B^0-$\bar{B}^0$ mixing parameters with $N_f = 2 + 1$ sea quarks in lattice QCD

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HPQCD Collaboration

· Fermilab, 28th June 2007 ·

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Fermilab Lattice and MILC Collaborations

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1. Introduction: $B_0 - \bar{B}_0$ mixing parameters

$$|B^0_{s/d}(H)\rangle = p|B^0_{s/d}\rangle + q|\bar{B}^0_{s/d}\rangle \qquad \Delta M_{s/d} = M_{s/d}(H) - M_{s/d}(L)$$

$$|B^0_{s/d}(L)\rangle = p|B^0_{s/d}\rangle - q|\bar{B}^0_{s/d}\rangle \qquad \Delta\Gamma_{s/d} = \Gamma_{s/d}(H) - \Gamma_{s/d}(L)$$

- experimentally: very well measured

$$\boxed{\Delta M_d|_{exp.} = 0.508 \pm 0.004}$$ World average

Two-sided bound on ΔM_s from **DØ** quickly followed by a precise measurement from **CDF**

$$\boxed{\Delta M_s|_{exp.} = 17.77 \pm 0.10(stat) \pm 0.07(syst) ps^{-1}}$$

Unofficial world average (**R.v.Kooten, FP& CP, April 2006**)

$$\Delta\Gamma_s = 0.097^{+0.041}_{-0.042}\, ps^{-1} \Longrightarrow \boxed{\left(\frac{\Delta\Gamma}{\Gamma}\right)_s \simeq 0.15 \pm 0.06}$$

• B_0 mixing parameters determined by the off diagonal elements of the mixing matrix

$$i\frac{d}{dt}\left(\begin{array}{c} |B_{s/d}(t)\rangle \\ |\bar{B}_{s/d}(t)\rangle \end{array}\right) = \left(M^{s/d} - \frac{i}{2}\Gamma^{s/d}\right)\left(\begin{array}{c} |B_{s/d}(t)\rangle \\ |\bar{B}_{s/d}(t)\rangle \end{array}\right)$$

$$\Delta M_{s/d} \propto |M_{12}^{s/d}| \qquad\qquad \Delta\Gamma_{s/d} \propto |\Gamma_{12}^{s/d}|$$

$$\boxed{\textbf{New physics can significantly affect } M_{12}^{s} \propto \Delta M_s}$$

* Γ_{12} dominated by `CKM-favoured` $b \to c\bar{c}s$ tree-level decays.

- theoretically: In the Standard Model

$$\Delta M_s|_{theor.} = \frac{G_F^2 M_W^2}{6\pi^2} \underbrace{|V_{ts}^* V_{tb}|^2}_{5\%} \eta_2^B S_0(x_t) M_{B_s} \underbrace{f_{B_s}^2 \hat{B}_{B_s}}_{\geq 30\%}$$

where $x_t = m_t^2/M_W^2$, η_2^B is a perturbative QCD correction factor and $S_0(x_t)$ is the `Inami-Lim` function.

Need accurate theor. calculation of $f_{B_s}^2 \hat{B}_{B_s}$ to match experimental accuracy

\# Non-perturbative input

$\frac{8}{3} f_{B_s}^2 B_{B_s}(\mu) M_{B_s}^2 = \langle \bar{B}_s^0 | O_L | B_s^0 \rangle(\mu)$ with $O_L \equiv [\overline{b^i}\, s^i]_{V-A} [\overline{b^j}\, s^j]_{V-A}$

For $\Delta\Gamma_s$ one needs either O_S and O_L, or O_3 and O_L

$$\begin{aligned} O_S &\equiv [\overline{b^i}\, s^i]_{S-P} [\overline{b^j}\, s^j]_{S-P} \\ O_3 &\equiv [\overline{b^i}\, s^j]_{S-P} [\overline{b^j}\, s^i]_{S-P} \end{aligned}$$

Precise determination of CKM matrix elements

$$\left|\frac{V_{td}}{V_{ts}}\right| = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}}\sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}}$$

* Experimentally known with an accuracy better than 1%

* Many uncertainties in the theoretical (lattice) determination cancel totally or partially in the ratio

Calculating $\xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}}$ **with a** `few per-cent` **error**

2. Lattice formulations for light and heavy quarks

MILC $N_f^{sea} = 2 + 1$ configurations

* **Light quarks** (sea and valence): improved staggered quarks (`Asqtad` action)

* **(Heavy) b quarks**: Non Relativistic QCD (`NRQCD`) **HPQCD coll.**

* **(Heavy) b quarks**: `Fermilab` action **Fermilab and MILC coll.**

* **Improved gluon action** $\rightarrow$ further reduction of discretization errors

As in previous HPQCD studies of B leptonic and semileptonic decays, all action parameters fixed via light and heavy-heavy simulations prior embarking on `B` physics

$$\begin{array}{rcl} \Upsilon \quad 2S - 1S\,\texttt{splitting} & \rightarrow & a^{-1} \\ \Upsilon & \rightarrow & m_b \\ \texttt{Kaon} & \rightarrow & m_s \end{array}$$

Staggered action

(for light u, d and s valence and sea quarks)

$\longrightarrow$ Advantages of staggered fermions

* **good chiral properties**

* **computationally efficient** numerical simulations with **realistic sea quarks** $\longrightarrow$ chiral extrapolation using ChPT

$\longrightarrow$ Disadvantage: four **tastes** of doublers

* **Continuum limit**: they are degenerate

$\longrightarrow$ they can be removed by hand

* **Finite spacing**: quark-gluon interactions violate taste symmetry

$\longrightarrow$ large $\mathcal{O}(a^2)$ discretization errors

$\longrightarrow$ large one-loop corrections

These **problems** can be **reduced** by using **improved staggered fermion actions**

J.F.Lagae and D.K.Sinclair

G.P.Lepage

Improved staggered Asqtad action

(for light u, d and s valence and sea quarks)

(S.Naik, the MILC collaboration, P. Lepage)

- removal of $\mathcal{O}(a^2)$ unphysical taste-changing interactions at tree level

$\Longrightarrow$ $\boxed{\text{errors } \mathcal{O}(a^2\alpha_s)}$

- HISQ action: Highly improved staggered action.

