Constructing 5D Orbifold GUTs from Heterotic Strings

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In 2007, the LHC turns on; hopefully opening up a new age of discovery. String theory is by far the best candidate for a self-consistent quantum theory of gravity. Moreover, it provides a framework for a true theory of everything.

Estimates suggest there are more than 10^{300} string vacua. The string vacuum fixes the gauge group, the number of families, fermion masses and supersymmetry breaking. Many have searched for phenomenologically acceptable string vacua.

Yet we do not know whether even one string

Phenomenological Guidelines

1. Preserve gauge coupling unification
2. Low energy SUSY $\Rightarrow$ $M_Z << M_G$
3. Quarks and leptons in 16 of SO(10)
4. Higgs in 10
5. Preserve GUT relations for 3rd family Yukawa
6. "Natural" See-Saw scale $\sim M_G$
7. <u>Use intuition from Orbifold GUTs</u>

Outline

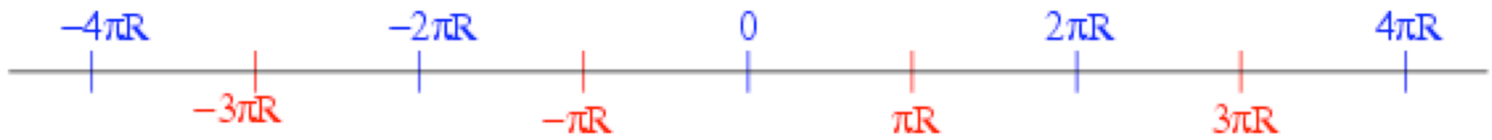
3 Family Orbifold GUT on $M_4 \times S_1/(Z_2 \times Z_2')$

Heterotic string compactified on $[T_2]^3/Z_6$

+ Wilson lines

Consider E_6 example

5D orbifold field theory



- $S^1/\mathbb{Z}_2 = \mathbb{R}/\Omega$

  orbifold space group $\Omega \supset \mathcal{P}, \mathcal{T}$

  $\mathcal{P}$: $y \to -y$ space reversal

  $\mathcal{T}$: $y \to y + 2\pi R$ translation

  $\mathcal{P}' = \mathcal{P}\mathcal{T}$ is also a $\mathbb{Z}_2$ action.

- Two conjugacy classes of fixed points

  $$y = 2n\pi R, \quad (2n+1)\pi R, \qquad n \in \mathbb{Z}$$

- Fundamental domain $[0, \ \pi R]$

  $y = 0$ – fixed point of $\mathcal{P}$;
  $y = \pi R$ – fixed point of $\mathcal{P}'$

- A general 5D field $\phi(x,y)$ transforms as

$$\mathcal{P}: \qquad \phi(x,y) \to \phi(x,-y) = P\phi(x,y)$$

$$\mathcal{T}: \qquad \phi(x,y) \to \phi(x,y+2\pi R) = T\phi(x,y)$$

P, T realize $\mathcal{P}$, $\mathcal{T}$. (T – Wilson line.)

Equivalently, ϕ is characterized by orbifold parities P, P'.

- ϕ has a Kaluza-Klein mode expansion

$$\phi(x,y) = \sum_{m\in\mathbb{Z}} \phi_m(x)\psi_m(y)$$

Depending on the orbifold parities P, P', the wave function $\psi_m(y)$ and KK state masses are

P	P'	Wave functions	Masses
$+$	$+$	$\cos\left(\frac{my}{R}\right)$	$\frac{m}{R}$
$+$	$-$	$\cos\frac{(m+1/2)y}{R}$	$\frac{(m+1/2)}{R}$
$-$	$+$	$\sin\frac{(m+1/2)y}{R}$	$\frac{(m+1/2)}{R}$
$-$	$-$	$\sin\frac{(m+1)y}{R}$	$\frac{(m+1)}{R}$

5D $SO(10)$ orbifold GUT

- $SO(10)$ "bulk"

  Vector multiplet $V(\mathbf{45}) + \Sigma(\mathbf{45})$
  Hyper multiplet $H(\mathbf{10}) + H^c(\mathbf{10})$

