QCD and a Holographic Model of Hadrons

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Motivation and plan

- Large N_c:
  - planar diagrams dominate
  - resonances are infinitely narrow
- Effective theory in terms of resonances is weakly coupled?
- What is this effective theory?
- String theory explicit examples suggest it is a 5d effective theory.
- Bottom-up approach (AdS from QCD vs QCD from AdS).

A holographic model: J. Erlich, E. Katz, D. Son and M.S., hep-ph/0501128.
(L. Da Rold, A. Pomarol, hep-ph/0501218)

This talk:

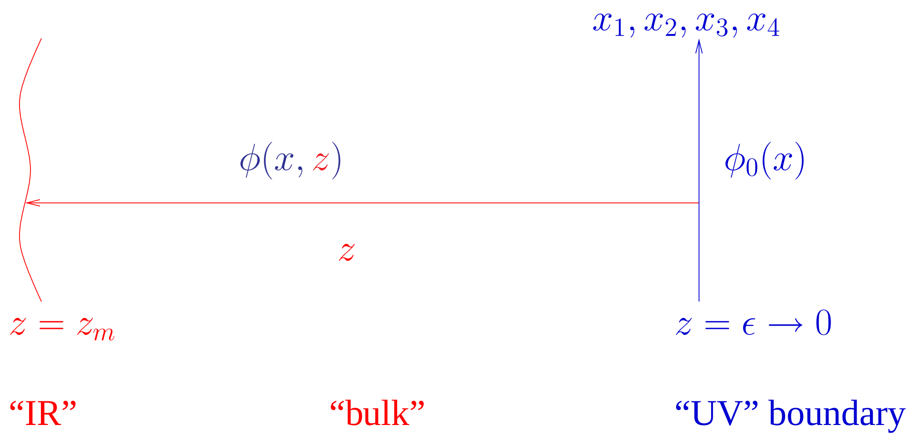
- ABC of AdS/CFT (holography)
- A simple model – generic features:
  - chiral symmetry breaking
  - quark-hadron duality, sum rules
  - VMD, etc.
- Excited meson spectrum: $m^2 \sim n$ (hep-ph/0602229)

AdS/CFT correspondence: formulation

- Begin with $S_4[G,q] = \int d^4x\, \mathcal{L}[G,q]$.
  Generating functional for correlators of an operator $\mathcal{O}$ (examples of $\mathcal{O}$: $G^a_{\mu\nu}G^{\mu\nu a}$, $\bar{q}q$, $\bar{q}\gamma^\mu t^a q, \ldots$):

$$Z_4[\phi_0(x)] = \int \mathcal{D}[G,q]\ \exp\left\{ iS_4 + i\int_{x^4} \phi_0 \mathcal{O} \right\}.$$

- 5d bulk metric:
  $ds^2 = z^{-2}\left(-dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$.
  $\eta_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$.
  (Note: $x^m \to \lambda x^m$).
  AdS "slice"



- $$Z_5[\phi_0(x)] = \int_{\phi(x,\epsilon)=\phi_0(x)} \mathcal{D}[\phi] e^{iS_5[\phi]}$$

$$Z_4 = Z_5$$

(Generating functnl) [4d sources $\phi_0(x)$] = (Effective action) [fields $\phi_0(x)$].

Example: conserved current

- Let $\mathcal{O}$ be a current: $J^{\mu a} = \bar{q}\gamma^\mu t^a q$.
  ϕ_0: source for $J^{\mu a}$ is a vector potential $V_0^{\mu a}$. I.e.,

$$Z_4[V] = \int \mathcal{D}[G,q] \exp\left\{ iS_4 + i\int_{x^4} V_0 \cdot J \right\}.$$

- We shall look at

$$\int d^4x e^{iqx} \langle J^{\mu a}(x) J^{\nu b}(y) \rangle = \delta^{ab}(q^\mu q^\nu - q^2 \eta^{\mu\nu})\Pi(-q^2).$$

  In QCD, scale invariance in the UV means $\Pi(Q^2) \sim \ln(Q^2)$.

