

Warped Extra Dimensions and Partial Compositeness *

Brian Batell

University of Minnesota

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- *with Tony Gherghetta
 - arXiv:0706.0890
 - arXiv:0710.1838

5D Warped Dimension = 4D Strong Dynamics

- AdS/CFT duality: Extra dimension is a calculational tool
- Randall-Sundrum models \iff Standard Model partial compositeness
- How to quantify elementary/composite mixing?
 - Understand structure and phenomenology of 4D dual theory
- Answer: The Holographic Basis:

$$\Phi(x, y) = \varphi^s(x)g^s(y) + \sum_{n=1}^{\infty} \varphi_{CFT}^n(x)g^n(y)$$

Outline

- Randall-Sundrum models and geometrical hierachies
- AdS/CFT and Holography
- The Kaluza-Klein Basis
- The Holographic Basis
- Elementary/composite content of SM fields
- Explain warped phenomenology in a 4D language
(e.g. Fermion masses, GIM mechanism)

A slice of AdS

Randall, Sundrum '99

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

- Warped geometry \implies
Energy scales depend on location

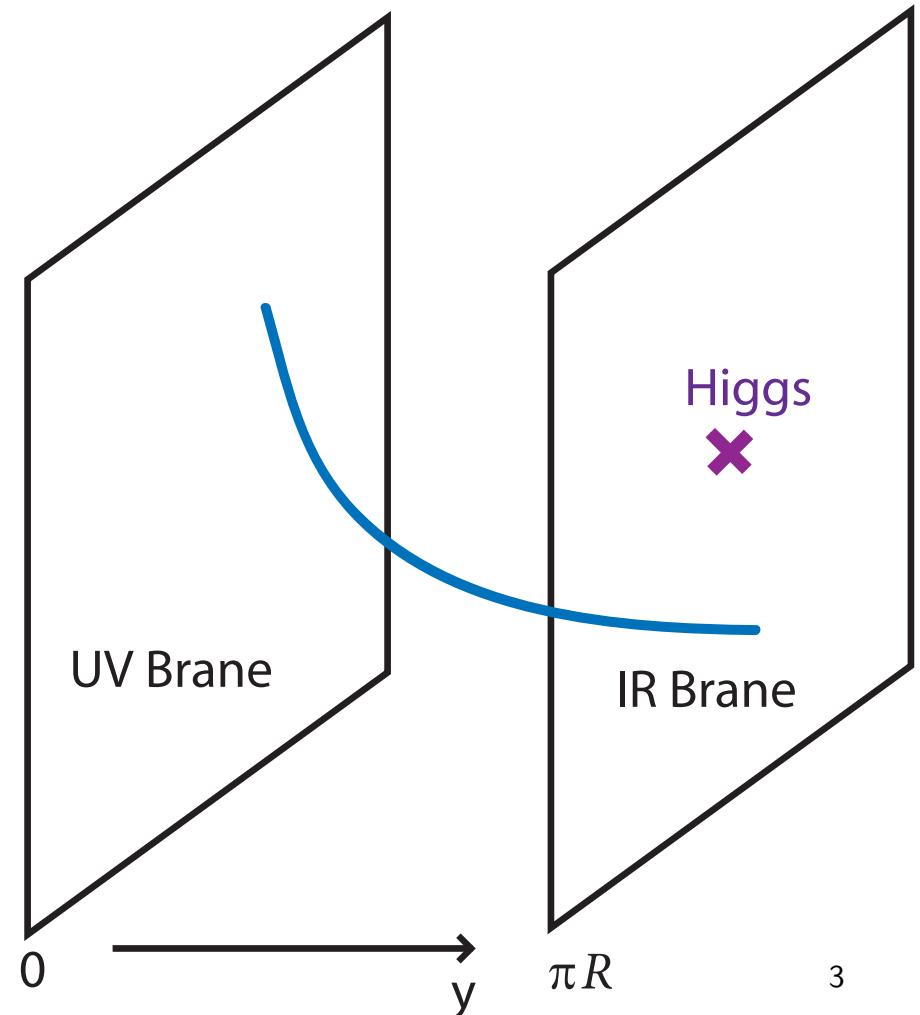
- Planck/Weak scale hierarchy:

$$\Lambda_{weak} \sim M_P e^{-\pi k R}$$

$$k \sim \mathcal{O}(M_P), \quad \pi k R \sim \mathcal{O}(30)$$

- R can be naturally stabilized

Goldberger, Wise '99



Standard Model in the bulk

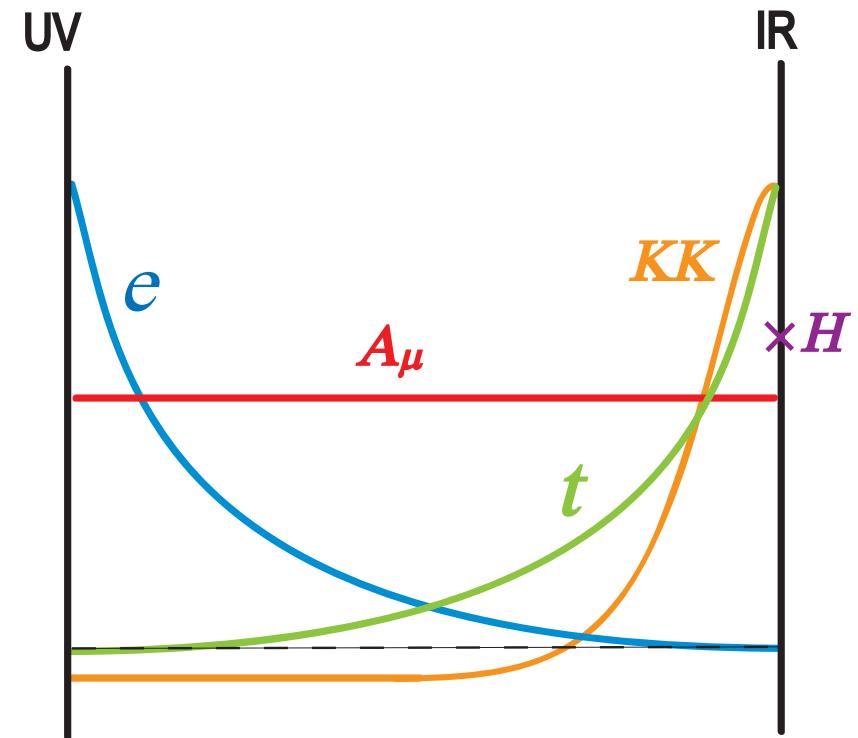
Davoudiasl, Hewett, Rizzo '99; Pomarol '99

Grossman, Neubert '99

Chang et al. '99; Gherghetta, Pomarol '00

Theory of Flavor!

- Natural Yukawa hierarchies
- FCNC suppressed



Bulk fields

Scalar field with tuned bulk and boundary masses

$$S = \int d^5x \sqrt{-g} \left[-\frac{1}{2}(\partial_M \Phi)^2 - \frac{1}{2}ak^2\Phi^2 - bk\Phi^2 (\delta(y) - \delta(y - \pi R)) \right]$$

Tuning:

$$b = 2 \pm \sqrt{4 + a}$$

Why consider this toy model?

- Tuning allows for a localized zero mode: $\tilde{f}^0(y) \sim e^{(b-1)ky}$
 \implies Holographic interpretation depends on b
- special values for b mimic bulk graviton and gauge boson

AdS/CFT duality

Maldacena '97

is for our purposes . . .

Weakly coupled gravity in warped 5D	dual	Strongly coupled gauge theory (CFT) in 4D*
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* Large N_c gauge theory

$$\left\langle \exp \left(- \int \varphi_0 \mathcal{O} \right) \right\rangle_{\text{CFT}} = \exp [- \Gamma(\varphi_0)]$$

Gubser, Klebanov, Polyakov '97; Witten '97

“Dictionary”

<u>5D</u>		<u>4D</u>
bulk field $\Phi(x, y)$	\iff	CFT operator $\mathcal{O}(x)$
BC $\Phi(x, y_0) = \varphi_0(x)$	\iff	source: $\varphi_0(x)\mathcal{O}(x)$
bulk mass	\iff	dimension Δ of \mathcal{O}

e.g.

Bulk gauge field

global symmetry current

$$A_\mu(x, y) \iff$$

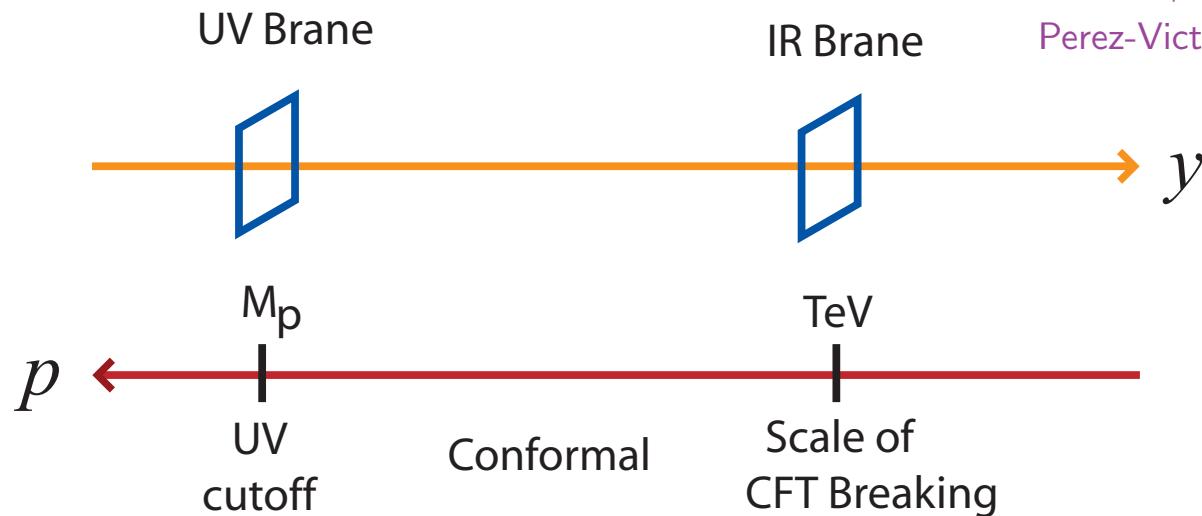
$$J_\mu^{CFT}$$

$$m_A^2 = 0 \iff$$

$$\Delta_J = 3$$

Holography for RS1

Arkani-Hamed, Poratti, Randall '00
Rattazzi, Zaffaroni '00
Perez-Victoria '00



zero mode	\sim	source field (elementary)
KK modes	\sim	CFT bound states (composites)

but wait ...