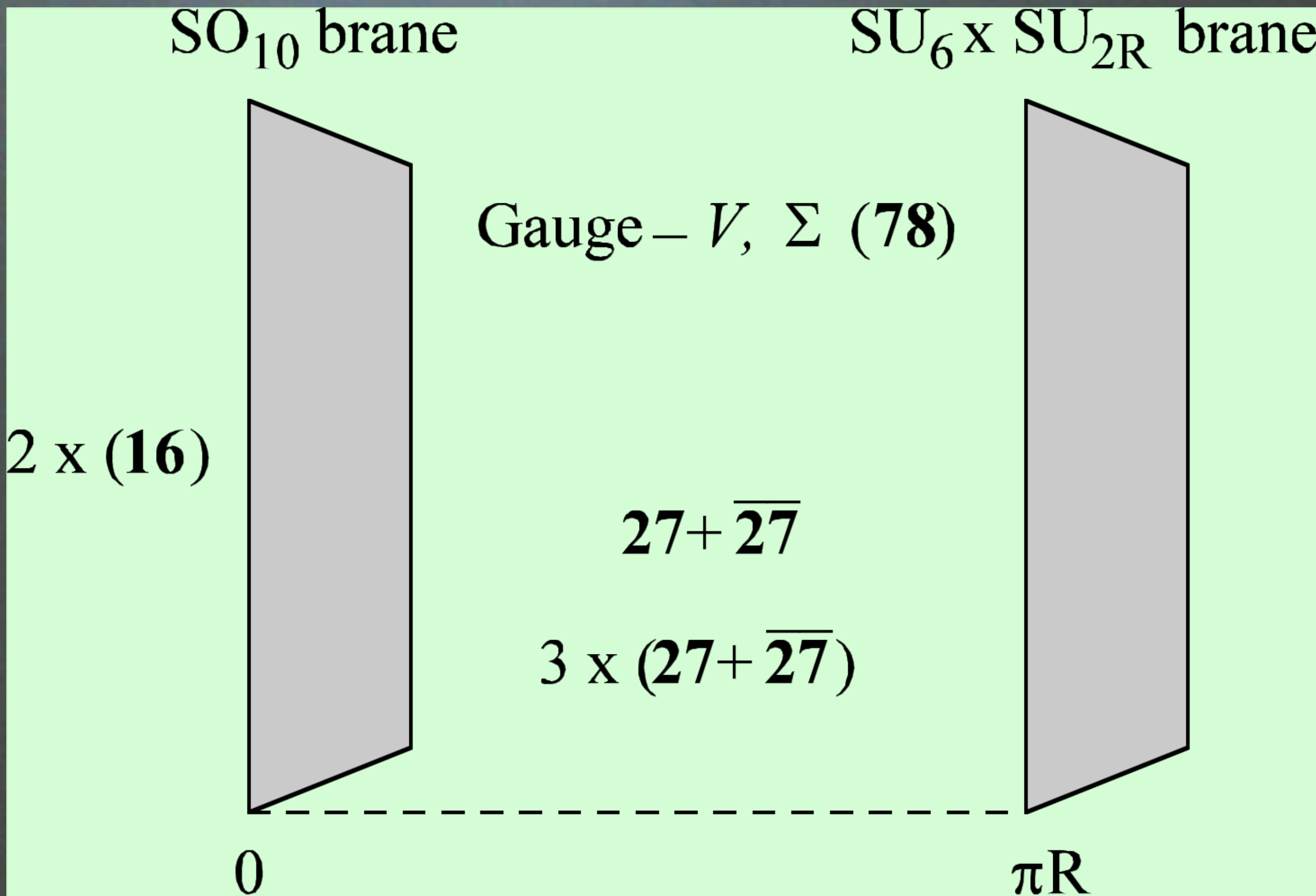
States	P	P'	States	P	P'
$V(\mathbf{15},1,1)$	$+$	$+$	$\Sigma(\mathbf{15},1,1)$	$-$	$-$
$V(1,\mathbf{3},1)$	$+$	$+$	$\Sigma(1,\mathbf{3},1)$	$-$	$-$
$V(1,1,\mathbf{3})$	$+$	$+$	$\Sigma(1,1,\mathbf{3})$	$-$	$-$
$V(\mathbf{6},\mathbf{2},\mathbf{2})$	$+$	$-$	$\Sigma(\mathbf{6},\mathbf{2},\mathbf{2})$	$-$	$+$
$H(\mathbf{6},1,1)$	$+$	$-$	$H^c(\mathbf{6},1,1)$	$-$	$+$
$H(1,\mathbf{2},\mathbf{2})$	$+$	$+$	$H^c(1,\mathbf{2},\mathbf{2})$	$-$	$-$