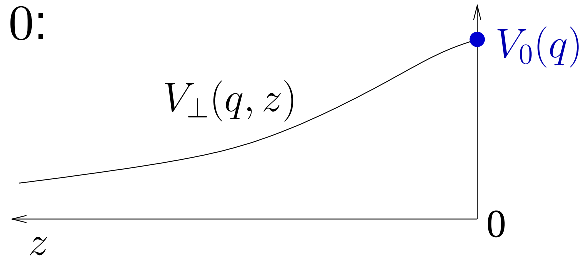
- 5d action for V_m^a? Let us take

$$S_5 = -\frac{1}{4g_5^2} \int d^5x \sqrt{g}\, V_{mn}^a V^{amn}.$$

- At tree level, we need to minimize S_5 wrt V with the b.c. $V(x,\epsilon) = V_0(x)$.
  Then take 2 variational derivatives wrt $V_0(x)$ and $V_0(y)$ to find $\langle J(x)J(y)\rangle$.

Example: current (contd)

- $V_5 = 0$ gauge; linearize; Fourier $x^\mu \to q^\mu$; $q_\mu V_\perp^\mu = 0$:



$$\partial_z \left(\frac{1}{z} \partial_z V_\perp \right) + \frac{q^2}{z} V_\perp = 0.$$

- Action to quadratic order in V, on eqs. of motion (int. by parts):

$$S_5 = -\frac{1}{2g_5^2} \int d^4x \, \frac{1}{z} \, V_\mu^a \partial_z V^{a\mu} \Big|_{z=\epsilon}.$$

- Let $V(q,z)$ be a solution with $V(q,z) = 1$. We need $V_\perp(q,z) = V_0(q) V(q,z)$.

$$\Pi(Q^2) = -\frac{1}{g_5^2} \, \frac{1}{Q^2} \, \frac{\partial_z V(q,z)}{z} \Big|_{z=\epsilon}.$$

$$V(q,z) = (Qz) K_1(Qz) = 1 + \frac{Q^2 z^2}{2} \ln(Qz) + \mathcal{O}(z^2). \qquad (z, Q^{-1} \ll z_m)$$

Thus

$$\Pi(Q^2) = -\frac{1}{g_5^2} \ln(Q z_m) + \text{contact terms}$$

5d coupling and N_c

In QCD

$$\Pi(Q) = -\frac{N_c}{24\pi^2} \ln Q^2 + \ldots$$

In AdS$_5$:

$$\Pi(Q^2) = -\frac{1}{2g_5^2} \ln Q^2 + \ldots$$

Thus

$$g_5^2 = \frac{12\pi^2}{N_c}$$

Large N_c $\Leftrightarrow$ small coupling

Example: current (endnotes)

- The role of 5d gauge inv:

$$\partial_z \left(\frac{1}{z} \partial_z V_\| \right) = 0 \quad \Rightarrow \quad \frac{\delta S_{5,\mathrm{eff}}}{\delta (V_0)_\|} = 0 \qquad \text{corresponds to} \qquad \partial_\mu J^\mu = 0.$$

- $\log Q^2$ in UV $\quad \Leftarrow \quad$ scale inv. of the 5d theory near $z = 0$.

$$[x^m] = -1; \quad [g_{mn}] = [z^{-2}] = 2 :$$

$$S_5 = -\frac{1}{4g_5^2} \int \underbrace{d^5x \sqrt{g}}_{0} \, \underbrace{V_{mn}^a V^{amn}}_{0}$$

$$[g_5] = 0$$

Dictionary

4d	$\leftrightarrow$	5d
generating func. W_4	$\leftrightarrow$	eff. action $S_{5,\mathrm{eff}}$
operator $\mathcal{O}(x)$ (ϕ_0 – source)	$\leftrightarrow$	field $\phi(x,z)$ (ϕ_0 – boundary value)
scale invariance ($\log Q$)	$\leftrightarrow$	scale invariance
$\partial_\mu J^\mu = 0$	$\leftrightarrow$	gauge invariance
large N_c	$\leftrightarrow$	small g_5
large Q	$\leftrightarrow$	small z
dimension of $\mathcal{O}$	$\leftrightarrow$	mass of ϕ

