

PHASES IN $V(\lambda)$:

EXAMPLE → $U_{MNSP} = U_{CKM}^{\dagger}\, \mathcal{F}$ REVISITED

LEFT SHIFT (NO MAJORANA $\not{CP}$)

NAIVELY → USE WOLFENSTEIN PARAM FOR U_{CKM}

+ COMPUTE →

$$J = \frac{1}{4} A \lambda^3 \eta \cos\eta_{\oplus} \sin 2\eta_{\oplus} \sin 2\eta_{\odot}$$

RECALL MIXING ANGLES

$\theta_{\oplus}, \theta_{\odot} \sim \eta_{\oplus}, \eta_{\odot}$; $\theta_{13} \sim O(\lambda)$

$\therefore$ $J \sim \sin 2\theta_{\oplus} \sin 2\theta_{\odot} \sin 2\theta_{13} \sin\delta_{MNSP}$

$\Rightarrow$ $\delta_{MNSP} \sim O(\lambda^2)$

$$U_{MNSP} = \begin{pmatrix} \cos\theta_{\odot} & \sin\theta_{\odot} & \underset{\downarrow}{\overset{O(\lambda)}{\epsilon_R}} + i \underset{\downarrow}{\overset{O(\lambda^3)}{\epsilon_I}} \\ & & \sin\theta_{\oplus} \\ & & \cos\theta_{\oplus} \end{pmatrix}$$

(*) → IMPLICATIONS FOR OBSERVABILITY OF $\not{CP}$ GIVEN θ_{13} MEASURED

BUT- CHOICE OF PHASE IN 1-3 MIXING

ONLY A CONVENTION FOR $\mathcal{U}_{CKM}$

$$\mathcal{U}_{CKM} \sim \begin{pmatrix} 1 & a_1\lambda & c_3\lambda^3 \\ & 1 & b_2\lambda^2 \\ & & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

WOLFENSTEIN: $a_1 = 1$

$b_2 = A$

$c_3 = A(\rho - 1/2 - i\eta)$

$$\rightarrow \boxed{J_{CKM} = \lambda^6 \, \mathrm{Im}(a_1 b_2 c_3^*) + \mathcal{O}(\lambda^7)}$$

OUR CLASSIFICATION:

PLACEMENT OF PHASES IN $V(\lambda)$

PHYSICALLY SIGNIFICANT

EX. $\mathcal{U}_{MNSP} = \mathcal{U}^{\dagger}_{CKM} \mathcal{F} \longrightarrow$

$\mathrm{Im}(a_1) \neq 0 \rightarrow J_{MNSP} \sim \mathcal{O}(\lambda)$

$\delta_{MNSP} \sim \mathcal{O}(1)$

$\mathrm{Im}(a_1) = 0$,

$\mathrm{Im}(b_2)$ OR $\mathrm{Im}(c_3) \neq 0 \rightarrow J_{MNSP} \sim \mathcal{O}(\lambda^3)$

$\delta_{MNSP} \sim \mathcal{O}(\lambda^2)$

CHOOZ US THE WAY...

KEY TO SINGLING OUT PARAMETRIZATIONS →

CHOOZ ANGLE θ_{13}

(IMPORTANT FOR ~~Δm^2~~ TOO)

CURRENT BOUND → $\theta_{13} \lesssim \theta_c$

$\sin^2\theta_{13} < 0.05$

$\sin^2 2\theta_{13} < 0.2$

(UNLIKE ATMOSPHERIC ANGLE?)

~~DISCOVE~~ MEASUREMENT OR IMPROVED BOUNDS ON θ_{13}

EXPECTED IN FUTURE / PLANNED EXPTS

($\lesssim$ 10yr)

E.G. DOUBLE CHOOZ, ...

WILL PROBE θ_{13} DOWN TO ~ $\theta(\lambda^2)$

* $\sin^2 2\theta_{13} \lesssim 0.01 - 0.03$ 90% c.l.

$\theta_{13} \sim 3^\circ$ ($\theta(\lambda^3)$)

THIS CLASSIFICATION SCHEME →

DEMONSTRATES HOW MUCH CAN BE LEARNED

FROM KNOWING IF $\theta_{13} \sim \theta(\lambda)$ OR $\theta(\lambda^2)$

BEYOND PHENOMENOLOGICAL APPROACH →

ORIGIN OF LARGE ANGLES

◂ CABIBBO HAZE

- LARGE MIXINGS OF LEPTON SECTOR →

  CHALLENGE FOR TOP-DOWN FLAVOR MODEL BUILDING

STRATEGIES →

- NON-ABELIAN DISCRETE SYMMS IN ℓ SECTOR

  (MOTIVATION NOT STRONG IN Q SECTOR)

- CONTINUOUS FLAVOR SYMM →

  PERTURBATION → LARGE MIXINGS

  FOR DEGENERATE ν's

EXPLORE SIMPLE TOY MODELS →

SO(2) FAMILY SYMM

SYMM LIMIT →

2 FAMILIES : HIERARCHY — IN DIRAC MASSES

DEGENERATE — SEESAW'ED ν MASSES

(W/O HIERARCHICAL MASS TERMS FOR ν_R)

PERT → MAXIMAL MIXING + HAZE

H. DAVOUDIASL, L.E.
hep-ph/
0512188

ν MASS FROM QCD CONFINEMENT

THOMAS et al 192.

