Exploring the origin of the $\Delta I = 1/2$ rule in lattice QCD

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Thanks to my collaborators:

L. Giusti, M. Laine, C. Pena, P. Weisz, J. Wennekers and H. Wittig

hep-lat/0407007,hep-ph/0407086, hep-lat/0607027, hep-lat/0607028, hep-ph/0607220

The $\Delta I = 1/2$ rule

One of the most striking hierarchies in low-energy QCD remains to be understood:

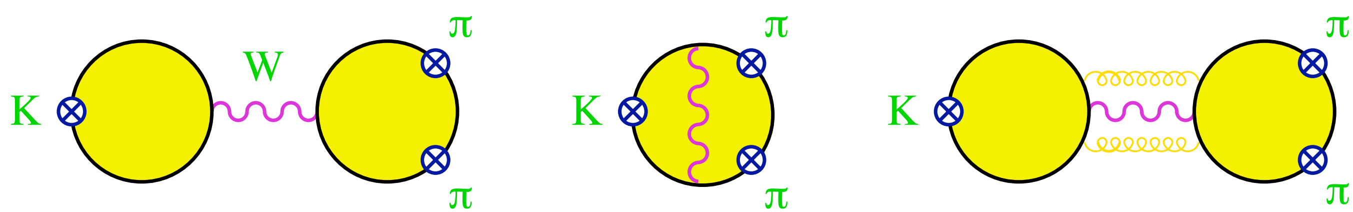
$$\begin{aligned}
T(K^+ \to \pi^+\pi^0) &= \frac{\sqrt{3}}{2}\, A_2\, e^{i\delta_2} \\
T(K^0 \to \pi^+\pi^-) &= \frac{1}{\sqrt{6}}\, A_2\, e^{i\delta_2} + \frac{1}{\sqrt{3}}\, A_0\, e^{i\delta_0} \\
T(K^0 \to \pi^0\pi^0) &= \frac{\sqrt{2}}{\sqrt{3}}\, A_2\, e^{i\delta_2} - \frac{1}{\sqrt{3}}\, A_0\, e^{i\delta_0}
\end{aligned}$$

$$T\left(K^0 \to \pi\pi|_{I=\alpha}\right) = A_\alpha e^{i\delta_\alpha} \qquad \frac{A_0}{A_2} = 22.1$$

The $\Delta I = 1/2$ rule

A notorious failure of large N_C :

$$\mathcal{H}_{|\Delta S|=1} \sim G_F J_W^\mu J_W^\mu$$



$O(N_C^2)$ $O(N_C)$ $O(1)$

$$T(K^0 \to \pi^0 \pi^0) = 0 \to \left[\frac{A_0}{A_2}\right]_{N_c} = \sqrt{2}$$

Fukugita *et al* (1977); Chivukula, Flynn, Georgi (1986)

The $\Delta I = 1/2$ rule

M_W $\quad \mathcal{H}_{SM} \to \mathcal{H}^{N_f=4}_{\Delta S=1} = \sqrt{2} G_F V^*_{us} V_{ud} (k_+ Q_+ + k_- Q_-)$

$$Q_\pm \equiv [\bar{s}u]_{V-A}[\bar{u}d]_{V-A} \pm [\bar{s}d]_{V-A}[\bar{u}u]_{V-A} - (u \leftrightarrow c)$$

$SU(4)_L \times SU(4)_R$: $Q_+ \to (84,1) \quad Q_- \to (20,1)$

m_c $\quad \mathcal{H}^{N_f=4}_{\Delta S=1} \to \mathcal{H}^{N_f=3}_{\Delta S=1} = \sqrt{2} G_F V^*_{us} V_{ud} \sum_{\sigma=1,10} C_\sigma Q_\sigma$

$$Q_\sigma : ..., [\bar{s}d]_{V-A}[\bar{q}q]_{V+A}, ...$$

$SU(3)_L \times SU(3)_R$: $(27,1) \to A_2, A_0, (8,1) \to A_0$

Λ_χ $\quad \mathcal{H}^{N_f=3}_{\Delta S=1} \stackrel{large N_C}{\longrightarrow} \mathcal{H}^{N_f=3}_{\chi PT}$

The $\Delta I = 1/2$ rule

The old lore:

- Resummation of $O(1/N_C)\log(\mu/M_W)$ up to $\mu > m_c$ "gives a moderate enhancement"

- Charm threshold: $\mu < m_c \rightarrow$ Penguins

- Penguin matrix elements can be large compared to that of left-left operators

Shifman, Zakharov,Vainstein (1977); Bardeen, Buras, Gerard (1986)

After so many years, we still do not have a satisfactory explanation...

The $\Delta I = 1/2$ rule on the lattice

Cabibbo, Martinelli, Petronzio (1984); Brower, Gavela, Gupta, Maturana (1984)

$\chi PT \oplus$ lattice

Bernard, Draper, Soni, Politzer, Wise (1985)

$$\begin{array}{rcl} (27,1) & \to & g_{27}\,\frac{F^4}{8} T_{ijkl} \left(\partial_\mu U U^\dagger\right)_{ki} \left(\partial_\mu U U^\dagger\right)_{lj} \\ (8,1) & \to & -g_8\,\frac{F^4}{8} \mathrm{Tr}\left[\Lambda \partial_\mu U \partial_\mu U^\dagger\right] \\ & \to & g_8'\,\frac{F^2 \Sigma}{2} \mathrm{Tr}\left[\Lambda (U M + M^\dagger U^\dagger)\right] \end{array}$$

At LO: $$\frac{A_0}{A_2} = \frac{1}{\sqrt{2}} \left(\frac{1}{5} + \frac{9}{5} \frac{g_8}{g_{27}} \right)$$

g_{27}, g_8, g_8' can be measured from two and three-point correlators:

$$\langle K | Q^\pm | \pi\pi \rangle \quad \text{from} \quad \langle K | Q^\pm | \pi \rangle \text{ and } \langle K | Q^\pm | 0 \rangle$$

The $\Delta I = 1/2$ rule on the lattice

Turned out to be a very difficult problem:

P1 Breaking of chiral symmetry: mixing with wrong-chirality four-quark operators and with lower dimensional ones resulting in power-diverging mixing coefficients

Maiani, Martinelli, Rossi, Testa (1987)

Dawson, Martinelli, Rossi, Sachrajda, Sharpe, Talevi, Testa (1997)

P2 Power-diverging mixings occur also with chirally-invariant regularizations if the charm quark is integrated out:

$$\mathcal{O}_m \quad \rightarrow \quad m_d \bar{s} P_+ d + m_s \bar{s} P_- d$$

P3 At large quark masses higher orders in ChPT are needed: many new couplings $\rightarrow$ no longer a direct relation between $K \rightarrow \pi\pi$ and $K \rightarrow \pi$

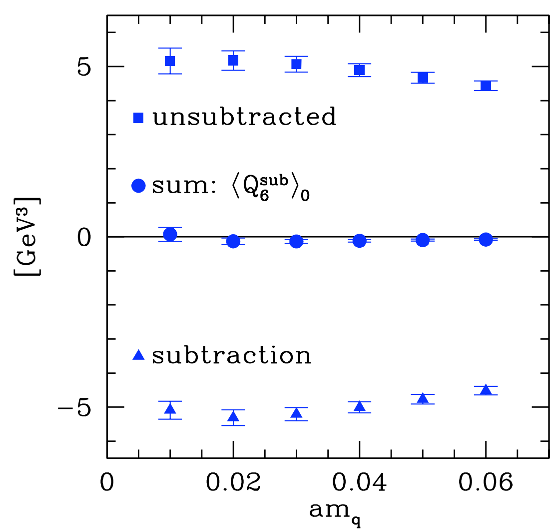
Kambor, Missimer, Wyler (1990); Bijnens, Pallante, Prades (1998); Golterman, Pallante (2000)