  * Highly reduce $\mathcal{O}(a^2\alpha_s)$ errors (an order of magnitude)

  * No tree-level $\mathcal{O}((am)^4)$ at first order in the quark velocity v/c

NRQCD action

(for b valence quarks (HPQCD)

Problem is discretization errors ($\simeq m_Q a, (m_Q a)^2, \cdots$) if $m_Q a$ is large.

Heavy quark is non-relativistic in bound states

$\longrightarrow$ $m_b a$ is not an important dynamical scale

(radial and orbital splittings in spectrum of HH and Hl $<<$ masses)

$\longrightarrow$ Use a discretized non-relativistic effective theory: NRQCD

Non-relativistic expansion of the Dirac lagrangian: improved by adding higher order in $v/c << 1$

$$\mathcal{L}_Q = \psi^\dagger \left(D_t - \frac{\vec{D}^2}{2m_Q a} - c_4 \frac{\vec{\sigma}\cdot\vec{B}}{2m_Q a} + \dots \right) \psi$$

* c_i fixed pert. or non-pert. matching to QCD

* Quark and anti-quark fields decouple
  $\longrightarrow$ ψ is a 2-component spinor

Much faster calculation of quark propagators

$$G(\vec{x},t+1) = \left(1-\frac{a\delta H}{2}\right)\left(1-\frac{aH_0}{2n}\right)^n U^\dagger(\vec{x},t)\left(1-\frac{aH_0}{2n}\right)^n \left(1-\frac{a\delta H}{2}\right) G(\vec{x},t)$$

$$G(\vec{x},t=0) = S(\vec{x})$$

Smearing function $S(\vec{x})$: minimize overlap with radial excitations

Our `NRQCD` lattice hamiltonian is (improved through $\mathcal{O}(1/M^2)$, $\mathcal{O}(a^2)$ and leading relativistic $\mathcal{O}(1/M^3)$):

$$aH_0 = -\frac{\Delta^{(2)}}{2(aM_0)} \quad \mathbf{non-relat.\,kinetic\,energy\,oper.}$$

$$a\delta H = -\boxed{c_1}\frac{(\Delta^{(2)})^2}{8(aM_0)^3} + \boxed{c_2}\frac{i}{8(aM_0)^2}\left(\nabla\cdot\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\cdot\nabla\right)$$

relativistic and

$$-\boxed{c_3}\frac{1}{8(aM_0)^2}\boldsymbol{\sigma}\cdot(\tilde{\nabla}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\tilde{\nabla})$$

discretization

$$-\boxed{c_4}\frac{1}{2(aM_0)}\boldsymbol{\sigma}\cdot\tilde{\mathbf{B}} + \boxed{c_5}\frac{\Delta^{(4)}}{24(aM_0)} - \boxed{c_6}\frac{(\Delta^{(2)})^2}{16n(aM_0)^2} + \cdots$$

corrections

Fermilab action

(for b valence quarks (MILC/Fermilab)

(El-Khadra, Kronfeld, Mackenzie)

$$
\begin{aligned}
S \;=\; & a^4 \sum_x \Bigg\{ m_0 \bar{\psi}(x)\psi(x) + \bar{\psi}(x) \left[\frac{1}{2}\left(1+\gamma_0\right) D_0^- + \frac{1}{2}\left(1-\gamma_0\right) D_0^+ \right] \psi(x) \\
& + \bar{\psi}(x)\vec{\gamma}\cdot\vec{D}\psi(x) - \frac{1}{2} a r_s \bar{\psi}(x)\Delta^{(3)}\psi(x) \\
& - \frac{i}{2} a c_B \psi(x)\vec{\Sigma}\cdot\vec{B}\psi(x) - \frac{1}{2} a c_E \bar{\psi}(x)\vec{\alpha}\cdot\vec{E}\psi(x) \Bigg\}
\end{aligned}
$$

- Improvement coefficients calculated at (tadpole improved)
  tree-level: $C_B = C_E = r_s$
- Errors: $\mathcal{O}(\alpha_s \Lambda_{QCD}/M)\,, \mathcal{O}(\left(\Lambda_{QCD}/M\right)^2)$

3. Relevant four fermion operators

(for ΔM_s and $\Delta\Gamma_s$)

$$\left.\begin{array}{lcl} O_L & \equiv & [\overline{b^i}\, s^i]_{V-A}[\overline{b^j}\, s^j]_{V-A} \\ O_S & \equiv & [\overline{b^i}\, s^i]_{S-P}[\overline{b^j}\, s^j]_{S-P} \\ O_3 & \equiv & [\overline{b^i}\, s^j]_{S-P}[\overline{b^j}\, s^i]_{S-P} \end{array}\right\} \text{lowest order in } 1/M$$

$$O_L^{j1} \equiv \frac{1}{2am_b}\left\{[\vec{\nabla}\overline{b^i}\cdot\vec{\gamma}\, s^i]_{V-A}[\overline{b^j}\, s^j]_{V-A} + [\overline{b^i}\, s^i]_{V-A}[\vec{\nabla}\overline{b^j}\cdot\vec{\gamma}\, s^j]_{V-A}\right\}$$

$$O_S^{j1} \equiv \frac{1}{2am_b}\left\{[\vec{\nabla}\overline{b^i}\cdot\vec{\gamma}\, s^i]_{S-P}[\overline{b^j}\, s^j]_{S-P} + [\overline{b^i}\, s^i]_{S-P}[\vec{\nabla}\overline{b^j}\cdot\vec{\gamma}\, s^j]_{S-P}\right\}$$

$$O_3^{j1} \equiv \frac{1}{2am_b}\left\{[\vec{\nabla}\overline{b^i}\cdot\vec{\gamma}\, s^j]_{S-P}[\overline{b^j}\, s^i]_{S-P} + [\overline{b^i}\, s^j]_{S-P}[\vec{\nabla}\overline{b^j}\cdot\vec{\gamma}\, s^i]_{S-P}\right\}$$

with `i`, `j` colour indices and am_b the bare b mass in lattice units.