Mixing through operator $\varphi_0(x)\mathcal{O}(x)$

\implies Mass eigenstates are elementary/composite mixtures

The “Holographic Recipe”

Step 1: Evaluate bulk action for arbitrary boundary condition
 $\Phi(x, y_0) = \varphi_0(x)$ to obtain $\Gamma(\varphi_0)$

Step 2: Take functional derivatives to compute correlation functions of CFT operators

$$\begin{aligned}\langle \mathcal{O} \mathcal{O} \rangle(p) &= \frac{\delta^2}{\delta \varphi_0^2} \left\langle \exp \left(- \int \varphi_0 \mathcal{O} \right) \right\rangle_{\text{CFT}} = \frac{\delta^2}{\delta \varphi_0^2} \exp[-\Gamma(\varphi_0)] \\ &= \mp ip \frac{J_{b-1} \left(\frac{ip}{k} \right) Y_{b-1} \left(\frac{ipe^{\pi k R}}{k} \right) - Y_{b-1} \left(\frac{ip}{k} \right) J_{b-1} \left(\frac{ipe^{\pi k R}}{k} \right)}{J_{b-2} \left(\frac{ip}{k} \right) Y_{b-1} \left(\frac{ipe^{\pi k R}}{k} \right) - Y_{b-2} \left(\frac{ip}{k} \right) J_{b-1} \left(\frac{ipe^{\pi k R}}{k} \right)}\end{aligned}$$

Step 3: Interpret $\langle \mathcal{O} \mathcal{O} \rangle(p)$

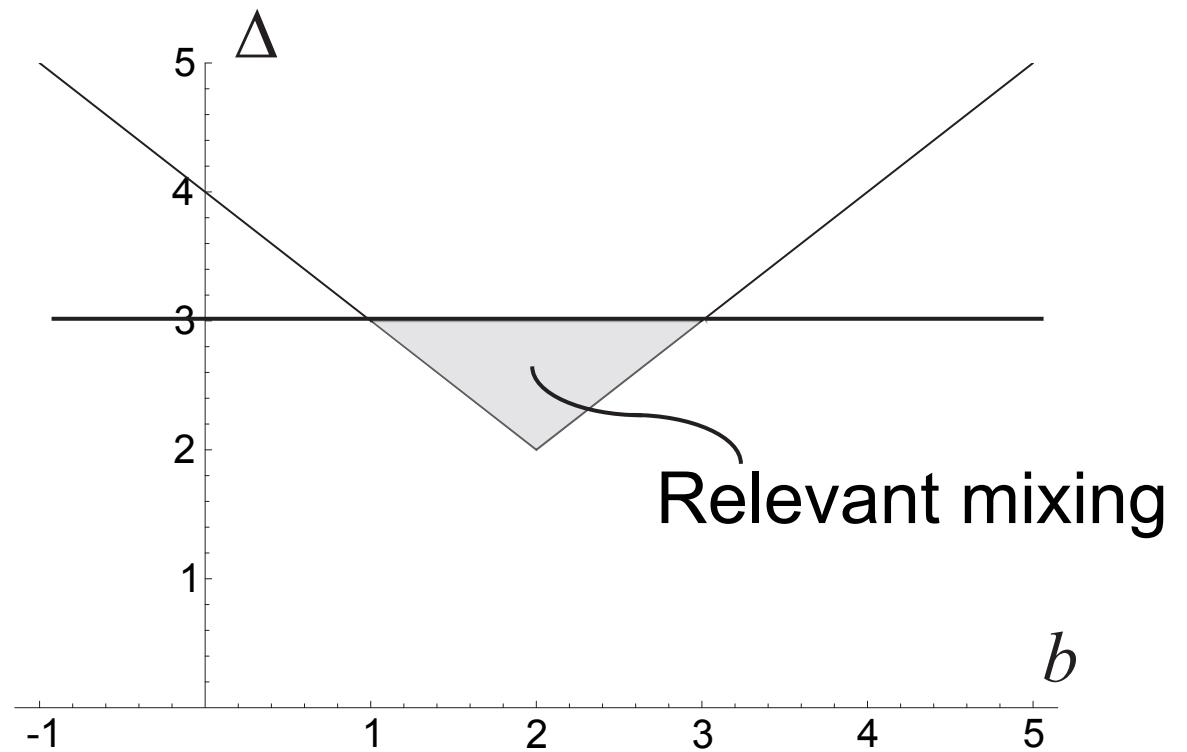
Operator Dimension

$$\Delta = 2 + \sqrt{4 + a} = 2 + |b - 2|$$

$$\varphi_0(x) \quad \overset{1}{\overbrace{}} \quad 2 + |b - 2| \quad \overset{2 + |b - 2|}{\overbrace{}} \quad \mathcal{O}(x)$$

$b < 1$ or $b > 3$
 \Rightarrow irrelevant mixing

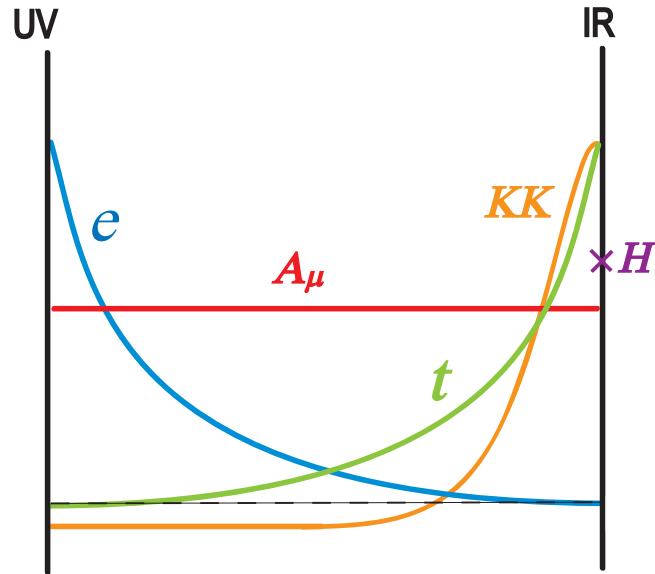
$1 < b < 3$
 \Rightarrow relevant mixing



$$\Delta = 2 + |b - 2|$$

- $b < 2$:
 - source field $\varphi_0(x)$ massless
 - zero mode primarily elementary
 - Nearly all RS phenomenological examples are described by $b < 2$ (fermions too!)
- $b > 2$:
 - source field $\varphi_0(x)$ massive $M_0 \sim k$
 - zero mode primarily composite
 - Higgs; perhaps t_R in some models