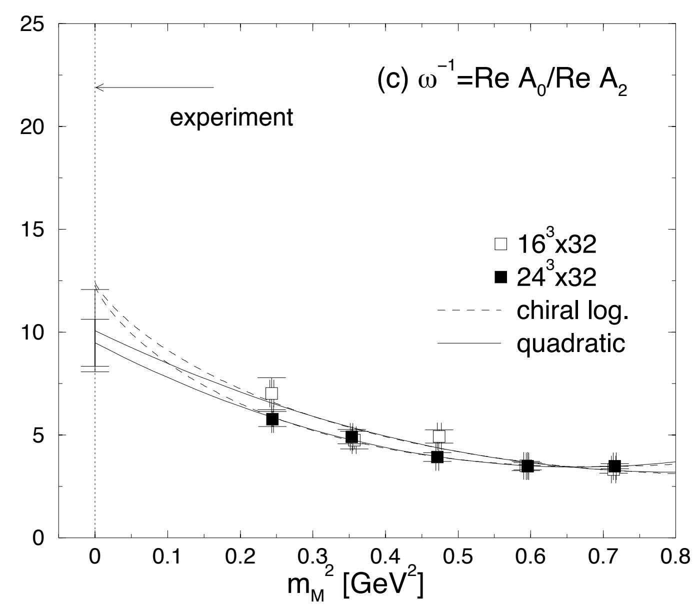
The $\Delta I = 1/2$ rule on the lattice

Blum, *et al*, (2001); Noaki, *et al*, (2001)

- Use finite N_s DW fermions $\longrightarrow$ no exact chiral symmetry
- No active charm $\longrightarrow$ power-divergent subtractions necessary
- Large quark masses $\longrightarrow$ very large uncertainty in chiral extrapolations
- Further quenched ambiguities

$\longrightarrow$ Golterman, Pallante (2006)



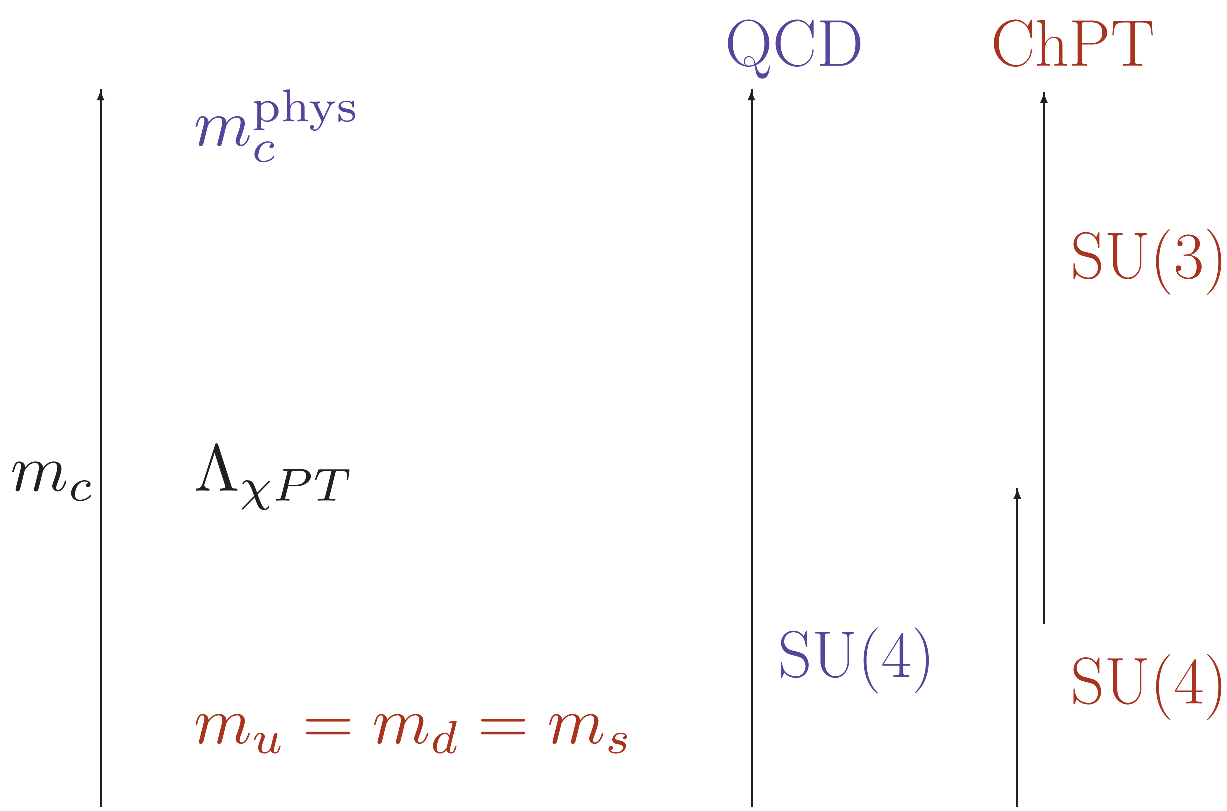


New Strategy

Lüscher and Giusti, PH, Laine, Weisz, Wittig (2004)

Lattice QCD can investigate in a well defined way the role of the different scales that enter in the problem, in particular the role of m_c

If the large enhancement is due to the large separation between $m_c \gg \Lambda_{QCD}$ or $m_c \gg m_u$ there should be no effect in the theory with a light charm quark!



Revealing the role of the charm

- Step 1: $m_c = m_u = m_d = m_s$

  $SU(4)$-lattice QCD matched to $SU(4)$-ChPT in to extract the low-energy couplings that mediate kaon decays near the chiral limit

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- Step 2: $\Lambda_{\chi PT} \gg m_c \gg m_u = m_d = m_s$

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- Step 3: $m_c > \Lambda_{ChPT} \gg m_u = m_d = m_s$

  $SU(4)$-lattice QCD matched to $SU(3)$-ChPT to extract $g_{27}(m_c)$, $g_8(m_c)$

SU(4) on the lattice

Below M_W the OPE (chiral flavour symmetry + CPS symmetry):

$$\mathcal{H}_{\mathrm{w}}^{QCD} = \frac{g_{\mathrm{w}}^2}{4M_W^2}(V_{us})^* V_{ud} \sum_{\sigma=\pm} k_1^\sigma Q_1^\sigma + k_2^\sigma Q_2^\sigma \quad \pm \leftrightarrow (84,1)/(20,1)$$

$$Q_1^\pm = \left\{ (\bar{s}\gamma_\mu P_- u)(\bar{u}\gamma_\mu P_- d) \pm (\bar{s}\gamma_\mu P_- d)(\bar{u}\gamma_\mu P_- u) \right\} - (u \to c),$$

$$Q_2^\pm = \left(m_u^2 - m_c^2 \right) \left\{ m_d \, (\bar{s}P_+ d) + m_s \, (\bar{s}P_- d) \right\}$$

Therefore in any regularization that preserves chiral symmetry and CP:

$$Q_1^\pm = \mathcal{Z}_{11}^\pm Q_1^{\pm,\mathrm{bare}} + \mathcal{Z}_{12}^\pm Q_2^{\pm,\mathrm{bare}}$$
$$Q_2^\pm = \mathcal{Z}_{21}^\pm Q_1^{\pm,\mathrm{bare}} + \mathcal{Z}_{22}^\pm Q_2^{\pm,\mathrm{bare}}$$

$Q_2^\pm$ do not contribute to physical matrix elements and nicely vanishes in the $SU(4)$ limit!

With Ginsparg-Wilson (overlap) fermions this pattern is mantained on the lattice !

