An Effective Higgsless Model

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Outline

1) Motivation

2) Precision Electroweak

3) Generation Mixing

4) Top Quark

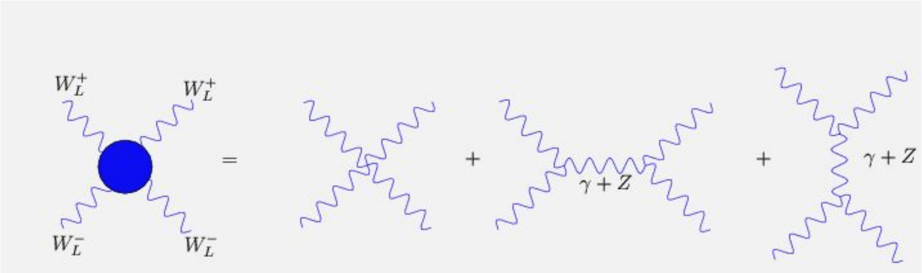
5) Conclusions

MOTIVATION

Motivation

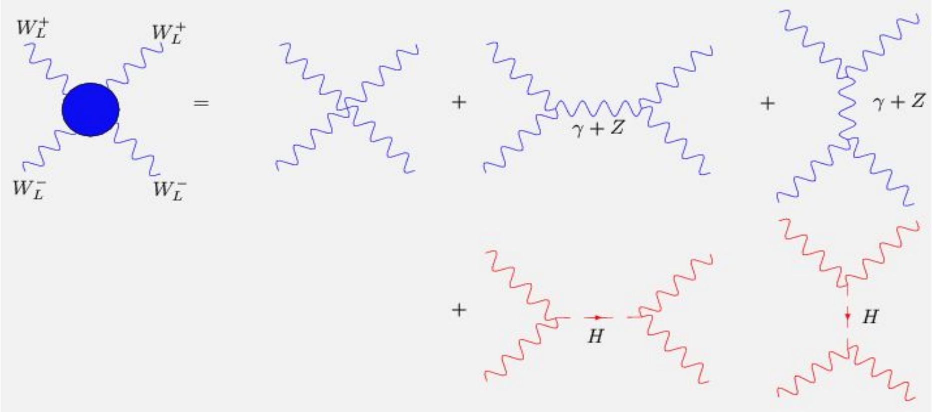
- The Standard Model with known particles only violates unitarity at the electroweak symmetry breaking scale.

- Example: $W_L^+ W_L^- \to W_L^+ W_L^-$
  Violation at $\sqrt{s} \simeq 1.6$ TeV.



- There must be new particles in the 10^2GeV range, which either restore unitarity or delay unitarity violation.

- Example: The SM Higgs boson. (Lee, Quigg, Thacker, 1977)



Motivation

• Is there any alternative to scalar particles as a tool to unitarize amplitudes ?

• YES! 5D gauge theories on an interval $\longrightarrow$ exchange of KK modes cancels the divergent behavior. (Chivukula, Dicus, He, 2001 - Csaki, Grojean, Murayama, Pilo, Terning, 2003)

• To be realistic, a 5D gauge theory must:

(I) Break the electroweak symmetry to E&M.

(II) Reproduce the SM gauge boson mass spectrum.

(III) Have the first KK excitation in the 10^2 GeV range.

Motivation

- Simplest choice: $SU(2)_{brane} \times SU(2)_{bulk} \times U(1)_{brane}$

(Foadi, Gopalakrishna, Schmidt, 2003)

$$\mathcal{S}_{gauge} = \int d^4x \int_0^{\pi R} dy \left[-\frac{1}{4\hat{g}_5^2 \pi R} W^{aMN} W^a_{MN} \right.$$
$$\left. -\delta(y) \frac{1}{4g^2} W^{a\mu\nu} W^a_{\mu\nu} - \delta(\pi R - y) \frac{1}{4g'^2} W^{3\mu\nu} W^3_{\mu\nu} \right]$$

$W^1_\mu = W^2_\mu = 0$ at $y = \pi R$

$$g^2,\ g'^2 \ll \hat{g}_5^2$$

Motivation

- KK expansion of the 5D fields:

massive 4D
charged bosons

$$W_\mu^\pm(x,y) = \sum_{n=0}^{\infty} f_n(y) W_{n\mu}^\pm(x)$$

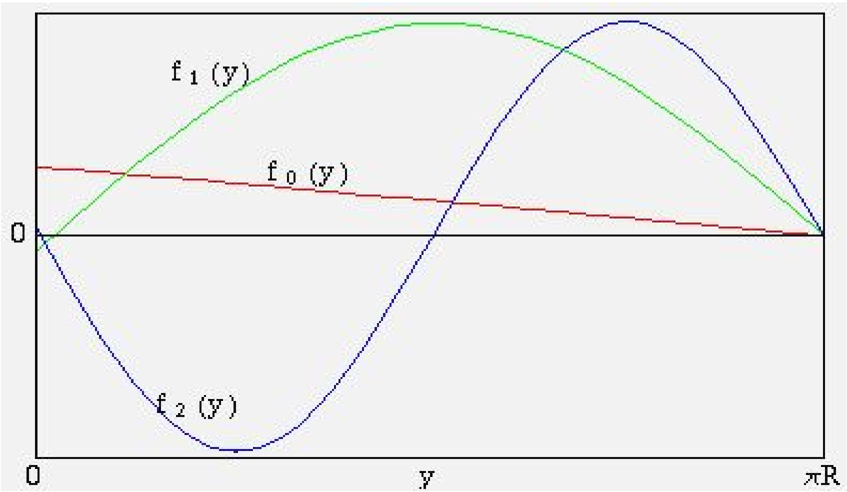
massive 4D
neutral bosons

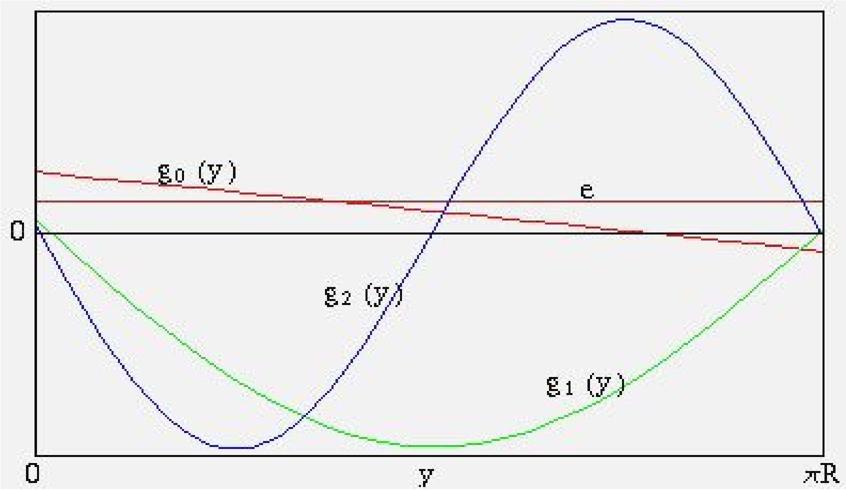
$$W_\mu^3(x,y) = eA_\mu(x) + \sum_{n=0}^{\infty} g_n(y) Z_{n\mu}(x)$$

$$W_0 \equiv W, \;\; Z_0 \equiv Z$$

Motivation

- Gauge boson wavefunctions:





- Photon wavefunction flat
  $\longrightarrow$ massless particle.