- $P = +$: complete multiplet of $SO(10)$
  $\rightarrow y = 0$ – $SO(10)$ "brane"

  $P' = +$: complete multiplet of $SU(4) \times SU(2)_L \times SU(2)_R$
  $\rightarrow y = \pi R$ – PS "brane"

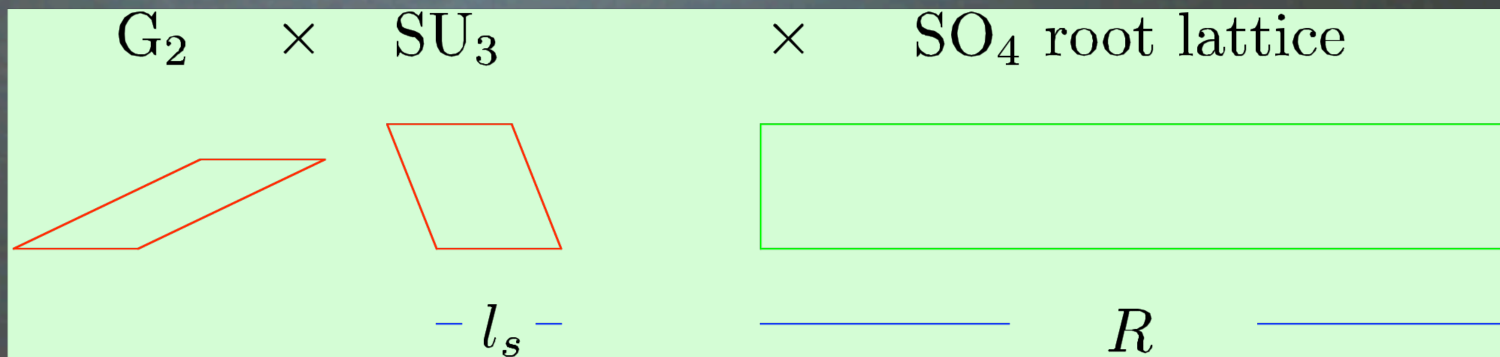
- Solve the doublet-triplet splitting problem.

E_6 Orbifold GUT on $M_4 \times S_1/(Z_2 \times Z_2')$



$E_8 \times E_8$ Heterotic string in 10D

Compactify 6D on $(T^2)^3$



Then mod by $Z_6 = (Z_3 \times Z_2)$

and

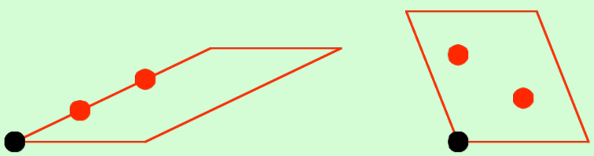
Add Wilson lines

Z_6 embedded in $E_8 \times E_8$ gauge lattice

as shift V_6 : consistent with mod. inv. !