Ginsparg-Wilson fermions

Lattice Dirac operators can be constructed which are local, do not suffer from the doubling problem and satisfy the Ginsparg–Wilson (GW) relation:

$$\boxed{\{D,\gamma_5\} = aD\gamma_5 D \quad \leftrightarrow \quad \{D^{-1}_{xy},\gamma_5\} = a\gamma_5\delta_{xy}}$$

Ginsparg and Wilson (1982)

Nielsen-Ninomiya no-go theorem : $(i) - (iv)$ cannot be simultaneously fulfilled

(i) D is local (bounded by $Ce^{-\gamma/a|x|}$)

(ii) $\tilde{D}(p) = i\gamma_\mu p_\mu + \mathrm{O}(ap^2)$ for $p \ll \pi/a$

(iii) $\tilde{D}(p)$ is invertible for all $p \neq 0 \leftrightarrow$ no doublers

(iv) chiral symmetry: $\{\gamma_5, D\} = 0$

Ginsparg-Wilson fermions

The way out is by generalizing the chiral transformation $\rightarrow$ modifying the requirement (iv) at finite lattice spacing

$$\{D, \gamma_5\} = aD\gamma_5 D$$

but this is enough to ensure an exact symmetry :

$$\delta_\chi \Psi = \epsilon\, \gamma_5 (1 - aD)\Psi \qquad \delta_\chi \bar{\Psi} = \epsilon \bar{\Psi}\gamma_5 \quad \rightarrow \delta_\chi S_f = 0$$

Lüscher (1998)

The first explicit implementation was the overlap operator:

$$aD_{ov} = 1 - \gamma_5 \mathrm{sign(Q)} \qquad \mathrm{Q} \equiv \gamma_5 (1 - \mathrm{aD_W})$$

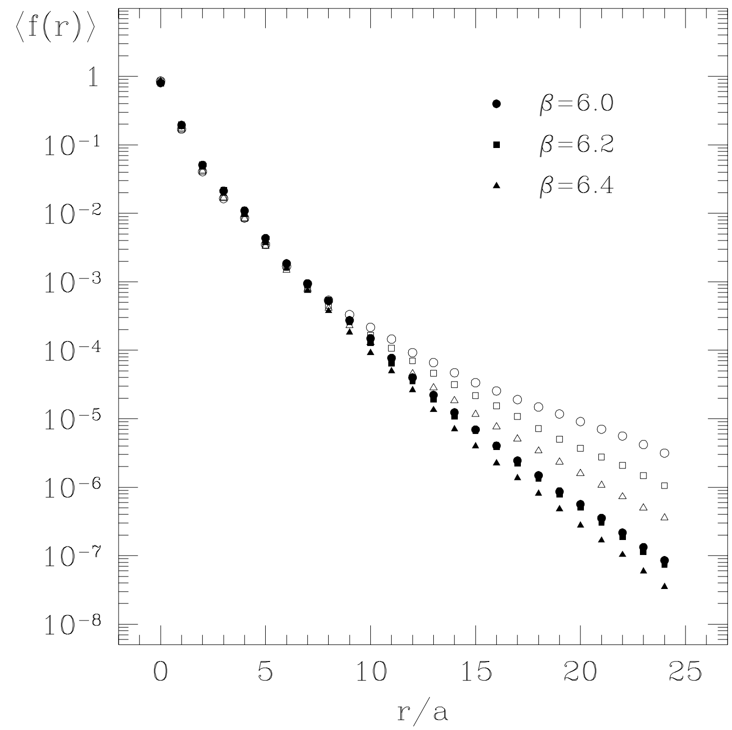
Neuberger (1998)

Overlap Operator

In spite of its looks, it is a local operator!

PH, Jansen, Lüscher (1999)

$$||D_{ov}(0,r)|| \leq e^{-\gamma|r|/a}$$

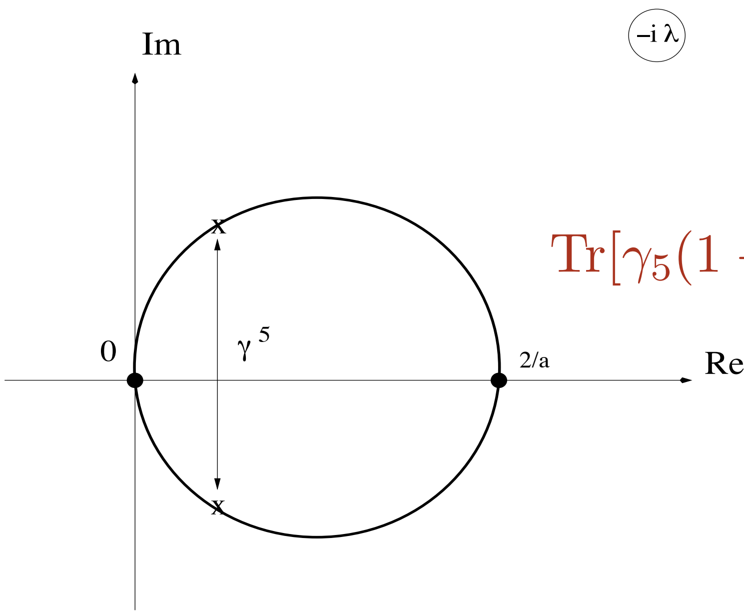


$$D_N = \frac{1+s}{a}\left\{1 - \gamma_5 sign(Q)\right\}, \quad Q = \gamma_5(1 + s - aD_w)$$

Exact index theorem

$U_A(1)$ anomaly is recovered due to the non-invariance of the fermion measure under a singlet chiral rotation:

$$\langle \delta_\chi O \rangle_F = \mathrm{Tr}\left[\gamma_5(1 - aD/2)\right] \langle O \rangle_F$$



$$\mathrm{Tr}[\gamma_5(1 - aD/2)] = N_f \times index(D)$$

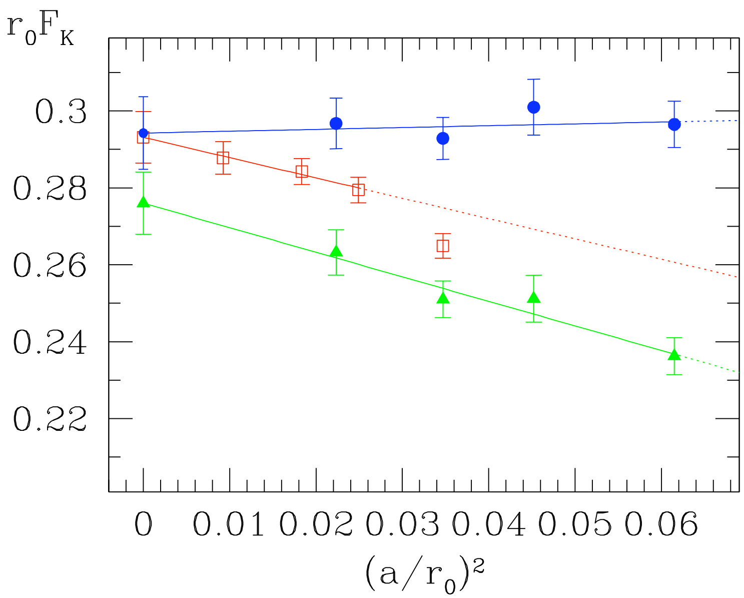
A GW operator satisfies an exact index theorem and topological sectors can be distinguished!

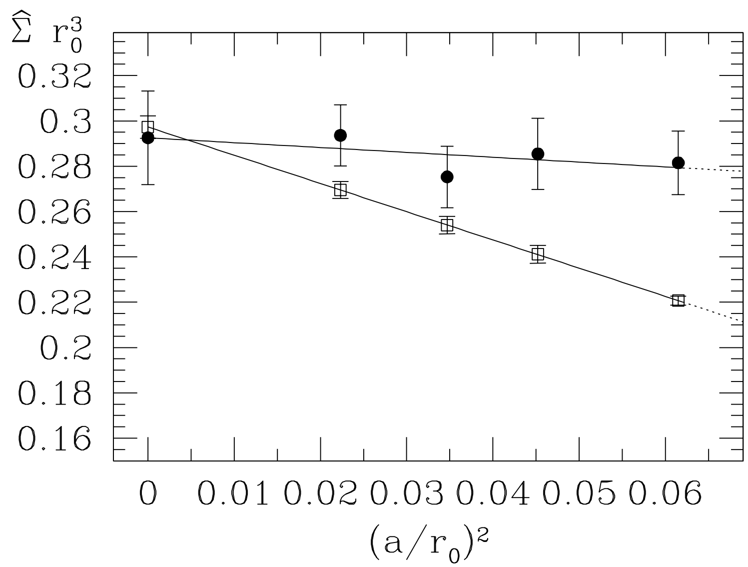
Hasenfratz, Laliena, Niedermayer (1998)

Small $\mathcal{O}(a^2)$ violations

The exact chiral symmetry ensures that the approach to the continuum limit is faster...

