QCD Factorization for Heavy Quarkonium Production at Collider Energies

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Based on work with Z.-B. Kang, G.C. Nayak, and G. Sterman
Outline

- Why heavy quarkonium?
- Successes and difficulties in quarkonium production
- Connect pQCD factorization to NRQCD factorization
- Failure of conventional NRQCD factorization at NNLO
- Color transfer in associated production of heavy quarkonium
- Summary and conclusions
Why heavy quarkonium?

A good system for studying the confinement

Heavy quarkonium provides a non-relativistic system, potentially, very similar to a QED bound state:

Two intrinsic scales: large mass and small binding energy

Charm: \( \frac{v^2}{c^2} \sim \frac{k_Q^2}{m_Q^2} \sim \frac{|M^2 - 4m_c^2|}{4m_c^2} \sim 0.3 \)  

Bottom: \( \frac{v^2}{c^2} \sim 0.1 \)

Heavy quark potential: \( V_{\bar{Q}Q}(r) \)

Offers a unique perspective to the hadronization

Production of heavy quarks is effectively perturbative:

Heavy quark pairs are produced at a distance scale much less than \( \text{fm} \)

\[ \Delta r \sim \frac{1}{2m_Q} \leq 0.1 \text{ fm} \quad \text{(for a charm-quark pair)} \]
\[ \leq 0.025 \text{ fm} \quad \text{(for a b-quark pair)} \]
The basic production mechanism

- Production of a heavy quark pair:
  \[ \Delta r \leq \frac{1}{2m_Q} \]

- Hadronization of the pair models:
  \[
  \sigma_{AB \rightarrow h} = \sum_{states} \int d\Gamma_{Q\bar{Q}} \frac{d\sigma_{AB \rightarrow states(Q\bar{Q})}}{d\Gamma_{Q\bar{Q}}} F_{states(Q\bar{Q}) \rightarrow h} \left( p_Q, p_{\bar{Q}}, p_h \right)
  \]

Different models ↔ Different assumptions/treatments on how the heavy quark pair becomes a quarkonium
Color singlet model

- color singlet pair
- right quantum numbers for the quarkonium
- same wave function for production and decay

\[ \sigma_{AB \rightarrow \psi} \propto \sigma_{AB \rightarrow (Q \bar{Q})} \left| R_\psi (0) \right|^2 \]

Works well for \( J/\psi \) production in photo-production and others
But, one order of magnitude too small for CDF data, ...

Einhorn and Ellis (1975), ...
Color Evaporation Model

- all pairs with invariant mass less than open flavor threshold
- color and spin average

\[ \sigma_{AB \to \psi} = f_\psi \int dm_{Q\bar{Q}}^2 \frac{d\sigma_{AB \to (Q\bar{Q})}}{dm_{Q\bar{Q}}^2} \]

Works well for total cross sections, not perfect for distributions, Predicts zero polarization for quarkonium production

Fritsch (1978); Halzen; …
Non-relativistic QCD (NRQCD) model

- Work in the heavy quark pair’s rest frame
- “Integrate out” heavy quark dynamics: \( \mathcal{O}(\alpha_s^n(m_Q)) \)
- Factorize the hadronization: \( \mathcal{O}(v_{rel}^n) \)
- IR divergences \( \rightarrow \) universal local matrix elements
  \[
  \sigma_{AB \rightarrow J/\psi}(M_{J/\psi}) \approx \sum_{[O]} \sigma_{AB \rightarrow [O]}(2m_{c\bar{c}} = M_{J/\psi}) \langle O_{J/\psi}(0) \rangle
  \]
- Quantum states \([O]\) separated by spin and color
- Color octet and color singlet \(QQ\bar{b}ar\) \(\rightarrow\) quarkonium
- Approximations/assumptions – velocity expansion
  \[
  \left\langle p_Q - p_{\bar{Q}} \right\rangle \ll 2m_Q
  \]
  It has been the most successful model