First consider $(T^2)^3/Z_3$ + W_3
[Wilson line in SU_3 torus]

G_2 SU_3 $SO(4)$



$V,\ \Sigma\ \in E(6)$

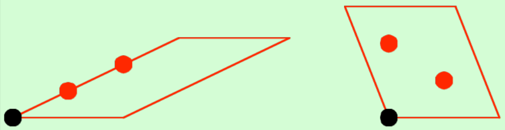
$(\mathbf{27} \oplus \overline{\mathbf{27}})$

$3(\mathbf{27} \oplus \overline{\mathbf{27}})$

Massless modes
From Untwisted sector +
(G_2, SU_3) twisted sector

E_6 GUT
in 5D

Then add Z_2 orbifold + W_2
[in long direction]



$V \in PS \quad (F_3^c + \bar{\chi}^c) \in \Sigma$

$$\boxed{F_3 \in \mathbf{27} + \mathcal{H} \in \overline{\mathbf{27}}}$$

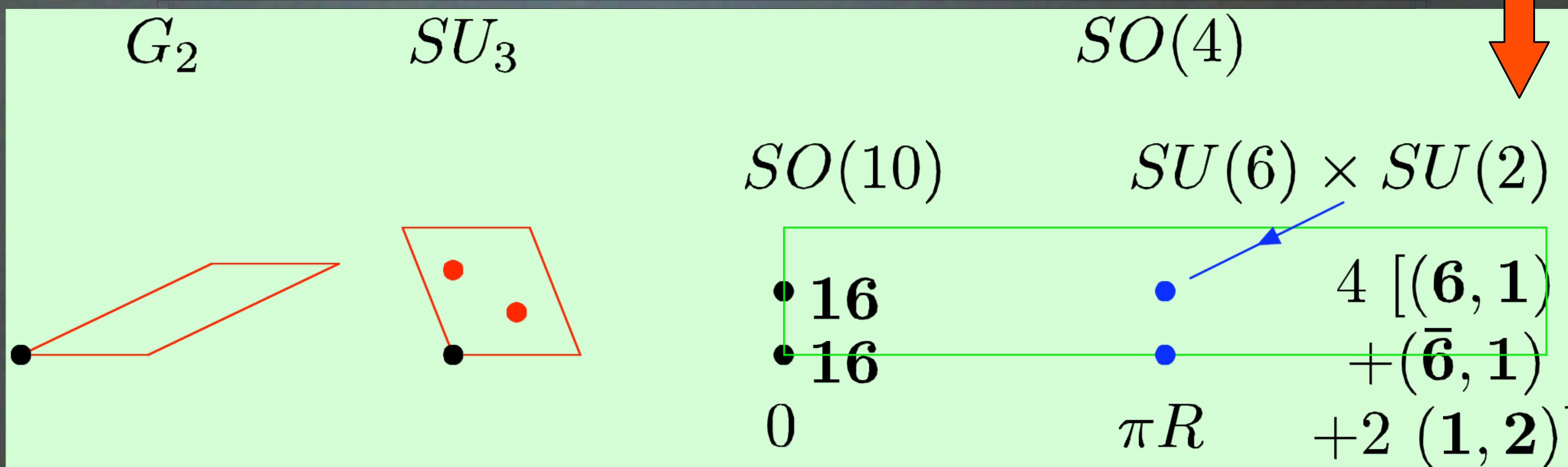
$2(\chi^c) + \bar{\chi}^c + 3C \in 3(\mathbf{27} \oplus \overline{\mathbf{27}})$

P (V_2) P' (V_2+ W_2)

$E_6 \Rightarrow SO(10) \Rightarrow SU(4)xSU(2)_LxSU(2)_R$

F_3 = (4,2,1), $F_3{}^c$ = (4^c,1,2) , H = (1,2,2)

Two families sit on Z_2 fixed points !! Exotics



$\Rightarrow$ D_4 family symmetry !!