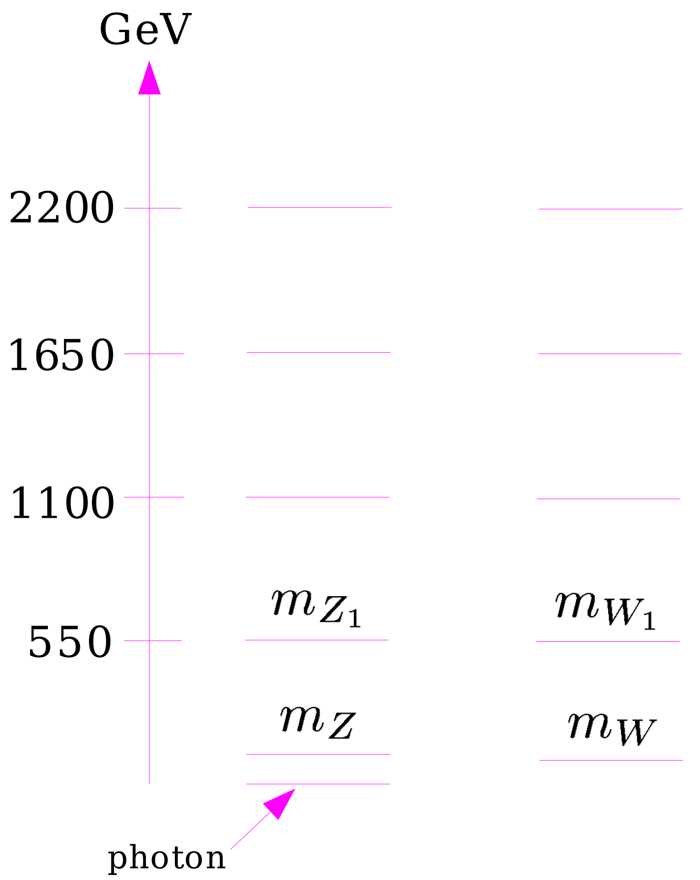
- $W^{\pm}$ boson and Z boson normalization suppressed.

$$\longrightarrow \quad m_W^2,\ m_Z^2 \ll \frac{1}{R^2}$$

- SM bosons mainly “brane fields”, KK excitations mainly “bulk fields”.

Motivation

- Mass spectrum:



$$1/R = 550 \text{ GeV}$$

$$m_W^2 \simeq \frac{\lambda^2}{\pi^2 R^2} \qquad m_Z^2 \simeq \frac{\lambda^2 + \lambda'^2}{\pi^2 R^2}$$

$$m_{W_n} \simeq m_{Z_n} \simeq \frac{n}{R}$$

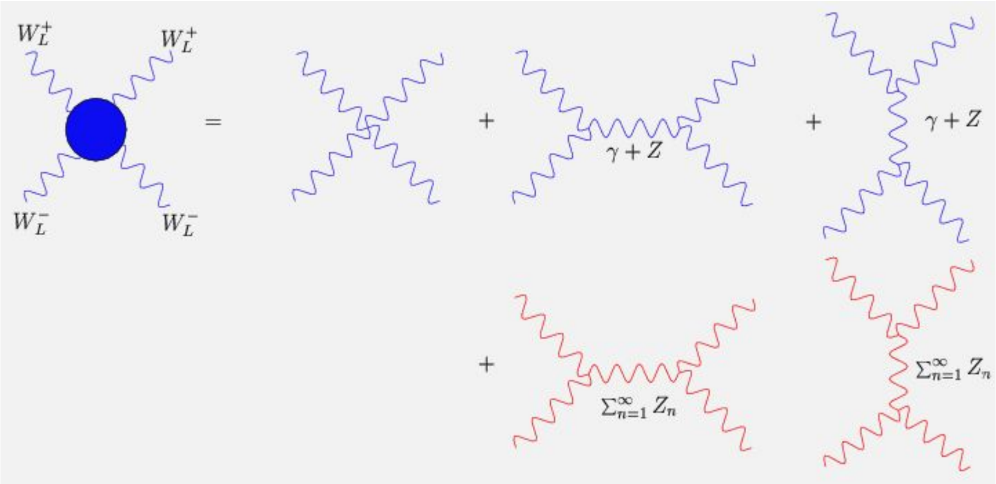
$$\lambda \equiv g/\hat{g}_5 \qquad \lambda' \equiv g'/\hat{g}_5$$

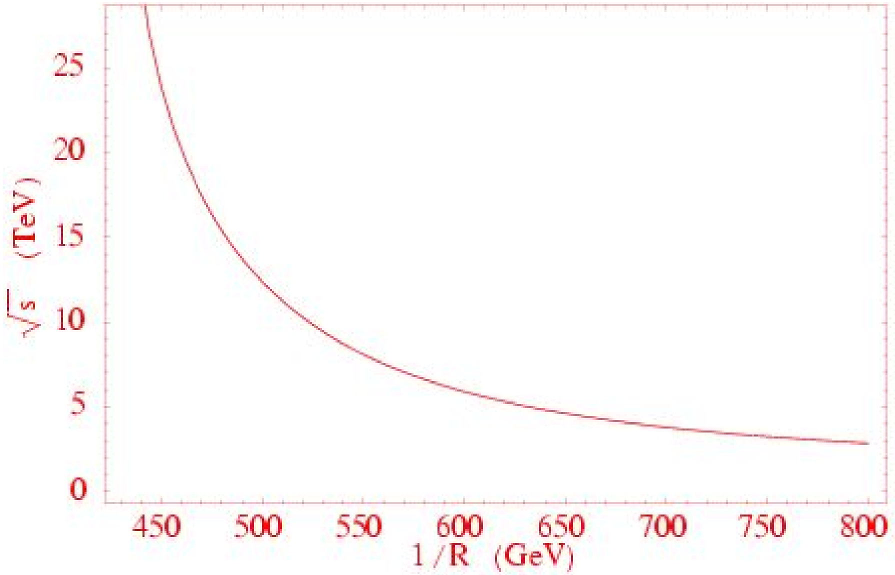
$$\lambda^2,\ \lambda'^2 \ll 1$$

strength of SM gauge boson
leakage into the bulk

Motivation

- Back to unitarity of $W_L^+ W_L^- \to W_L^+ W_L^-$:



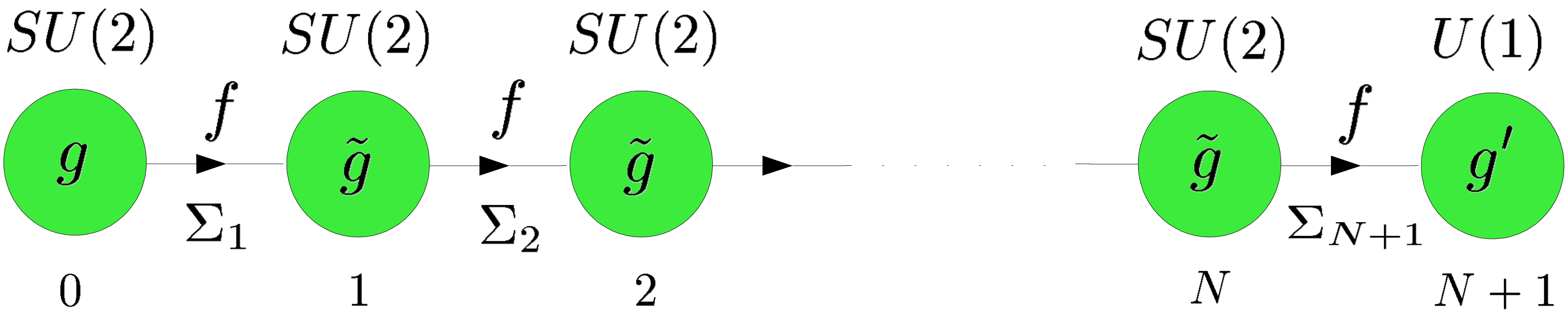


- Unitarity violation only delayed: as $\sqrt{s}$ grows, the # of exchanged KK modes is allowed to increase.