Dimension of 4d operator and 5d mass

- $$\mathcal{L}_5 = \frac{1}{2}\sqrt{g}\left(g^{mn}\partial_m\phi\,\partial_n\phi - m_5^2\,\phi^2\right)$$

  $z \to 0$ ($qz \ll 1$) extremum: $\phi \sim z^{\Delta_\phi}$ with $(\Delta_\phi - 4)\Delta_\phi - m_5^2 = 0.$

  $$m_5^2 = 0 \quad \Rightarrow \quad \Delta_\phi = 0 \quad \Rightarrow \quad \phi \longrightarrow \text{const} = \phi_0 \qquad \text{OK;}$$

  $$m_5^2 \neq 0 \quad \Rightarrow \quad \Delta_\phi \neq 0 \quad \Rightarrow \quad \underline{\phi\, z^{-\Delta_\phi} \longrightarrow \text{const} = \phi_0}.$$

- $$[\phi] = 0 \qquad \Rightarrow \qquad [\phi_0] = +\Delta_\phi \qquad ([x] = -1)$$

  Thus $[\mathcal{O}] = 4 - \Delta_\phi \equiv \Delta_\mathcal{O}$ and

  $$m_5^2 = (\Delta_\phi - 4)\Delta_\phi = \Delta_\mathcal{O}(\Delta_\mathcal{O} - 4)$$

- For example,

  $$T^\mu_\mu \quad : \quad m_5^2 = 0; \qquad \bar{\psi}\psi \quad : \quad m_5^2 = -3;$$

  $$J^\mu \quad : \quad m_5^2 = (\Delta - 3)(\Delta - 1) = 0 \qquad \text{— protected.}$$

Spontaneous symmetry breaking

- $$S_5 = \frac{1}{2}\int d^5x\sqrt{g}\,g^{mn}\partial_m\phi\,\partial_n\phi + \dots$$

  with b.c. at $z = 0$: $\phi\, z^{-\Delta_\phi} = \phi_0$. The extremum:

  $$\phi_{\rm sol} = \phi_0\, z^{\Delta_\phi} + A\, z^{\Delta_{\mathcal{O}}} \qquad (\Delta_\phi + \Delta_{\mathcal{O}} = 4).$$

- Vary the source: $\phi_0 \to \phi_0 + \delta\phi_0$: Klebanov-Witten

  $$\delta S_5 = \int d^4x\; z^{-3}\;\delta\phi\;\partial_z\phi\Big|_{z=0} + \dots = (\Delta_{\mathcal{O}} - \Delta_\phi)\int d^4x\;\delta\phi_0\;A$$

- Compare to W_4:

  $$\delta W_4 = \int d^4x\;\delta\phi_0\;\langle\mathcal{O}\rangle$$

  Therefore

  $$A = \frac{1}{2\Delta_{\mathcal{O}} - 4}\langle\mathcal{O}\rangle$$

  $A \nrightarrow 0$ as $\phi_0 \to 0$: spontaneous symmetry breaking

The model

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	$\Delta_{\mathcal{O}}$	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z) X^{\alpha\beta}$	0	3	-3

$$S = \int_0^{z_m} d^5x \sqrt{g}\, \mathrm{Tr}\left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

Symmetries: $X \to L X R^\dagger$, $F_L \to L F_L L^\dagger$, $F_R \to R F_R R^\dagger$.

Boundary conditions at $z = z_m$: $F_{z\mu} = 0$.