Partial compositeness of SM fields



- UV localized \iff mostly elementary
- IR localized \iff mostly composite

Can we quantify source/CFT (elementary/composite) mixing?

Kaluza-Klein mass eigenbasis

KK decomposition:

$$\Phi(x, y) = \sum_{n=0}^{\infty} \phi^n(x) f^n(y), \quad \text{BC : } (++)$$
$$(\partial_5 - bk) f^n(y) \Big|_{0, \pi R} = 0$$

Localized massless mode:

$$\tilde{f}^0(y) \sim e^{(b-1)ky}, \quad -\infty < b < \infty$$

The fields $\phi^n(x)$ are the mass eigenstates

- Spectrum:

$$J_{b-1} \left(\frac{m_n}{k} \right) Y_{b-1} \left(\frac{m_n e^{\pi k R}}{k} \right) - Y_{b-1} \left(\frac{m_n}{k} \right) J_{b-1} \left(\frac{m_n e^{\pi k R}}{k} \right) = 0$$

Holographic basis

Basic idea:

Expand the bulk field directly in terms of a source field $\varphi^s(x)$ and composite CFT states $\varphi_{CFT}^n(x)$:

$$\Phi(x, y) = \varphi^s(x)g^s(y) + \sum_{n=1}^{\infty} \varphi_{CFT}^n(x)g^n(y)$$

- Leads to kinetic and mass mixing in 4D effective theory
- Mass eigenstates will be a mixture of $\varphi^s(x)$ and $\varphi_{CFT}^n(x)$

Source profile $g^s(y)$

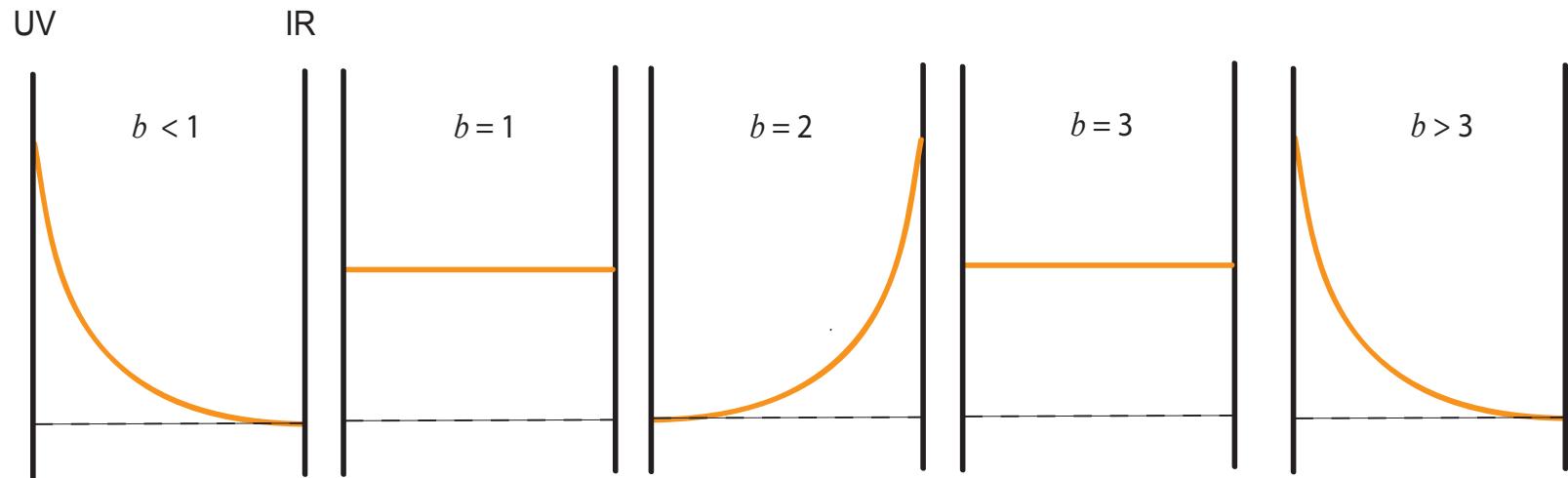
$g^s(y)$ can be determined from mass of source

$$M_s^2 = \begin{cases} 0 & \text{for } b < 2 \\ 4(b-2)(b-3)k^2 & \text{for } b > 2 \end{cases}$$

$$\implies \tilde{g}^s(y) \sim e^{-ky} e^{(4-\Delta)ky} = \begin{cases} e^{(b-1)ky} & \text{for } b < 2 \\ e^{(3-b)ky} & \text{for } b > 2 \end{cases}$$

Source profiles mimic operator dimensions:

$$\Delta = 2 + |2 - b|$$



- Indicates when mixing is relevant, marginal, or irrelevant

CFT composite profiles $g^n(y)$

CFT spectrum obtained from poles in 2-point function:

$$J_{b-2} \left(\frac{M_n}{k} \right) Y_{b-1} \left(\frac{M_n e^{\pi k R}}{k} \right) - Y_{b-2} \left(\frac{M_n}{k} \right) J_{b-1} \left(\frac{M_n e^{\pi k R}}{k} \right) = 0$$

Note different from KK spectrum!