(Chivukula, Dicus, He, 2001)

Motivation

- Deconstruction: (Cheng, Hill, Pokorsky, Whang, 2001 - Arkani-Hamed, Cohen, Georgi, 2001)



$$\tilde{g} = \hat{g}_5 \sqrt{N+1} \qquad f = \frac{2}{\hat{g}_5\ \pi R}\sqrt{N+1}$$

$$\Sigma_k(x) = \exp\left[\frac{2i}{f} W_5^a\left(x, (k+\frac{1}{2})\frac{\pi R}{N+1}\right) T^a\right] \quad k = 1 \ldots N+1$$

Motivation

• Advantages: (i) 4D better known than 5D. (ii) More freedom for model building.

• Disadvantages: (i) Non-linearity. (ii) Complicated eigenvalue eqs. and recurrence relations.

• A continuum theory space approach allows to retain (some of) the advanteges while avoiding the disadvantages: eigenvalue eqs. $\longrightarrow$ trascendental eqs., recurrence relations $\longrightarrow$ differential eqs..

• Useful especially for the fermion sector.

• Feel uncomfortable ? Go back to deconstructed !

PRECISION ELECTROWEAK

- Now couple fermions. Simplest set up
  → Brane-Localized Fermions

$$\mathcal{S} = \int d^4x \int_0^{\pi R} dy \left[-\frac{1}{4\hat{g}_5^2 \pi R} W^{aMN} W^a_{MN} + \delta(y) \left(\frac{1}{4g^2} W^{a\mu\nu} W^a_{\mu\nu} + \bar{\psi}_L i\gamma^\mu D_\mu \psi_L \right) \right.$$

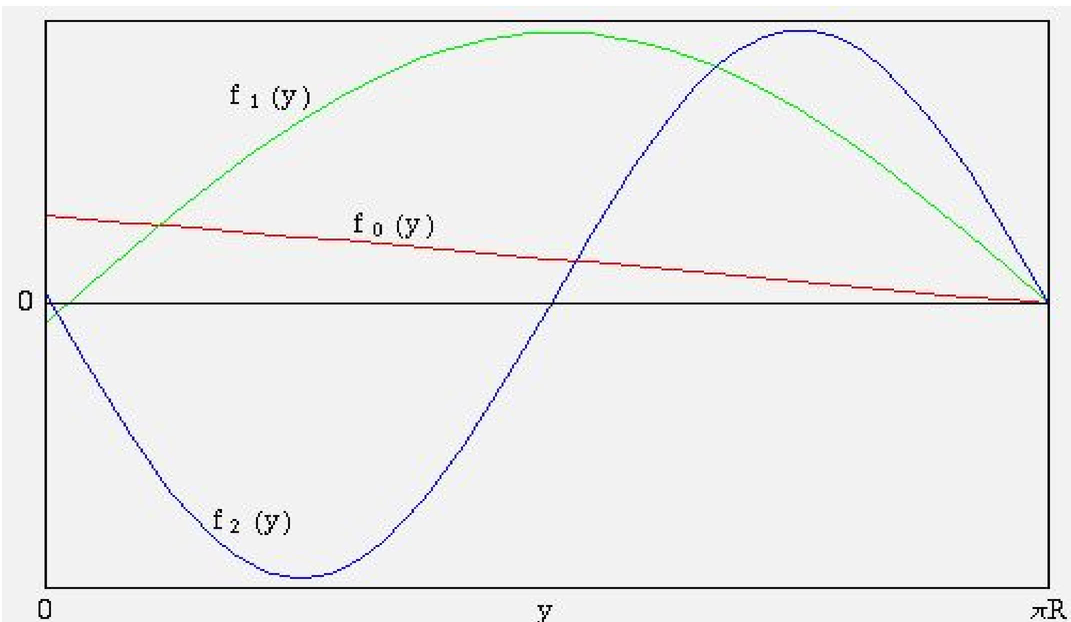
$$\left. + \delta(\pi R - y) \left(\frac{1}{4g'^2} W^3_{\mu\nu} W^{3\mu\nu} + \bar{u}_R i\gamma^\mu D_\mu u_R + \bar{d}_R i\gamma^\mu D_\mu d_R \right) \right]$$

$$\psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad D_\mu = \partial_\mu - iW^a_\mu(y) T^a - iW^3_\mu(\pi R) Y_L$$

- Mass term from Wilson line, and coupling of LH fields to $y = 0$ and $y = \pi R$: not allowed in 5D, legitimate in theory space.

Precision Electroweak

- Charged Sector



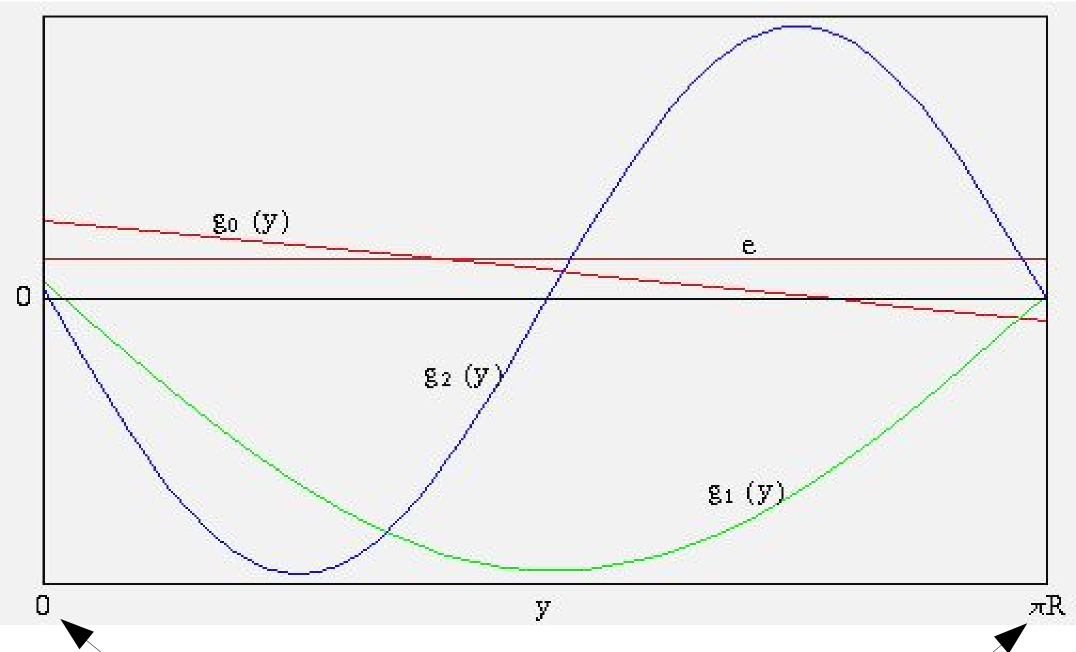
fermions only here !

- Coupling to $W^{\pm} \sim g$

- Coupling to $W_n^{\pm} \sim \lambda g$
  ⟶ No four-fermion operators from the charged sector, at the order of λ^2.

- No anomalous RH couplings in the charged sector.