* Dimension 7 operators O_X^{j1} required at $\mathcal{O}(\Lambda_{QCD}/M)$

* O_3 and O_L lead to smaller theoretical uncertainties in the calculation of $\Delta\Gamma_s$ than O_S and O_L (**Lenz & Nierste**):

$$\langle O_3\rangle = -\langle O_S\rangle - 1/2\langle O_L\rangle + \mathcal{O}(1/M)$$

4. One-loop matching

The input for the SM prediction for ΔM_s is

$$\langle O_L \rangle^{\overline{MS}}(\mu) \equiv \frac{8}{3} f_{B_s}^2 B_{B_s}^{\overline{MS}}(\mu) M_{B_s}^2$$

that is related to the lattice operators through $\mathcal{O}(\alpha_s)$, $\mathcal{O}\left(\frac{\Lambda_{QCD}}{M}\right)$ and $\mathcal{O}\left(\frac{\alpha_s}{aM}\right)$ for the `Asqtad-NRQCD` calculation (HPQCD coll.) by

$$\frac{a^3}{2M_{B_s}} \langle O_L \rangle^{\overline{MS}}(\mu) = [1 + \alpha_s \cdot \rho_{LL}] \langle O_L \rangle(a) + \alpha_s \cdot \rho_{LS} \langle O_S \rangle(a) + \left[\langle O_L^{j1} \rangle(a) - \alpha_s \left(\zeta_{10}^{LL} \langle O_L \rangle(a) + \zeta_{10}^{LS} \langle O_S \rangle(a) \right) \right]$$

* $\langle O_X \rangle$: operator's matrix elements in the lattice theory

* One-loop renormalization coefficients $\rho_{XY} = \rho_{XY}^{\overline{MS}}(\mu, m_b) - \rho_{XY}^{latt.}(am_b)$

* ζ_{10}^{XY} are necessary to subtract $\mathcal{O}\left(\frac{\alpha_s}{aM}\right)$ power law cont. from $\langle O_L^{j1} \rangle$

* $\alpha_s = \alpha_V(q^*) \rightarrow q^* = 2/a$, very close to q^*s for heavy-light currents

4. One-loop matching

Slightly different relation for `Asqtad-Fermilab` calculation (Fermilab/MILC coll.)

$$\frac{a^3}{2M_{B_s}}\,\langle O_L\rangle^{\overline{MS}}(\mu) \quad = \quad [1+\alpha_s\cdot\widetilde{\rho}_{LL}]\left(\langle O_L\rangle(a)+\mathbf{d_1}\,2am_b\langle O_L^{j1}\rangle(a)\right)$$
$$+\alpha_s\cdot\widetilde{\rho}_{LS}\left(\langle O_S\rangle(a)+\mathbf{d_1}\,2am_b\langle O_S^{j1}\rangle(a)\right)$$

* $\langle O_L\rangle(a)+\mathbf{d_1}\,2am_b\langle O_L^{j1}\rangle(a)$ calculated by rotating the heavy fermion field according to

$$b(x)\rightarrow\left(1+a\mathbf{d_1}\vec{\gamma}\cdot\vec{D}\right)\,b(x)$$

* $\mathbf{d_1}$ is a function of am_b, $\mathcal{O}(1/am_b)$ when am_b is large, and known at tree level (universal value)

* One-loop renormalization coefficients $\widetilde{\rho}_{XY}=\rho_{XY}^{\overline{MS}}(\mu,m_b)-\widetilde{\rho}_{XY}^{latt.}(am_b)$

$\rightarrow$ **Calculation in progress**

4. One-loop matching (MILC/Fermilab)

Partially nonperturbative matching calculation?

Rewrite the renormalization factor for any current J^{ac} as

$$Z_{J^{ac}} = \sqrt{Z^{aa}_{V_4} Z^{cc}_{V_4}} \rho_{J^{ac}}$$

It has been shown that for Fermilab currents and Fermilab-Asqtad currents

* $Z^{aa}_{V_4}$ and $Z^{cc}_{V_4}$ calculated nonperturbatively
* $\rho_{J^{ac}}$ calculated perturbatively $\longrightarrow$ **very close to 1 at one-loop**

Important **reduction** of **matching uncertainties**

We want to investigate to what extent a similar method reduces uncertainties in the renormalization of four fermion operators

$\Longrightarrow$ Similarly one can define bag parameters for the operators O_S and O_3 entering in the calculation of $\Delta\Gamma_S$

$$\langle O_S \rangle^{\overline{MS}}_{(\mu)} \equiv -\frac{5}{3} f^2_{B_s} \frac{B^{\overline{MS}}_S(\mu)}{R^2} M^2_{B_s} \,; \qquad \langle O_3 \rangle^{\overline{MS}}_{(\mu)} \equiv \frac{1}{3} f^2_{B_s} \frac{\tilde{B}^{\overline{MS}}_S(\mu)}{R^2} M^2_{B_s}$$

with $\frac{1}{R^2} \equiv \frac{M^2_{B_s}}{(\overline{m}_b + \overline{m}_s)^2}$

* Analogous matching relations

* Renormalization of these operators at one-loop does not involve new lattice operators

5. Numerical simulations and Fitting

We calculate both 3-point (for any $\hat{Q} = Q_X, Q_X^{1j}$) and 2-point correlators

* **HPQCD** calculation

$$C^{(4f)}(t_1, t_2) = \sum_{\vec{x}_1, \vec{x}_2} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}_1, t_1) \left[\hat{Q}\right](0) \Phi_{\bar{B}_q}{}^{\dagger}(\vec{x}_2, -t_2) | 0 \rangle$$

$$C^{(B)}(t) = \sum_{\vec{x}} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}, t) \Phi^{\dagger}_{\bar{B}_q}(\vec{0}, 0) | 0 \rangle$$

* In adittion for the **Fermilab/MILC calculation**

$$C^{(A_4)}(t) = \sum_{\vec{x}} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}, t) \bar{q}(0) \gamma_0 \gamma_5 b(0) | 0 \rangle$$

$\downarrow$

Used to calculate f_B and isolate B_B

- $\Phi_{\bar{B}_q}(\vec{x}, t) = \bar{b}(\vec{x}, t) \gamma_5 q(\vec{x}, t)$ is an interpolating operator for the B_q^0 meson.