Identical to the spectrum obtained with the following BC for $g^n(y)$:

$$\begin{aligned} \text{BC : } & (-+) \\ g^n(y) \Big|_0 &= 0 \\ (\partial_5 - bk)g^n(y) \Big|_{\pi R} &= 0 \end{aligned}$$

Effective 4D Lagrangian in the holographic basis

$$\mathcal{L} = \frac{1}{2} \vec{\varphi}^T \mathbf{Z} \square \vec{\varphi} - \frac{1}{2} \vec{\varphi}^T \mathbf{M}^2 \vec{\varphi},$$

where $\vec{\varphi}^T = (\varphi^s, \varphi_{CFT}^1, \varphi_{CFT}^2, \dots)$

$$\mathbf{Z} = \begin{pmatrix} 1 & z_1 & z_2 & z_3 & \cdots \\ z_1 & 1 & 0 & 0 & \cdots \\ z_2 & 0 & 1 & 0 & \cdots \\ z_3 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad \mathbf{M}^2 = \begin{pmatrix} M_s^2 & \mu_1^2 & \mu_2^2 & \mu_3^2 & \cdots \\ \mu_1^2 & M_1^2 & 0 & 0 & \cdots \\ \mu_2^2 & 0 & M_2^2 & 0 & \cdots \\ \mu_3^2 & 0 & 0 & M_3^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Notice kinetic mixing \implies nonorthogonal basis

z_n and μ_n^2 computed from wavefunction overlap integrals

Diagonalization leads to KK basis

$\gamma - \rho$ mixing in SM

Vector Meson Dominance

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}(\rho_{\mu\nu})^2 - \frac{1}{2}z_{\gamma\rho}F_{\mu\nu}\rho^{\mu\nu} - \frac{1}{2}m_\rho^2\rho_\mu\rho^\mu$$

“Physical” photon can be viewed as partly composite

Graviton $h_{\mu\nu}$

$$\tilde{f}^0(y) \sim e^{-ky}$$

$b = 0; \Delta = 4 \implies$ irrelevant mixing

$$\begin{pmatrix} h^0 \\ h^1 \\ \vdots \end{pmatrix} = \begin{pmatrix} 1 & \sim e^{-\pi kR} & \dots \\ 0 & \sim -1 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} h^s \\ h^{1(CFT)} \\ \vdots \end{pmatrix}$$

- 4D graviton $h_{\mu\nu}^0(x) \sim$ elementary source ; compositeness negligible
- KK modes are purely composite

Gauge field A_μ

$$\tilde{f}^0(y) = \frac{1}{\sqrt{\pi R}}$$

$b = 1; \Delta = 3 \implies$ marginal mixing

$$\begin{pmatrix} A_\mu^0 \\ A_\mu^1 \\ A_\mu^2 \\ \vdots \end{pmatrix} = \begin{pmatrix} 1 & -0.19 & 0.13 & \cdots \\ 0 & -0.98 & -0.03 & \cdots \\ 0 & 0.01 & -0.99 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} A_\mu^s \\ A_\mu^{1(CFT)} \\ A_\mu^{1(CFT)} \\ \vdots \end{pmatrix}$$

- massless eigenstate $A_\mu^0(x)$ is primarily elementary
- KK modes are purely composite

Bulk Fermions

- Bulk mass $m_\psi = ck$
- KK mass eigenbasis:

$$\psi_{\pm}(x, y) = \sum_{n=0}^{\infty} \psi_{\pm}^n(x) f_{\pm}^n(y),$$

- Chiral zero mode $\psi_+^0(x)$; wavefunction $\tilde{f}_+^0(y) \sim e^{(\frac{1}{2}-c)ky}$

Fermion holography

Contino, Pomarol '04

- Operator dimension:

$$\Delta_- = \frac{3}{2} + \left| c + \frac{1}{2} \right|$$

- If $\Delta_- < 5/2 \implies$ relevant mixing
- Holographic basis

$$\psi_+(x, y) = \psi^s(x)g^s(y) + \sum_{n=1}^{\infty} \lambda_+^n(x)g_+^n(y),$$

$$\psi_-(x, y) = \chi(x)g^\chi(y) + \sum_{n=1}^{\infty} \lambda_-^n(x)g_-^n(y),$$

Two branches in dual theory

$$\Delta_- = \frac{3}{2} + \left| c + \frac{1}{2} \right|$$

- $c > -1/2$:
 - source field $\psi^s(x)$ chiral; $\chi(x)$ absent from theory
 - zero mode primarily elementary
 - Nearly all bulk fermions described by $c > -1/2$
- $c < -1/2$:
 - field $\chi(x)$ marries with source field $\psi^s(x)$ to become massive $M_0 \sim k$
 - zero mode primarily composite
 - perhaps t_R in some models