Bodwin, Braaten, Lapage (1994); …
Successes of the production models

- Unpolarized J/ψ at the Tevatron: M. Kramer, 2001
  - Data is not consistent with color singlet model
  - Data is consistent with NRQCD, with matrix elements fixed by the data
  - CEM predicts a similar \( p_T \) distribution

\[ \text{BR}(J/\psi \rightarrow \mu^+ \mu^-) \frac{d\sigma(p\bar{p} \rightarrow J/\psi + X)}{dp_T} \text{(nb/GeV)} \]
\( \sqrt{s} = 1.8 \text{ TeV}; |\eta| < 0.6 \)

Diagram showing the differential cross section for the production of J/ψ with data points and theoretical predictions for different models.
Works for other states too:

Difficulties

- Transverse polarization at high $p_T$?

NRQCD: Cho & Wise, Beneke & Rothstein, 1995, ...

KT-fact: Baranov, 2002

$$\alpha = \frac{\sigma_T - \sigma_L}{\sigma_T + \sigma_L}$$

Same problem for other states:


Braaton & Lee, PRD63, 071501 (2001)
Double $c\bar{c}$ production in $e^+e^-$

**Exclusive production:** [4] Li, He, and Chao, [6] Braaten and Lee

<table>
<thead>
<tr>
<th>$J/\psi$ $c\bar{c}$</th>
<th>$\eta_c(1S)$</th>
<th>$\chi_{c0}$</th>
<th>$\eta_c(2S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BABAR</strong></td>
<td>17.6 ± 2.8$^{+1.5}_{-2.1}$</td>
<td>10.3 ± 2.5$^{+1.4}_{-1.8}$</td>
<td>16.4 ± 3.7$^{+2.4}_{-3.0}$</td>
</tr>
<tr>
<td>Belle [14]</td>
<td>25.6 ± 2.8 ± 3.4</td>
<td>6.4 ± 1.7 ± 1.0</td>
<td>16.5 ± 3.0 ± 2.4</td>
</tr>
<tr>
<td>NRQCD [6]</td>
<td>2.31 ± 1.09</td>
<td>2.28 ± 1.03</td>
<td>0.96 ± 0.45</td>
</tr>
<tr>
<td>NRQCD [4]</td>
<td>5.5</td>
<td>6.9</td>
<td>3.7</td>
</tr>
</tbody>
</table>

**Possible resolution for $J/\psi + \eta_c$:**

- **NLO correction:** $K_{\text{factor}} = 1.96$
- **Relativistic Correction:**
  - X-section: $K_{\text{factor}} = 1.34$
  - Wave func: $K_{\text{factor}} = 1.32$
- Combined: $K_{\text{factor}} = 4.15$

$$\sigma[\bar{e}^+ e^- \rightarrow J/\psi + \eta_c] = 17.5 \pm 5.7 \text{ fb}$$

Bodwin et al. hep-ph/0611002

Zhang, Gao, Chao
Double $c\bar{c}$ production in $e^+e^-$

- **Inclusive production:**
  \[ \sigma(e^+e^- \rightarrow J/\psi c\bar{c}) \]
  - **Belle:** $(0.87^{+0.21}_{-0.19} \pm 0.17)$ pb
  - **NRQCD:** $\sim 0.07$ pb

- **Ratio to light flavors:**
  \[ \frac{\sigma(e^+e^- \rightarrow J/\psi c\bar{c})}{\sigma(e^+e^- \rightarrow J/\psi X)} \]
  - **Belle:** $0.59^{+0.15}_{-0.13} \pm 0.12$

**Message:**
Production rate of $e^+e^- \rightarrow J/\psi c\bar{c}$ is larger than all these channels: $e^+e^- \rightarrow J/\psi gg$, $e^+e^- \rightarrow J/\psi q\bar{q}$, ... combined?
Large NLO correction

- Color singlet at NLO: Campbell, Maltoni, Tramontano, PRL98, 2007

Huge enhancement of NLO at high $p_T$
Questions

Is the NRQCD factorization valid for these observables?