Furthermore scaling studies show that $\mathcal{O}(a^2)$ corrections are small for several quantities such as F_K, Σ





Wennekers, Wittig (2005)

SU(4) in χPT

$$\mathcal{H}_{\rm w}^{\rm ChPT} = \frac{g_{\rm w}^2}{4M_W^2}(V_{us})^* V_{ud} \sum_{\sigma=\pm} g^\sigma [\mathcal{O}^\sigma]$$

$$\mathcal{O}^\pm = \frac{F^4}{4}\left[\left(U\partial_\mu U^\dagger\right)_{us}\left(U\partial_\mu U^\dagger\right)_{du} \pm \left(U\partial_\mu U^\dagger\right)_{uu}\left(U\partial_\mu U^\dagger\right)_{ds} - (u \to c)\right]$$

In contrast with $SU(3)$, only two operators appear in $SU(4)$-ChPT at LO:

$$\frac{A_0}{A_2} = \frac{1}{\sqrt{2}}\left(\frac{1}{2} + \frac{3}{2}\frac{g^-}{g^+}\right) \qquad [g^+]_{N_c} = [g^-]_{N_c} = 1$$

In the quenched case no Golterman-Pallante ambiguities at LO!

The Matching

By equating certain correlation functions in lattice QCD and in the chiral theory: three-point functions of the bare operators and two left currents

Lattice QCD	χPT
$Z_A^2(g_0)\sum_{\mathbf{x}}\langle[J_{L0}(x)]_{\alpha\beta}[J_{L0}(0)]_{\beta\alpha}\rangle$	$\int d^3x\,\langle[\mathcal{J}_{L0}(x)]_{\alpha\beta}[\mathcal{J}_{L0}(0)]_{\beta\alpha}\rangle$
$\downarrow$	$\downarrow$
$Z_A^2 C(x_0)$	$\mathcal{C}(x_0)$
$Z_A^2(g_0)Z^{\pm}(g_0)\sum_{\mathbf{x},\mathbf{y}}\Big\langle[J_{L0}(x)]_{du}\,Q^{\pm}(0)\,[J_{L0}(y)]_{us}\Big\rangle$	$g^{\pm}\int d^3x\int d^3y\,\Big\langle[\mathcal{J}_{L0}(x)]_{du}\,\mathcal{O}^{\pm}(0)\,[\mathcal{J}_{L0}(y)]_{us}\Big\rangle$
$\downarrow$	$\downarrow$
$Z_A^2 Z^{\pm}(g_0)C^{\pm}(x_0,y_0)$	$g^{\pm}\mathcal{C}^{\pm}(x_0,y_0)$

More concretely:

$$R^{\sigma}(x_0,y_0)\equiv\frac{C^{\sigma}(x_0,y_0)}{C(x_0)C(y_0)}\qquad\mathcal{R}^{\sigma}(x_0,y_0)\equiv\frac{\mathcal{C}^{\sigma}(x_0,y_0)}{\mathcal{C}(x_0)\mathcal{C}(y_0)}$$

The Matching

$$g^\sigma \quad [\mathcal{R}^\sigma(m, V, LECS)] = \quad [k^\sigma(M_W)]_{RGI} \quad \left[\frac{Z^\sigma(g_0)}{Z_A^2}\right]_{RGI} \quad R^\sigma$$

$$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ \chi\mathrm{P}T & \mathrm{P}.T. - 2\ loop & \mathrm{N}.P. & \mathrm{L}attice \end{array}$$

Each of the needed elements was not trivial:

- The possibility to approach the $m \to 0$ limit, to reduce the number of low-energy constants involved in $\mathcal{R}^\sigma$

  $\longrightarrow$ ϵ-regime of χPT $\oplus$ p-regime

- Taming the large fluctuations observed in the ϵ-regime

  $\longrightarrow$ Low-mode averaging

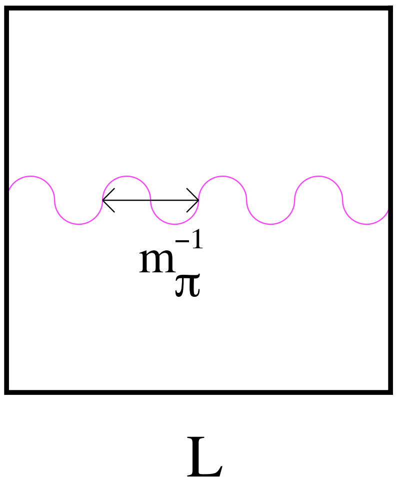
- Non perturbative renormalization

  $\longrightarrow$ Matching to an intermediate scheme (SF- tmQCD)

$\mathcal{R}^\sigma$ in ChPT

In a finite volume we can distinguish two regimes of χPT:

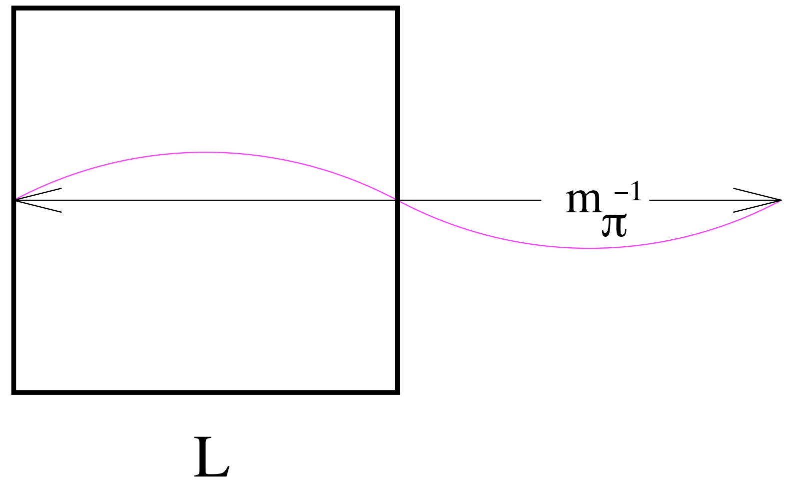
p-regime: $m\Sigma V \gg 1$



Standard χPT in finite V:

$m \sim p^2 \qquad L^{-1}, T^{-1} \sim p$

ϵ-regime: $m\Sigma V \leq 1$



Zero-modes of pions are not perturbative!