Right-handed top t_R

$$\tilde{f}^0(y) = e^{(\frac{1}{2}-c)ky} \quad m_\psi = ck$$

Take e.g. $c = -0.7$; $\Delta = 1.7 \Rightarrow$ relevant mixing

$$\begin{pmatrix} t_R^{(0)} \\ t_R^{(1)} \\ t_R^{(2)} \\ t_R^{(3)} \\ \vdots \end{pmatrix} = \begin{pmatrix} 0.9796 & \sim -1 & \sim 0 & \sim 0 & \dots \\ -0.1816 & \sim 0 & \sim -1 & \sim 0 & \dots \\ 0.0514 & \sim 0 & \sim 0 & \sim -1 & \dots \\ 0.0471 & \sim 0 & \sim 0 & \sim 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} t_R^s \\ t_R^{CFT(1)} \\ t_R^{CFT(2)} \\ t_R^{CFT(3)} \\ \vdots \end{pmatrix}$$

- massless eigenstate $t_R^0(x)$ roughly equal mixture of source/CFT
- KK modes contain elementary component

Phenomenology in the holographic basis

- Yukawa hierarchies
- FCNC suppression (GIM mechanism)

Important point on inverse transformation:

- Source field contains zero mode

$$\psi^s(x) = \psi_+^0(x) + \sum_{m=1}^{\infty} \omega_+^{sm} \psi_+^m(x)$$

- Composite modes do not; entirely composed of KK modes

$$\lambda_+^n(x) = \sum_{m=1}^{\infty} \omega_+^{nm} \psi_+^m(x)$$

Yukawa hierarchies

Consider Higgs on IR brane:

$$\begin{aligned} S_{Yukawa} = & \lambda_{ij}^5 \int d^4x \left\{ \left(\zeta_{ss} \bar{\psi}_{iL}^s H \psi_{jR}^s + \text{h.c.} \right) \right. \\ & + \sum_n \left[\left(\zeta_{sn} \bar{\psi}_{iL}^s H \lambda_{jR}^n + \text{h.c.} \right) + \left(\zeta_{ns} \bar{\lambda}_{iL}^n H \psi_{jR}^s + \text{h.c.} \right) \right] \\ & \left. + \sum_{n,m} \left(\zeta_{nm} \bar{\lambda}_{iL}^n H \lambda_{jR}^m + \text{h.c.} \right) \right\}. \end{aligned}$$

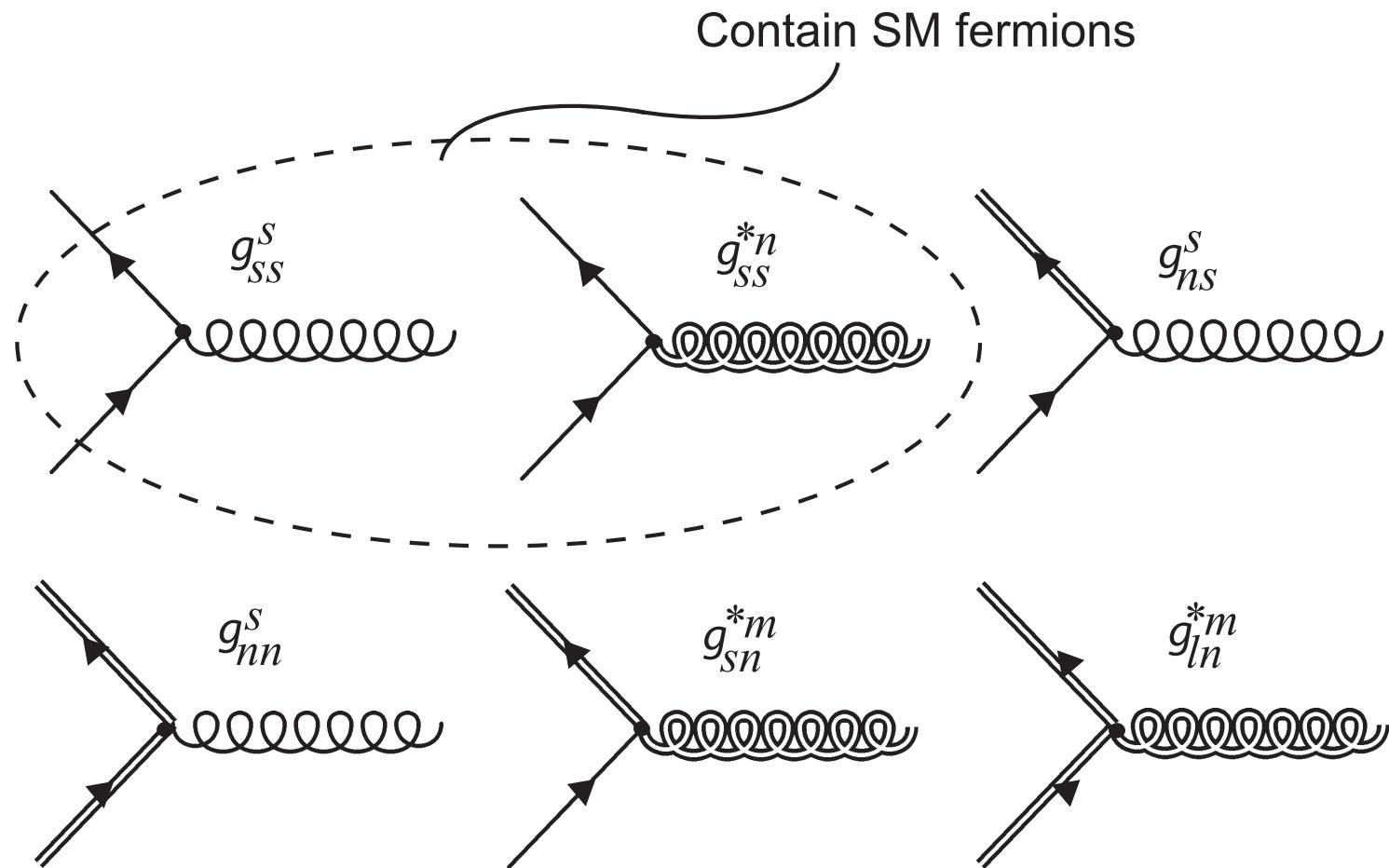
- Only 2-source/Higgs vertex ζ_{ss} contains SM coupling since $\lambda^n(x)$ contains no zero mode