Fitting

We carried out **simultaneous** fits of the 3-point and 2-point correlators using bayesian statistics to the forms

$$\begin{aligned}
C^{(4f)}(t_1,t_2) &= \sum_{j,k=0}^{N_{exp}-1} A_{jk}\,\xi_i\,\xi_j\,(-1)^{j\cdot t_1}\,(-1)^{k\cdot t_2}\,e^{-E_B^{(j)}(t_1-1)}\,e^{-E_B^{(k)}(t_2-1)} \\
C^{B}(t) &= \sum_{j=0}^{N_{exp}-1} \xi_j\,(-1)^{j\cdot t}\,e^{-E_B^{(j)}(t-1)} \\
C^{A_4}(t) &= \sum_{j=0}^{N_{exp}-1} \xi_j\,A_4\,(-1)^{j\cdot t}\,e^{-E_B^{(j)}(t-1)}
\end{aligned}$$

* The hadronic matrix element of any 4-fermion operator $\hat{Q} = O_X, O_X^{1j}$ defined before is given by

$$\boxed{\langle\hat{Q}\rangle_{eff.} \equiv \langle\bar{B}_s|\hat{Q}|B_q\rangle_{eff.} = A_{00}}$$

* Fit directly to $C^{(4f)}$ and C^{B} rather than take ratios

$\longrightarrow$ `smaller statistical errors`

HPQCD Calculation

Published results: B_s^0 mixing

Matrix elements needed in the calculation of ΔM_s and $\Delta\Gamma_s$ with
  * **Asqtad** `light` (u, d and s) valence and sea quarks
  * **NRQCD** valence b quarks

Details of simulations

  * We work with $1 \leq t_1, t_2 \leq 16$.
  * No smearing (minimize overlap with radial excitations).
  * Physical valence s and b quarks (fixed from $Kaon$ and Υ masses).
  * Two ensembles of MILC configurations (560 and 414 conf.) with $(m_u^{sea} = m_d^{sea})/m_s = 0.25, 0.50$ and $a^{-1} = 1.6\text{GeV}$.

Fitting details

  * Use range $2 \leq t_1, t_2 \leq 16$
  * We let $N_{exp} \leq 7 - 9$

Fitting

Fits more challenging than in previous work with B leptonic and semileptonic decay matrix elements

Try to increase the number of exp. until central values, fit errors and χ^2/dof stabilizes.

$\Longrightarrow$ **Problem:** Very good fits interlaced with worse fits (worse χ^2/dof)

$\Longrightarrow$ Instead of simultaneous 2-pt and 3-pt fits, fix E_B^0 and E_B^1 from 2-pt fits and then fit 3-pt correlators.

(taking very narrow prior widths)

$\Longrightarrow$ ''statistical+fitting'' errors were inflated to take into account what happens when prior widths are relaxed

In progress: Improving statistics and fitting stability by using smearings, more configurations and more time sources.

Results: sources of error

	$m_f/m_s = 0.25$	$m_f/m_s = 0.50$
$f^2_{B_s} B^{\overline{MS}}_{B_s}(m_b)$ $[GeV^2]$	0.051(8)	0.054(8)

Main Errors in $f^2_{B_s} B_{B_s}(m_b)$

Statistical + Fitting	**9 %**
Higher Order Matching	**9 %**
Discretization	4 %
Relativistic	3 %
Scale (a^{-3})	5 %
Total	15 %

Light sea quark mass dependence smaller than statistical errors

$\rightarrow$ use the $m_f/m_s = 0.25$ results as our central values

HPQCD Calculation

NEW (preliminary) results: B_s^0 and B_d^0 mixing

New data for **4 light sea quark masses** (4 MILC ensembles) and **strange and down valence quarks**. One lattice spacing $a^{-1} \simeq 1.6\mathrm{GeV}$

$\Longrightarrow$ For each light sea quark mass $\rightarrow$
mixing parameters (ΔM and $\Delta\Gamma$) for B_s^0 and B_d^0 with

* $m^b_{valence}$ and $m^s_{valence}$ fixed to their physical values
* $m^d_{valence} = m^d_{sea}$
* Fits: $2 \leq t_1, t_2 \leq 24$
* Several time sources $\rightarrow$ **improved statistics**
* Using **local** and **smeared** (1S) **heavy propagators**