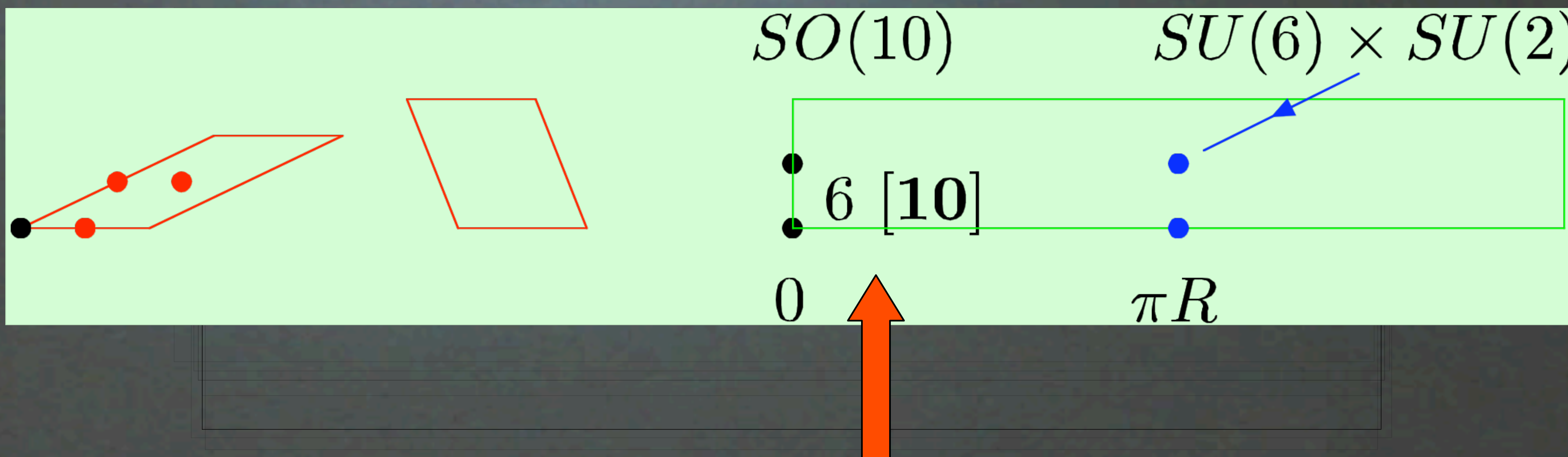
D_4 family symmetry

$$D_4 = \left\{\pm 1, \pm\sigma_1, \pm\sigma_3, \mp i\sigma_2\right\}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} : f_1 \leftrightarrow f_2 \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} : f_2 \leftrightarrow -f_2$$