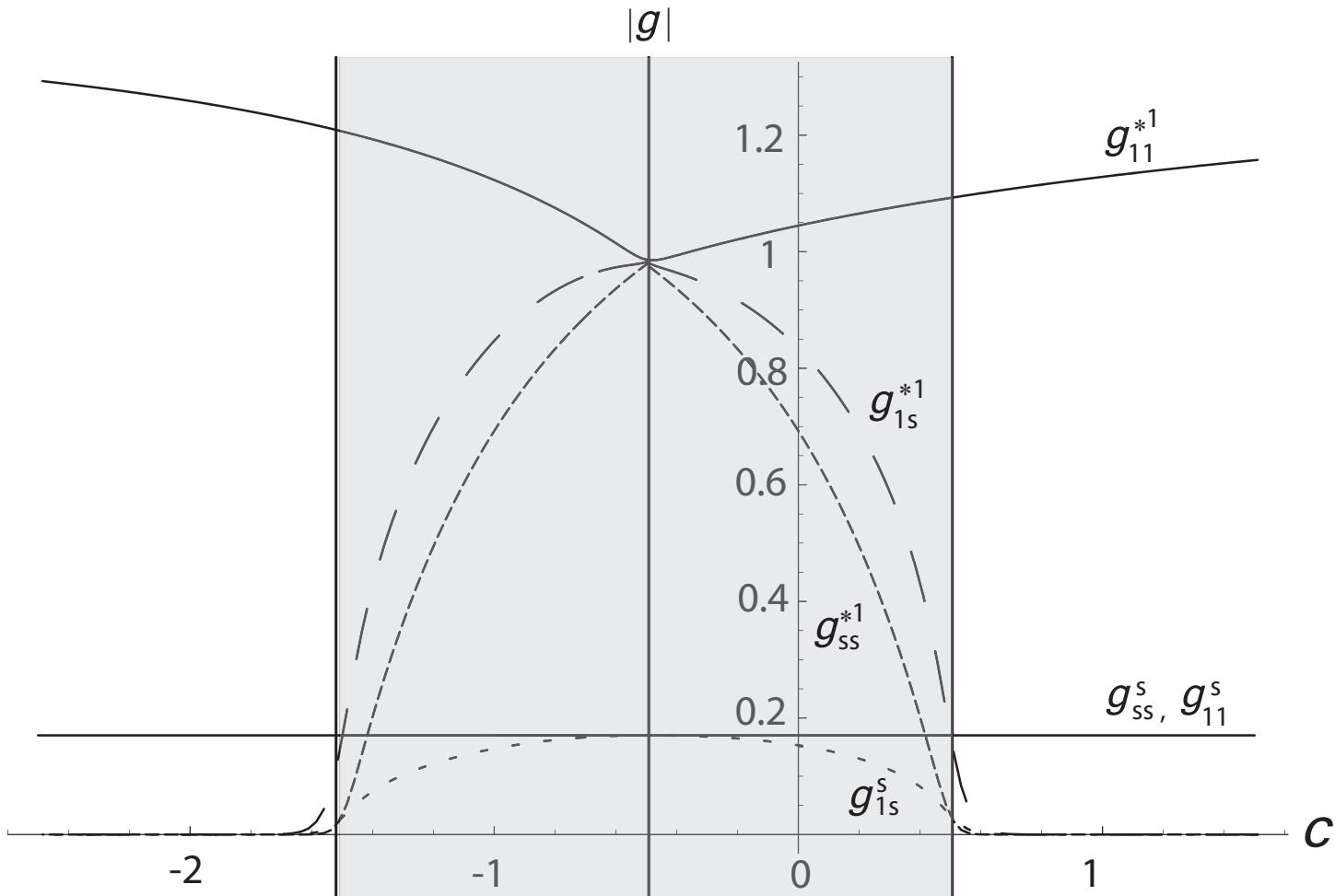
Yukawa hierarchies -cont'd

$$\begin{aligned}\zeta_{ss} &= \int_0^{\pi R} dy \frac{e^{ky}}{k} g_{iL}^s(y) g_{jR}^s(y) \delta(y - \pi R) \\ &= \sqrt{\frac{5 - 2\Delta_i}{1 - e^{-(5-2\Delta_i)\pi kR}}} \sqrt{\frac{5 - 2\Delta_j}{1 - e^{-(5-2\Delta_j)\pi kR}}}.\end{aligned}$$