- NRQCD factorization is valid for heavy quarkonium decay
  Bodwin, Braaten, Lepage, 1995

- But, there is no rigorous proof for production of heavy quarkonium
  Nayak, Qiu, Sterman, 2005/6
What we did on factorization

- Study soft gluons in heavy quarkonium production at high pt
- Find uncancelled infrared poles at NNLO not matched by conventional NRQCD matrix elements
- NNLO fix:
  - Gauge invariance → modification of NRQCD operators
- Get nonabelian phases: Wilson lines
- Factorization at high orders?
  - current state: can neither prove nor disprove
None of the factorized production models, including NRQCD model, were proved theoretically.

Factorization of NRQCD model fails for low $p_T$.

Factorization of NRQCD model might work for large $p_T$.

Spectator interactions are suppressed by $(1/p_T)^n$.

Factorization is necessary for the predictive power.
Factorization into a fragmentation function at large $p_T$

Nayak, Qiu, Stermen, 2005

\[ d\sigma_{A+B\rightarrow H+X}(p_T) = \sum_i d\bar{\sigma}_{A+B\rightarrow i+X}(p_T/z, \mu) \otimes D_{H/i}(z, m_c, \mu) + O(m_H^2/p_T^2) \]

Fragmentation function – gluon to a hadron H (e.g., $J/\psi$):

\[ D_{H/g}(z, m_c, \mu) \propto \frac{1}{P^+} Tr_{color} \int dx^- e^{-ik^+x^-} \]
\[ \times \langle F^{+\lambda}(0) [\Phi^{(g)}(0)]^\dagger a_H(P^+) a_H^\dagger(P^+) \Phi^{(g)}(x^-) F^{+\lambda}_{\lambda}(y^-) \rangle \]
The proof works in two steps

- **Step 1:** Fragmentation factorizes from the rest

  Fragmentation function

  Reduced diagram

  Still has long-distance physics due to incoming hadrons
Step 2: Cancellation of remaining IR final state:

Note: Uncut loops are short distance

Remaining soft-interaction absorbed into the Wilson lines of PDFs

H is IR safe!
The Wilson line in $x^-$ direction ($n^\mu = \delta_{\mu-}$)

$$\Phi^{(g)}(x^-) = P \exp \left[ -i g \int_0^\infty n \cdot A^{(adj)} ((x^- + \lambda)n) \right]$$

Which depends on the “direction” vector: $n^\mu$

For the fragmentation function, or the jet, all that is left is gluon source:

A necessary condition for the factorization, or the universality of the fragmentation function is:

The fragmentation function is independent of the $n^\mu$
Connection to NRQCD Factorization

- Proposed NRQCD factorization:
  \[ d\sigma_{A+B\to H+X}(p_T) = \sum_n d\hat{\sigma}_{A+B\to cc[n]+X}(p_T) \langle O^H_n \rangle \]

- Proved pQCD factorization for single hadron production:
  \[ d\sigma_{A+B\to H+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B\to i+X}(p_T/z, \mu) \otimes D_{H/i}(z, m_c, \mu) + \mathcal{O}(m_H^2/p_T^2) \]

- Prove NRQCD Factorization

  To prove:
  \[ D_{H/i}(z, m_c, \mu) = \sum_n d_{i\to cc[n]}(z, \mu, m_c) \langle O^H_n \rangle \]

  with
  - \[ d_{g\to cc[n]}(z, \mu, m_c) \] IR safe
  - \[ \langle O^H_n \rangle \] gauge invariant and universal
  - independent of the direction of the Wilson lines
Gauge Invariance and Wilson lines

- Conventional operator definition (Q\Bar{Q} rest frame)
  \[ \mathcal{O}_n^H(0) = \chi^\dagger \mathcal{K}_n \psi(0) \left( a_H^\dagger a_H \right) \psi^\dagger \mathcal{K}_n' \chi(0) \]

- \( \psi, \chi \) are heavy quark, antiquark fields

- \( \mathcal{K}_n, \mathcal{K}_n' \): Products of color and spin matrices, covariant derivatives

- Fields at \( x = 0 \) but \( \mathcal{O}_n^H \) is not truly local

- Operator-valued gauge transformations (as to \( A^+ = 0 \) gauge) do not commute with \( a_H^\dagger a_H \)