$m \sim p^4 \qquad L^{-1}, T^{-1} \sim p$

The ϵ-regime

The χ expansion can be reordered by factoring out the constant field configurations and treating them as collective variables:

$$U = U_0 U_\xi = U_0\, e^{i\,2\xi(x)/F} \qquad \int dx \xi(x) = 0$$

$$\mathcal{Z} = \int_{SU(N_f)} dU_o \int d\xi \;\; e^{-\mathcal{S}_\chi(U_o,\xi)}$$

Gasser, Leutwyler (1987),Hansen (1990), Hansen, Leutwyler (1991)

- Implies a reordering of the chiral expansion: at any order less relevant couplings appear as compared to the usual chiral expansion
- The quenched or partially quenched version require some care, in particular to consider fixed topological sectors:

$$\mathcal{Z}_\nu = \int_{U(N_f)} dU_o \; \det(U_0)^\nu \int d\xi \;\; e^{-\mathcal{S}_\chi(U_o,\xi)}$$

Damgaard, Splittorff (2000); Damgaard, Diamantini, PH, Jansen (2002); Damgaard, PH, Jansen, Laine, Lellouch (2003)

$\mathcal{R}^\sigma$ in ϵ-regime

PH, Laine (2003), Giusti, PH, Laine, Weisz, Wittig (2004)

For the processes at hand at NLO in the ϵ-expansion no higher order operator neither strong nor weak enters!

$$\text{NLO: } \chi \equiv MU \quad \mathcal{L}_\mu \equiv i\partial_\mu U U^\dagger, \quad \mathcal{W}_{\mu\nu} = 2(\partial_\mu \mathcal{L}_\nu + \partial_\nu \mathcal{L}_\mu); \quad (\Delta_{ij})_{ab} = \delta_{ai}\delta_{bj}$$

		p-regime	ϵ-regime
Gasser, Leytwyler	$\mathcal{H}_{QCD}$	$L_4 \langle D_\mu U^\dagger D^\mu U\rangle \langle U^\dagger \chi + \chi^\dagger U\rangle$	$\times$
		$L_5 \langle D_\mu U^\dagger D^\mu U \left(U^\dagger \chi + \chi^\dagger U\right)\rangle$	$\times$
		$L_6 \langle U^\dagger \chi + \chi^\dagger U\rangle^2$	$\times$
		$L_8 \langle \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi\rangle$	$\times$
Kambor,Missimer,Wyler	$\mathcal{H}_{weak}$	$D_2^\pm t^\pm_{ij,kl} \langle \Delta_{ij}(\chi - \chi^\dagger)\rangle\langle \Delta_{kl}(\chi - \chi^\dagger)\rangle$	$\times$
		$D_4^\pm t^\pm_{ij,kl} \langle \Delta_{ij}\mathcal{L}_\mu\rangle\langle \Delta_{kl} \left\{\mathcal{L}^\mu, (\chi + \chi^\dagger)\right\}\rangle$	$\times$
		$D_7^\pm t^\pm_{ij,kl} \langle \Delta_{ij}\mathcal{L}_\mu\rangle\langle \Delta_{kl}\mathcal{L}_\mu\rangle\langle(\chi + \chi^\dagger)\rangle$	$\times$
		$D_{20}^\pm t^\pm_{ij,kl} \langle \Delta_{ij}\mathcal{L}_\mu\rangle\langle \Delta_{kl}\partial_\nu \mathcal{W}_{\mu\nu}\rangle$	$\times$
		$D_{24}^\pm t^\pm_{ij,kl} \langle \Delta_{ij}\mathcal{W}_{\mu\nu}\rangle\langle \Delta_{kl}\mathcal{W}_{\mu\nu}\rangle$	$\times$

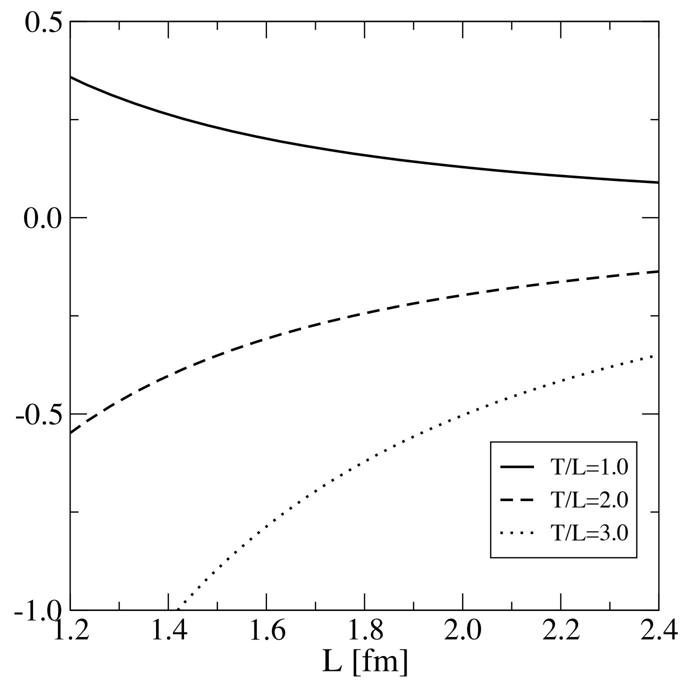
$\mathcal{R}^\sigma$ in ϵ-regime

$$2\,\mathcal{R}^{\pm}(x_0, y_0) = 1 \pm \frac{2}{(FL)^2}\left[\rho^{-1/2}\beta_1 - \rho k_{00}\right] = 1 \pm K$$

with $\rho \equiv T/L$ and β_1, k_{00} are shape coefficients of the box.

- same for all ν
- independent of x_0 and y_0
- same in (partially-)quenched theory
- no higher order weak or strong LECS

K



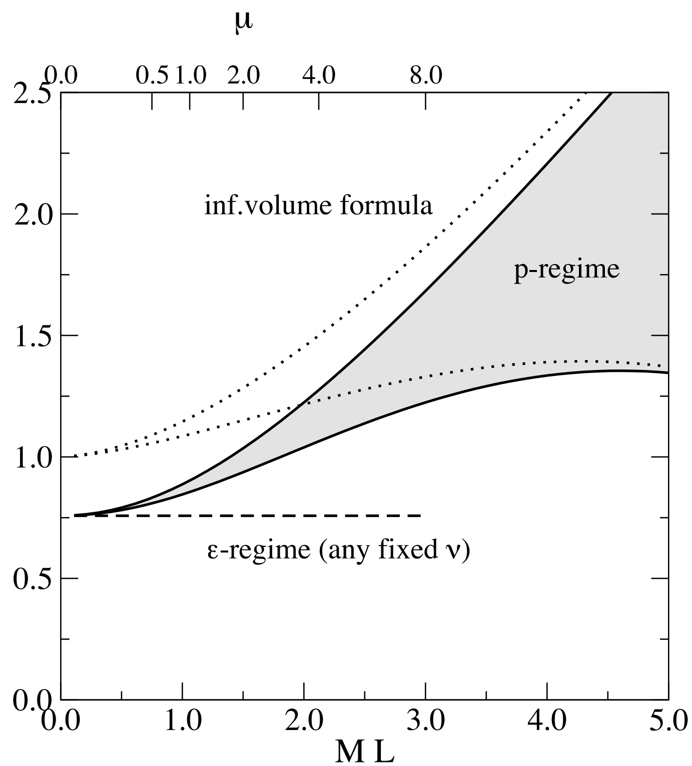
$\mathcal{R}^\sigma$ in p-regime ChPT

PH, Laine, hep-lat/0607027

For these observables the ϵ-regime and ∞-volume results can be smoothly reached from the p-regime expressions

$N_f = 3$, $L = 2$fm, $T/L = 2$, $\Lambda_+ = 500 - 2000$MeV :

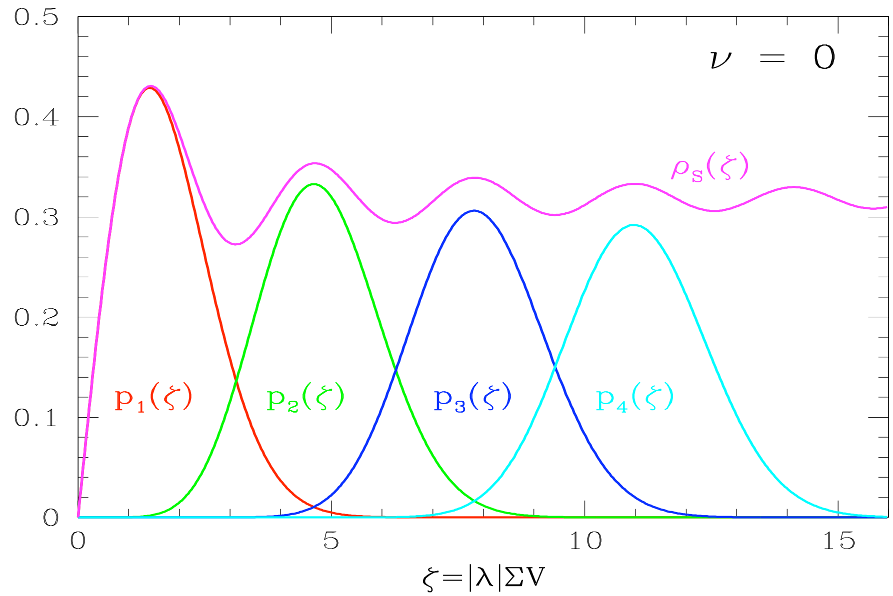
$2\mathcal{R}^+(-T/3, T/3)$



Deviations from the infinite volume expectation are significant for $ML \leq 5$

R^σ on the lattice

In the ϵ-regime: $m\Sigma V \leq 1$, large fluctuations in the observables are observed:



$$\langle \lambda_i \rangle_\nu = \frac{\mathcal{O}(1)}{\Sigma V}, \quad \Delta\lambda = \lambda_{i+1} - \lambda_i \sim \frac{\mathcal{O}(1)}{\Sigma V} \geq m$$