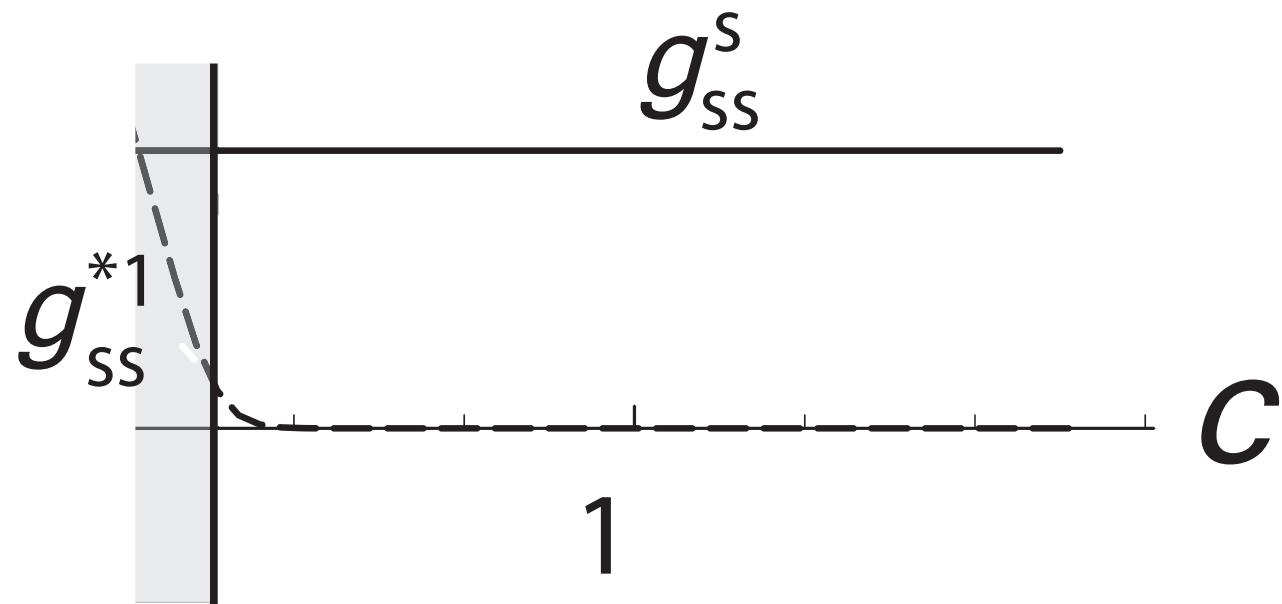
- Calculation is identical to KK basis calculation
- Bottom line: Yukawa hierarchies depend on coupling of elementary fermions to the CFT

Gauge interactions $g_{\psi\psi}^A$





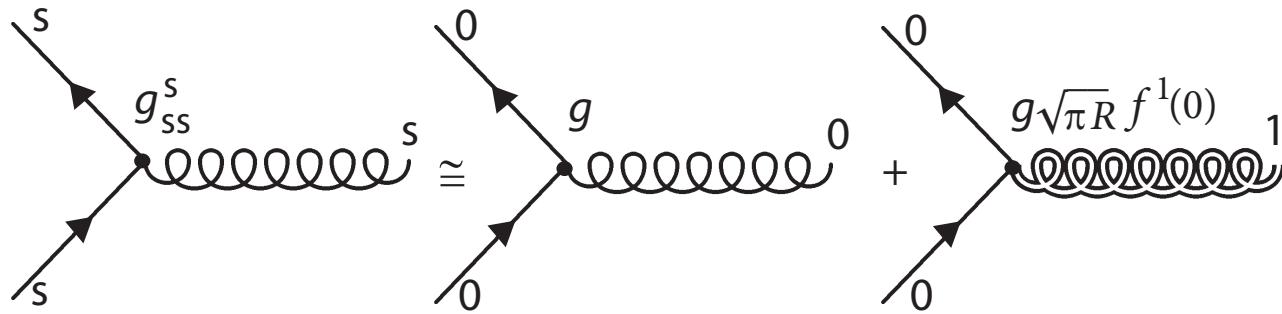
- Light fermions have exponentially suppressed couplings to composites



- 3-source vertex dominates

RS GIM mechanism

For light fermions, $c > 1/2$, 3-source vertex dominates:

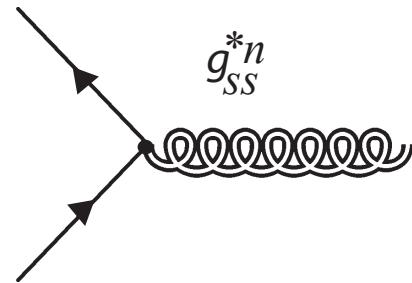


KK gauge boson couplings are approx. universal for light fermions

⇒ FCNCs suppressed

Flavor violation

Important to track nonuniversal contribution to coupling



$$g_{ss}^{*n} = \int_0^{\pi R} e^{ky} g^s(y) g^{*n}(y) g^s(y)$$

Sum over all composite modes \Rightarrow

$$g_{nonuniversal}^1 = \sum_{n=1}^{\infty} g_{ss}^{*n} \omega^{n1} = g^1 - g_{universal}^1$$

Flavor violation - cont'd

Near $c \sim 1/2$, first composite mode saturates nonuniversal piece:

$$\begin{aligned} g_{nonuniversal}^1 &\simeq g_{ss}^{*1} \omega^{11} \\ &\simeq g \sqrt{2\pi kR} \left(\frac{2c-1}{2-2c} \right) e^{(1-2c)\pi kR}. \end{aligned}$$

- Works well - order few % - for $c < 0.6$ (e.g. tau, muon)
- Deviates for $c > 0.6$ (e.g. electron), but nonuniversal contributions smaller anyway*

* Thanks to K. Agashe for discussions

Conclusions

- Holographic basis: bulk field expanded in source and CFT resonances
- Quantitatively describe elementary/composite mixing in warped duals
- Explain warped physics in terms of strong gauge dynamics
- Things to do:
 - Other applications: Higgsless models, warped SUSY, Gauge-Higgs models (QCD?)
 - Loop diagrams
 - important for EWPT, gauge coupling unification etc.
 - Brane localized kinetic terms - could modify composite content
 - More general geometries?