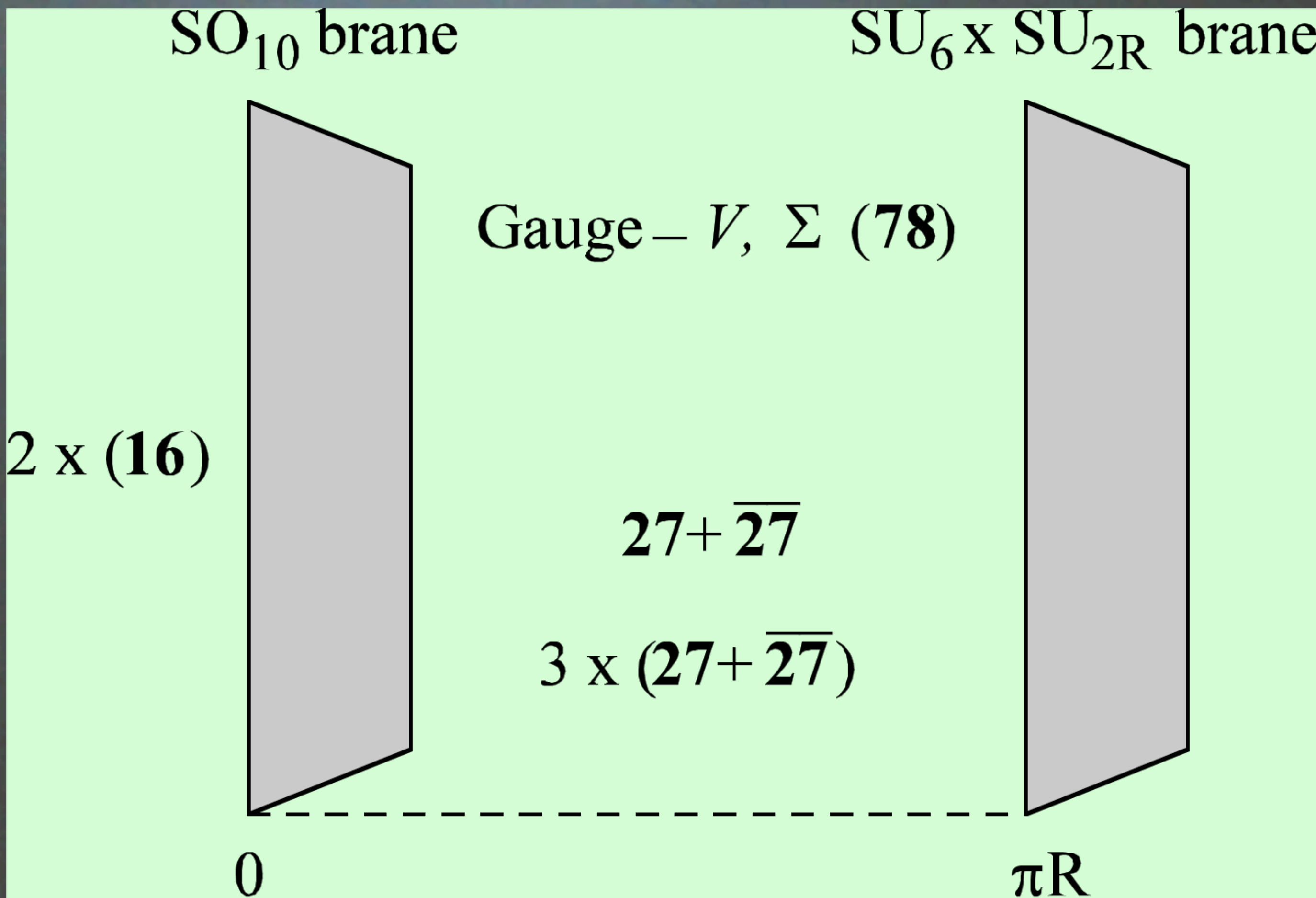
$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$ doublet $\qquad f_3$ singlet

Z_2 twisted sector



Extra color triplets !!

E_6 Orbifold GUT on $M_4 \times S_1/(Z_2 \times Z_2')$



To summarize

- 3^{rd} family in bulk → gauge-Yukawa unification

$$\frac{g_5}{\sqrt{\pi R}}\int_0^{\pi R} dy\,\overline{27}\,\Sigma\,27 = gHF_3^c F_3$$

- 1^{st} and 2^{nd} families on Z_2 fixed points

→ D_4 family symmetry

- $$2\left(\chi^c + \overline{\chi^c}\right) + 3\;C\left(= T + \overline{T}\right)$$

→ Higgs for PS symmetry breaking

PS symmetry breaking

$$W = SC_1C_4 + C_4\left(\chi_1^c\chi_1^c + \overline{\chi_1^c}\,\overline{\chi_1^c}\right) + S'\overline{\chi_2^c}\chi_2^c$$

F and D flat direction breaks
PS to SM
+ gives mass to all SM non-singlets

Gauge coupling unification

4D ? GUT

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha_G} + b_i^{MSSM} \log\frac{M_G}{\mu} + 6\delta_{i3}$$

5D Orbifold GUT

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha_{string}} + b_i^{MSSM} \log\frac{M_{PS}}{\mu} + \left(b_{++}^{PS} + b_{brane}\right)_i \log\frac{M_{string}}{M_{PS}}$$

$$-\frac{1}{2}\left(b_{++}^{PS} + b_{--}^{PS}\right)_i \log\frac{M_{string}}{M_c} + b^{E_6}\left(\frac{M_{string}}{M_c} - 1\right)$$

Gauge coupling unification

++ and -- modes are in overlap of

SO_{10} and $SU_6 \times SU_{2R}$ ⇒

$$\left(b_{++}^{PS}\right)_2 = \left(b_{++}^{PS}\right)_3 \qquad \left(b_{--}^{PS}\right)_2 = \left(b_{--}^{PS}\right)_3$$

$$\left(b_{brane}\right)_2 = \left(b_{brane}\right)_3$$

$$\boxed{\alpha_2^{-1} - \alpha_3^{-1}} \Rightarrow \boxed{M_{PS} = e^{-3/2} M_G \approx 7 \times 10^{15}\ GeV}$$