- Neutral Sector



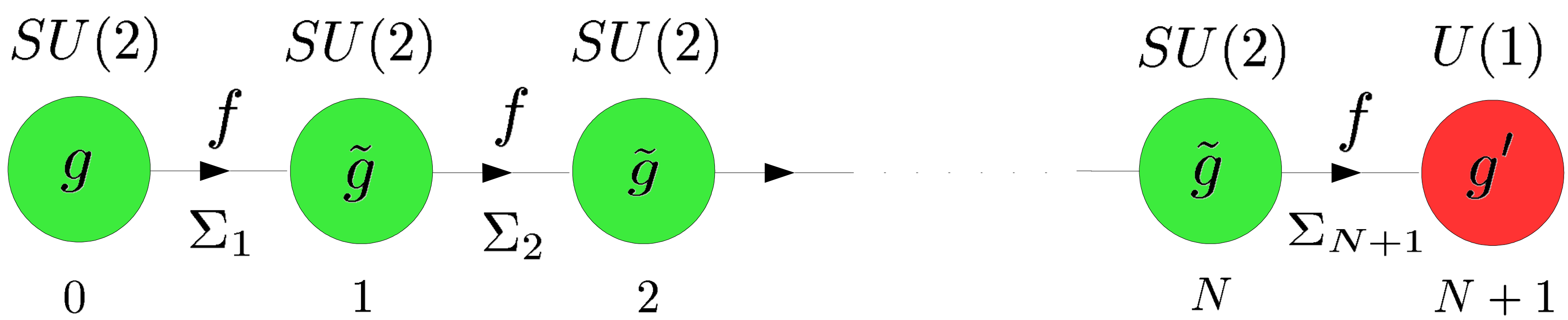
fermions only here !

- Coupling to $Z \sim g, g'$
- Coupling to $Z_n \sim \lambda g$
  $\longrightarrow$ No four-fermion operators from the neutral sector, at the order of λ^2.
- u_R and d_R coupled to Y_R not T^3 : no anomalous RH couplings in the neutral sector.

- Photon profile flat $\longrightarrow$ universal coupling (due to $U(1)_Q$).

Precision Electroweak

- Also, dimension-5 operators suppressed.
- Therefore, to order λ^2, the contribution of the KK towers on the low-energy effective Lagrangian is entirely parametrized by S, T, U.
- Custodial symmetry $\longrightarrow$ $T = U = 0$ to order λ^2.



- Take α, m_Z, m_W as fundamental input observables.
  → Independent of fermion profiles.

- Define $c \equiv m_W / m_Z, \ s \equiv \sqrt{1 - c^2}$. With $T = U = 0$:

$$g^{CC} = c \cdot g_3^{NC} = \frac{e}{s} \left[1 + \frac{\alpha S}{4 s^2} \right]$$

- Direct calculation:

$$g^{CC} = c \cdot g_3^{NC} = \frac{e}{s} \left[1 + \frac{\lambda^2}{6} + O(\lambda^4) \right]$$

Precision Electroweak

- Therefore, $S \neq 0$ to order λ^2.

- Unitarity constraint (at $\sqrt{s} = 5$ TeV):

$$\lambda^2 \simeq (m_W \pi R)^2 \gtrsim \frac{m_W^2 \pi^2}{720^2 GeV^2} \simeq 0.12$$

- Precision electroweak constraint:

$$\lambda^2 = \frac{3\alpha S}{2s^2} \simeq 0.053 \cdot S \ll 10^{-1}$$

- Brane-localized fermions $\longrightarrow$ unitarity constraints and precision EW constraints incompatible. (Chivukula, Simmons, He, Kurachi, Tanabashi, 2004)

Precision Electroweak

- Slightly Delocalized Fermions (Cacciapaglia, Csaki, Grojean Terning, 2004 – Foadi, Gopalakrishna, Schmidt, 2004)

$$\mathcal{S}_{fermion} = \int d^4x \int_0^{\pi R} dy \left[\frac{1}{\pi R} \left(\frac{1}{2} \bar{\psi} i \Gamma^M D_M \psi + h.c. - M \bar{\psi} \psi \right) \right.$$

$$\left. + \delta(y) \frac{1}{t_L^2} \bar{\psi}_L i \gamma^\mu D_\mu \psi_L + \delta(\pi R - y) \left(\frac{1}{t_{u_R}^2} \bar{u}_R i \gamma^\mu D_\mu u_R + \frac{1}{t_{d_R}^2} \bar{d}_R i \gamma^\mu D_\mu d_R \right) \right]$$

$$\psi = \psi_L + \psi_R$$

$$t_L^2, \; t_{u_R}^2, \; t_{d_R}^2 \ll 1$$

$$\psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \psi_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

strength of SM fermions leakage into the bulk

$$D_M = \partial_M - i W_M^a(y) T^a - i W_M^3(\pi R) Y_L$$

- Still non-local covariant derivative, but no need for Wilson line.

- KK expansion of the fermion fields:

$$u_L(x,y) = \sum_{n=0}^{\infty} \alpha_n^{(u)}(y)\, u_{n_L}(x)$$

$$u_R(x,y) = \sum_{n=0}^{\infty} \beta_n^{(u)}(y)\, u_{n_R}(x)$$

$$\begin{pmatrix} u_{n_L} \\ u_{n_R} \end{pmatrix}$$

4D massive Dirac fermion

- The lowest KK mode, $n = 0$, is a SM fermion.

• More overlap of light fermion wavefunctions with heavy boson wavefunctions ($\sim 1/\lambda^2$), but less probability of a light fermion to be in the bulk ($\sim t_L^2$).

⟶ Four-fermion operators still suppressed.

• Anomalous RH couplings go to zero as $t_{u_R}^2, t_{d_R}^2 \to 0$.

⟶ Very small amomalous RH couplings for light fermions.

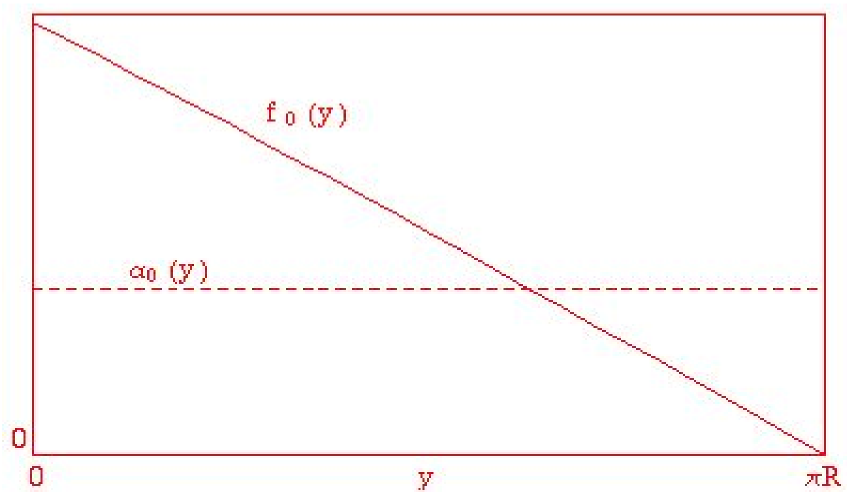
• Therefore, the low-energy effective Lagrangian is still parametrized entirely by S, T, U.

Precision Electroweak

- Still $T = U = 0$, to order λ^2 (custodial symmetry not affected by delocalization).

- As before, let α, m_Z, m_W be the input observables.

$$\longrightarrow \quad g^{CC} = c \cdot g_3^{NC} = \frac{e}{s}\left[1 + \frac{\alpha S}{4s^2}\right] \quad \text{to order } \lambda^2$$



- Now there is a bulk contribution as well, which vanishes as $t_L^2 \to 0$.

$\longrightarrow$ Negative contribution to S proportional to t_L^2 .

- It is possible to tune S to zero if t_L^2 has the same size of λ^2.

- For zero bulk mass, S is set to zero by:

$$t_L^2 = \frac{\lambda^2}{3}$$

- With t_L fixed, the fermion masses are determined by t_{u_R}, t_{d_R}.