- Only color-singlet \( \mathcal{K}'s \) give gauge invariant \( \mathcal{O}'s \) or, the color-octet operators are not gauge invariant
Resolution: supplement fields by Wilson lines:

\[ \Phi_l[x, A] = \exp \left[ -ig \int_0^\infty d\lambda \ l \cdot A(x + \lambda l) \right] \]

Our new, gauge invariant operators:

\[ O_n^H(0) \rightarrow \chi^\dagger K_{n,c} \psi(0) \Phi_l^\dagger[0, A]_{cb} \left( a_H^\dagger a_H \right) \Phi_l[0, A]_{ba} \chi^\dagger K_{n,a} \psi(0) \]

Two remaining questions for NRQCD factorization:

\[ D_{H/i}(z, m_c, \mu) = \sum_n d_{i \rightarrow c\bar{c}[n]}(z, \mu, m_c) \langle O_n^H \rangle \]

- Are the “coefficient” functions \( d_{g \rightarrow c\bar{c}[n]}(z, \mu, m_c) \) IR safe?
  
  Our NNLO answer is no The lines are necessary

- Do the lines absorb all IR divergences?
  
  Can’t tell yet for sure. OK at NNLO

An all-order proof of NRQCD factorization at high pT is still lacking, and urgently needed
Factorization works to NLO at $v^2$

- **LO:**
  - Short-distance
  - Eikonal line
  - $n = [\bar{Q}Q]_{\text{singlet}}$

- **Velocity expansion:**
  - $\left\{\begin{array}{c}
  \left\langle \mu + \frac{\mu}{\mu} \right\rangle =
  \left\langle \frac{1}{\mu} + O(v^2) \right\rangle
  
  \end{array}\right.$

- **NLO:**
  - Topologically-factorized the matrix element $\langle O_n^H \rangle$

  
  $\begin{align*}
  n & = \frac{16}{3} \frac{\alpha_s}{\pi} \frac{q^2}{P^2} \frac{1}{-\varepsilon} + \ldots
  
  \end{align*}$

  - Color neutralization is IR divergent – nonperturbative!

  - IR divergences cancel between real and virtual diagrams
Factorization fails at NNLO

- Diagrams:

- All IR divergences cancel between real and virtual diagrams, except
Explicit calculation at NNLO at $v^2 - 1$

- The infrared divergent expression to order $q^2 \sim v^2$:

\[
\begin{align*}
\Sigma^{(2c)}(P, q, l) &= -16i \, g^4 \mu^{4\varepsilon} \int \frac{d^D k_1}{(2\pi)^D} \frac{d^D k_2}{(2\pi)^D} \, 2\pi \delta(k_1^2) l^\lambda V_{\nu\mu\lambda}[k_1, k_2] \\
&\times [q^\mu (P \cdot k_1) - (q \cdot k_1) P^\mu] \, [q^\nu (P \cdot k_1) - (q \cdot k_2) P^\nu] \\
&\times \frac{1}{[P \cdot k_1 + i\varepsilon]^2 \, [P \cdot k_2 - i\varepsilon]^2} \\
&\times \frac{1}{[k_2^2 - i\varepsilon] \, [(k_2 - k_1)^2 - i\varepsilon] \, [l \cdot (k_1 - k_2) - i\varepsilon]} \, ,
\end{align*}
\]

- The result is:

\[
\Sigma^{(2)}(P, q, l) = \alpha_s^2 \frac{4}{3\varepsilon} \left[ \frac{(P \cdot q)^2}{P^4} - \frac{q^2}{P^2} \right]
\]
Explicit calculation at NNLO at $v^2$ - II

- In heavy quark pair’s rest frame:

\[
\Sigma(P, q, l) = \alpha_s^2 \frac{4}{3\varepsilon} \frac{\bar{q}^2}{4m_c^2} = \alpha_s^2 \frac{1}{3\varepsilon} \frac{\bar{v}^2}{4}
\]

- IR poles would appear in coefficient function at NNLO unless we have eikonal interactions to absorb them.