Low-lying spectrum of D_m is discrete: $\Delta\lambda \geq \lambda_k + m$

Space-time fluctuations in the wave-functions of the low-lying spectrum $\rightarrow$ large fluctuations in point-to-all propagators!

Two strategies to tame these fluctuations:

- Low-mode averaging: applicable to these observables
- Physics from zero-mode wave-functions: involves new observables

Low Mode Averaging (LMA)

Giusti, PH, Laine, Weisz, Wittig (2004); Degrand, Schaefer (2004)

Fluctuations in the wave functions of low modes will be averaged out if

$$C^I(t_x - t_y) = \sum_{\vec{y}} \mathrm{Tr}[\Gamma_I S(x,y)\Gamma_I S(y,x)] \Rightarrow \sum_{x,y,|t_x-t_y|=fixed} \mathrm{Tr}[\Gamma_I S(x,y)\Gamma_I S(y,x)],$$

When only a few eigenfunctions give the largest contribution:

$$S(x,y) = S_h(x,y) + S_l(x,y), \quad S_l(x,y) = \frac{1}{V}\sum_{k=1}^{n} \frac{v_k(x) v_k(y)^\dagger}{\lambda_k + m}$$

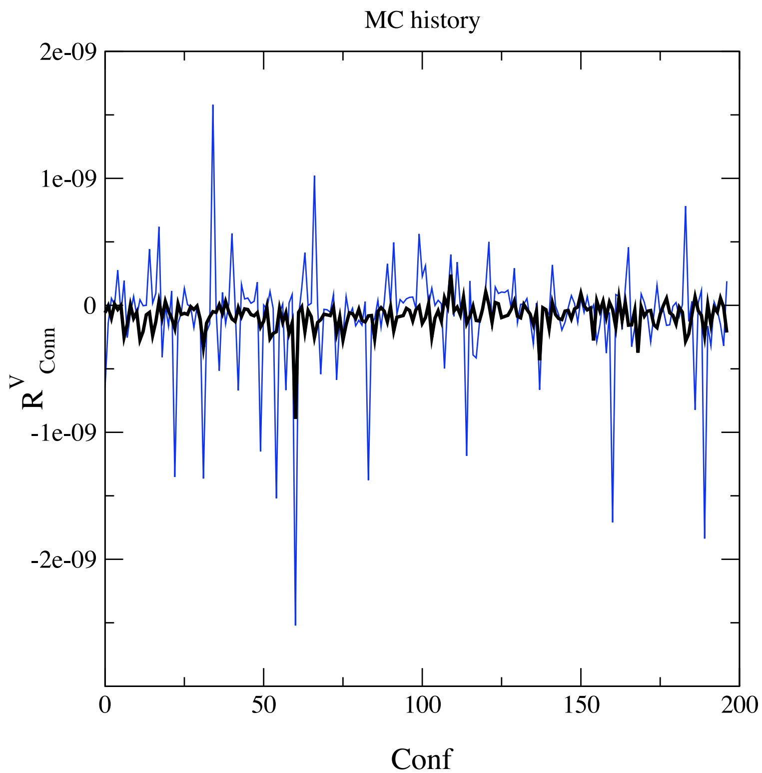
$$\begin{array}{cccc} C^I(t_x - t_y) = C^I_{hh}(t_x - t_y) & +C^I_{hl}(t_x - t_y)+ & C^I_{ll}(t_x - t_y) & \\ \downarrow & \downarrow & \downarrow & \\ \sum_{\vec{y}} & \sum_{\vec{x},\vec{y}} & \sum_{t_x,t_y,|t_x-t_y|=fixed} & \sum_{\vec{x},\vec{y}} \end{array}$$

Similarly most of the contributions containing S_l to the three-point functions are averaged

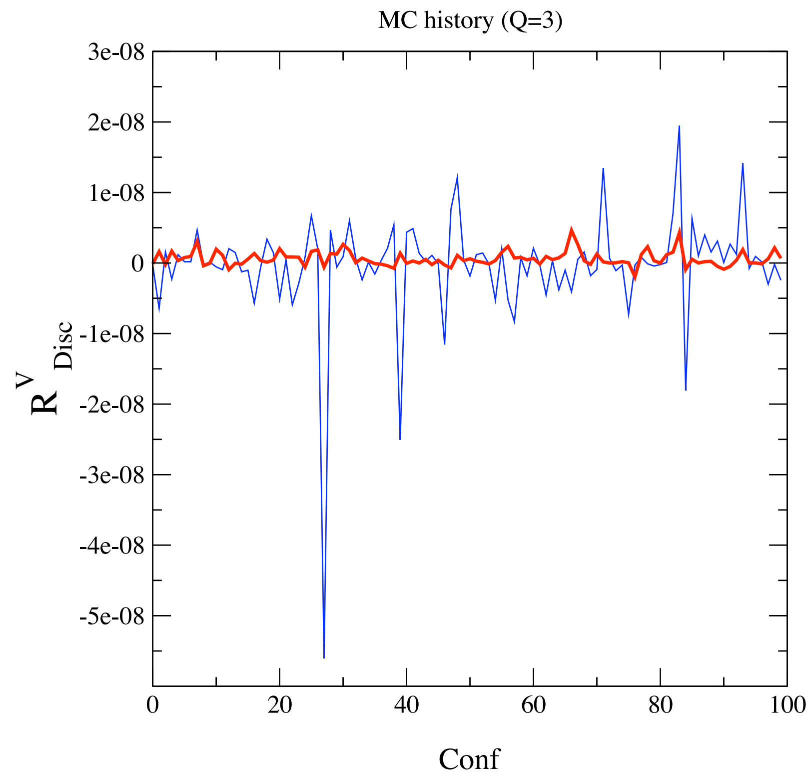
Low Mode Averaging (LMA)

Important variance reduction in the three point functions both in the p and ϵ regimes !

p-regime



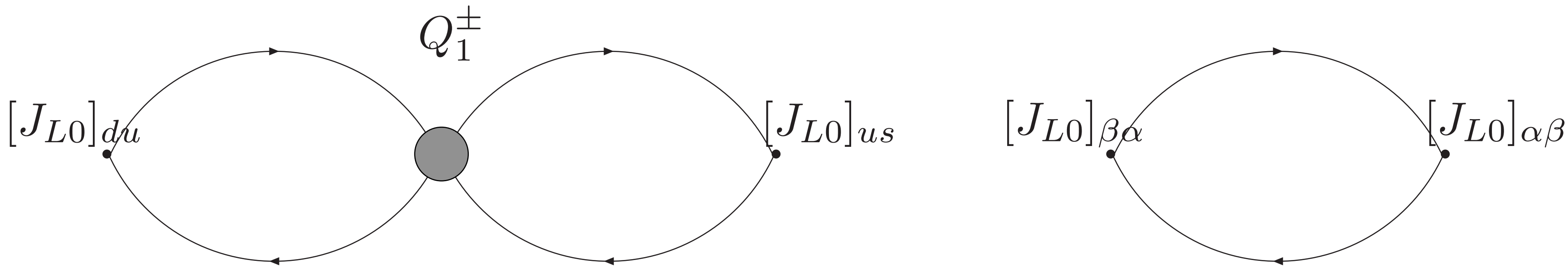
ϵ-regime



$$n_{low} = 20!$$

The simulation

Giusti, PH, Laine, Pena, Wennekers, Wittig, hep-ph/0607220



We use the overlap operator:

$$D_N = \frac{1+s}{a}\left\{1 - \frac{A}{(A^\dagger A)^{1/2}}\right\}, \quad A = 1 + s - aD_w$$