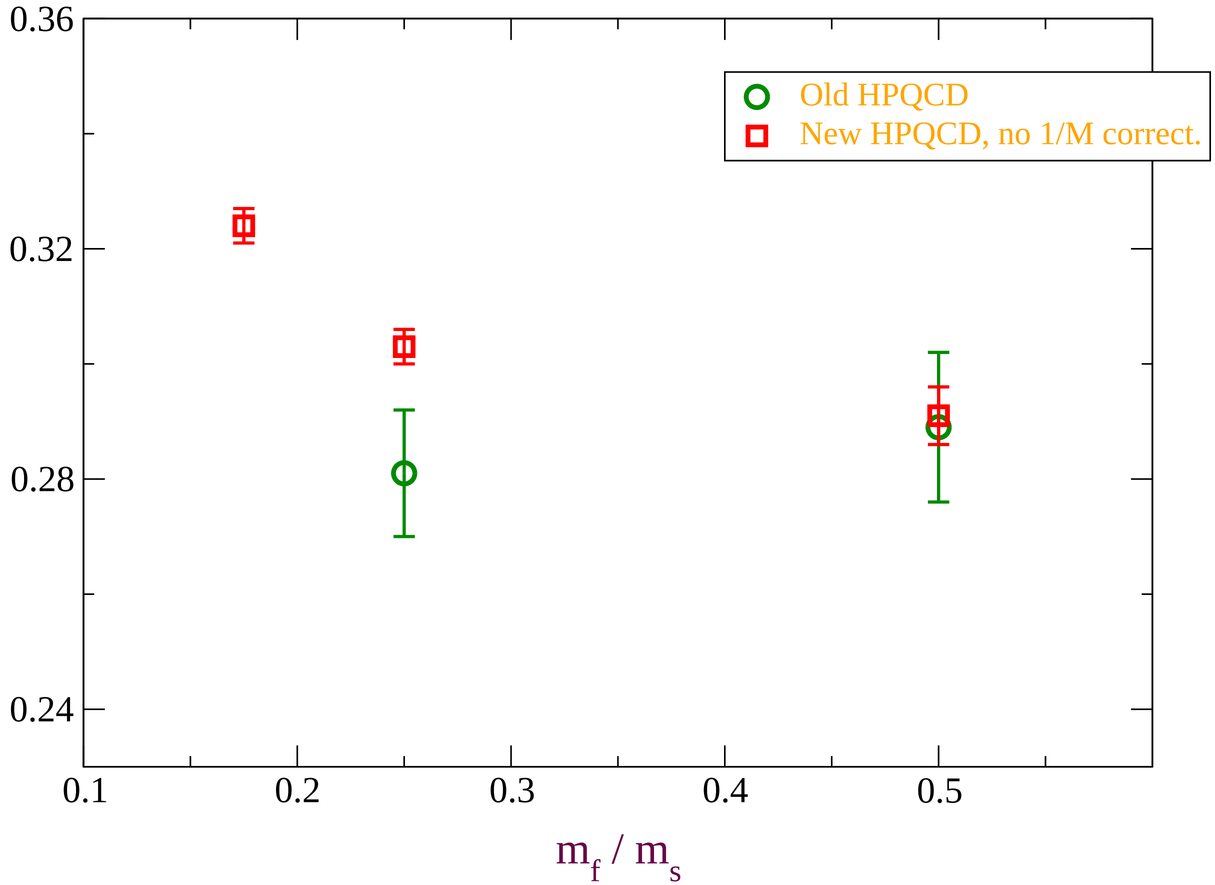
$\downarrow$

Very stable simultaneous 2pt + 3pt fits

HPQCD Calculation

NEW (preliminary) results: B_s^0 and B_d^0 mixing

$f_{B_s}\sqrt{B_{B_s}}$ (only stat. errors)



$$m_f/m_s \equiv (m_{sea}^{up} = m_{sea}^{down})/m_{phys.}^s$$

* No $1/M$ corrections included yet in the new numbers

HPQCD Calculation

NEW (preliminary) results: B_s^0 and B_d^0 mixing

Statistical+fitting errors reduced from 4.5% to $1-2\%$

for both $f_{B_s}\sqrt{\hat{B}_{B_s}}$ and $f_{B_d}\sqrt{\hat{B}_{B_d}}$.

* **Random wall sources** for the light propagators could reduce this error another factor of 2-3

$\longrightarrow$ testing with HISQ light valence quarks

HPQCD Calculation

Preliminary results for ξ: No $1/M$ corrections

	$m_f/m_s = 0.50$	$m_f/m_s = 0.25$	$m_f/m_s = 0.175$
ξ	1.084(27)	1.142(32)	1.164(29)

$\xi = \dfrac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}}$ with $m_d^{sea} = m_d^{valence}$.

- Statistics+fitting errors $\simeq 2.5\%$ (no correlations considered)
- Complete cancellation of scale a^{-3} uncertainties
- Large cancellation of perturbative and $1/M$ corrections.
  - * Difference between `tree level` and `one-loop` results $< 1\%$
  - * Results nearly unchanged for f_{B_s}/f_{B_d} when adding one-loop and $1/M$ corrections

- Discretization, relativistic and Higher order operator matchings corr.

`Corresponding errors in` $f_{B_q}\sqrt{B_q} \times a(m_s - m_q)$

MILC/Fermilab Calculation

B_s^0 **and** B_d^0 **mixing parameters**

\# Matrix elements needed in the calculation of $\Delta M_{d/s}$ and $\Delta\Gamma_{d/s}$ with

* **Asqtad** `light` (u, d and s) valence and sea quarks
* **Fermilab** valence b quarks