- This non-topological IR divergence cannot be absorbed into the conventional NRQCD matrix elements.

- Can be absorbed into the modified matrix elements.
Factorization at a finite $v$?

- Velocity expansion is not efficient for charmonium
  - Large phase space available for gluon radiation:
    \[ Q^2 - 4M_C^2 \Rightarrow 4M_D^2 - 4M_C^2 \approx 6 \text{ GeV}^2 \]
  - Large possible velocity in production:
    \[ u_{\text{prod}} \sim \frac{|k_c|}{M_c} \sim \sqrt{\frac{4M_D^2 - 4M_C^2}{4M_C^2}} \sim 0.88 \]
  - Very different from decay:
    \[ u_{\text{decay}} \sim \sqrt{\frac{4M_{J/\psi}^2 - 4M_C^2}{4M_C^2}} \sim 0.48 \]

- High order terms are very important

Still no solution for the polarization data even if NRQCD factorization is valid
Factorization at NNLO and all order in $v^2$

- Calculation with a finite $v$

\[
\mathcal{I}^{(8\rightarrow 1)} = \frac{\alpha_s^2}{4\varepsilon} \left[ 1 - \frac{1}{2f(|\vec{v}|)} \ln \left( \frac{1 + f(|\vec{v}|)}{1 - f(|\vec{v}|)} \right) \right]
\]

with

\[f(v) = \frac{2v}{1 + v^2}, \quad \vec{v} = \frac{q}{E^*}\]

$2E^*$ is the total energy of the heavy quark pair

(Q\bar{Q} rest frame)

- Reproduce the $v^2$ result when expanded
Significance?

- The IR poles at all orders of $v$-expansion at NNLO are independent of the direction of the Wilson line and universal - consistent with factorization.

- Although limited to NNLO, our result suggests that the decoupling of light parton dynamics from heavy quark pair production is robust in perturbation theory at the level of infrared divergence - high orders?

- Although the eikonal approximation do not cover many terms in general NRQCD velocity expansion, in particular, those dealing with spin, it should cover all perturbative infrared divergences.
Associate production

- Observation from NNLO calculation:
  Double IR poles may not be canceled for a massive eikonal line

- Color transfer in associated heavy quarkonium production:
  A heavy quark as a color source to enhance the rate for an octet pair to become a singlet pair
Inclusive double $c\bar{c}$ production in $e^+e^-$

- **Recall:**
  \[ \sigma(e^+e^- \rightarrow J/\psi c\bar{c}) \]
  - Belle: $(0.87^{+0.21}_{-0.19} \pm 0.17)$ pb
  - NRQCD: $\sim 0.07$ pb
  \[ \frac{\sigma(e^+e^- \rightarrow J/\psi c\bar{c})}{\sigma(e^+e^- \rightarrow J/\psi X)} \]
  - Belle: $0.59^{+0.15}_{-0.13} \pm 0.12$

- **Associated production is enhanced:**
Soft gluon between heavy quarks

- **Active pair:** $P_1, P_2$
- **Spectators:** $P_3, P_4$

- NRQCD does not work when 3 heavy (anti-)quarks are close together:

  There are now 3 “velocities”:

  $$ \beta_{ij} \equiv \sqrt{1 - 4m^2 / (P_i + P_j)^2} $$

- **Soft gluon:**

  $$ - i \int \frac{d^D k}{(2\pi)^D} \frac{4P_i \cdot P_j}{[2P_i \cdot k + k^2 + i\epsilon][-2P_j \cdot k + k^2 + i\epsilon][k^2 + i\epsilon]} $$

  $$ = \frac{\alpha_s}{2\pi} \left[ - \frac{1}{2\epsilon} \left( \frac{1}{\beta_{ij}} + \beta_{ij} \right) (2\beta_{ij} - i\pi) + \ldots \right] \quad \Longrightarrow \quad i \frac{1}{\epsilon} \frac{\pi}{\beta_{ij}} $$
NNLO enhancement