B.C.s

Weak

$$2\pi/\alpha_s = \pi/4\left(M_{Planck}/M_s\right)^2$$

$$M_s(Max) = e^2 M_G \approx 2\times10^{17}\,GeV \Rightarrow \alpha_s^{-1} \approx 450$$

Strong

$$\frac{2\pi}{\alpha_s} = \frac{1}{2(4\pi)^{5/3} M\rho}\left(\frac{M_{Pl}}{M}\right)^2$$

$$M_s \approx M = 2M_G$$

$$M_c \cong M_{PS} = e^{-3/2} M_G \approx 7\times10^{15}\ GeV$$

$$\rho = M_G^{-1}$$

Successe

✓ Gauge coupling unification

$$\tau_{\left(p\to e^+\pi^0\right)} \approx 3\times 10^{33}\left(\frac{0.015\ GeV^3}{\beta_{lattice}}\right)^2 yrs > 5.4\times 10^{33}\ yrs.\ [S-K]$$

✓ 3rd generation and Higgs in bulk
-> gauge-Yukawa unif.

✓ D_4 family symmetry : family hierarchy
+ suppress flavor violation

D_4 family symmetry

PS breaking VEVs

$$O_i = \langle \chi^c_\alpha \; \bar{\chi}^c_i \rangle, \quad i = 1, 2$$

- Fermion mass matrix [simple form]

$$(f_1 \;\; f_2 \;\; f_3) \; h \; \mathcal{M} \begin{pmatrix} f^c_1 \\ f^c_2 \\ f^c_3 \end{pmatrix}$$

$$\mathcal{M} = \begin{pmatrix} (O_2 \; \tilde{S}_e + S_e) & (O_2 \; \tilde{S}_o + S_o) & (O_1 \; O_2 \; \phi_e + \tilde{\phi}_e) \\ (O_2 \; \tilde{S}_o + S_o) & (O_2 \; \tilde{S}_e + S_e) & (O_1 \; O_2 \; \phi_o + \tilde{\phi}_o) \\ \phi'_e & \phi'_o & 1 \end{pmatrix}$$

So what's the problem ?

- Exotics

BUT Vector-like OK ! IF massive

- NO R parity !!

May be OK ?

So what now ?
Search for R parity inv. theory
Z_3 x Z_2 or Alternate orbifold construct

New Search $Z_2 \times Z_4$ orbifold

w/ Wingerter & Vaudrevange

$V_2 \times V_4$ 120 inequivalent possibilities

Narrow the search

W_1 and/or W_2 500,000 possible models

Consider eg. MSSM in 4D

Why bother with GUTs ☺

Demand -

3 (3, 2) + 6 (3* , 1) + > 5 (1 , 2)

TYPICALLY have Chiral Exotics

i.e. Wrong Hypercharge

☹

Solution - Demand PS or SU(5) intermediate sym. & identify hypercharge

$\frac{1}{2}(B - L) + T_{3R}$ in PS or Y in SU(5)