→ STANDARD SEESAW ⟶ MAJORANA ν_L's:

$$\sim \frac{1}{\Lambda} HL\ HL$$

$\Lambda \sim M_R \sim \mathcal{O}(M_{GUT})$

ALTERNATIVE → LOWER SCALE Λ

$\frac{1}{\Lambda} HL\ HL$ FORBIDDEN BY EXTRA SYMMETRY

m_ν FROM HIGHER-D OPERATORS

INTRIGUING POSSIBILITY →

$\langle q\bar{q} \rangle$ AND $\langle H \rangle$ →

SAME SM QUANTUM #s

$$\frac{1}{\Lambda^3} (q_L q_R L)(HL)$$

$$\longrightarrow m_\nu \sim \frac{\langle q_L \bar{q}_R \rangle \langle H \rangle}{\Lambda^3}$$

$m_\nu \sim \mathcal{O}(.1\ eV)$ for $\Lambda \sim \mathcal{O}(TeV)$

MORE PRECISELY →

CONSIDER ν MASS GENERATION USING

$$O_u^M = \frac{y_{ijk}}{\Lambda^3}(\bar{Q}_L^i u_R \cdot L^j)(H L^k)$$

SYMMETRY TO FORBID STD SEESAW OP:

Z_3

L_i:	+1
H	0
u_R	+1
e_R:	+1
Q_i:	0
d_R:	0

$\langle q\bar{q} \rangle \rightarrow$ NONTRIVIAL CHARGE

NOTE → FORBIDS $Q_L \bar{U}_R H$ ("MASSLESS" UP QUARK)

ANOMALY → MIXED $Z_3 \cdot SU(3)_C$

↓ SEEMS UNAVOIDABLE!

IMPLICATIONS →

NEARLY MASSLESS UP QUARK:

- CONSISTENT WITH PHENOMENOLOGY ?

  (DISFAVORED)

- STRONG CP PROBLEM →

  LOOP-INDUCED $m_u \sim \langle H \rangle \frac{m_v}{\Lambda}$

  → $\sim 10^{-8}$-10^{-10} SUPPRESSION

  θ_{QCD} PHYSICAL, BUT NO STRONG FINE-TUNING REQUIRED BY n EDM

  ∴ SOLVED FOR ALL PRACTICAL PURPOSES

BOUNDS ON θ_u^M : NON-HELICITY SUPPRESSED CONTRIBUTIONS TO

$\pi \to e\nu$, $K \to e\nu$, $B \to e\nu$

→ FLAVOR-DEP HIERARCHY ↓ $\Lambda \sim$ 3-10 TeV

CONCLUSIONS + OUTLOOK

- LESSONS FROM GRAND UNIFICATION →

  LEPTONIC CABIBBO SHIFTS

QUARK AND LEPTON MIXINGS →

EXPANSION IN λ

"WOLFENSTEIN-LIKE" PARAMS OF U_{MNSP}

CLASSIFICATION BASED ON PHYSICS OF $\lambda \to 0$ LIMIT:

- 2 LARGE ANGLES

  MYRIAD POSSIBILITIES, CATALOGUED BY PREDICTIONS FOR $\boxed{\theta_{13}}$ + J_{MNSP}

- 1 LARGE ANGLE (ATM.)

  TIGHT CONSTRAINTS → $\theta_\odot$

- 3 LARGE ANGLES → ANARCHY HYPOTHESIS MORE WORKABLE?

MANY CASES → CAN HAVE $\delta_{MNSP} << \delta_{CKM}$

TAXONOMY → FIRST STEP. NEXT STAGE →

CONNECTION WITH TOP-DOWN FLAVOR MODELS

→ MASSES?

EXPERIMENTS WILL "CHOOZ" US THE WAY →

PROBE $\theta(\lambda)$ EFFECTS →

SOLAR ANGLE : KNOWN AT $\theta(\lambda^3)$

ATMOSPHERIC ANGLE : KNOWN ONLY TO $\theta(1)$

EXPT → DIFFICULT TO IMPROVE CURRENT BOUNDS?

QUESTION → $\theta(\lambda)$ HAZE (FROM MAXIMAL?)

CHOOZ ANGLE : $\theta(\lambda)$ BOUND

MAY BE DOMINANTLY CABIBBO EFFECT

CURRENTLY → $\sin^2 2\theta_{13} < 0.2$

NEXT GENERATION EXPTS →

MAY REACH $\sin^2 2\theta_{13} \sim 0.01$

$\theta(\lambda^2)$

IMPORTANT RANGE FOR THEORY

- BEYOND CABIBBO HAZE PARAMETRIZATIONS

  → ADDRESS ORIGIN OF LARGE ANGLES

SEESAW + UNIF. FRAMEWORK ?

2 FAM. → 1 LARGE ANGLE ; GENERALIZE TO 3 FAMILIES

- SEESAW ALTERNATIVE →

  ν MASS FROM QCD CONFINEMENT

INTRIGUING FRAMEWORK,

MODEL-BUILDING CHALLENGES

CHALLENGE → UV COMPLETION, FLAVOR PUZZLE

CAN WE MAKE IT WORK ?

WITH IMPROVED DATA + THEORY →

PERHAPS FLAVOR PATTERNS

WILL EMERGE THROUGH THE

CABIBBO HAZE