- NLO correction to the amplitude:

\[ \text{Im} [\mathcal{A}_{13} + \mathcal{A}_{23}] = \frac{\alpha_s}{4\varepsilon} A^{(0)}(P_i) \left[ \frac{1 + \beta_{13}^2}{\beta_{13}} - \frac{1 + \beta_{23}^2}{\beta_{23}} \right] \]

- Expansion of relative velocity:

\[
\frac{1}{\beta_{13}} - \frac{1}{\beta_{23}} \sim -\frac{2}{\beta_S^3} \frac{q_S \cdot q}{m^2} \sim \frac{2}{\beta_S^2} \nu \cos \phi_S
\]

\[ 2q_S = P_3 - (P_1 + P_2)/2 \]

\[ \beta_S \sim \sqrt{-q_S^2/m} \]

- Enhancement factor from NNLO:

\[
\left| A_{\text{Singlet}}^{\text{NNLO}} \right|^2 \sim \left( C_{8\to1} \frac{\alpha_s^2 \nu^2}{\varepsilon^2} \right) \left( \frac{\pi^2}{\beta_S^4} \right) \left| A_{\text{Octet}}^{\text{LO}} \right|^2
\]
Estimate for the NNLO enhancement

- **LO hard parts with color factor:**

  \[ (P + p_3)^2 \]

- **Singlet**

  \[ (P + p_4)^2 \]

- **Octet**

- **Matrix elements:**

  \[ \frac{\pi^2 \alpha_s^2 v^2}{\epsilon^2} \Rightarrow \left\langle O_8 \right\rangle \]

  \[
  d\sigma_{e^+e^- \rightarrow H+X}^{\text{tot}}(p_H) \sim d\hat{\sigma}_{e^+e^- \rightarrow Q\bar{Q}[S_1]+Q'(\beta_S)}(p_H) \left\langle \frac{3}{\beta_s^4} S_{1H}^H \right\rangle
  \]

  \[
  + d\hat{\sigma}_{e^+e^- \rightarrow Q\bar{Q}[S_8]+Q'(\beta_S)}(p_H) \left\langle \frac{3}{\beta_s^4} S_{8H}^H \right\rangle
  \]

  Two terms are equally important if \( \beta_s \sim 0.3 \)

  Same feature for heavy quark fragmentation, enhances long. Pol.
Summary and conclusions

- Predictive power of pQCD calculation relies on the factorization:
  - Infrared Safety of the short-distance part
  - Universality of the parton distribution/fragmentation

- None of the existing factorized production models, including NRQCD model, were proved theoretically.

- We show that “NRQCD”-type factorization is valid up to the NNLO order in $\alpha_s$ for the fragmentation function.

- Effective velocity in quarkonium production could be much larger than that in quarkonium decay.
Summary and conclusions

- Associated production of heavy quarkonium could be strongly enhanced due to soft color transfer.
- NRQCD does not work for associated production due to multiple “velocities”.
- After more than 30 years, since the discovery of $J/\psi$, we still have not been able to fully understand the production mechanism of heavy quarkonia.
- A tough question, but should have an interesting answer.

Thank you!
Backup slices
Could be a good probe for Quark Gluon Plasma (QGP)

- The transition from a heavy quark pair to a quarkonium should be sensitive to the soft physics or medium properties. The quarkonium binding energy is given by
  \[ \frac{|M^2 - 4m_Q^2|}{4m_Q^2} \ll 1 \]

- Color screening in QGP suppresses the formation of \( J/\psi \)
  - Potential: \( V_{Q\bar{Q}}(r) \Rightarrow V_{Q\bar{Q}}(r, T) \)  
  - Wave function: \( \Phi_{Q\bar{Q}}(r) \Rightarrow \Phi_{Q\bar{Q}}(r, T) \)
  - \( J/\psi \) formation rate \( \propto \left| \Phi_{Q\bar{Q}}(r, T) \right|^2 \)

\( J/\psi \) suppression \( \Leftrightarrow \) medium properties

But, do we understand the production mechanism of \( J/\psi \) well enough to calibrate the production rate and to extract the information on QGP?
LEP data on $J/\psi$ photo-production: $\gamma\gamma \rightarrow J/\psi + X$