- Chiral symmetry breaking:

$$X_0(z) = \frac{1}{2} M z + \frac{1}{2} \Sigma z^3.$$

Matching to QCD: $\Sigma^{\alpha\beta} = \langle \bar{q}^\alpha q^\beta \rangle$. We take $M = m_q \mathbf{1}$ and $\Sigma = \sigma \mathbf{1}$.

- Four free parameters: m_q, σ, z_m and g_5.

Compared to three in QCD: m_q, Λ_{QCD} and N_c.

Hadrons and QCD sum rule

- $$\partial_z \left(\frac{1}{z} \partial_z V_\perp \right) + \frac{q^2}{z} V_\perp = 0.$$

  Normalizable modes: $\psi_\rho(\epsilon) = 0$, $\partial_z \psi_\rho(z_m) = 0$, $\int (dz/z)\, \psi_\rho(z)^2 = 1$.
  Exist only for discrete values of $q^2 = m_\rho^2$.

- Expanding $V(q, z) = \sum_\rho a_\rho(q) \psi_\rho(z)$,

  $$\Pi_V(-q^2) = -\frac{1}{g_5^2} \frac{1}{Q^2} \frac{\partial_z V(q,z)}{z}\Big|_{z=\epsilon} = -\frac{1}{g_5^2} \sum_\rho \frac{[\psi_\rho{}'(\epsilon)/\epsilon]^2}{(q^2 - m_\rho^2) m_\rho^2} \,.$$

  $$F_\rho = \frac{1}{g_5} \frac{\psi_\rho'(\epsilon)}{\epsilon} \,.$$

- QCD sum rule:

  $$\Pi_V(Q^2) = -\frac{1}{2 g_5^2} \ln Q^2 + \ldots$$

Chiral symmetry breaking and pion mass

$$\boxed{m_\pi^2 f_\pi^2 = 2m_q\sigma + \mathcal{O}(m_q^2)}$$

- with $A = (A_L - A_R)/2$, $\quad A_\mu = A_{\mu\perp} + \partial_\mu\varphi$, $\quad v(z) = m_q z + \sigma z^3$

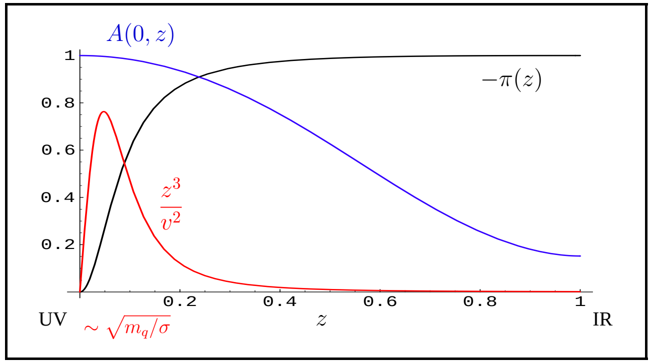
$$\delta A_\perp : \qquad \partial_z\left(z^{-1}\partial_z A_\perp\right) + z^{-1}q^2 A_\perp - z^{-3}g_5^2 v^2 A_\perp = 0;$$

$\Pi_A(-q^2) \overset{q\to 0}{\to} \frac{f_\pi^2}{-q^2}$. As for V: $A(q,\epsilon) = 1$. $\qquad f_\pi^2 = -\frac{1}{g_5^2}\frac{\partial_z A(0,z)}{z}\Big|_{z=\epsilon}$.