Precision Electroweak

- Fermion Masses. For zero bulk mass:

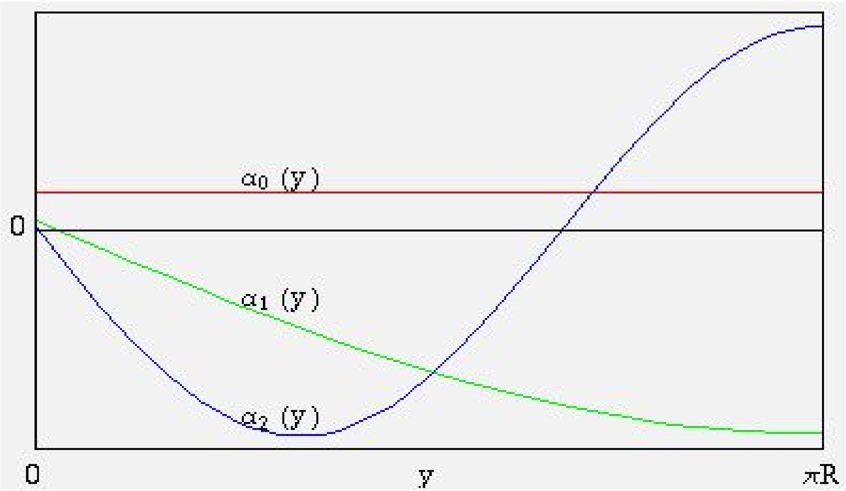
$$m_u \simeq \frac{1}{\pi R} \frac{t_L t_{u_R}}{\sqrt{1+t_{u_R}^2}} \qquad t_L^2 \ll 1$$

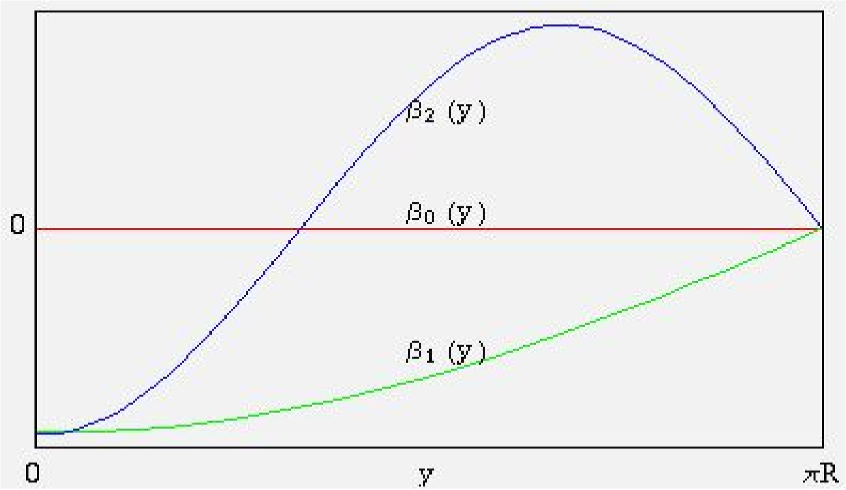
$$m_{u_n} \simeq \left(n - \frac{1}{2}\right) \frac{1}{R} \qquad n = 1, 2, \ldots.$$

- Since $t_L \sim \lambda \sim 10^{-1}$, t_{u_R}, t_{d_R} must be very small, for light fermions.

Precision Electroweak

- Fermion wavefunctions:





- Light modes nearly flat
  → small mass.

- Wavefunctions of light fermions suppressed by large normalization factor. RH wavefunctions highly suppressed.

- LH light fermions mainly “brane fields”, LH KK modes Mainly “bulk fields”

- RH light fermions mainly “brane fields”, RH KK modes Mainly “bulk fields”

GENERATION MIXING

Generation Mixing

- In the Standard Model:

$$\mathcal{L}_{quark} = \bar{Q}^i_L i\gamma^\mu D_\mu Q^i_L + \bar{u}^i_R i\gamma^\mu D_\mu u^i_R + \bar{d}^i_R i\gamma^\mu D_\mu d^i_R$$
$$- \left(Q^i_L \lambda^D_{i,j} \phi d^j_R + Q^i_L \lambda^U_{i,j} \tilde{\phi} u^j_R + \ h.c. \right)$$

(I) Chiral fermions $\longrightarrow$ LH and RH fields can be transformed independently.

(II) Diagonalization of Yukawa terms.

(III) Kinetic terms $\longrightarrow$ $[U(3)]^3$ symmetry.

# of physical parameters $= \underbrace{18}_{\lambda^U} + \underbrace{18}_{\lambda^D} - (\underbrace{9 \times 3}_{[U(3)]^3} - \underbrace{1}_{B}) = 10 = 6 + 4$

masses CKM

Generation Mixing

- In the Higgsless model:

$$\mathcal{S}_{quark} = \int d^4x \int_0^{\pi R} dy \left[\frac{1}{\pi R} \left(\frac{1}{2} \bar{Q}^i i \Gamma^M D_M Q^i + h.c. - M \bar{Q}^i Q^i \right) \right.$$

$$\left. + \delta(y) \frac{1}{t_L^2} \bar{Q}_L^i i \gamma^\mu D_\mu Q_L^i + \delta(\pi R - y) \left(\bar{u}_R^i K_{i,j}^U i \gamma^\mu D_\mu u_R^j + \bar{d}_R^i K_{i,j}^D i \gamma^\mu D_\mu d_R^j \right) \right]$$

(I) Vector-like fermions $\longrightarrow$ rotation of LH and RH fields not independent.

(II) Flavor physics on the $y = \pi R$ brane only.

(III) Diagonalization of $y = \pi R$ brane kinetic terms.

(IV) Bulk and $y = 0$ terms $U(3)$ symmetric.

Generation Mixing

$$\mathcal{S}_{quark} = \int d^4x \int_0^{\pi R} dy \left[\frac{1}{\pi R} \left(\frac{1}{2} \bar{Q}^i i \Gamma^M D_M Q^i + h.c. - M \bar{Q}^i Q^i \right) \right.$$

$$\left. + \delta(y) \frac{1}{t_L^2} \bar{Q}_L^i i \gamma^\mu D_\mu Q_L^i + \delta(\pi R - y) \left(\bar{u}_R^i K_{i,j}^U i \gamma^\mu D_\mu u_R^j + \bar{d}_R^i K_{i,j}^D i \gamma^\mu D_\mu d_R^j \right) \right]$$

\# of physical parameters $= 9 + 9 - (9 - 1) = 10 = 6 + 4$

$K^U \quad K^D \quad U(3) \quad B$

masses CKM

Generation Mixing

- After diagonalization and redefinitions:

$$\mathcal{S}_{quark} = \int d^4x \int_0^{\pi R} dy \left[\frac{1}{\pi R} \left(\frac{1}{2} \bar{Q}^i i \Gamma^M D_M Q^i + h.c. - M \bar{Q}^i Q^i \right) \right.$$

$$\left. + \delta(y) \frac{1}{t_L^2} \bar{Q}_L^i i \gamma^\mu D_\mu Q_L^i + \delta(\pi R - y) \left(\frac{1}{t_{u_R^i}^2} \bar{u}_R^i i \gamma^\mu D_\mu u_R^i + \frac{1}{t_{d_R^i}^2} \bar{d}_R^i i \gamma^\mu D_\mu d_R^i \right) \right]$$

$$Q^i = Q_L^i + Q_R^i \qquad Q_L^i = \begin{pmatrix} u_L^i \\ V^{ij} d_L^j \end{pmatrix}, Q_R^i = \begin{pmatrix} u_R^i \\ V^{ij} d_R^j \end{pmatrix}$$

- No flavor-changing neutral currents.
- Easy implementation of the lepton sector.