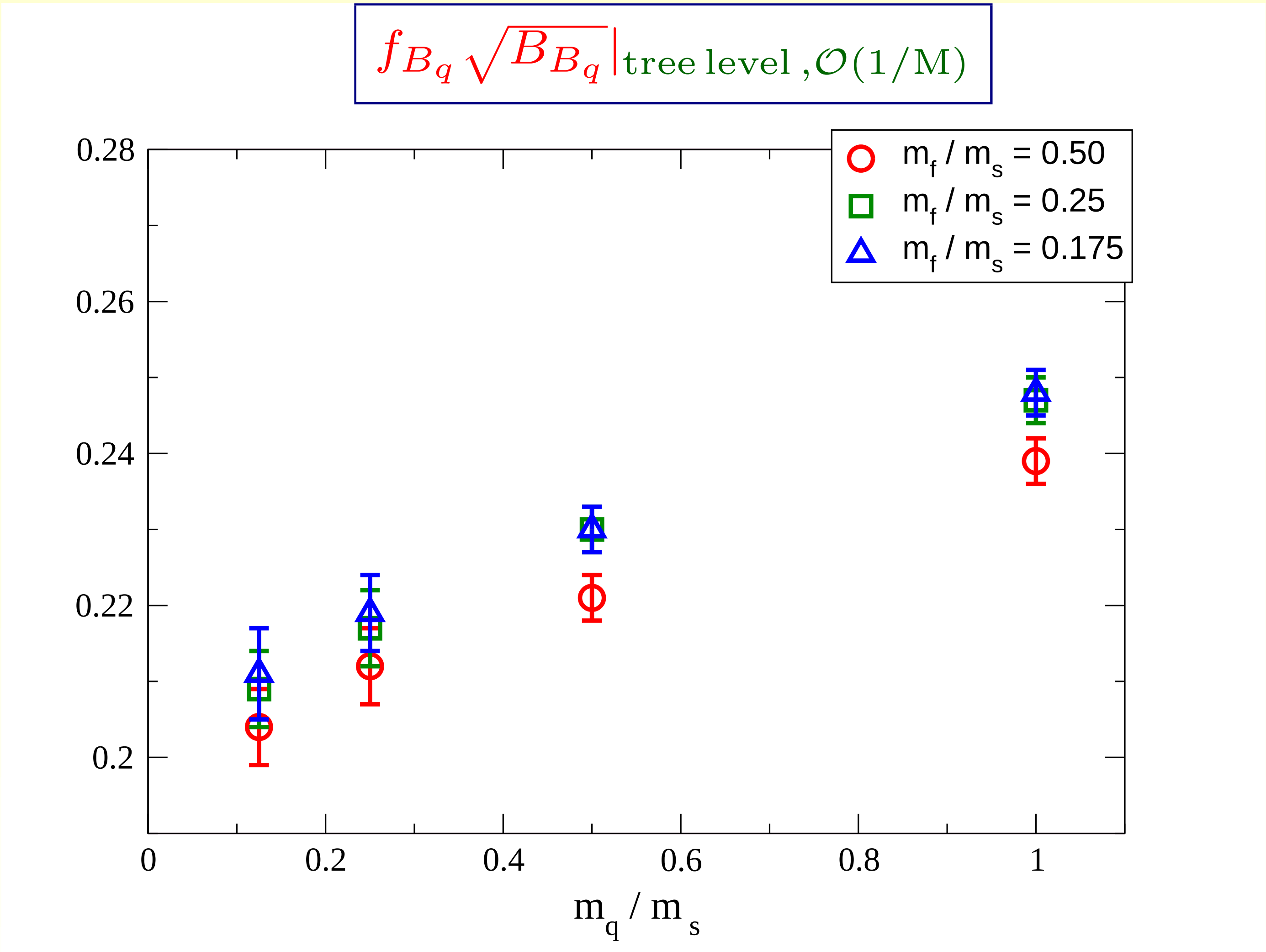
\# Matrix elements calculated (so far) for **3 light sea quark masses** and **6 valence quark masses**. One lattice spacing $a^{-1} \simeq 1.6\mathrm{GeV}$.

$\Longrightarrow$ For each light sea quark mass $\rightarrow$ mixing param. (ΔM_q and $\Delta\Gamma_q$) for B_q^0 with $m_q = m^s_{phys.}$, $\frac{m^s_{phys.}}{2}$, $\frac{m^s_{phys.}}{4}$ and $\frac{m^s_{phys.}}{8}$

* $m^s_{valence}$ and $m^b_{valence}$ tuned to give correct K and B_s^0 masses
* Fits: $3 \leq t_1, t_2 \leq 20$
* `Four time sources` for each $m_{sea}/m_{valence}$ pair
* `Smearing` techniques (`1S`) used to reduce overlap with radial excitations

MILC/Fermilab Calculation

One loop matching calculation is underway $\Longrightarrow$



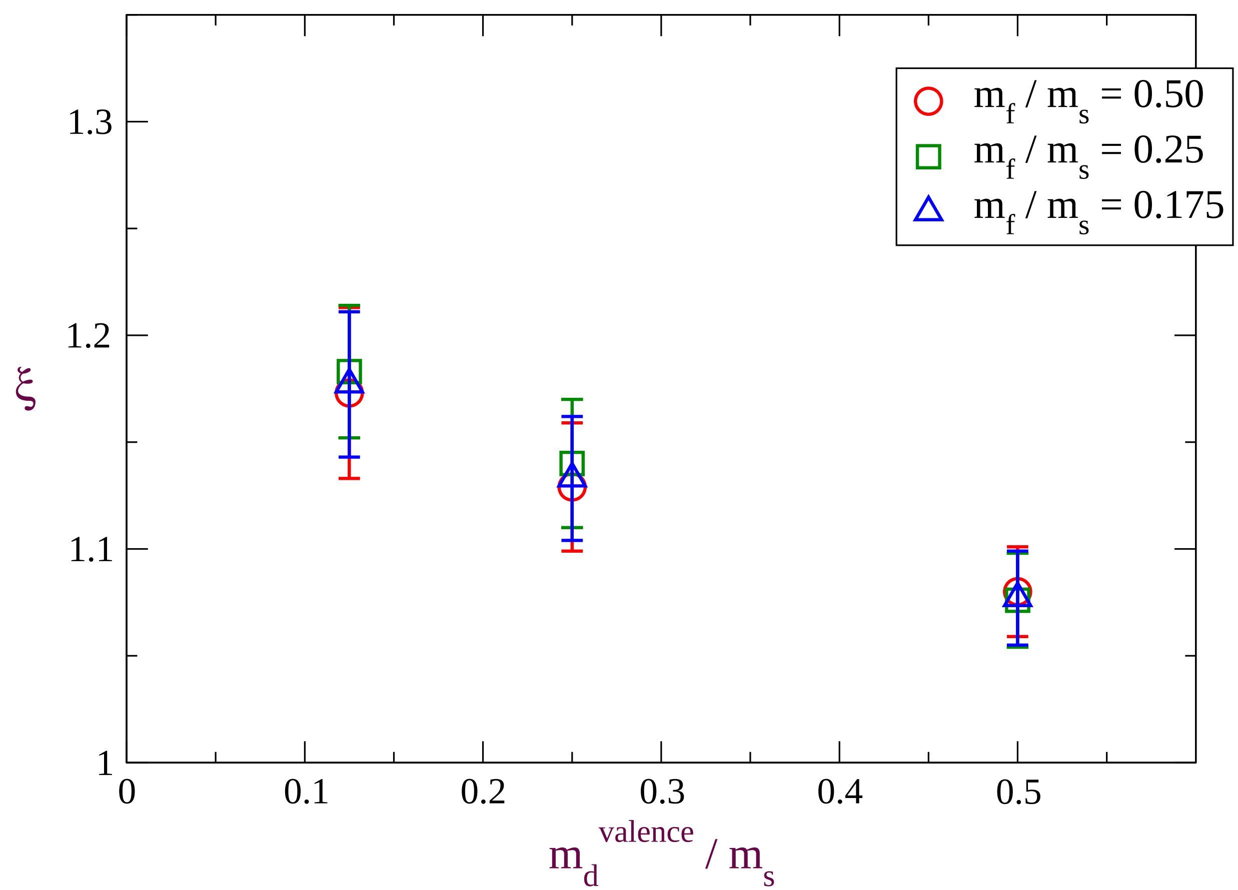
with $m_f = m_{light}^{sea}$, $m_s = m_s^{physical}$ and $m_q = m_q^{valence}$.