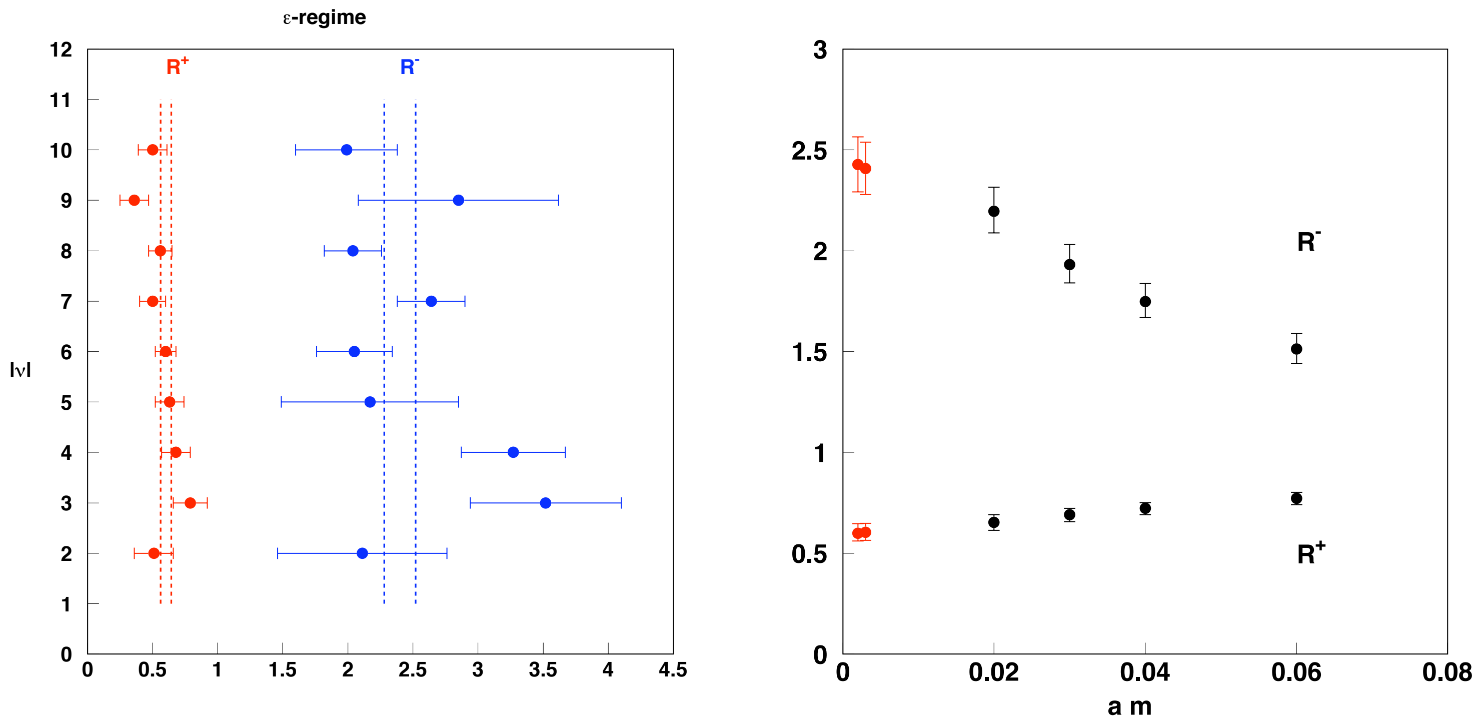
Neuberger 1997

We are in the quenched approximation...

	β	L/a	T/a	n_{low}	$L[\mathrm{fm}]$	m	# cfgs
ϵ-regime	5.8485	16	32	20	2	$m_s/40, m_s/60$	$O(800)$
p-regime	5.8485	16	32	20	2	$m_2/2 - m_s/6$	$O(200)$

R^σ

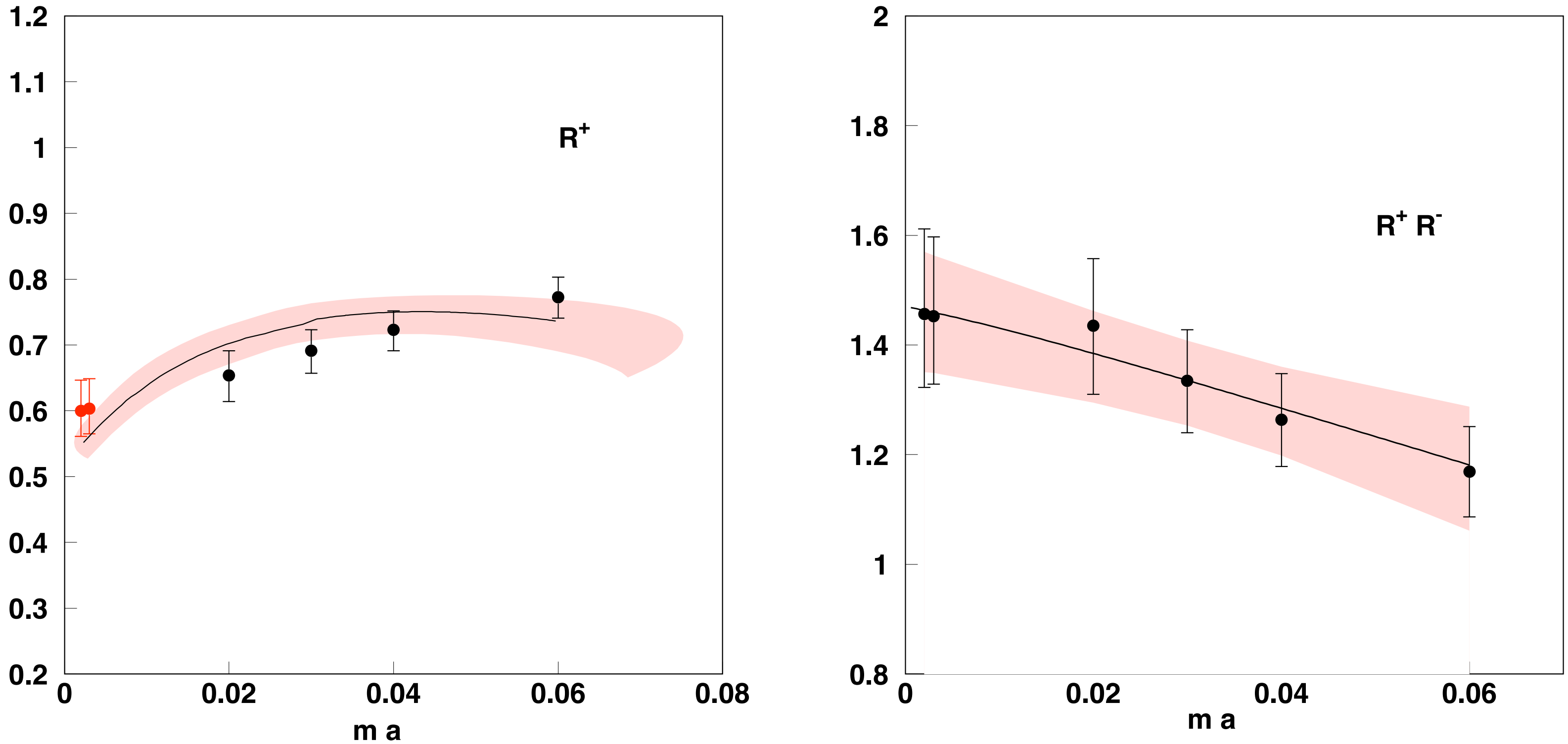
For the masses in the ϵ-regime we bin in $|\nu|$:



The expected features of the $R^\sigma(x_0, y_0)$ in the ϵ-regime: independence on x_0, y_0, m and ν are well reproduced by the data

Fitting strategy

We choose two combinations that have smaller mass corrections: R^{+} and $R^{+}R^{-}$ and fit the NLO ChPT expressions to extract $g^{\pm}$ and $\Lambda^{\pm}$ using all masses



There is some tension in fitting only p or only ϵ regime which could indicate non-negligible higher order corrections, we include a systematic error to account for this

Non-perturbative renormalization factors

Matching $R^{\pm}$ to tm-Wilson fermions at some reference pion mass where the Z factors are known non-perturbatively at M_K

$$\frac{\widehat{Z}^{+}_{\rm ov}}{Z^{2}_{\rm A;ov}}(g_0) \equiv \frac{\widehat{B}_K}{B^{\rm ov}_K(g_0)} = \frac{\widehat{B}_K}{B^{\rm tm}_K(g_0')} \cdot \frac{B^{\rm tm}_K(g_0')}{B^{\rm ov}_K(g_0)} = \left[\lim_{g_0'\to 0} \frac{\widehat{Z}^{+}_{\rm tm}(g_0')}{Z^{2}_{\rm A;tm}(g_0')} \cdot B^{\rm tm}_K(g_0')\right] \cdot \frac{1}{B^{\rm ov}_K(g_0)}$$

At $\beta = 5.8485$:

	bare P.T.	MFI P.T.	N.P.
$\widehat{Z}^{-}/\widehat{Z}^{+}$	0.525	0.582	0.584(62)
$\widehat{Z}^{+}/Z^{2}_{\rm A}$	1.242	1.193	1.15(12)
$\widehat{Z}^{-}/Z^{2}_{\rm A}$	0.657	0.705	0.561(61)

Dimopoulos, Giusti, PH, Palombi, Pena, Vladikas, Wennekers and Wittig, hep-lat/0607028

Results $K \to \pi\pi$ in the $SU(4)$ chiral limit

Giusti, PH, Laine, Pena, Wennekers, Wittig, hep-ph/0607220

	g^+	g^-
This work	$0.51(3)(5)(6)$	$2.6(1)(3)(3)$
"Exp"	~ 0.5	~ 10.4
Large N_c	1	1

- $\Delta I = 3/2$ very close to the "experimental" value
- $\Delta I = 1/2$ amplitude a factor ~ 4 too small
- A significant enhancement $A_0/A_2 \sim 6$: problem with large N_c that cannot be explained by penguins!

Step 2: Decoupling the charm quark

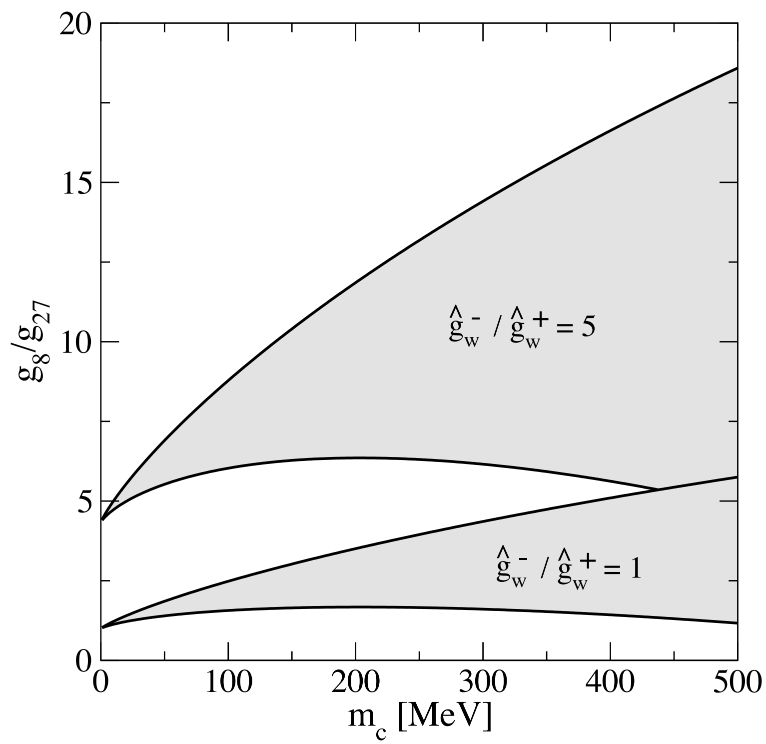
As the charm quark mass is increased, the first interesting observation is that for $\frac{m_u \Sigma}{F^2} \ll \frac{m_c \Sigma}{F^2} \ll (4\pi F)^2$ the charm can be integrated out analytically in the effective theory:

$$g_8(m_c) = \frac{1}{2}\left[\frac{1}{5}g^+\left(1 + 15\frac{M_c^2}{(4\pi F)^2}\ln\frac{\Lambda_\chi}{M_c}\right) + g^-\left(1 + 3\frac{M_c^2}{(4\pi F)^2}\ln\frac{\Lambda_\chi}{M_c}\right)\right]$$

$$g_{27}(m_c) = \frac{3}{5}g^+$$

- Logarithm enhancement of octect!
- Many unknown NLO couplings: bands $1GeV \leq \Lambda_\chi \leq 4GeV$

PH, Laine (2004)



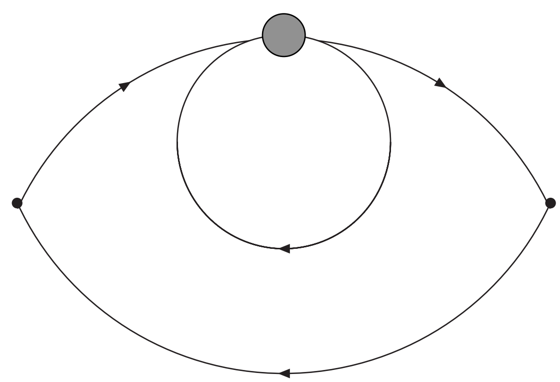
Step 3: Decoupling the charm quark

Giusti, PH, Koma[2], Laine, Necco, Pena, Wennekers, Wittig

Matching the lattice QCD theory with a massive but active charm to a $SU(3)$ theory to get $g_{27}(m_c), g_8(m_c)$

There are new challenges:

- penguin contractions are more noisy but we expect that LMA can tame the fluctuations also there...



- Quenched artifacts on the effective theory side, which are however rather mild in the ϵ-regime

more resuls soon...

Conclusions

- The $\Delta I = 1/2$ remains a big challenge in QCD: large N_c fails quite miserably and lattice QCD is only starting to be able to tackle this difficult problem

- The role of the charm quark mass (penguins) in the enhancement can be addressed in an unambiguous way in lattice QCD, by comparing the results in an $SU(4)$ light flavour theory to the physical $SU(3)$ light flavour theory

- The $SU(4)$ theory is easier and the amplitudes have been computed for the first time near the (quenched) chiral limit: an enhacement of $A_0/A_2 \sim 6$ is found
  - not enough to explain the physical result
  - already a challenge for large N_c and the standard lore...

Outlook

- Next step: monitor the dependence on m_c as it is increased

- The relevance of final state interactions could eventually be addressed through a direct computation of $K \to \pi\pi$

- Of course we would like to go unquenched... JLQCD presented in Lattice 06 remarkable progress towards large scale dynamical overlap fermions simulations!

hep-lat/0607020, Fukaya *et al*