- with $X = X_0 \exp(i2\pi^a t^a)$

$$\delta A_\| : \qquad \partial_z\left(z^{-1}\partial_z\varphi\right) + z^{-3}g_5^2 v^2(\pi - \varphi) = 0;$$

$$\delta A_z : \qquad -q^2\partial_z\varphi + z^{-2}g_5^2 v^2\partial_z\pi = 0.$$

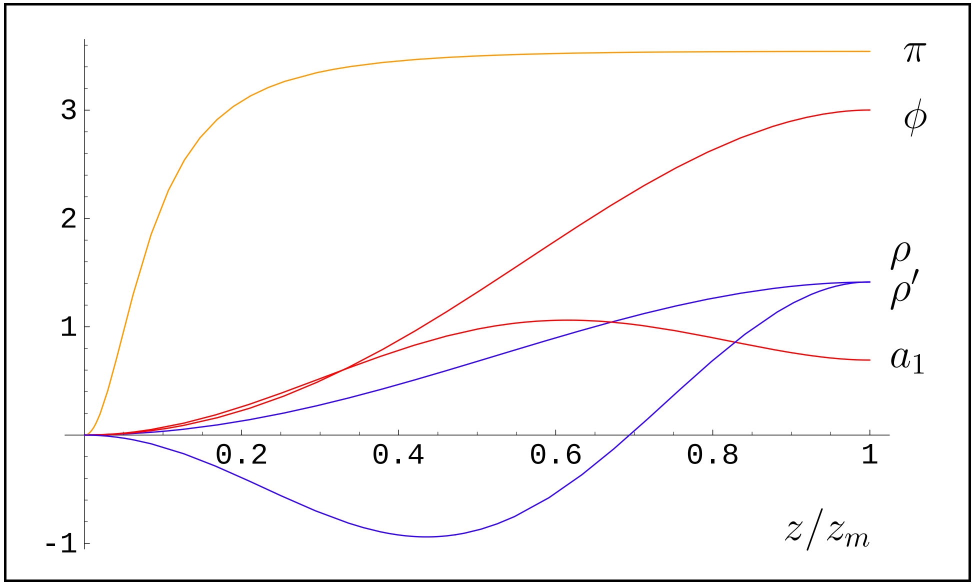


For $q \to 0$: $\quad \varphi(z) = A(0,z) - 1, \quad \pi(z) = -1.$
$\phi(0) = 0$, but $\pi(0) = 0$ requires $q \neq 0$ and

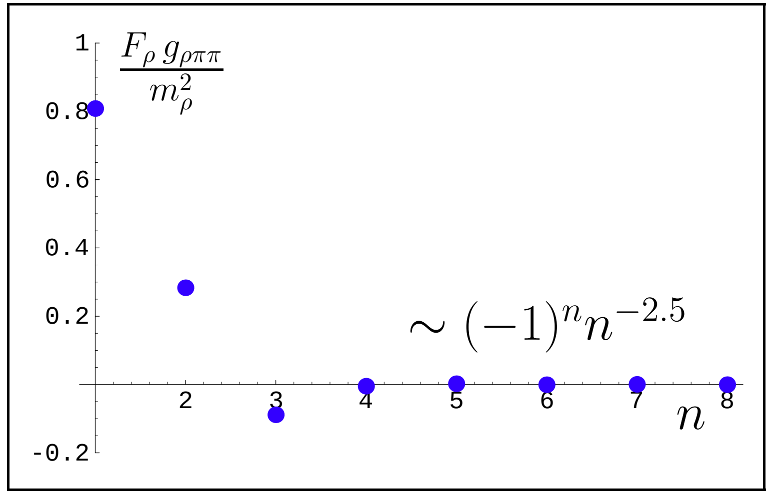
$$\pi(z') = q^2\int_0^{z'} dz\,\frac{z^3}{v(z)^2}\cdot\frac{1}{g_5^2 z}\partial_z A(0,z) = -q^2 f_\pi^2\frac{1}{2m_q\sigma} = -1 \quad \text{for} \quad z \gg \sqrt{m_q/\sigma}.$$

(Continues to hold for $\Delta_\sigma \neq 3$, or deformed AdS.)