statistics+fitting errors $\sim$ 1-3%

MILC/Fermilab Calculation

Preliminary results for ξ: `tree level`, $\mathcal{O}(1/M)$

$$(f_{B_s}\sqrt{\hat{B}_{B_s}})/(f_{B_d}\sqrt{\hat{B}_{B_d}})$$

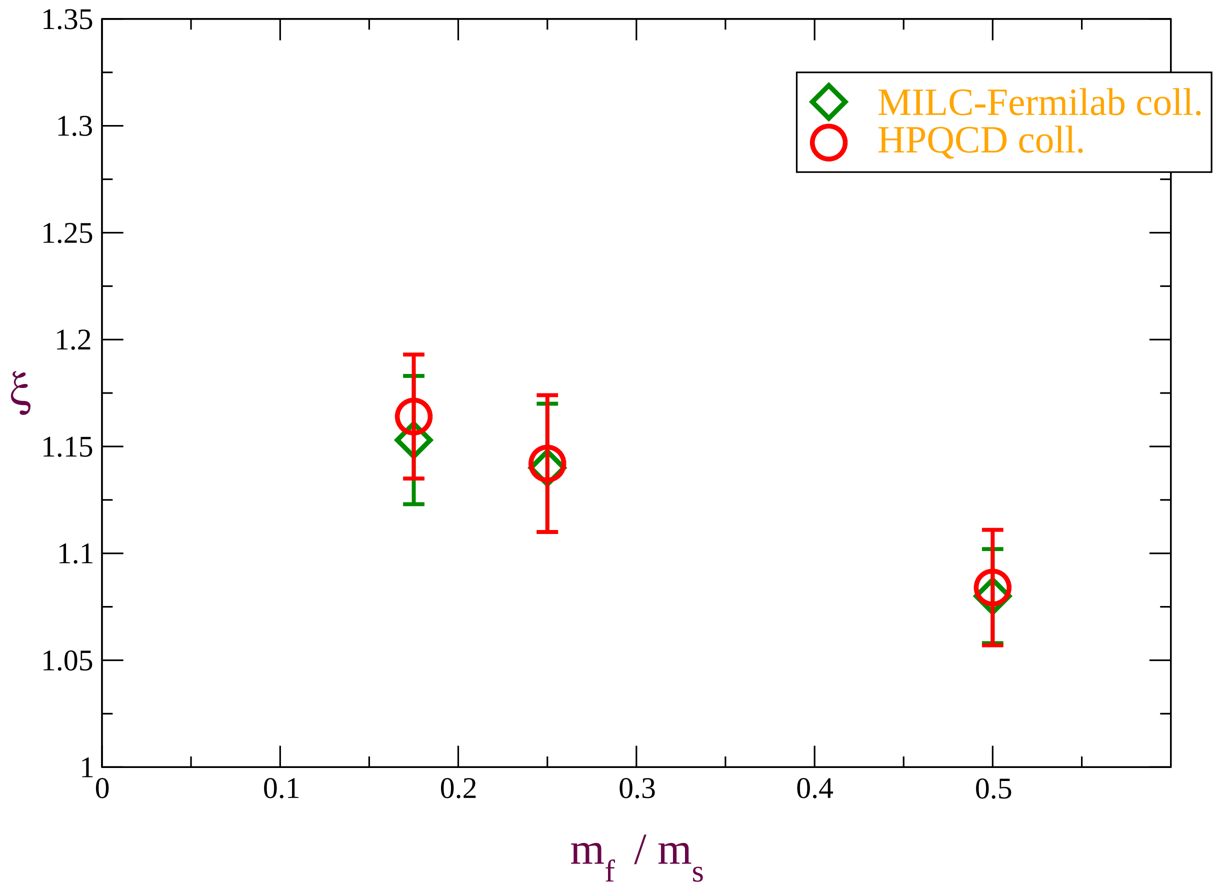


with $m_f = m_{light}^{sea}$ and $m_s = m_s^{physical}$.

Comparison of full QCD ξ results

* MILC/Fermilab results do not include one-loop corrections

* HPQCD results do not include $1/M$ corrections



full QCD $\equiv$

$(m^{valence} = m^{sea})$

Results in complete agreement

Results in complete agreement

We expect very small $1/M$ and `one-loop` corrections

Sizeable dependence on light quark masses $\rightarrow$ Need ChPT machinery to extrapolate to the physical point

* Chiral extrapolation dominated by f_{B_s}/f_{B_d} $\rightarrow$ errors of the same order

(`statist.`+`fitting`+`chiral ext.` $= 2.5\%$ for f_{B_s}/f_{B_d} with NRQCD b quarks)

Preliminary

Taking as central value for ξ, the one with smallest m_d

$$\left|\frac{V_{td}}{V_{ts}}\right| = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}}\sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}} = \begin{cases} 0.199(5) \text{ HPQCD} \\ 0.197(5) \text{ MILC/Fermilab} \end{cases}$$

* Only statistical errors included (no correlation considered)

Recent experimental results on branching ratios $R_{\rho/\omega} = \bar{\mathcal{B}}(B \to (\rho, \omega), \gamma)/\bar{\mathcal{B}}(B \to K^* \gamma)$, and QCD sum rules

$$\left|\frac{V_{td}}{V_{ts}}\right|_{BaBar} = 0.199^{+0.023}_{-0.025}(exp) \pm 0.014(th)$$

$$\left|\frac{V_{td}}{V_{ts}}\right|_{Belle} = 0.207^{+0.028}_{-0.033}(exp)^{+0.014}_{-0.015}(th)$$

(Patricia Ball)