I. MSSM in 4D
w/ 3 families & NO chiral exotics

Find PS -> SM - 6 models
SU(5) -> SM - 40 models

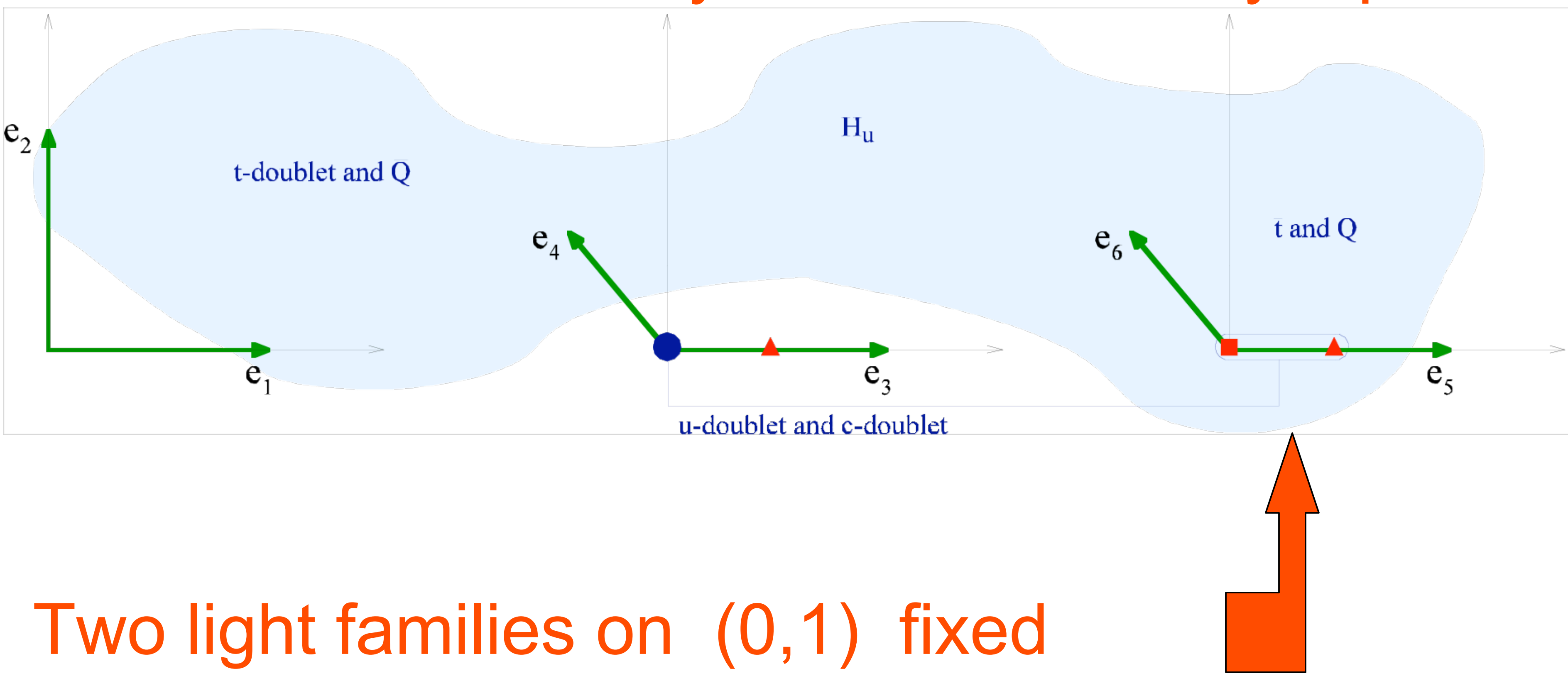
II. Pati – Salam in 4D
w/ 3 families, PS Higgs
& NO chiral exotics

Find ~ 220 models

Z_2 x Z_4 orbifold construction

One example : preliminary

Part of third family in bulk => heavy top



Two light families on (0,1) fixed poiints

Spectrum

$$3\,Q + \left(Q + \overline{Q}\right)$$

$$3\left(\overline{u} + \overline{e} + \overline{d}\right) + 7\left(\overline{d}_{2/3} + d_{-2/3}\right)$$

$$3\,L + 7\left(H_u + H_d\right)$$

$$8\left(E_1 + \overline{E}_{-1}\right) + 8\left(q_{1/3} + \overline{q}_{-1/3}\right) + \ldots$$

Order S^0 : top mass

S^2 : bottom + tau mass

μ term

Conclusions

- New 3 family E_6 orbifold GUT
- 3^{rd} generation -- gauge-Yukawa unif.
- D_4 family symmetry : family hierarchy
  + suppress flavor violation
- Few vector-like exotics
- NO R parity ☹

$Z_2 \times Z_4$ & $Z_2 \times Z_3$ search in progress