TOP QUARK

Top Quark

- For zero bulk mass, the mass of the lightest fermion is

$$m_u \simeq \frac{1}{\pi R} \frac{t_L t_{u_R}}{\sqrt{1+t_{u_R}^2}} \qquad t_L^2 \ll 1$$

- The cancellation of the S parameter requires

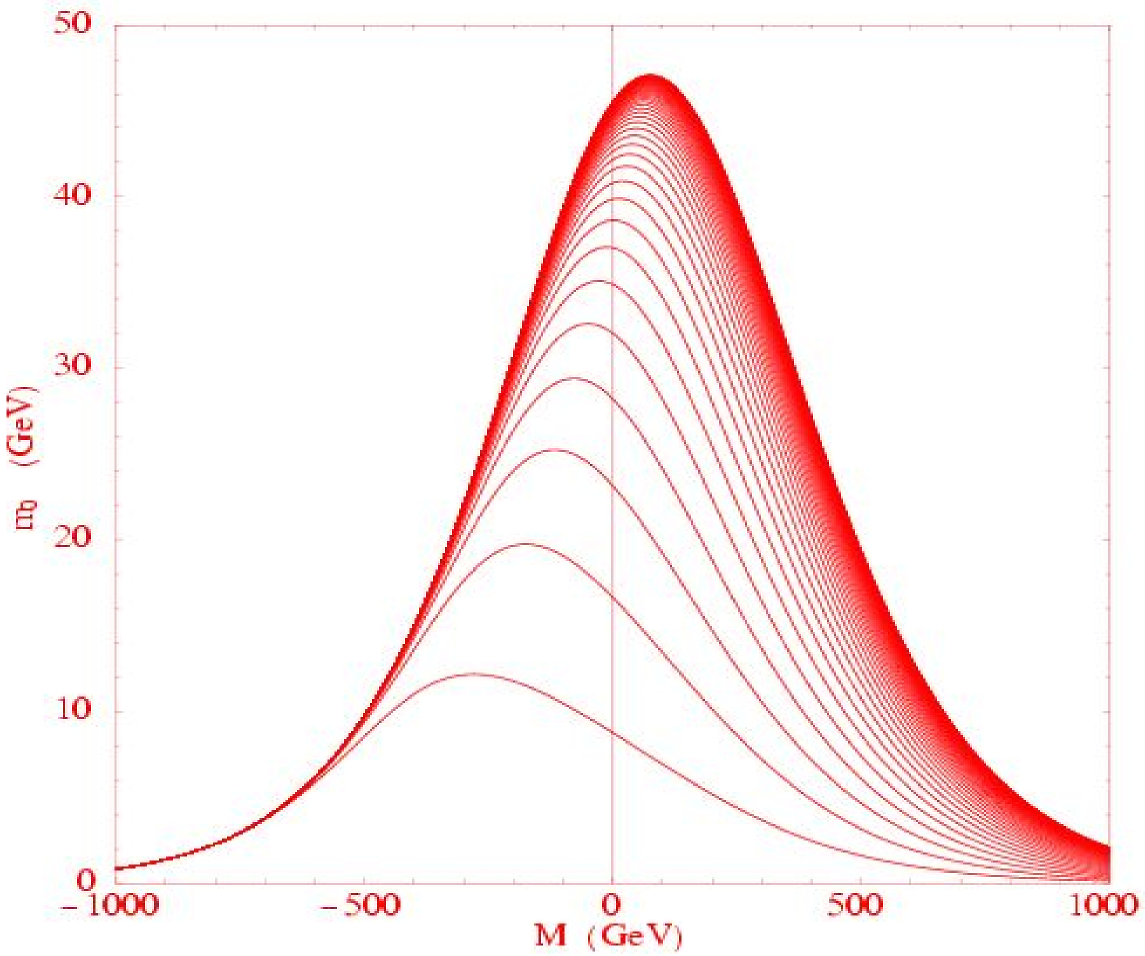
$$t_L^2 = \frac{\lambda^2}{3}$$

$$\longrightarrow \quad m_u \simeq \frac{\lambda}{\sqrt{3}\pi R} \frac{t_{u_R}}{\sqrt{1+t_{u_R}^2}} \leq \frac{\lambda}{\sqrt{3}\pi R} = \frac{m_W}{\sqrt{3}}$$

Top Quark

$$1/R = 500 \text{ GeV}$$

- Top quark mass unachievable.
- Allowing a non-zero bulk mass doesn't help much.
- Value of $1/R$ tightly constrained by unitarity.



Top Quark

- However we have not taken the most general $SU(2)_{brane} \times SU(2)_{bulk} \times U(1)_{brane}$ model from theory space.

- We can introduce a Lorentz-5D breaking term. (Foadi, Schmidt, 2005 – Panico, Serone, Wulzer, 2005)

$$\mathcal{S}_{fermion} = \int d^4x \int_0^{\pi R} dy \left[\frac{1}{\pi R} \left(\bar{\psi} i \Gamma^\mu D_\mu \psi + \kappa \left(\frac{1}{2} \bar{\psi} i \Gamma^5 D_5 \psi + h.c. \right) - M \bar{\psi} \psi \right) \right.$$
$$\left. + \delta(y) \frac{1}{t_L^2} \bar{\psi}_L i \gamma^\mu D_\mu \psi_L + \delta(\pi R - y) \left(\frac{1}{t_{u_R}^2} \bar{u}_R i \gamma^\mu D_\mu u_R + \frac{1}{t_{d_R}^2} \bar{d}_R i \gamma^\mu D_\mu d_R \right) \right]$$

- Now we have two compactification scales:

gauge sector $\longrightarrow \dfrac{1}{R_g} \equiv \dfrac{1}{R}$

fermion sector $\longrightarrow \dfrac{1}{R_f} \equiv \dfrac{\kappa}{R}$

- Fermion masses:

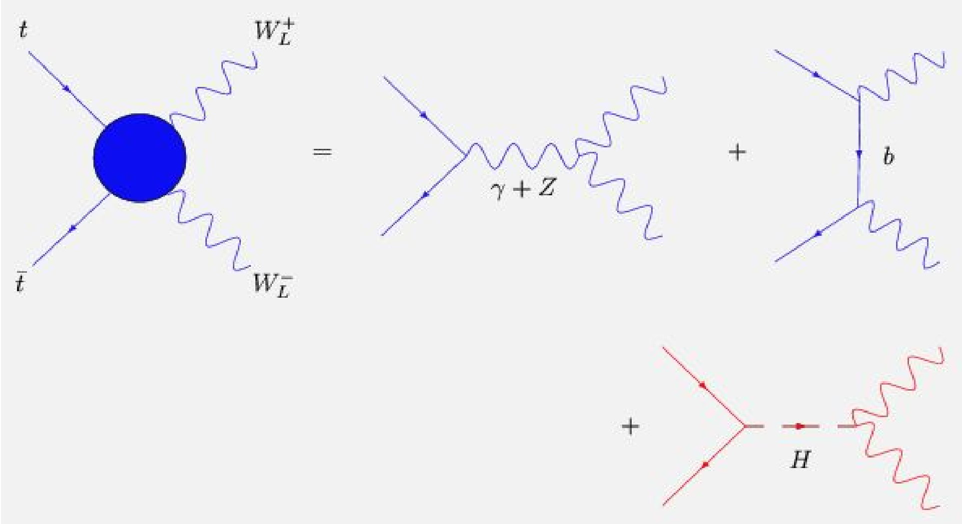
$$m_u \simeq \frac{1}{\pi R_f} \frac{t_L t_{u_R}}{\sqrt{1+t_{u_R}^2}} \qquad t_L^2 \ll 1$$

$$m_{u_n} \simeq \left(n - \frac{1}{2}\right) \frac{1}{R_f} \qquad n = 1, 2, \ldots.$$

- Now $1/R_f$ is no longer bounded by unitarity of $W_L^+ W_L^- \to W_L^+ W_L^-$.

- Bound on $1/R_f$ comes from unitarity of $t\bar{t} \to W_L^+ W_L^-$.

Top Quark



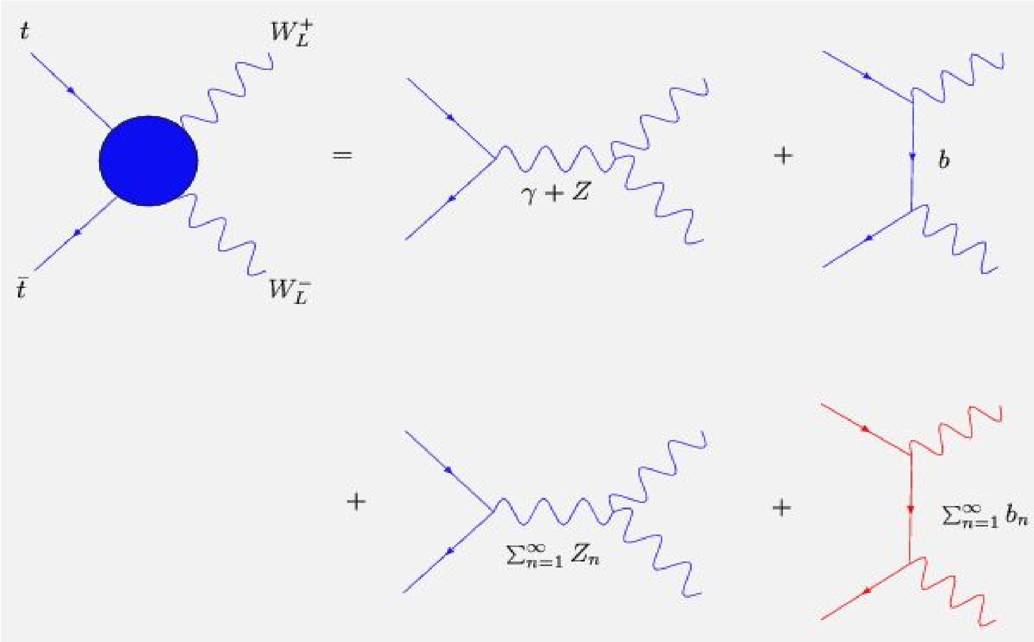
Standard Model

- Opposite helicities $\longrightarrow$ γ and Z produce a quadratically divergent $J = 1$ amplitude unitarized by the b-quark. $\longrightarrow$ No role played by the Higgs boson $\longrightarrow$ no bound expected in the Higgsless model.

- Same helicities $\longrightarrow$ b produces a linearly divergent $J = 0$ amplitude unitarized by the Higgs boson. (Lee, Quigg, Thacker, 1977)

- Higgsless model $\longrightarrow$ unitarity expected from the b_n's.

Top Quark



Higgsless Model

- As expected, no bounds on $1/R_f$ from the $J=1$ amplitude.

- The b_n's exchange diagrams exactly cancel the linearly growing term.

- There is an upper bound on $1/R_f$ due to the increase of the # of KK modes contributing, as the energy grows.

- The bound on $1/R_f$ is expected to be weaker than the bound on $1/R_g$, because the growth is only linear.

Top Quark

- There are also lower bounds on $1/R_f$.
- From the top mass:

$$m_t \simeq \frac{1}{\pi R_f} \frac{t_L t_{t_R}}{\sqrt{1+t_{t_R}^2}} \leq \frac{t_L}{\pi R_f} = \frac{m_W}{\sqrt{3}} \frac{R_g}{R_f}$$

$$\longrightarrow \quad \frac{1}{R_f} \geq \sqrt{3} \frac{m_t}{m_W} \frac{1}{R_g}$$

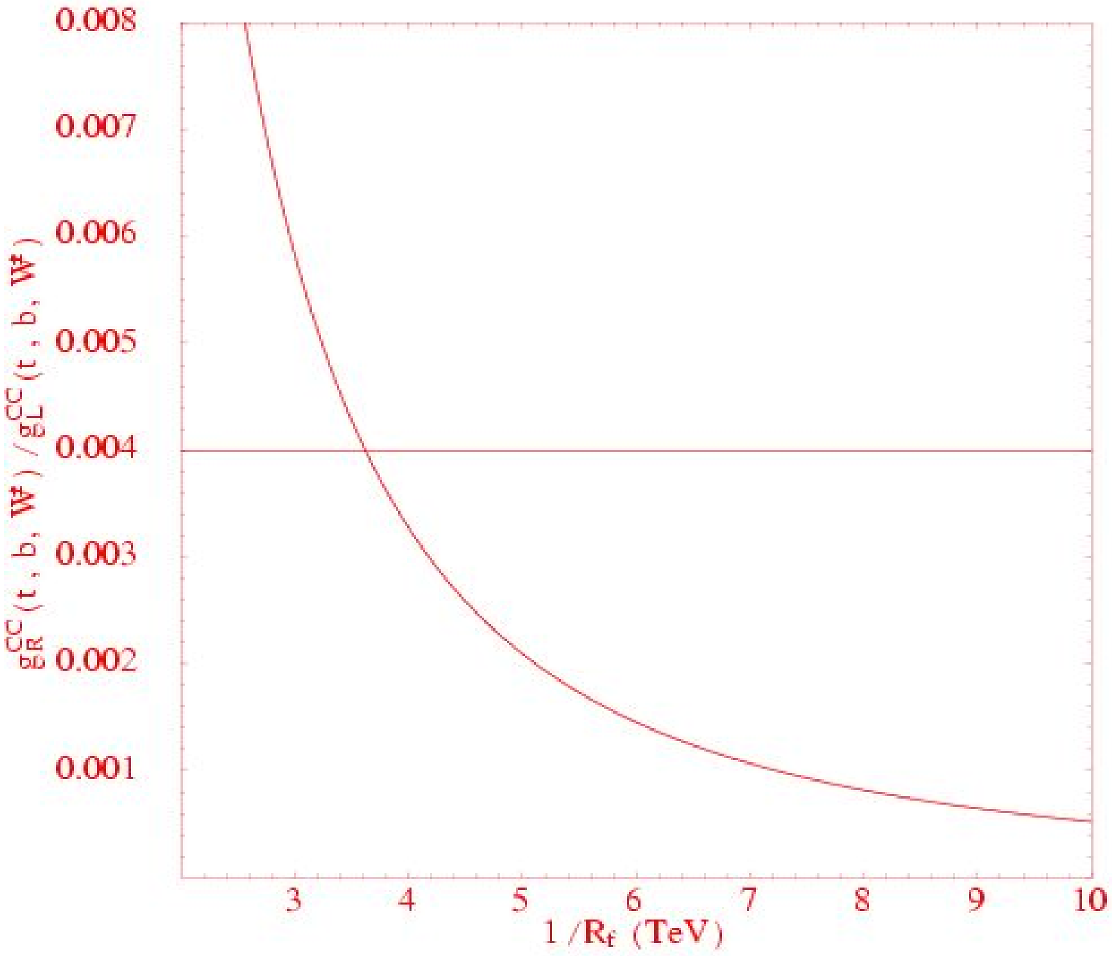
Top Quark

$$1/R_g = 550 \text{ GeV}$$

- From RH tbW .

$$\frac{g_R^{CC}(tbW)}{g_L^{CC}(tbW)} \simeq \frac{t_{t_R} t_{b_R}}{2\sqrt{1+t_{t_R}^2}}$$

$$\simeq \frac{3}{2}\frac{m_b m_t}{m_W^2}\frac{R_f^2}{R_g^2}$$



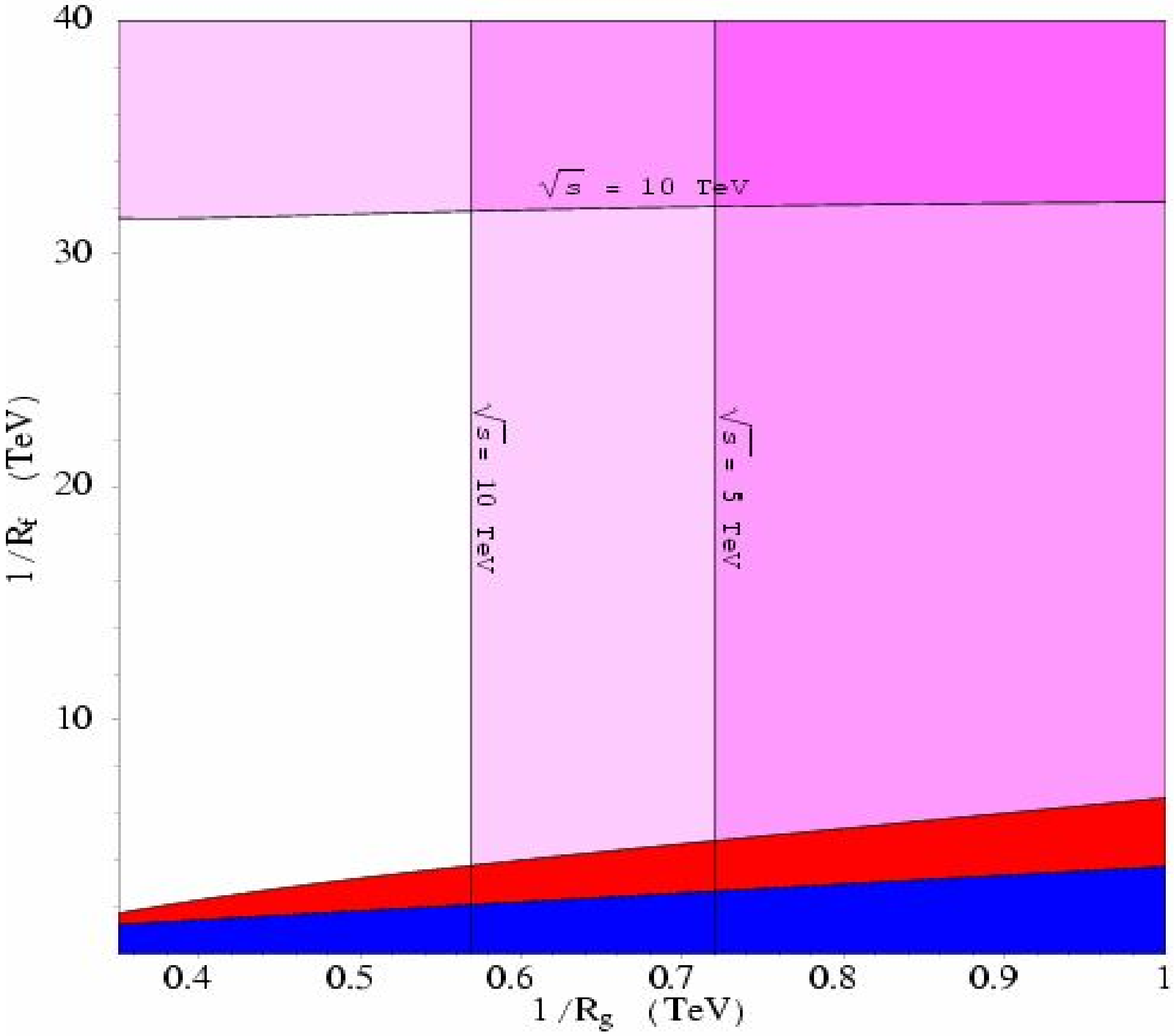
- Example: $b \to s\ \gamma$

(Larios, Perez, Yuan, 1999)

Top Quark

- Stronger bounds on $1/R_f$ from the one-loop correction to T are possible.

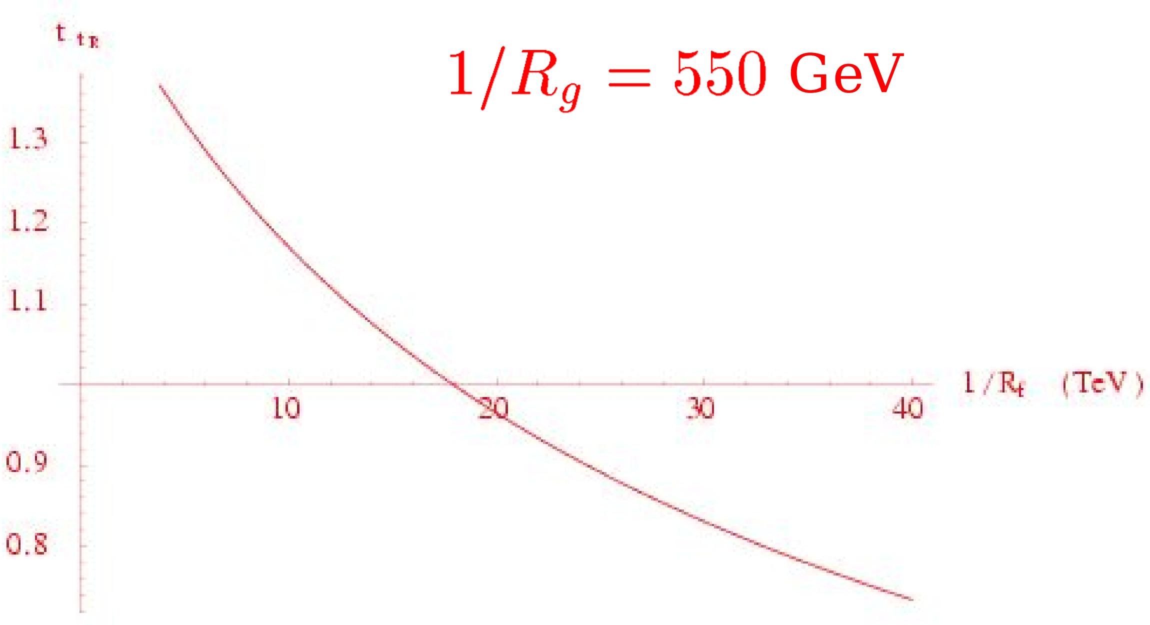
- Lower bounds on $1/R_g$ from direct search lowered by delocalization and numerical factors.



Top Quark

- The amount of RH field leakage into the bulk is very small except for the top quark.

- However t_{t_R} decreases as $\kappa \equiv R_g/R_f$ grows.



- $\kappa = 4 \rightarrow \;\; t_{t_R} \sim 1, \;\; t_{b_R} \sim 10^{-2}, \;\; t_{e_R} \sim 10^{-6}, \;\; t_{\nu_R} \sim 10^{-12}$

- No significant discrepancy with the SM tree level prediction for $Z \rightarrow b\bar{b}$.

CONCLUSIONS

- We presented an effective Higgsless theory which:

  (I) Satisfies the unitarity bounds imposed by longitudinal gauge boson scattering.

  (II) Satisfies the constraints imposed by electroweak precision data, at tree level.

  (III) Accommodates a heavy top quark.

  (IV) Satisfies the constraints on anomalous right-handed couplings.

Conclusions

(V) Allows generation mixings without introducing flavor-changing neutral currents.

- What to do next:

(I) Refine the analysis on precision electroweak constraints, by including loop effects.

(II) LHC phenomenology.

(III) Verify the stability of the model under loop corrections.