Comparison with experiment: ΔM_s

CDF measurement:

$$\boxed{\Delta M_s|_{exp.} = 17.77 \pm 0.10(stat) \pm 0.07(syst)\, ps^{-1}}$$

Standard Model expression

$$\Delta M_s|_{theor.} = \frac{G_F^2 M_W^2}{6\pi^2} |V_{ts}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_s}^2 \hat{B}_{B_s}$$

* $|V_{ts}^* V_{tb}| \simeq |V_{cs}^* V_{cb}| \simeq 4.1(1) \cdot 10^{-2}$ from measured $|V_{cb}|$ + unitarity

* HPQCD result for $f_{B_s}\sqrt{\hat{B}_{B_s}} = 0.281(21)\text{GeV}$

$$\boxed{\Delta M_s|_{theor.} = 20.3 \pm 3.0 \pm 0.8\, ps^{-1}}$$

* first error: $f_{B_s}^2 \hat{B}_{B_s}$ error

* second error: other uncertainties dominated by $|V_{ts}^* V_{tb}|^2$ error estimate

Conversely, one can use $\Delta M_s|_{exp.}$ and our value of $f_{B_s}^2 \hat{B}_{B_s}$ to get

$$\boxed{|V_{ts}^* V_{tb}| = (3.8 \pm 0.3 \pm 0.1) \times 10^{-2}}$$

Comparison with experiment: $\Delta\Gamma_s/\Gamma_s$

Unofficial experimental world average **(R.v.Kooten, FPCP, Vancouver, April 2006)**

$$\Delta\Gamma_s^{exp.} = 0.097^{+0.041}_{-0.042}\, ps^{-1} \Longrightarrow \boxed{\left(\frac{\Delta\Gamma}{\Gamma}\right)^{exp.}_{B_s} \simeq 0.15 \pm 0.06}$$

Use NLO formula of **Lenz& Nierste**

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)^{theor.}_{B_s} = \left(\frac{1}{245\mathrm{MeV}}\right)^2 \left[0.170\left(f^2_{B_s} B_{B_s}\right) + 0.059 R^2 \left(\frac{f^2_{B_s}\tilde{B}_S}{R^2}\right) - 0.044 f^2_{B_s}\right]$$

Inserting **HPQCD's** $f_{B_s} = 0.260(29)\mathrm{GeV}$, $R^2 \equiv \frac{(\overline{m}_b + \overline{m}_s)^2}{M^2_{B_s}} = 0.652$ and our results for $f_B B^2_B$

$$\boxed{\left(\frac{\Delta\Gamma}{\Gamma}\right)^{theor.}_{B_s} = 0.16 \pm 0.03 \pm 0.02}$$

Comparison with other (lattice) work

	$m_f/m_s = 0.25$	$m_f/m_s = 0.50$	JLQCD ($N_f = 2$)
$B_{B_s}^{\overline{MS}}(m_b)$	0.76(11)	0.80(12)	-
$B_{B_s}^{\overline{MS}}(m_b)$ (no 1/M correc.)	0.88(13)	0.92(14)	0.85(6)
$\hat{B}_{B_s}$	1.17(17)	1.23(18)	1.30(9)

	$m_f/m_s = 0.25$	$m_f/m_s = 0.50$	Hashimoto et al. (quenched)
$\frac{B_S^{\overline{MS}}(m_b)}{R^2}$	1.29(19)	1.34(20)	1.24(16)
$\frac{\tilde{B}_S^{\overline{MS}}(m_b)}{R^2}$	1.38(21)	1.42(21)	-
			Becirevic et al. (quenched)
$B_S^{\overline{MS}(m_b)}$	0.84(13)	0.87(13)	0.84(2)(4)
$\tilde{B}_S^{\overline{MS}}(m_b)$	0.90(14)	0.93(14)	0.91(3)(8)

7. Summary and future work

Previous HPQCD work

Results are presented for the B_s meson mixing parameters $f_{B_s}^2 B_{B_s}$, $f_{B_s}^2 \frac{B_S}{R^2}$ and $f_{B_s}^2 \frac{\tilde{B}_S}{R^2}$

* **MILC** collaboration $N_f = 2+1$ configurations

* NRQCD b-quarks

* Staggered (Asqtad) light quarks

Standard Model predictions using these parameters are consistent with recent experimental determinations of ΔM_s and $\left(\Delta\Gamma/\Gamma\right)_{B_s}$

Using the HPQCD value $f_{B_s} = 0.260(29)\mathrm{GeV}$, the extracted bag parameters B_{B_s}, B_S and $\tilde{B}_S$ are consistent with previous $N_f = 2$ and quenched results.

Need a reduction of the error dominated by `statistical+fitting` and `higher order matching`

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`statistical+fitting` and `higher order matching`

Preliminary MILC/Fermilab and HPQCD results for B_s^0 and B_d^0 mixing parameters

Explore different smearings and better fitting approaches

Improve statistics

$\Longrightarrow$ **Statistics+fitting** `errors reduced from 9% to` **2-6%**

* Work in progress involving more sophisticated smearing and fitting methods to reduce this error further

Work on finer lattices (smaller a)
$\rightarrow$ reduction of `discretization` and `perturbative` error ($9\% \rightarrow 6\%$)

Work on higher order matching $\rightarrow$ reduction of `perturbative` error

Partial non-perturbative renormalization using Fermilab action ?

MILC/Fermilab and HPQCD results

Very premilinary results for the ratio $\xi = [f_{B_s}\sqrt{\hat{B}_{B_s}}]/[f_{B_d}\sqrt{\hat{B}_{B_d}}]$. Much better determined theoretically.

- * Statistical errors $\simeq 2.5\%$
- * Complete cancellation of scale a^{-3} corrections
- * (Almost complete) cancellation of `relativistic` and `matching uncertainties`
- * (Partial) cancellation of `chiral` and `discretization` corrections.

Main sources of error reduced $\rightarrow$ **Chiral extrapolation** to the physical point using Staggered χPT (incorporates discretization and perturbative corrections).

(**J. Laiho and R. Van de Water**)

* More relevant for B_d^0 mixing parameters since we need an extrapolation in both valence and sea quark masses.