Meson wavefunctions and VMD



VMD



Couplings:

$$g_{\rho\pi\pi} = g_5 \int dz\, \psi_\rho(z) \left(\frac{\phi'(z)^2}{g_5^2\, z} + \frac{v(z)^2(\pi - \phi)^2}{z^3} \right).$$

Isospin sum rule for the pion formfactor:

$$G(q^2 = 0) = \sum_\rho \frac{F_\rho g_{\rho\pi\pi}}{m_\rho^2} = 1$$

Comparison with experiment

Observable	Measured	Model	Units
m_π	139.6± 0.0004	139.6*	MeV
m_ρ	775.8± 0.5	775.8*	MeV
m_{a_1}	1230± 40	1363	MeV
f_π	92.4± 0.35	92.4*	MeV
$F_\rho^{1/2}$	345± 8	329	MeV
$F_{a_1}^{1/2}$	433± 13	486	MeV
$g_{\rho\pi\pi}$	6.03± 0.07	4.48	–
$m_{\rho'}$	1465± 25	1450	MeV

- $N_c \quad \Rightarrow \quad g_5 = \sqrt{12\pi^2/N_c} = 2\pi$.
- $m_\rho = 2.405/z_m \quad \Rightarrow \quad z_m = (323 \text{ MeV})^{-1}$.
- f_π and $m_\pi \quad \Rightarrow \quad \sigma = (327 \text{ MeV})^3$ and $m_q = 2.29 \text{ MeV}$.

Linear confinement: $m_n^2 \sim n$

- High n mesons are large $\Rightarrow$ IR regime. Thus high n $\Leftrightarrow$ large z.
- $$I = \int d^5x \sqrt{g}\, e^{-\Phi}\, \mathcal{L},$$

  A. Karch et al., hep-ph/0602229

  Instead of hard cutoff at z_m ($m_n^2 \sim n^2$) — $\Phi \to z^2$ at large z
- Mode equation

  $$\partial_z \left(e^{-B} \partial_z v_n \right) + m_n^2 e^{-B} v_n = 0,$$

  where $B = \Phi(z) - A(z)$, with $e^{A(z)} = z^{-1}$. Substitute $v_n = e^{B/2}\psi_n$

  $$-\psi_n'' + V(z)\psi_n = m_n^2 \psi_n, \qquad V(z) = \frac{1}{4}(B')^2 - \frac{1}{2}B''.$$
- With $\Phi = z^2$: $V = z^2 + \frac{3}{4z^2}$ — 2d harmonic oscillator (radial, $m = 1$).

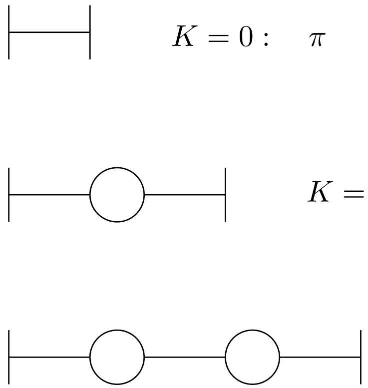
  $$m_n^2 = 4(n+1)$$
- For spin S mesons: $V = z^2 + \frac{S^2 - 1/4}{z^2} + 2(S-1)$ and $m^2 = 4(n+S)$.
- (Classical) Nambu-Gotto string also predicts: $\frac{dm^2}{dn} = \frac{dm^2}{dS}$.

Outlook

- $h(z,x) \longrightarrow$ (scalar) glueball spectrum (Polchinski-Strassler, Boschi-Filho-Braga)
- strange mesons (Shock-Wu)
- higher order chiral Lagrangian (DaRold-Pomarol, Hirn-Sanz)
- chiral anomaly (WZW) (Hill-Zachos, Sakai-Sugimoto)
- Baryons (Teramond-Brodsky)
  - Dirac fields or skyrmions/instantons?
  - finite density?
- running of α_s — log corrections to warp factor in UV.
- power corrections to OPE — higher order terms in 5d (Hirn-Rius-Sanz).

Appendix

Holographic model vs open moose



$K = 0: \quad \pi$

$K = 1: \quad \pi, \rho$

$K = 2: \quad \pi, \rho, a_1$

Folded: